Documentation

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The function `ode.m` provides an implementation of the Optimal Mode Decomposition (OMD) algorithm developed in [1, 2]. For comparison, the function will also implement the Dynamic Mode Decomposition (DMD) algorithm described in [3, 4].

Usage: `[L,M] = (A,B,r,L0,method)` solves the rank-constrained optimization problem

$$\begin{aligned} \min \quad & \left\| A - LML^\top B \right\|^2 \\ \text{s.t.} \quad & L^\top L = I \\ & M \in \mathbb{R}^{r\times r}, L \in \mathbb{R}^{p\times r} \end{aligned} \tag{1}$$

Inputs:

- `A,B`: data matrices of size $p \times n$
- `r`: integer specifying the desired output size
- `L0`: initial condition for `L`. If `L0` is not specified, then the initial iterate will be based on the first k singular vectors of the matrix `[A B]`
- `method`: Specifies which optimization method to use. Must be one of `‘alteranating’` (the default), `‘conjgrad’` or `‘gradient’` – which are implementations of the OMD algorithm – or `‘dmd’` which implements the DMD algorithm.

Examples

Example code is provided to show how the OMD algorithm can be used to extract eigenvalue information from experimental data. We consider the sinosoidal flow

$$f(x,t) := \sin(kx - \omega t)e^{\gamma t} \tag{2}$$

studied in [3, 2]. Data matrices A, B are selected to contain snapshots of the flow corrupted with Gaussian noise, as explained in [2, Section 4]. The aim is to identify the true eigenvalues $\lambda_i^{\text{true}} = \gamma \pm i\omega$ of the flow using only the noisy snapshot data contained in matrices A, B.

The OMD algorithm may be used to estimate λ_{true} by the following proceedure:

(i) run `[L,M] = omd(A,B,2,[],'conjgrad')`

(ii) calculate the OMD eigenvalues

$$\lambda_i^{\text{OMD}} := \frac{\log \lambda_i(M)}{\Delta t}$$

where $\lambda_i(M)$ are the eigenvalues of M and Δt is the timestep between successive snapshots.

(iii) The OMD eigenvalues then provide an approximation

$$\lambda_i^{\text{OMD}} \approx \lambda_i^{\text{true}}$$

to the true system eigenvalues.

Quality of the eigenvalue approximation is quantified by the *fractional growth rate error* statistic ϵ_{OMD} [3] given by

$$\epsilon_{\text{OMD}} := \min_{i=1,2} \frac{|\text{Re}(\lambda_i^{\text{OMD}}) - \text{Re}(\lambda_i^{\text{true}})|}{\text{Re}(\lambda_i^{\text{true}})}.$$

`example1.m`: calculates the OMD eigenvalues λ_i^{OMD} for snapshot data taken from the sinosoidal flow (2) with parameters

$$k = 1, \quad \omega = 2, \quad \gamma = 1$$

and data corrupted with Gaussian noise with a range of covariances $\sigma \in [0, 1]$. For comparison, the eigenvalues λ_i^{DMD} calculated using Dynamic Mode Decomposition (DMD) [3, 4] are also calculated.

`example2.m`: displays the fractional growth rate errors of the DMD and OMD eigenvalues when applied to the sinosoidal flow for the range of frequency and covariance pairs (ω, σ) studied in [2, Section 4].

References

[1] Goulart, P. J., Wynn, A., and Pearson, D., 'Optimal mode decomposition for high-dimensional systems. In 51st IEEE Conference on Decision and Control. Maui, Hawaii, Dec. 2012. Available at `http:\\control.ee.ethz.ch\~goularpa\`

[2] Wynn, A., Pearson, D., Ganapathisubramani, B., and Goulart, P. J., 'Optimal mode decomposition for unsteady and turbulent flows', June 2012. Submitted to Journal of Fluid Mechanics. Available at `http:\\control.ee.ethz.ch\~goularpa\`

[3] Duke, D., Soria, J., and Honnery, D., 'An error analysis of the dynamic mode decomposition', 2012 *Exps. Fluids* **52** (2), 529–542.

[4] Schmid, P. J., 'Dynamic mode decomposition of numerical and experimental data', 2010 *J. Fluid Mech.* **656** (5-28).