Some selected topics on

Similarity Renormalization Group

Are low-energy nuclear observables sensitive to high-energy phase shifts?

S.K. Bogner[1], R.J. Furnstahl[1], R.J. Perry[1], A. Schwenk[2]

The evolution or flow of the hamiltonian with a parameter s follows from a unitary transformation,

$$H_s = U(s) H U^\dagger(s) \equiv T_{\rm rel} + V_s \,, \qquad (1)$$

where $T_{\rm rel}$ is the relative kinetic energy and $H = T_{\rm rel} + V$ is the initial hamiltonian in the center-of-mass system. Eq. (1) defines the evolved potential V_s, with $T_{\rm rel}$ taken to be independent of s. Then H_s evolves according to

$$\frac{dH_s}{ds} = [\eta(s), H_s] \,, \qquad (2)$$

with

$$\eta(s) = \frac{dU(s)}{ds} U^\dagger(s) = -\eta^\dagger(s) \,. \qquad (3)$$

Choosing $\eta(s)$ specifies the transformation. As in Ref. [12], we restrict ourselves to the simple choice [11]

$$\eta(s) = [T_{\rm rel}, H_s] \,, \qquad (4)$$

which gives the flow equation,

$$\frac{dH_s}{ds} = [[T_{\rm rel}, H_s], H_s] = [[T_{\rm rel}, V_s], H_s] \,. \qquad (5)$$

In a momentum basis, this choice suppresses off-diagonal matrix elements, forcing the hamiltonian towards a band-diagonal form.

The evolution in Eq. (5) includes all many-body components of the hamiltonian. In the space of relative momentum NN states only, it means that the partial-wave momentum-space potential evolves as (with normalization so that $1 = \frac{2}{\pi}\int_0^\infty q^2\, dq\, |q\rangle\langle q|$ and in units where $\hbar = c = m = 1$ with nucleon mass m),

$$\begin{aligned}\frac{dV_s(k,k')}{ds} &= -(k^2 - k'^2)^2\, V_s(k,k') \\ &\quad + \frac{2}{\pi}\int_0^\infty q^2\, dq\, (k^2 + k'^2 - 2q^2)\, V_s(k,q)\, V_s(q,k') \,. \end{aligned} \qquad (6)$$

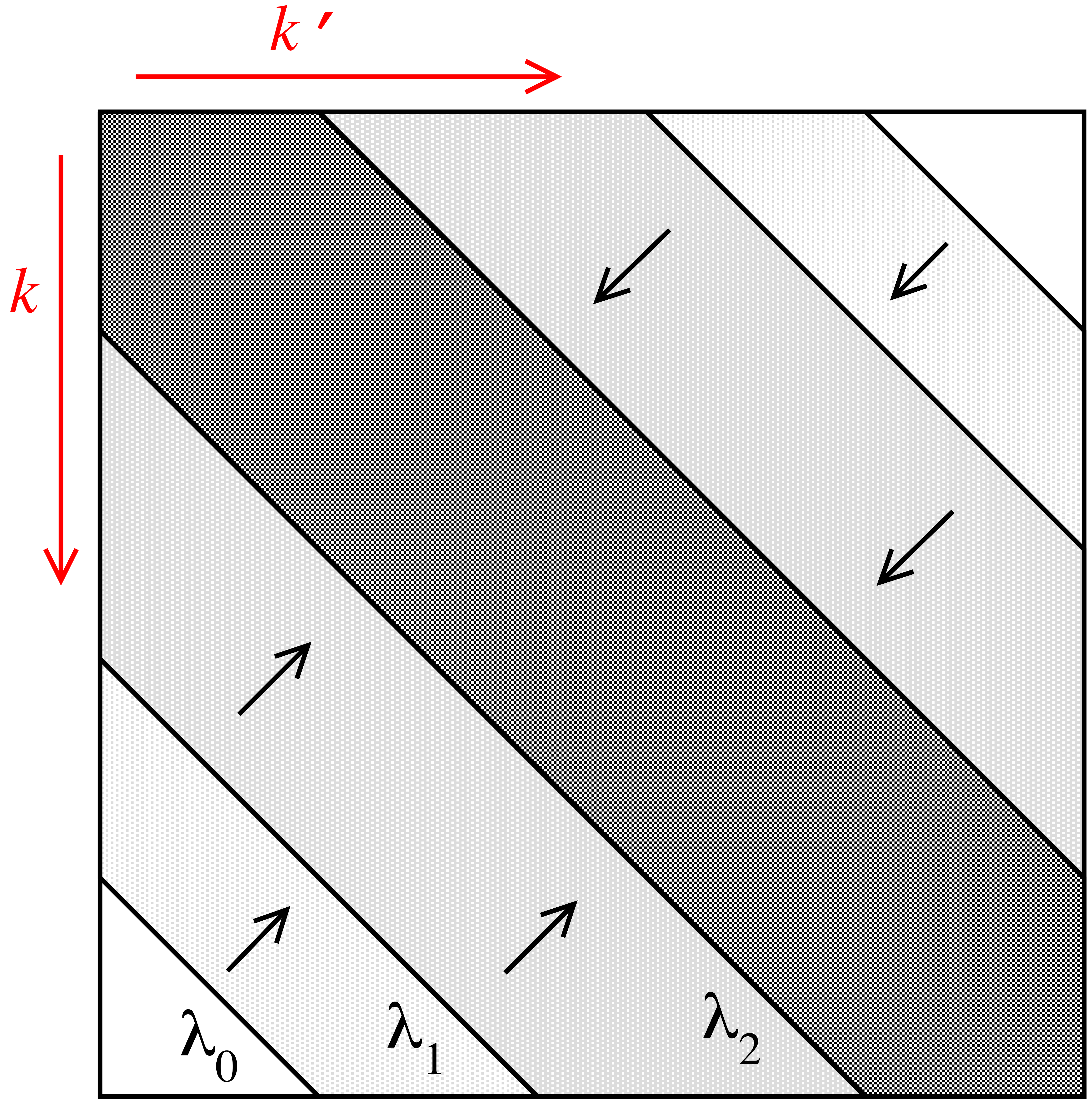
The evolution with s of any other operator O is given by the same unitary transformation, $O_s = U(s) O U^\dagger(s)$, which means that O_s evolves according to

$$\frac{dO_s}{ds} = [\eta(s), O_s] = [[T_{\rm rel}, V_s], O_s] \,. \qquad (7)$$

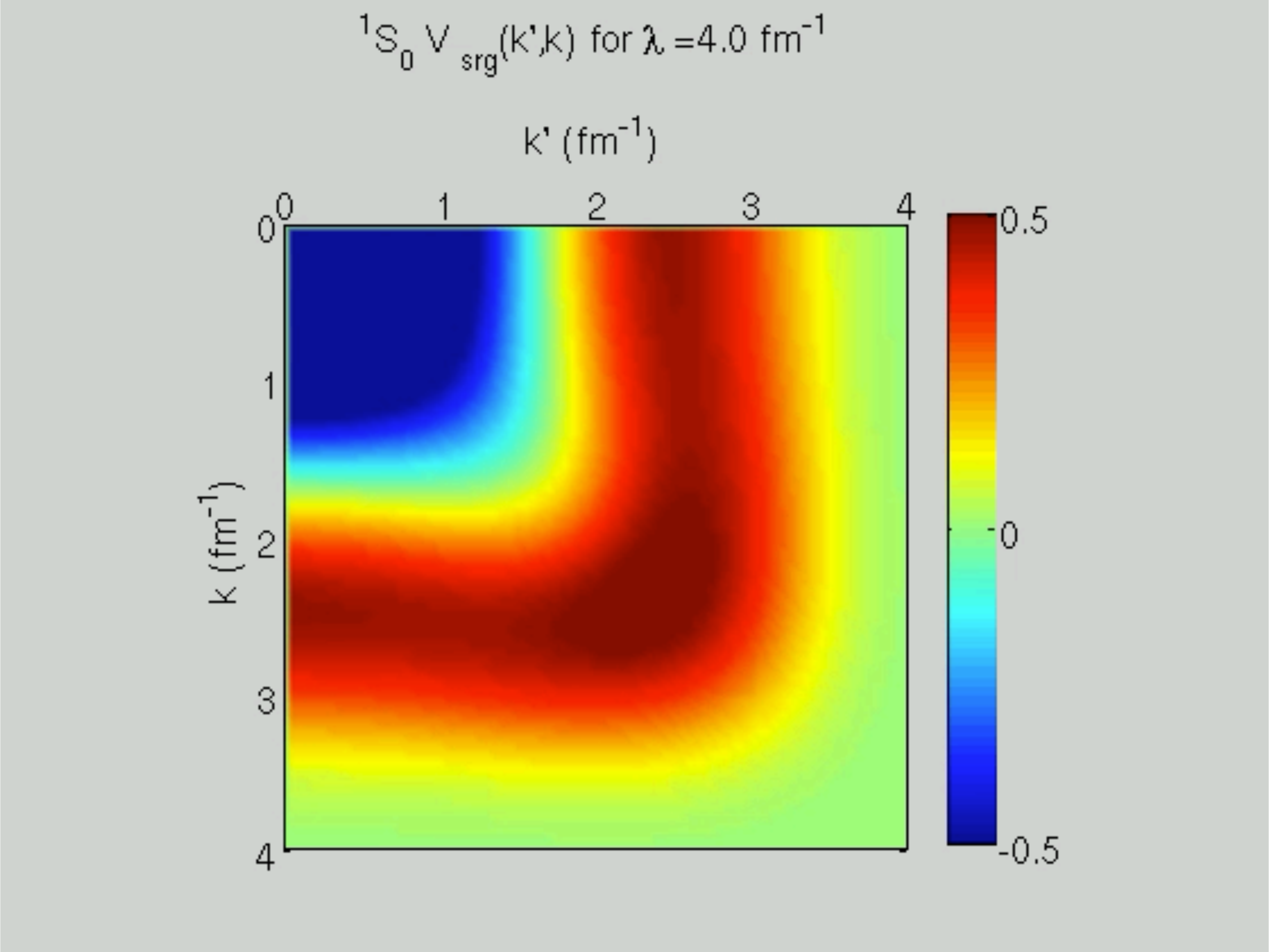
Just as with the hamiltonian H_s, this evolution will induce many-body operators even if the initial operator is purely two-body. If we restrict ourselves to the relative momentum NN space, we have

$$\begin{aligned}\frac{dO_s(k,k')}{ds} &= \frac{2}{\pi}\int_0^\infty q^2\, dq\, \Big[(k^2 - q^2)\, V_s(k,q)\, O_s(q,k') \\ &\quad + (k'^2 - q^2)\, O_s(k,q)\, V_s(q,k')\Big] \,. \end{aligned} \qquad (8)$$

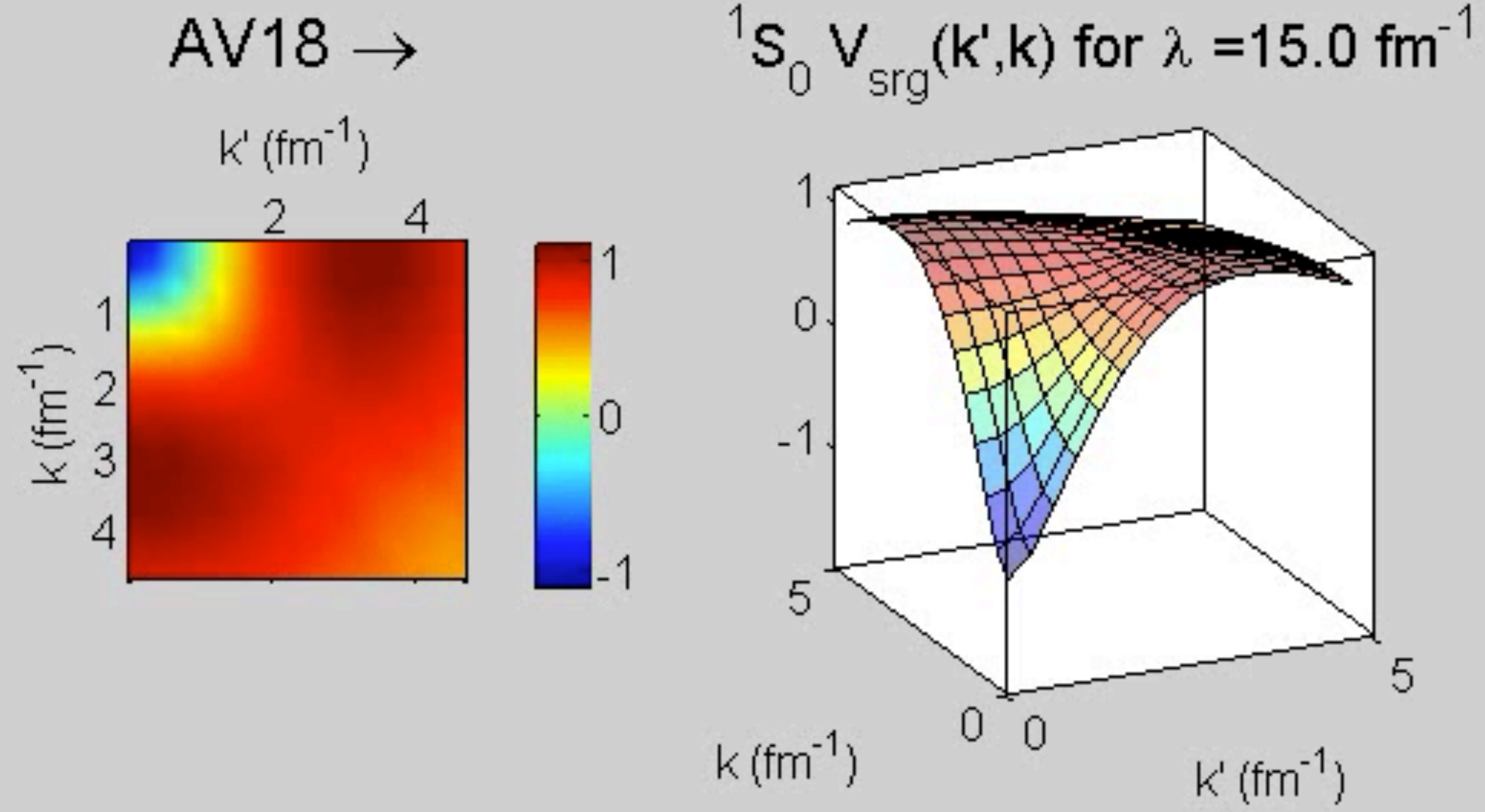
To evolve a particular O_s simultaneously with V_s, we simply include the discretized version of Eq. (8) as additional coupled first-order differential equations.

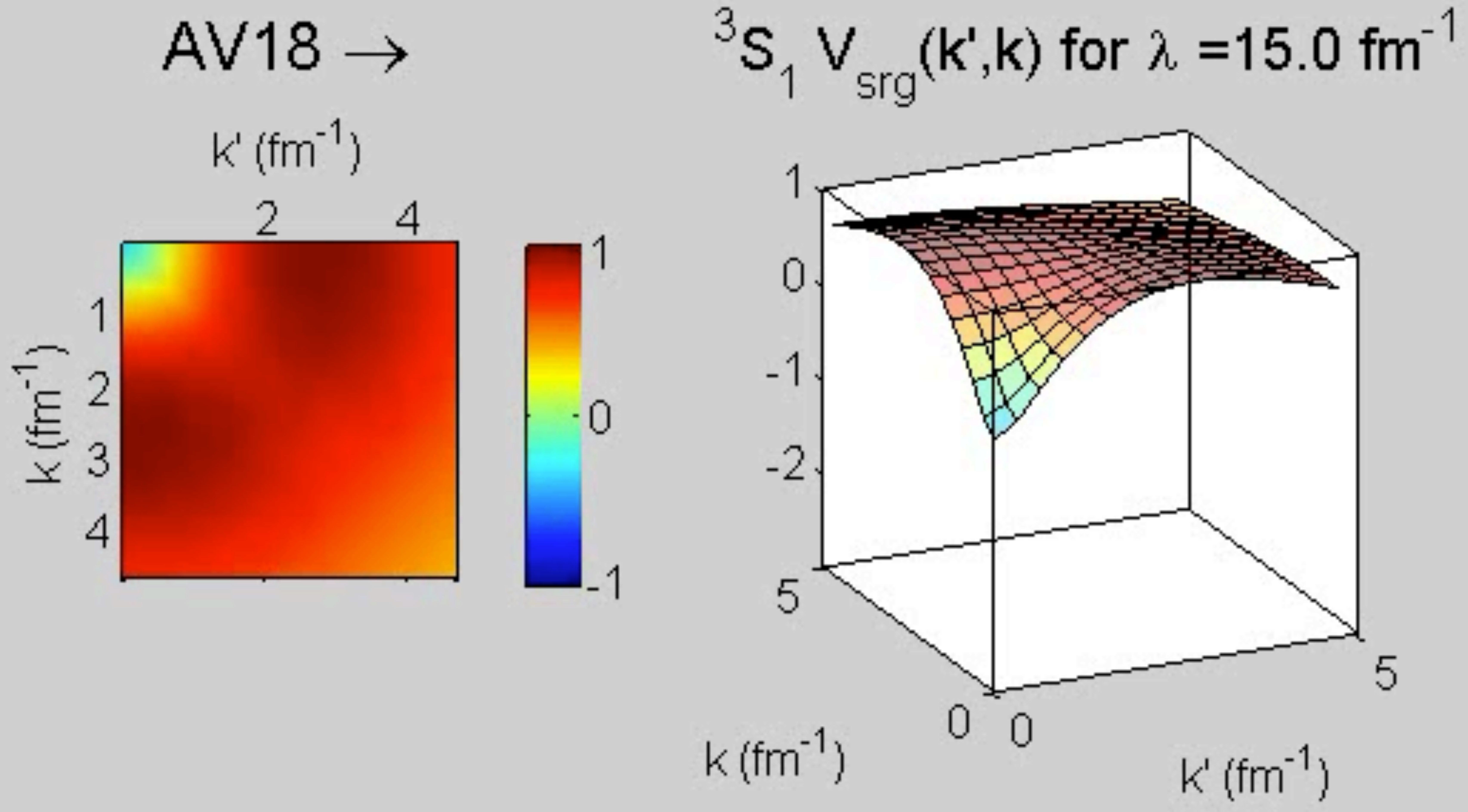


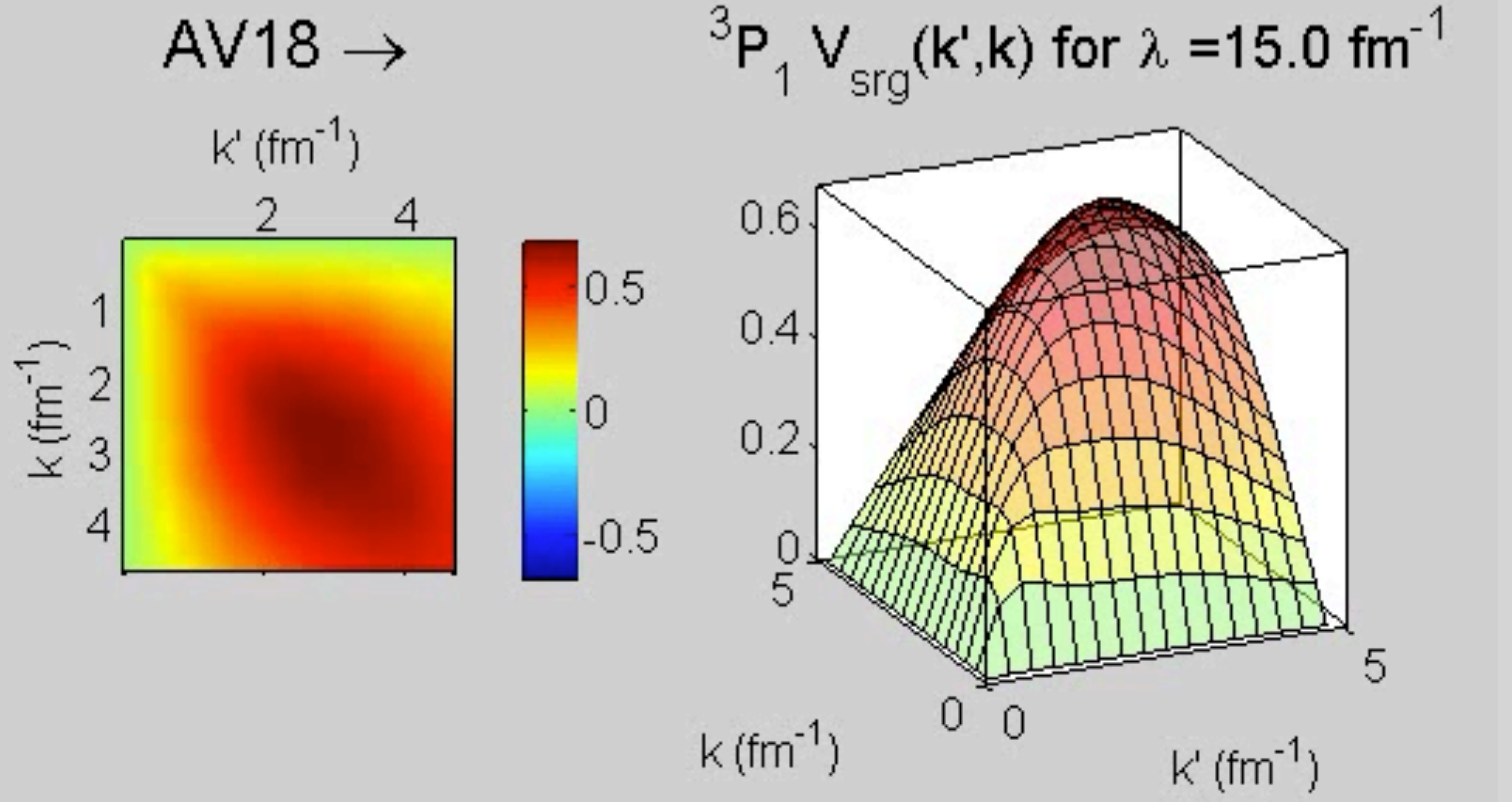
k'
k
λ_0
λ_1
λ_2



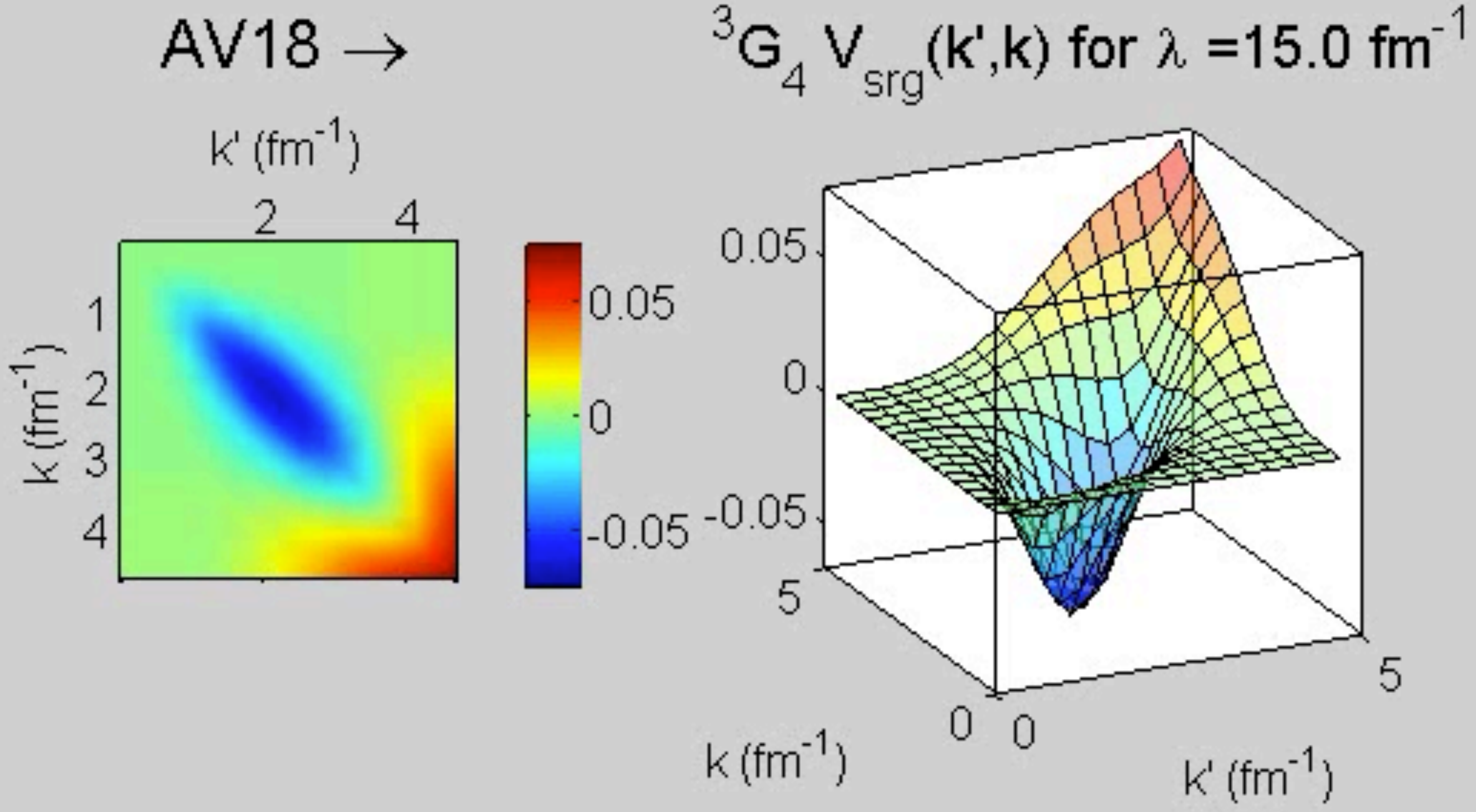
$^1S_0$ $V_{srg}(k',k)$ for λ =4.0 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
0.5
0
-0.5


AV18 →
$^1S_0$ $V_{srg}(k',k)$ for λ =15.0 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
2
4
1
2
3
4
1
0
-1
5
0
5
k (fm^{-1})
k' (fm^{-1})


AV18 →
$^3S_1$ $V_{srg}(k',k)$ for λ =15.0 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
2
4
1
2
3
4
1
0
-1
1
0
-1
-2
5
0
0
5
k (fm^{-1})
k' (fm^{-1})



AV18 →
$^3P_1$ $V_{srg}(k',k)$ for λ =15.0 fm^{-1}
k' (fm^{-1})
2
4
1
2
3
4
k (fm^{-1})
0.5
0
-0.5
0.6
0.4
0.2
0
5
0
0
5
k (fm^{-1})
k' (fm^{-1})


AV18 →
$^3G_4\ V_{srg}(k',k)$ for λ =15.0 fm^{-1}
k' (fm^{-1})
2
4
1
2
3
4
k (fm^{-1})
0.05
0
-0.05
0.05
0
-0.05
5
0
0
5
k (fm^{-1})
k' (fm^{-1})

Are low-energy nuclear observables sensitive to high-energy phase shifts?

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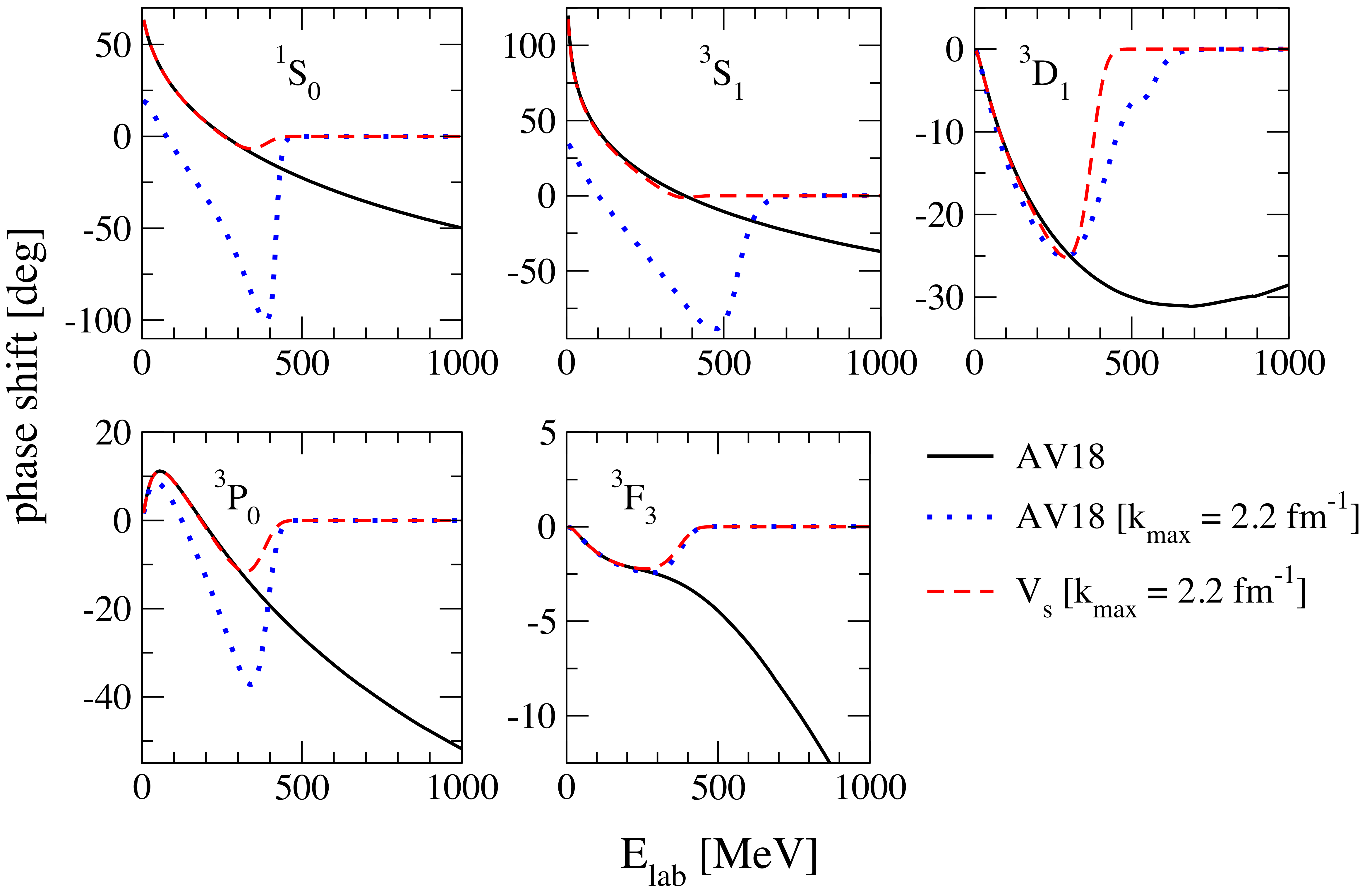

Fig. 2. Phase shifts in selected channels for the Argonne v_{18} potential [17] and when intermediate momenta $k > k_{\max} = 2.2\,\mathrm{fm}^{-1}$ are excluded. We contrast the latter results to the phase shifts obtained from the evolved V_s potential for $\lambda = 2\,\mathrm{fm}^{-1}$ and the additional constraint $k_{\max} = 2.2\,\mathrm{fm}^{-1}$.

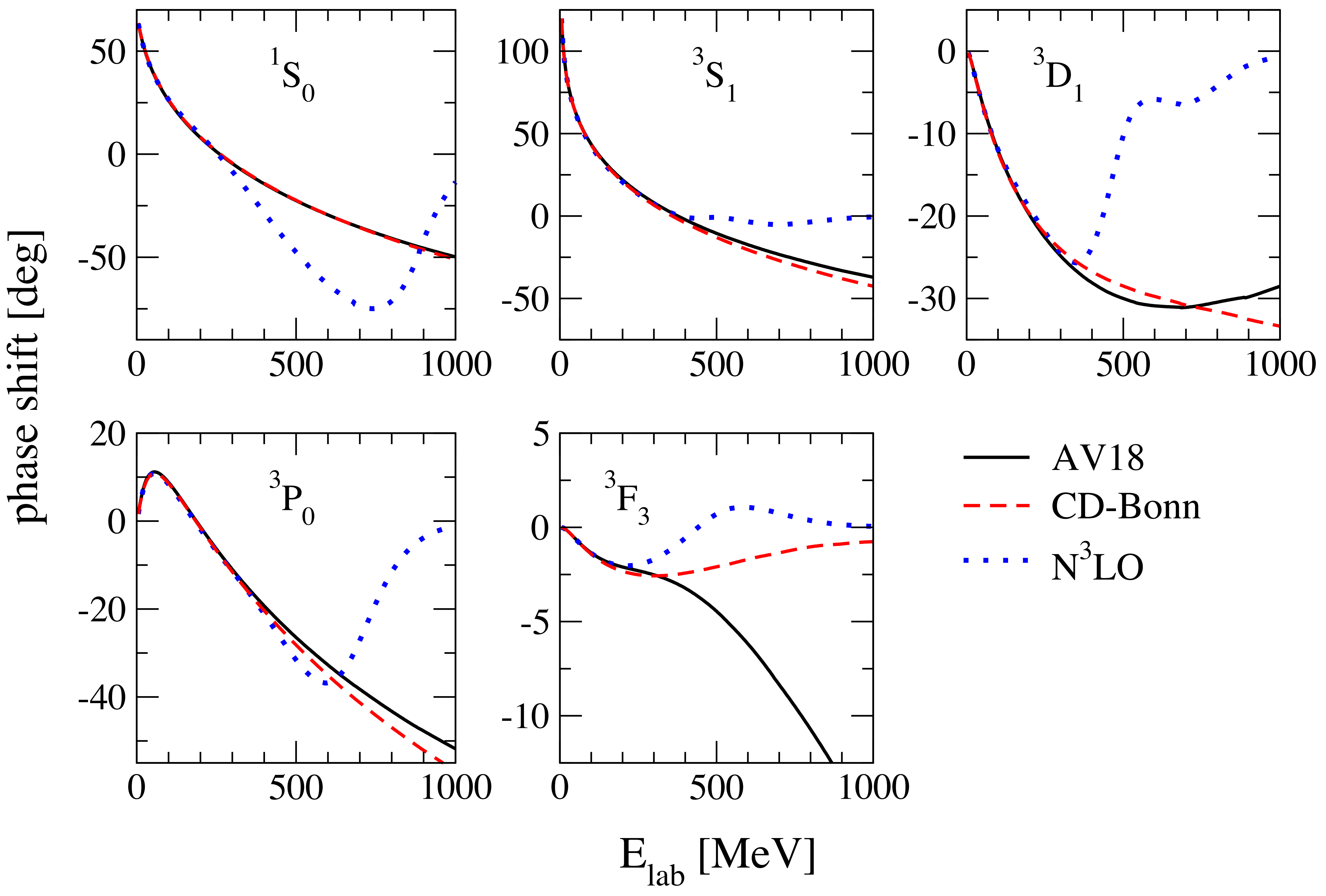

Fig. 1. Phase shifts for the Argonne v_{18} [17], CD-Bonn [18] and chiral N^{3}LO [14] potentials in selected channels (using nonrelativistic kinematics). The phase shifts after evolving in λ from each initial potential agree for all λ to within the widths of the lines at all energies.

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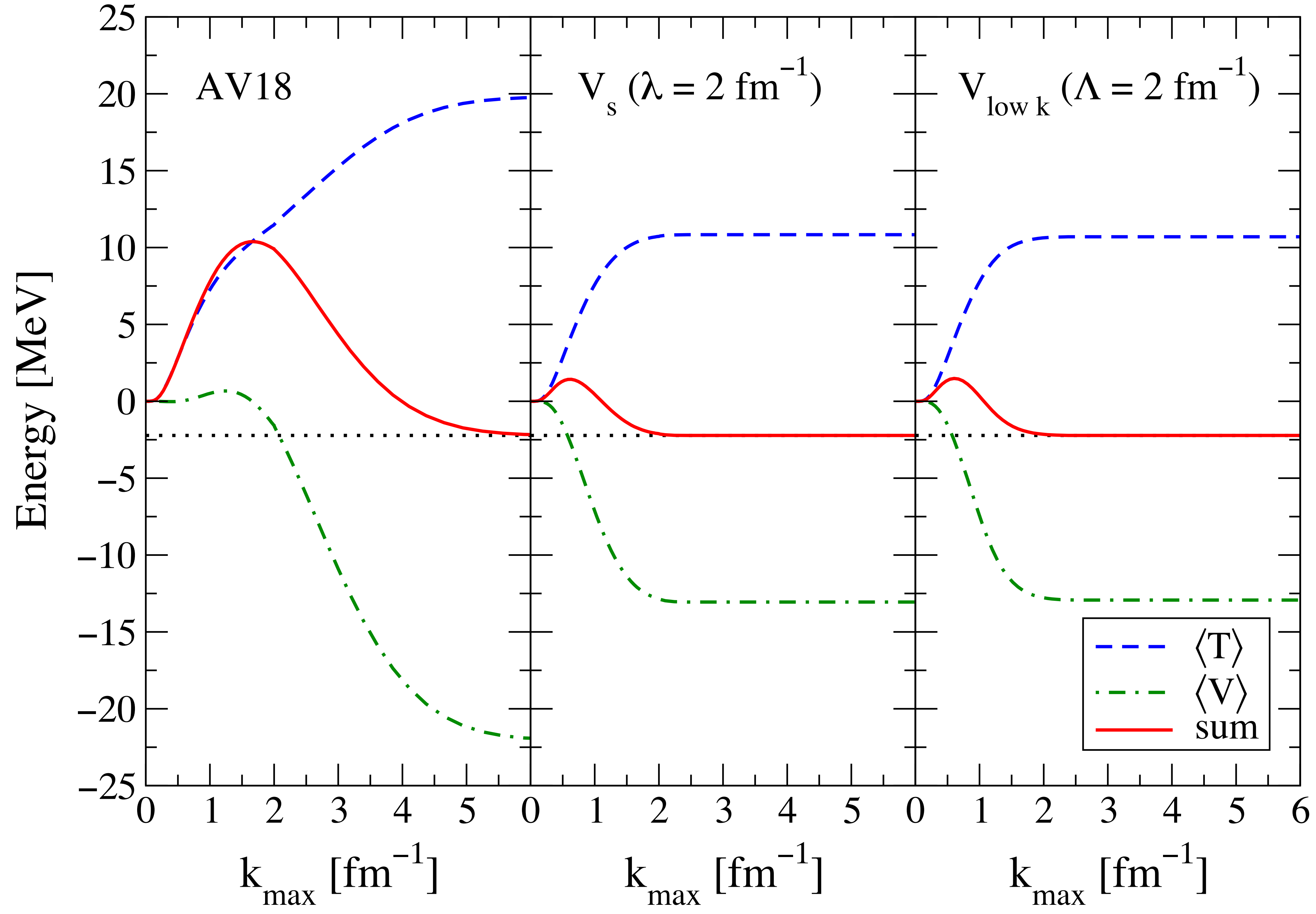


Fig. 3. Expectation values in the deuteron of the kinetic, potential, and total energy evaluated in momentum space as a function of the maximum momentum k_{max}, see Eq. (11). Results are shown for the Argonne v_{18} potential [17] (left), the evolved V_s potential for $\lambda = 2\,\text{fm}^{-1}$, and the smooth-cutoff $V_{\text{low}\,k}$ interaction with $\Lambda = 2\,\text{fm}^{-1}$ and exponential regulator $n_{\text{exp}} = 2$

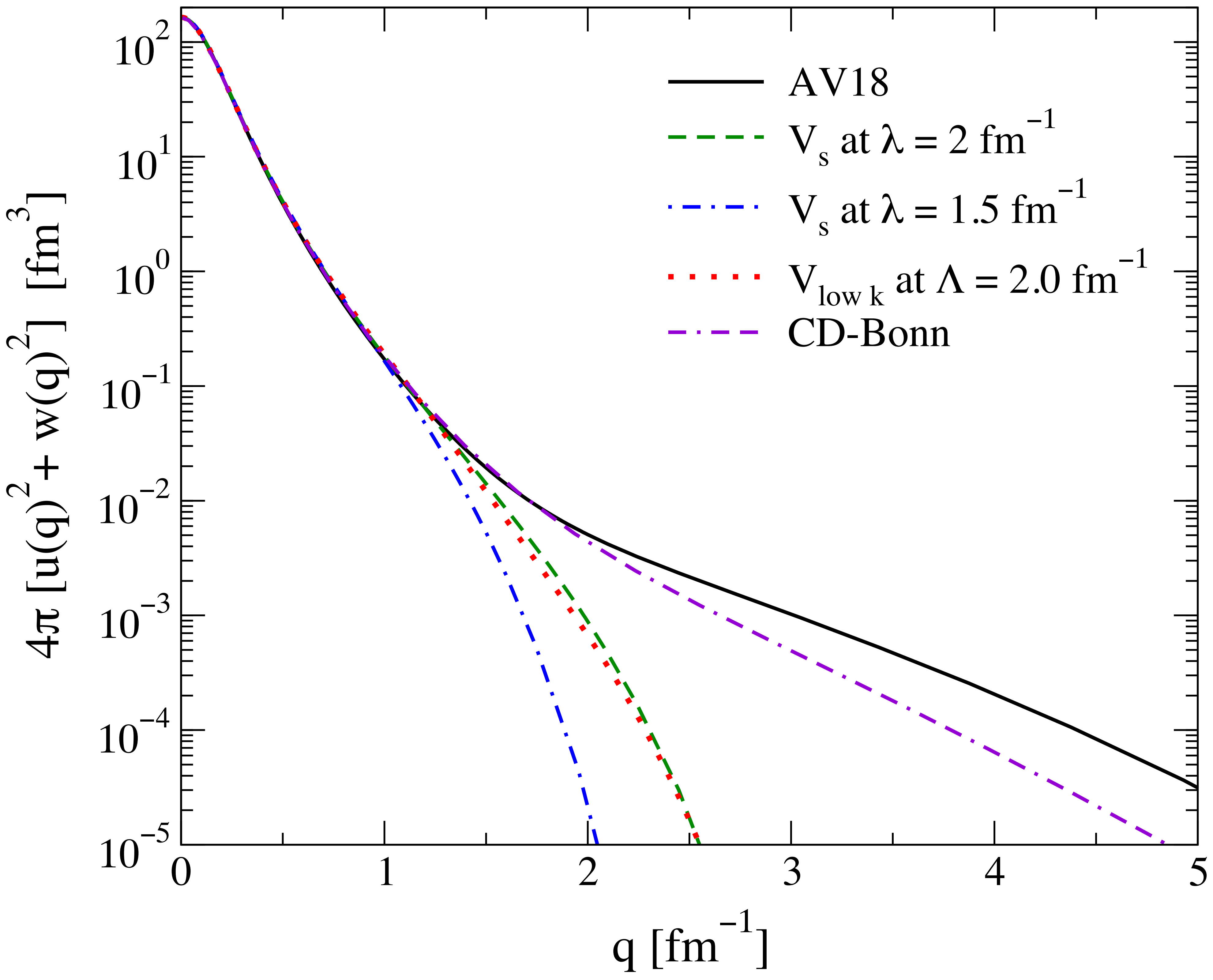
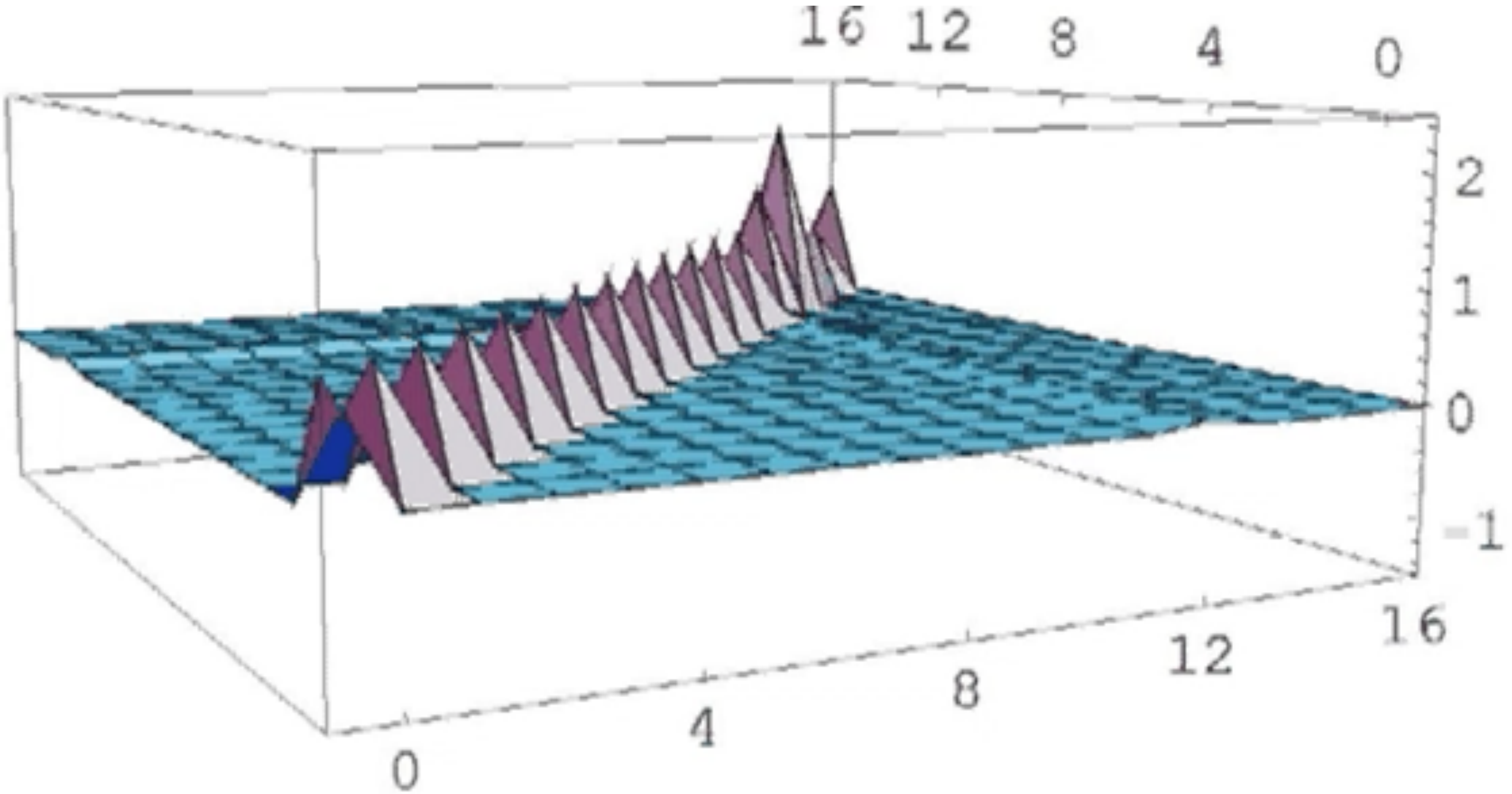


Fig. 4. Deuteron momentum distribution $\langle a^\dagger_{\mathbf{q}} a_{\mathbf{q}} \rangle_d \propto u(q)^2 + w(q)^2$ using the Argonne v_{18} [17], CD-Bonn [18] and SRG potentials evolved from Argonne v_{18} to $\lambda = 1.5\,\text{fm}^{-1}$ and $2\,\text{fm}^{-1}$ (but not evolving the operator), and a smooth-cutoff $V_{\text{low}\,k}$ interaction with $\Lambda = 2\,\text{fm}^{-1}$ and exponential regulator $n_{\text{exp}} = 2$.



Each and every grid point has coordinates **m** and **n** that are the subscripts of the matrix elements of the Hamiltonian **H**. The third coordinate, extending in the vertical direction from -1.5 to 2.5, is used to show the magnitude of the matrix elements. Namely, the vertical coordinate is equal to the ratio of the matrix element $\mathbf{H_{mn}}$ to the square root of the product of eigenvalues $\mathbf{E_m}$ and $\mathbf{E_n}$ of the free Hamiltonian $\mathbf{H_0}$. The films show the RG evolution of the real and imaginary parts of the Hamiltonian matrix elements

Limit cycles of effective theories [Phys. Rev. D75, 025005 (2007); hep-th/0611015]

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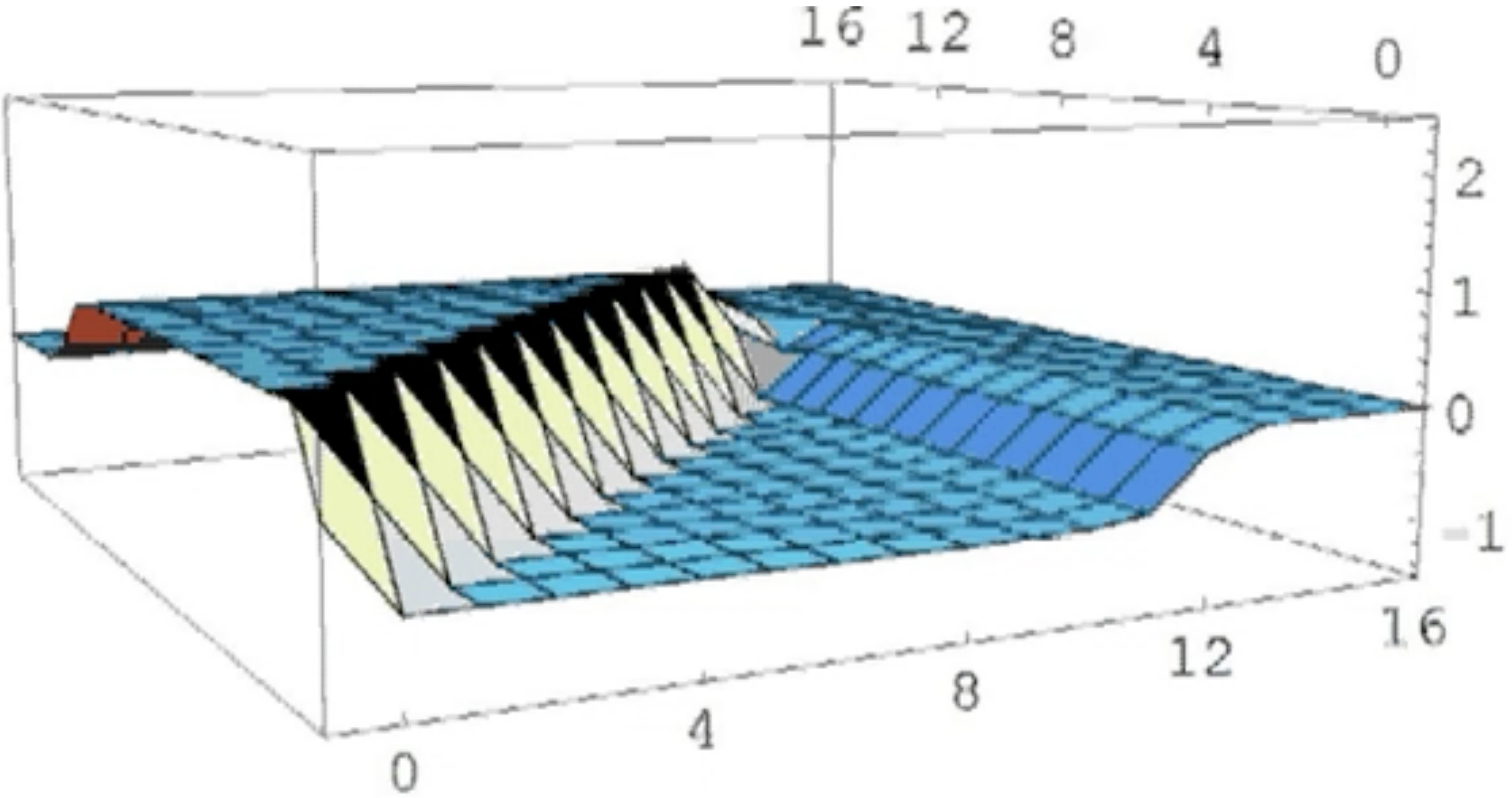

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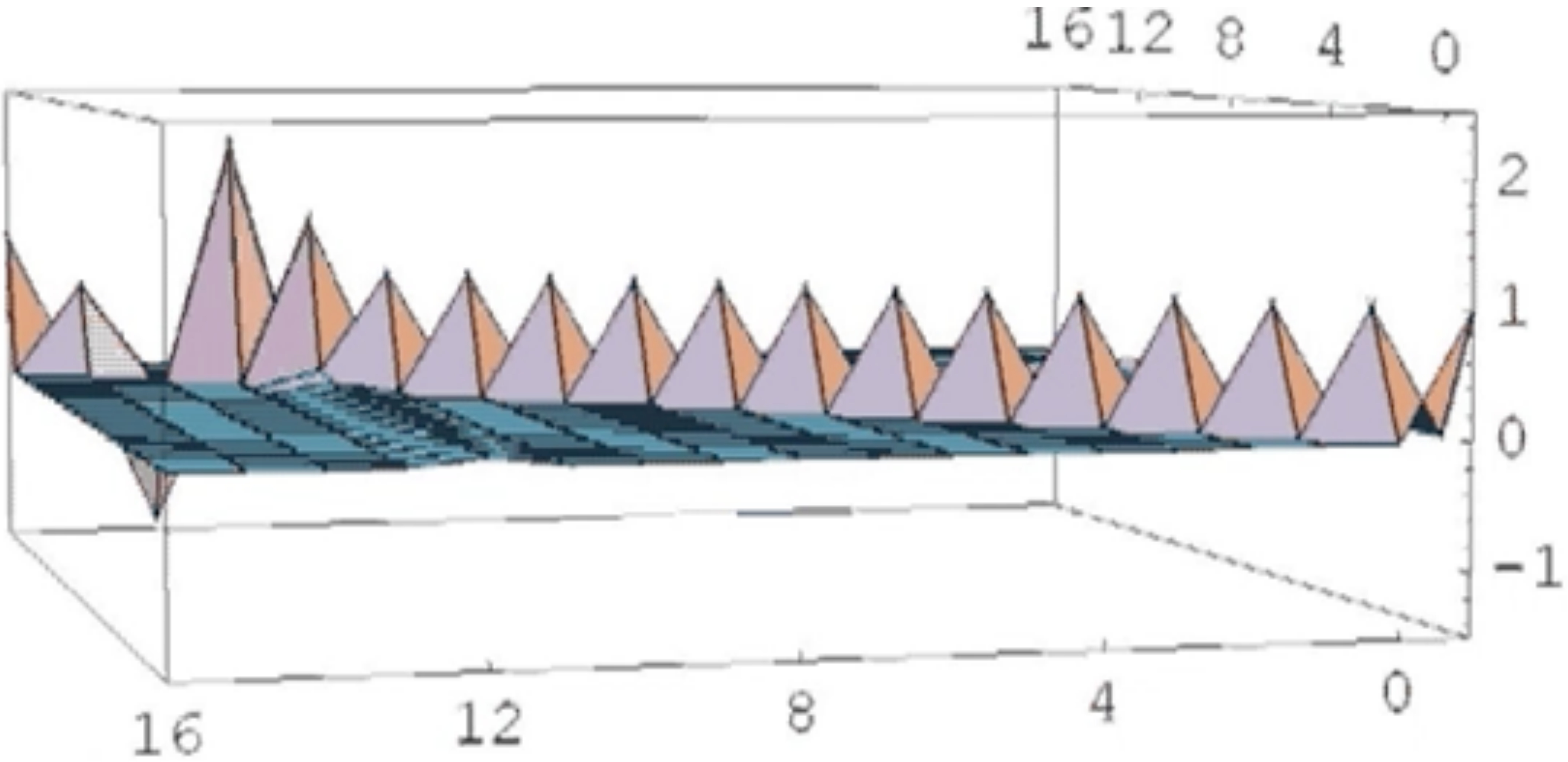

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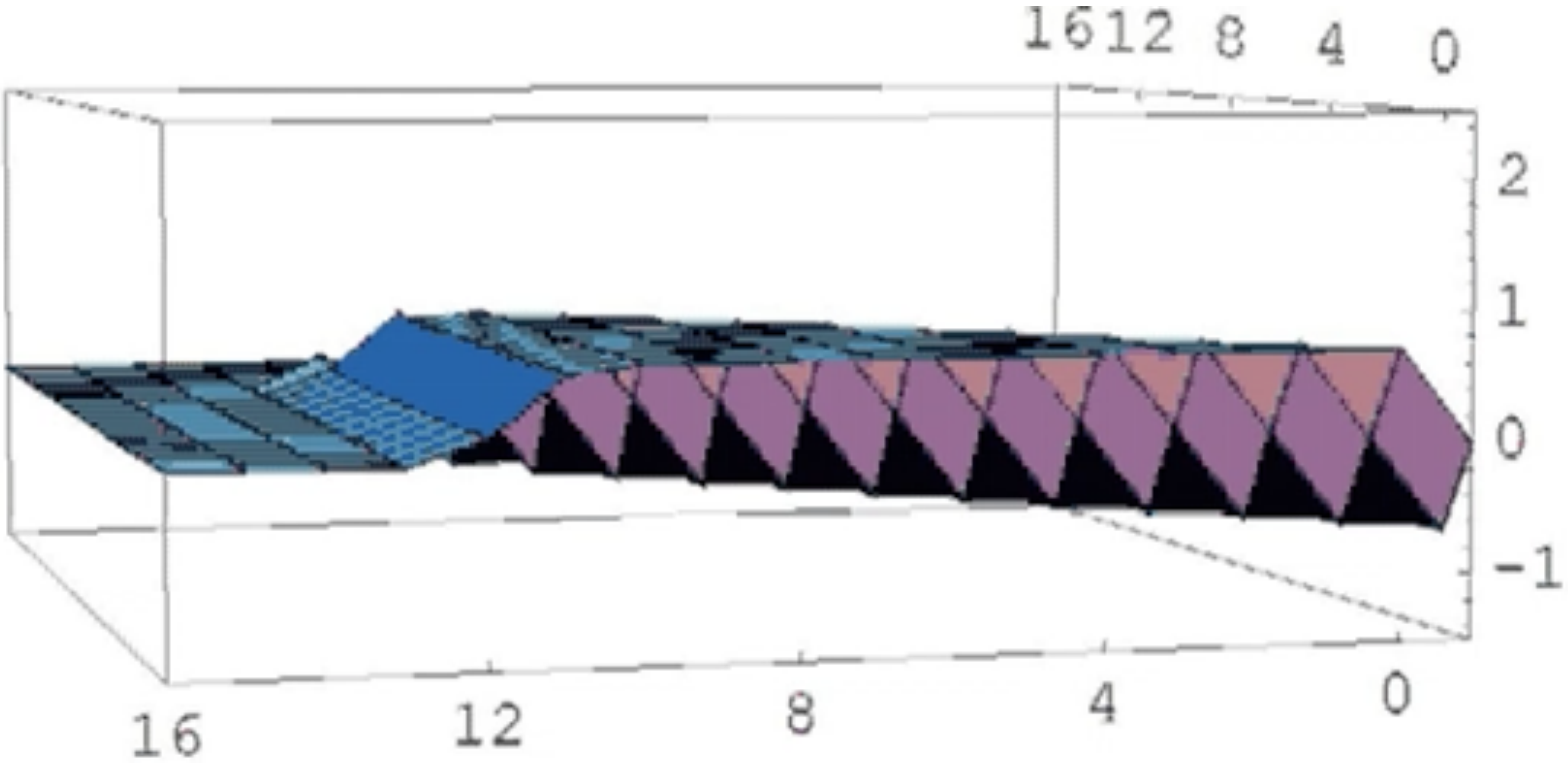

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