4. The Random-Phase Approximation for Collective Excitations

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4.1 Introduction

The random-phase approximation (RPA) is a theory of small-amplitude vibrations in the quantum many-body system. The name was coined in the first application of the method by Bohm and Pines [4.1] to the plasma oscillations of an electron gas. The theory is equivalent to a limit of time-dependent Hartree–Fock theory in which the amplitude of the motion is small. Consequently, the applicability of the theory is restricted to systems where a Hartree–Fock or some effective mean-field theory provides a good description of the ground state. Thus the theory is best for closed-shell nuclei. To the extent that open-shell nuclei can be described with Hartree–Fock–Bogoliubov theory, a corresponding generalization can be applied, the quasi-particle RPA. The first applications in nuclear physics were to collective quadrupole states in open-shell nuclei by Baranger [4.2] and to the collective octupole state in spherical nuclei by Brown, Evans, and Thouless [4.3]. Early applications to nuclear physics were partly phenomenological, owing to deficiencies in the interaction and numerical limitations in the size of the configuration space. With improved interactions and present computer resources, RPA has proven itself to be a robust theory, capable of predicting and describing in detail many properties of collective excitations.

There are two quite different mathematical representations of the RPA, namely the response-function formalism and the A, B-matrix formulation. These are equivalent in principle, but in practice one or the other may be more suited to the problem at hand. The response-function formalism allows excitations to be calculated in very large spaces of configurations – the computational effort is only linear in the number of configurations. However, the interaction must have a simple form, with very restricted possibilities for nonlocality. In particular, the exchange interaction can only be calculated approximately, in a zero-range approximation. In contrast, the matrix formulation puts no limitations on the interaction, but the computational effort is cubic in the number of configurations included in the space. The response-function formalism is particularly convenient for dealing with excitations in the continuum, where the states acquire a width due to the escape of particles. It is much harder to deal with the continuum by the matrix method. The program presented in this chapter has as its primary focus the resonances in the continuum, and it uses the response-function method. The program follows closely the one described in Ref. [4.4]. Other nuclear studies using this method may be found in Refs. [4.5–7]. The method has also been applied to the study of collective electron excitations in spherical

atomic clusters [4.8]. The matrix method has been applied extensively to nuclear excitations as in [4.9] and in [4.10]. The latter calculations in particular are self-consistent and use an interaction that is fully realistic as to range and exchange nonlocality.

As mentioned, the response-function method relies on a simple representation of the interaction. In fact, the interaction must be expressed by a sum of separable terms. The numerical difficulty in the evaluation of the response function is controlled by the number of terms in the interaction; the computation requires inversion of a matrix whose dimensionality is that number. In the program included in this chapter, the interaction is a δ-function, represented on a discrete mesh in coordinate space. The separability of the interaction is obvious from its numerical representation as $\delta(r_1 - r_2) \sim \sum_i \delta(r_1 - R_i)\delta(r_2 - R_i)\Delta R$, where R_i are mesh points and ΔR is the mesh spacing. As one of the compromises in the present program, no dependence of the interaction on currents or on spin is provided. However, the isospin degree of freedom is fully treated by evaluating neutron and proton responses, which are coupled together by the neutron–proton interaction. Other response programs including more general interactions are described in Ref. [4.5]. It is also possible to construct the interaction in momentum space and apply the response technique there [4.11]. However, the coordinate-space representation seems well-suited for interactions of the Skyrme type that are popular in Hartree–Fock calculations.

4.2 Derivation

The RPA may be derived from the small-amplitude limit of the time-dependent Hartree–Fock equations in an external field. The time-dependent Hartree-Fock equations have the structure

$$i\frac{\partial}{\partial t}\phi_i(t) = \left(-\frac{\nabla^2}{2m} + V[\rho] + \lambda V_{\text{ext}} \cos\omega t\right)\phi_i(t). \tag{4.1}$$

Here ϕ_i are the single-particle wave functions which depend on both coordinate and time. In (4.1) V is a potential due to the interacting particles, which in principle is a nonlocal functional of the single-particle density matrix. V_{ext} is an external potential that drives the system at angular frequency ω. In this term the real number λ is a smallness parameter to help us keep track of the first-order deviations from the equilibrium solution. This equilibrium solution is the set of wave functions $\{\phi_i^0\}$ which satisfy the static Hartree–Fock equations,

$$\left(-\frac{\nabla^2}{2m} + V[\rho_0]\right)\phi_i^0 \equiv H_0\phi_i^0 = \epsilon_i\phi_i \tag{4.2}$$

where ϵ_i are the single-particle energies. The perturbed wave function is expanded in the small-amplitude limit as

$$\phi_i = e^{-i\epsilon_i t}(\phi_i^0 + \lambda\delta\phi_i). \tag{4.3}$$

We also need to expand the density to first order in λ. The ordinary density is defined by

$$\rho(r,t) = \sum_i \phi_i^*(r,t)\phi_i(r,t).$$

The expansion is

$$\rho = \rho_0 + \lambda\delta\rho + O(\lambda^2)\,,$$

where

$$\rho_0 = \sum_i \phi_i^{0*}\phi_i^0$$

and

$$\delta\rho = \sum_i (\phi_i^{0*}\delta\phi_i + (\delta\phi_i)^*\phi_i^0). \tag{4.4}$$

We only consider here the simple case where the potential is a function of the ordinary densities (including spin and isospin densities). Then the perturbed potential may be expressed to order λ as

$$V[\rho] = V[\rho_0] + \lambda\frac{\partial V}{\partial\rho}[\rho_0]\,\delta\rho + O(\lambda^2). \tag{4.5}$$

The first term is just the equilibrium Hartree–Fock potential, and the second term, calculated by differentiating the function V with respect to ρ, behaves like a residual interaction. Inserting (4.4) and (4.5) in (4.1) and equating the terms linear in λ, we obtain the equation

$$i\frac{\partial}{\partial t}\delta\phi_i + \epsilon_i\delta\phi_i = H_0\,\delta\phi_i + \frac{\partial V}{\partial\rho}\,\delta\rho\,\phi_i^0 + V_{\text{ext}}\cos\omega t\,\phi_i^0. \tag{4.6}$$

Next $\delta\phi_i$ is separated into its real and imaginary parts, and (4.6) is written as separate equations for the two parts. A solution to the inhomogeneous equations is obtained by taking $\delta\phi_i$ to have the time dependence

$$\delta\phi_i(t) = \cos\omega t\,\mathrm{Re}\,\delta\phi_i + i\sin\omega t\,\mathrm{Im}\,\delta\phi_i.$$

We use the same symbol ϕ to denote both time-dependent and time-independent functions, but this should cause no confusion. The equations satisfied by the (time-independent) $\delta\phi_i$ are then

$$\omega\,\mathrm{Im}\,\delta\phi_i = [H_0 - \epsilon_i]\,\mathrm{Re}\,\delta\phi_i + \frac{\partial V}{\partial\rho}\delta\rho\phi_i^0 + V_{\text{ext}}\phi_i^0$$

$$\omega\,\mathrm{Re}\,\delta\phi_i = [H_0 - \epsilon_i]\,\mathrm{Im}\,\delta\phi_i. \tag{4.7}$$

We eliminate $\mathrm{Re}\delta\phi_i$ from the two equations to obtain

$$\mathrm{Re}\delta\phi_i = [\omega^2 - (H_0 - \epsilon_i)^2]^{-1}[H_0 - \epsilon_i]\left(\frac{\partial V}{\partial\rho}\rho\phi_i + V_{\mathrm{ext}}\phi_i\right). \tag{4.8}$$

The reciprocal operator is simplified using

$$(A^2 - B^2)^{-1}B = \frac{1}{2}[(A - B)^{-1} - (A + B)^{-1}].$$

The equation for $\mathrm{Re}\delta\phi$ is then

$$\mathrm{Re}\delta\phi_i = -\frac{1}{2}([H_0 - \omega - \epsilon_i]^{-1} + [H_0 - \epsilon + \omega]^{-1})\left(\frac{\partial V}{\partial\rho}\delta\rho + V_{\mathrm{ext}}\right)\phi_i^0. \tag{4.9}$$

We can eliminate $\delta\phi_i$ as a variable by multiplying (4.9) by ϕ_i^{0*} and summing over i, which is equal to half of $\delta\rho$. The resulting equation is expressed compactly as

$$\delta\rho = -G^0\left(\frac{\partial V}{\partial\rho}\delta\rho + V_{\mathrm{ext}}\right),$$

where G^0 is defined in coordinate space as

$$G^0(r, r') = \sum_i \phi_i^{0*}(r)([H_0 - \epsilon_i - \omega]^{-1}_{r,r'} + [H_0 - \epsilon_i + \omega)]^{-1}_{r,r'})\phi_i^0(r'). \tag{4.10}$$

This equation is formally solved for $\delta\rho$ as

$$\delta\rho = G^{\mathrm{RPA}}V_{\mathrm{ext}}, \tag{4.11}$$

where

$$G^{\mathrm{RPA}} = \left[1 + G^0\frac{\partial V}{\partial\rho}\right]^{-1} G^0. \tag{4.12}$$

This formal expression is the basis of the response-function method for obtaining the excitations of the system. The operators are approximated in some suitable basis and the resulting matrix equations are solved by matrix inversion. As mentioned in the beginning, a coordinate-space representation is very useful, and is the basis in the numerical program.

We next make an angular-momentum decomposition of the functions appearing in the response. This permits the RPA to be calculated from independent radial equations for each multipole, provided the Hartree–Fock state is spherically symmetric. We define the multipolar interaction v_L and response G_L as follows:

$$\frac{\partial V(r)}{\partial\rho(r')} = \sum_{LM} v_L(r_1, r_2)Y^*_{LM}(\hat{r}_1)Y_{LM}(\hat{r}_2)/r_1r_2\ ,$$

$$G^0(r_1,r_2) = \sum_{LM} G_L(r_1,r_2) Y^*_{LM}(\hat{r}_1) Y_{LM}(\hat{r}_2)/(r_1 r_2)^2 ,$$

where the Y_{LM} are the usual spherical harmonics. The single-particle Green's function is similarly expanded in a basis of the $(ls)j$-coupled single-particle states. Then the formula for the independent particle response is

$$\begin{aligned} G_L(r,r',\omega) &= \sum_{j,j'} \frac{(2j+1)(2j'+1)}{4\pi(2L+1)} \left(j'\tfrac{1}{2} j - \tfrac{1}{2}|L0\right)^2 \phi_j(r)^* \phi_j(r') \\ &\quad ([H_0 - \epsilon_j - \omega]^{-1}_{j';r,r'} + [H_0 - \epsilon_j + \omega]^{-1}_{j';r,r'}) , \end{aligned} \tag{4.13}$$

where the sum over single-particle states j, j' is restricted to those with orbital angular momentum l and l' such that $L + l + l'$ is even. In this equation, the ϕ_j are radial wave functions multiplied by r.

The radial Green's function is most conveniently evaluated in the coordinate representation by the usual Green's function formula for second-order linear differential equations,

$$[H_0 - \epsilon]^{-1}_{r,r'} = \frac{u(r_<)w(r_>)}{W}. \tag{4.14}$$

Here u, w are solutions of the radial differential equation that have appropriate boundary conditions, i.e. u must be regular at the origin and w must be a pure outgoing wave at infinity:

$$u(r) \to r^{L+1} \quad \text{for} \quad r \to 0,$$

$$w(r) \to \exp(ikr) \quad \text{for} \quad r \to \infty.$$

W in (4.14) is the Wronskian of the differential operator,

$$W = \frac{1}{2m}\left(w\frac{\mathrm{d}u}{\mathrm{d}r} - u\frac{\mathrm{d}w}{\mathrm{d}r}\right).$$

The program calculates G_L^{RPA} as a matrix in coordinate space. The radial transition density associated with the external field is then calculated by the relation

$$\delta\rho_L(r) = \int \mathrm{d}r' G_L^{\mathrm{RPA}}(r,r',\omega) V_{\mathrm{ext}}(r'). \tag{4.15}$$

This is related to the usual transition density by

$$\delta\rho(\boldsymbol{r}) = \delta\rho_L(r) Y_{LM}(\hat{r})/r^2.$$

Finally, the external field is integrated over the transition density to obtain the response function to the external perturbation of the system,

$$G_L(V_{\mathrm{ext}},\omega) = \int \mathrm{d}r \delta\rho_L(r) V_{\mathrm{ext}}(r) = \int \mathrm{d}r\mathrm{d}r' V_{\mathrm{ext}}(r) G_L^{\mathrm{RPA}}(r,r',\omega) V_{\mathrm{ext}}(r').$$

The real and imaginary parts of this function are the primary output quantities that are printed.

4.3 Strength Function and Transition Strengths

Information about the transition strengths between an initial state i and final states f is contained in the strength function

$$S(\omega) = \sum_f \langle i|V_{ext}|f\rangle^2 \delta(E_f - E_i - \omega).$$

The relation to the response-function is simply

$$S(\omega) = \frac{1}{\pi}\mathrm{Im}G_L(V_{\mathrm{ext}}, \omega). \tag{4.16}$$

This formula is often used in applications. For example, inelastic scattering in the continuum is conveniently calculated using (4.16) with V_{ext} being the transition field of the projectile, or some approximation to the projectile interaction [4.12]. The experimental strength function is often smoother than the RPA prediction, either because of experimental resolution or because of physical damping processes not described by the RPA. To facilitate comparison with smoothed strengths, it is useful to artificially add an imaginary part to the frequency. This is equivalent to convoluting the strength function with a Lorentzian. Otherwise, only the boundary condition on the single-particle Green's function, that particles be outgoing, provides an imaginary part to the response and a width to the resonances.

In the neighborhood of a resonance of the system, the response function may be expressed as

$$G(V_{\mathrm{ext}}, \omega) \sim \langle i|V_{ext}|R\rangle^2 \left(\frac{1}{E_R + \mathrm{i}\Gamma/2 - \omega}\right), \tag{4.17}$$

where E_R and $\Gamma/2$ are the real and imaginary parts of the frequency of the mode. Equation (4.17) is useful for extracting the properties of individual vibrational modes from the response. To find the squared matrix element for an individual resonance, one first calculates the response over a small frequency interval containing the mode in question. One may then either fit the imaginary part to the function $\mathrm{Im}(E_R + \mathrm{i}\Gamma/2 - \omega)^{-1} = \Gamma/2/((E_R - \omega)^2 + (\Gamma/2)^2)$ or simply integrate over an interval containing the resonance,

$$\langle i|V_{\mathrm{ext}}|R\rangle^2 \sim \int_{E_R-\Delta}^{E_R+\Delta} S(\omega)\mathrm{d}\omega.$$

The transition density associated with a resonance is defined in terms of the matrix element of the density operator as

$$\delta\rho = \langle i|\hat{\rho}|R\rangle.$$

It may be calculated from the above quantities by the equation

$$\delta\rho = \frac{G^{\mathrm{RPA}}\, V_{\mathrm{ext}}}{\langle i|V_{\mathrm{ext}}|R\rangle}.$$

For inelastic scattering of hadronic projectiles, the transition strengths are often expressed in terms of the collective-model parameter β_L. This model assumes that the nucleons in the nuclear interior move incompressibly, so that the transition density is surface peaked. The transition density is taken to have the form

$$\delta\rho_L \sim \frac{\beta_L}{\sqrt{2L+1}} R_0 \frac{\mathrm{d}\rho_0}{\mathrm{d}r} Y_{LM} , \tag{4.18}$$

where R_0 is an appropriately defined nuclear radius. The magnitude of the transition density is set by β_L. Since the RPA transition densities of collective states follow (4.18) quite well, the β_L parameters of the transitions can be extracted from the multipole matrix element by relating it to the equivalent matrix element from eq. (4.18). This yields

$$\beta_L = \frac{4\pi\sqrt{2L+1}\langle i|r^L Y_{LM}|R\rangle}{(L+2)AR_0\langle r^{L-1}\rangle}. \tag{4.19}$$

4.4 Sum Rules and the Spurious State

The integrated transition strengths obey sum rules which are respected by the RPA theory. It is an important check on calculations to see how well the sum rules are satisfied numerically. Also, it is convenient to express transition strengths as a fraction of the appropriate sum rule, which eliminates any possible ambiguity in the definition of the strength functions or the normalization of the operator matrix elements.

For the Hamiltonians considered here in which the potential only depends on density, the sum rules may be expressed generally as

$$\int S(\omega)\omega \mathrm{d}\omega = \int \mathrm{d}^3 r \frac{(\nabla V_{\mathrm{ext}})^2}{2m}\rho_0.$$

In the special case of multipole fields, $V_{\mathrm{ext}} = r^L Y_{LM}(\theta,\phi)$, the sum rule becomes [4.13]

$$\int S_L(\omega)\omega \mathrm{d}\omega = \frac{L(2L+1)}{8\pi m}\int \mathrm{d}^3 r\, r^{2L}\rho_0 = \frac{L(2L+1)}{8\pi m} A\langle r^{2L-2}\rangle. \tag{4.20}$$

This equation applies both to isoscalar and isovector fields, i.e. fields in which V_{ext} acts on both neutrons and protons with a positive or negative phase relationship, respectively. For a pure proton field, the number of particles A in (4.20) should be replaced by Z. In the program, the strength function is calculated for some range of energy, and the total strength in that interval is compared with the multipole sum rule.

Finally, we mention the spurious-state phenomenon. The fact that the Hartree–Fock state is translationally degenerate means that a fictitious excitation can be created simply by displacing the Hartree–Fock wave functions

by a small amount. In the RPA, this appears as an excitation at $\omega = 0$, having L=1. Thus a good test of complete consistency in the calculation is the presence of this zero-frequency mode. In practice, the consistency will not be perfect, but there will always be an isoscalar mode at a frequency close to zero.

The existence of the spurious state causes some difficulty with the dipole sum rule. Since the spurious state has practically all of the isoscalar dipole strength, the isoscalar and proton sum rules will be grossly violated unless the contribution from that state is included.

4.5 Numerics

At the core of the program is the computation of the radial single-particle Green's function (4.14) in the subroutine `GREEN`. Here the simplest possible second-difference algorithm is used to integrate the radial Schrödinger equation. This part of the program consumes relatively little time, so there is not much point in using fancier methods. A mesh in coordinate space of 0.1 to 0.25 fm is quite adequate to obtain accuracy of less than 0.1 MeV on the single-particle eigenenergies. The boundary condition on the Green's function is that it corresponds to an outgoing wave if the particle is unbound, and it vanishes at the coordinate space boundary if the particle is bound.

The input Hartree–Fock wave functions should have been calculated with the same numerical Hamiltonian as the Green's function for complete consistency; in fact the same algorithm should be used. In the program, a subroutine `STATIC` is provided which computes Woods–Saxon single-particle wave functions with the same algorithm. This subroutine requires as input the mass and charge of the nucleus and the quantum numbers of the occupied orbits. If the self-consistency is not strictly maintained, transition strength may appear at frequencies corresponding to transitions between occupied states. These Pauli-forbidden transitions in principle cancel out in the two terms in (4.10). The boundary condition for the particle Green's function in the subroutine `GREEN` is consistent with the particle spectrum of the subroutine `STATIC`.

The most time-consuming part of the calculation for light nuclei is the inversion of the polarization matrix, the first factor in (4.12). Since the matrix is complex, readily available library inversion routines cannot be used; a simple matrix-inversion subroutine, `MATR`, is included with the program. However, only the vector array `DRHRPA`, defined in (4.15), is needed in the program. If time is a real consideration, it is faster to obtain `DRHRPA` by directly solving the linear equations relating it to the vector array `DRHOF`. The mesh size for the matrix is set independently of the single-particle wavefunction mesh. The multipole strength function can be computed to about 10% accuracy with a mesh as coarse as 1 fm. Unless the external field is strongly varying, there is no useful gain in accuracy with mesh spacings

smaller than 0.5 fm. To give an example of the dimensionality, the response for ^{208}Pb requires a radial interval extending to about 9 fm. With a 1 fm mesh spacing and remembering that neutrons and protons are treated separately, the matrix size is 18 by 18.

The residual interaction should also be input consistently with the Hartree–Fock Hamiltonian. However, to make the program run as is, a subroutine **VRESID** is included that constructs a residual interaction based on the Skyrme parameterization. To alleviate the lack of consistency between this subroutine and **STATIC**, a parameter **VSCAL** is provided to renormalize the strength of the interaction. This may chosen to put the spurious state at zero frequency. As the program stands, the required renormalization factor is in the range 0.8–1.0.

The external field is determined in the subroutine **EXTV**. The spatial form of the field is a pure multipole, $V_{\rm ext} = r^L Y_{LM}$. There are three choices for the isospin character of the field: it can act on protons only, on protons and neutrons with equal amplitude (isoscalar), or on protons and neutrons with opposite amplitudes (isovector). The sum rule for the field (4.20) is calculated in the subroutine **SUMRLE**. Other fields besides multipole fields may be of interest. For example, electron scattering could be represented by a plane-wave field, which would have a Bessel-function radial dependence in the multipole expansion. If the user substitutes another field, the corresponding sum rule is automatically calculated by **SUMRLE**.

A detailed description of the input format is provided in Table 4.1.

4.6 Tests and Studies

A data set is provided for the response of ^{16}O. The first thing to examine is the consistency between the single-particle wave functions, the single-particle Hamiltonian H_0, and the residual interaction. If these are self-consistent, there will be a zero-frequency mode in the $L = 1$ response. The sample data set is set up to make this computation. What follows are some things to note from the output of this first test run. The free particle response will show characteristic behavior due to the existence of particle-hole excitations at a frequency corresponding to the shell-gap energy. In ^{16}O, the shell gap is about 14 MeV. The real part of the response is negative just above the transition frequency, and is large and positive just below it, as in (4.17). The imaginary part of the response has a peak just at the transition frequency. Of course, if the states are bound, the imaginary part will vanish, unless an imaginary component is imposed numerically on the single-particle Green's function. This may be accomplished by giving nonzero values to the parameter **GAM**.

The interacting response will have a peak near zero frequency, the spurious state. The real part of the response will be positive if the spurious state has positive energy and negative if the residual interaction is so strong

Table 4.1. Input to the RPA program. The numerical values are the data appropriate for the calculation of the dipole response of the nucleus ^{16}O.

Line	Variable	Explanation
1	DEL, NGRID	DEL=0.25 fm is the mesh size for the Schrödinger equation and single-particle Green's function. NGRID=50 is the number of mesh points. Thus, wave functions are calculated over the spatial range 0 to DEL*NGRID=12.5 fm.
2	A,Z	A=16 and Z=8 are the mass and charge of the nucleus, necessary to determine an appropriate Woods–Saxon potential to construct the (not Hartree–Fock) single-particle wave functions.
3.1 ... 3.n	L,J,NQ, NODE	These are the quantum numbers for the occupied orbitals, with J twice the $(ls)j$-coupled angular momentum and NQ the charge. For example, the lowest $p_{3/2}$ proton orbital has L=1,J=3,NQ=1,NODE=0. The orbital data is terminated by the line −1 0 0 0.
4	N, DEL2	The mesh for the response function has N=10 points spaced by DEL2=0.5 fm.
5	T0,T3, X, VSCAL	These are parameters of the residual interaction in the Skyrme formulation, with VSCAL an overall scaling factor. Self-consistent values for infinite nuclear matter are close to T0=−1100 MeV fm^3, T3=15,000 MeV fm^5. The spin exchange parameter X has the value X=0.5. Because the self-consistency is not implicit in the wave functions, VSCAL=0.93 to produce a spurious state close to zero frequency.
6	L,EX, EXM, DEX, GAM	Here L=1 is the multipolarity of the response. The response is calculated on a grid of energies starting from EX=0 MeV and ranging up to EXM=40 MeV in steps of DEX=1 MeV. GAM=1 MeV is an added imaginary part of the energy (actually i*GAM/2 is added) to smooth the strength function.
7	I	I is a character specifying the isospin of the external field. I=0 and I=1 correspond to isoscalar and isovector external fields. For any other value, e.g. I=−1, the external field acts only on the protons.

as to bring it below zero frequency. Assuming that the input is not completely self-consistent, vary the interaction scaling parameter VSCAL to put the spurious state near zero. Also, it is interesting to see how sensitive the frequency of the lowest state is to the interaction.

As mentioned before, it is important to check the sum rule. For this one needs a large range of energies, and one needs to sample the energies at small enough intervals to pick up all the states. Examine the proton L=1 response in ^{16}O, using several values of GAM and a step size of 1 MeV in the mesh of frequencies. Also, set VSCAL slightly smaller than the self-consistent value so that the spurious state in included in the sum. Why does GAM=0 give poor results? How large a GAM is required to smooth the strength function sufficiently for the 1 MeV frequency mesh? How high an energy interval must be included to get 90% of the sum rule?

Examine the isoscalar quadrupole response by setting L=2 in data line 6 and I=0 in line 7. You will see that the attractive interaction causes the quadrupole strength to be shifted down from the independent particle strength function. In ^{16}O, the RPA predicts that all of the strength is in a peak at about 20 MeV excitation, the giant quadrupole resonance. The energy agrees well with the empirical value of 22 MeV [4.14]. What is the width of the resonance? The RPA width will be small compared to the empirical width, which is of the order of 6 MeV. This is due to the neglect in RPA of damping into more complicated configurations than those having a 1-particle 1-hole structure. Do the same for the nucleus ^{208}Pb. Here the giant quadrupole is at 10.9 MeV, and furthermore there is significant quadrupole strength at lower excitation. What fraction of the sum rule is in the lower portion of the strength function? Experimentally, there is a 2^+ state at 4.086 MeV which has a strength given by $\beta_2 = 0.058$ [4.15]. Using (4.19), compare the predicted strength with the empirical.

It is also interesting to see how close the transition density is to the collective model. To get the radial transition density from the program, it is necessary to print out the vector array DRHRPA. It may then be compared with the collective model shape, $\frac{d\rho_0}{dr}$. Remember that the radial transition density includes an extra factor of r^2.

If a nucleus is unstable with respect to quadrupole deformation in the Hartree–Fock ground state, the L=2 RPA response of the spherical state will be negative at zero frequency (owing to the presence of an imaginary eigenfrequency). Are there any j-shell closures that are unstable with respect to quadrupole deformation? For example, the nucleus ^{28}Si with a filled $d_{5/2}$ shell is on the borderline of stability. If the strength of the spin–orbit potential in the single-particle Hamiltonian is decreased, it becomes deformed.

The isoscalar monopole resonance (L=0) is very interesting because of its close connection with the compression modulus of nuclear matter [4.10]. What is the predicted frequency in ^{208}Pb of this mode? Experimentally,

it lies at 14 MeV [4.16]. The RPA prediction with Skyrme interactions is at a higher excitation energy. The Skyrme interaction apparently has too strong a density dependence. Change the interaction (via VSCAL) to put the monopole resonance at the correct excitation. What is its width? This should be compared with the experimental width of $\sim$ 3 MeV.

There is a 0^+ state in ^{4}He at 20.1 MeV excitation having a width of 0.27 MeV [4.17]. Can this state be reasonably described as a monopole resonance? Although the use of RPA for a 4-particle nucleus is problematic, it is interesting to see whether the width is consistent with the monopole assumption. To make a fair test, the input parameters in the program need to be adjusted to give the correct separation energy of the ^{4}He nucleus as well as reproduce the energy of the monopole.

Examine the octupole response of the nucleus ^{208}Pb. The lowest L=3 state in this nucleus has a transition strength of 38 single-particle units, making it one of the most collective octupole states of any nucleus. The state has an excitation energy of 2.6 MeV. The square of its charge matrix element $er^2 Y_{LM}$ is given by the $B(E3, 3 \rightarrow 0)$, with an experimental value $B(E3, 3 \rightarrow 0) = 0.9 \times 10^5$ e^2 fm^6 [4.15]. Its β moment is 0.12. Determine the theoretical position and strength of this state, being sure that VSCAL is set to satisfy the consistency condition on the dipole state. It is also interesting to compare the shape of the transition density of this state with experiment [4.18].

The giant dipole resonance may be examined using the isovector field, setting the parameter I=1 in line 7. Determine where the center of gravity of the strength is for ^{16}O. Experimentally, it is much higher, at about 25 MeV excitation. The disagreement is partly due to an inadequate description of the interaction in the program. We have assumed that the potential field depends only on local density. In fact, the Hartree–Fock field is nonlocal, so that nucleons in a nucleus have an effective mass of about 0.75 times the free mass. This is not noticeable in the isoscalar motion, because when the nucleons move together the interaction restores the potential field to give back the free mass. An ad hoc way to circumvent this deficiency is to change the mass of the nucleon by hand in the program. Put in an effective mass of 0.75 the free mass, and see how well the giant dipole is reproduced. What is the width of the predicted dipole resonance? The experimental width of the dipole in light nuclei is attributed to the nucleon escape width, and so should be reproduced in the model. Try the same study in ^{208}Pb. Here the giant dipole resonance is at 13.5 MeV; it has a width of 4 MeV and has a very smooth shape [4.19]. In this case, the escape width is only a small fraction of the total width and the shape of the RPA resonance bears little resemblance to the actual line shape of the giant dipole resonance.

Are there threshold effects in the dipole response associated with nearly unbound s-wave orbitals? There have been speculations that nuclei near the limit of zero nucleon binding may have unusually strong dipole transition

strengths at low excitation. See for example Ref. [4.20]. Calculate the dipole response for a nucleus that is very neutron rich, such as ^{10}He (which does not exist in nature) or ^{28}O. Is there significant strength just above the particle emission threshold?

4.7 Technical Note

The program expects input to be read from the keyboard. Example values for the input parameters as well as a detailed description of the input can be found in Table 4.1.

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