Gell Mann-Low theorem

and

Goldstone theorem

Gell Mann-Low theorem

The Gell-Mann Low theorem states that if exists at every perturbative order such limit

$$\lim_{\epsilon\to 0} \frac{\hat{U}_\epsilon(0,-\infty)\left|\Phi_0\right\rangle}{\left\langle\Phi_0\right|\hat{U}_\epsilon(0,-\infty)\left|\Phi_0\right\rangle} = \lim_{\epsilon\to 0} \frac{\left|\Psi_0^\epsilon\right\rangle}{\left\langle\Phi_0|\Psi_0^\epsilon\right\rangle} \equiv \frac{\left|\Psi_0\right\rangle}{\left\langle\Phi_0|\Psi_0\right\rangle}$$

then $|\psi_0\rangle$ is an eigenstate of the following Hamiltonian

$$\hat{H}\frac{\left|\Psi_0\right\rangle}{\left\langle\Phi_0|\Psi_0\right\rangle} = E\frac{\left|\Psi_0\right\rangle}{\left\langle\Phi_0|\Psi_0\right\rangle}$$

where

$$E - E_0 = \frac{\left\langle\Phi_0\right|\hat{H}_1\left|\Psi_0\right\rangle}{\left\langle\Phi_0|\Psi_0\right\rangle}$$

Gell Mann-Low theorem

Let's assume that

$$\left(\hat{H}_0 - E_0\right)|\Psi_0^\epsilon\rangle = \left(\hat{H}_0 - E_0\right)\hat{U}_\epsilon(0,-\infty)|\Phi_0\rangle = \Big[\underbrace{\hat{H}_0, \hat{U}_\epsilon(0,-\infty)}\Big]|\Phi_0\rangle$$

nth order

$$\begin{aligned}\left[\hat{H}_0, \hat{H}_1(t_{p_1})...\hat{H}_1(t_{p_j})...\hat{H}_1(t_{p_n})\right] &= \left[\hat{H}_0, \hat{H}_1(t_{p_1})\right]\hat{H}_1(t_{p_2})...\hat{H}_1(t_{p_n}) \\ &+ ... + \hat{H}_1(t_{p_1})...\left[\hat{H}_0, \hat{H}_1(t_{p_j})\right]...\hat{H}_1(t_{p_n}) + ... \\ &+ \hat{H}_1(t_{p_1})...\hat{H}_1(t_{p_j})...\left[\hat{H}_0, \hat{H}_1(t_{p_n})\right].\end{aligned}$$

$$i\hbar\frac{\partial}{\partial t}\hat{H}_1(t) = \left[\hat{H}_1(t), \hat{H}_0\right]$$

$$\boxed{\left[\hat{H}_0, \hat{H}_1(t_{p_1})...\hat{H}_1(t_{p_j})...\hat{H}_1(t_{p_n})\right] = \frac{\hbar}{i}\left(\frac{\partial}{\partial t_{p_1}} + ... + \frac{\partial}{\partial t_{p_n}}\right)\hat{H}_1(t_{p_1})...\hat{H}_1(t_{p_j})...\hat{H}_1(t_{p_n})}$$

Gell Mann-Low theorem

Using the following relation, the previous equation holds for any time-ordering

$$\theta(t_{p_1}-t_{p_2})...\theta(t_{p_{n-1}}-t_{p_n}) \quad \frac{\hbar}{i}\left\{\sum_{i=1}^{n}\frac{\partial}{\partial t_{p_i}}\right\}\left[\hat{H}_1(t_{p_1})...\hat{H}_1(t_{p_n})\right]$$

$$=\frac{\hbar}{i}\left\{\sum_{i=1}^{n}\frac{\partial}{\partial t_{p_i}}\right\}\left[\theta(t_{p_1}-t_{p_2})...\theta(t_{p_{n-1}}-t_{p_n})\hat{H}_1(t_{p_1})...\hat{H}_1(t_{p_n})\right]$$

since

$$\left\{\sum_{i=1}^{n}\frac{\partial}{\partial t_{p_i}}\right\}\left[\theta(t_{p_1}-t_{p_2})\theta(t_{p_2}-t_{p_3})...\theta(t_{p_{n-1}}-t_{p_n})\right]=0$$

Time derivatives can be put outside the normal ordering

$$\mathcal{T}\left[\left\{\sum_{i=1}^{n}\frac{\partial}{\partial t_i}\right\}\hat{H}_1(t_1)...\hat{H}_1(t_n)\right]=\left\{\sum_{i=1}^{n}\frac{\partial}{\partial t_i}\right\}\mathcal{T}\left[\hat{H}_1(t_1)...\hat{H}_1(t_n)\right]$$

Gell Mann-Low theorem

to get

$$\left(\hat{H}_0 - E_0\right)|\Psi_0^\epsilon\rangle = -\sum_{n=1}^{\infty}\left(\frac{-i}{\hbar}\right)^{n-1}\frac{1}{n!}\int_{-\infty}^{0}dt_1 \ldots \int_{-\infty}^{0}dt_n\ e^{-\epsilon(|t_1|+\ldots+|t_n|)}\left\{\sum_{i=1}^{n}\frac{\partial}{\partial t_i}\right\}\mathcal{T}\left[\hat{H}_1(t_1)\ldots\hat{H}_1(t_n)\right]|\Phi_0\rangle$$
$$= -\sum_{n=1}^{\infty}\left(\frac{-i}{\hbar}\right)^{n-1}\frac{1}{(n-1)!}\int_{-\infty}^{0}dt_1 \ldots \int_{-\infty}^{0}dt_n\ e^{-\epsilon(|t_1|+\ldots+|t_n|)}\frac{\partial}{\partial t_1}\mathcal{T}\left[\hat{H}_1(t_1)\ldots\hat{H}_1(t_n)\right]|\Phi_0\rangle\ ,$$

All the time derivatives give the same contribution, so we need to perform only one evaluation and then multiply by *n*

$$\int_{-\infty}^{0}dt_1\ e^{-\epsilon(|t_1|)}\frac{\partial}{\partial t_1}\mathcal{T}\left[\hat{H}_1(t_1)\ldots\hat{H}_1(t_n)\right] = e^{-\epsilon(|t_1|)}\mathcal{T}\left[\hat{H}_1(t_1)\ldots\hat{H}_1(t_n)\right]\Big|_{-\infty}^{0} - \epsilon\int_{-\infty}^{0}dt_1\ e^{-\epsilon(|t_1|)}\mathcal{T}\left[\hat{H}_1(t_1)\ldots\hat{H}_1(t_n)\right]$$
$$= \mathcal{T}\left[\hat{H}_1(0)\ldots\hat{H}_1(t_n)\right] - \epsilon\int_{-\infty}^{0}dt_1\ e^{-\epsilon(|t_1|)}\mathcal{T}\left[\hat{H}_1(t_1)\ldots\hat{H}_1(t_n)\right]. \quad (51$$

Gell Mann-Low theorem

$$\left(\hat{H}_0 - E_0\right)|\Psi_0^\epsilon\rangle = -\hat{H}_1 \boxed{\left\{\sum_{n=1}^{\infty}\left(\frac{-i}{\hbar}\right)^{n-1}\frac{1}{(n-1)!}\int_{-\infty}^{0}dt_2 \ldots \int_{-\infty}^{0}dt_n\; e^{-\epsilon(|t_2|+\ldots+|t_n|)}\mathcal{T}\left[\hat{H}_1(t_2)\ldots\hat{H}_1(t_n)\right]|\Phi_0\rangle\right\}}$$

$$+\,\epsilon\sum_{n=1}^{\infty}\left(\frac{-i}{\hbar}\right)^{n-1}\frac{1}{(n-1)!}\int_{-\infty}^{0}dt_1 \ldots \int_{-\infty}^{0}dt_n\; e^{-\epsilon(|t_1|+\ldots+|t_n|)}\mathcal{T}\left[\hat{H}_1(t_1)\ldots\hat{H}_1(t_n)\right]|\Phi_0\rangle$$

$$\left|\Psi_0^\epsilon\right\rangle$$

?

$$\hat{H}_1(t) = \lambda\hat{\mathcal{H}}_1(t)$$

Gell Mann-Low theorem

To understand the physical interpretation of the second term, let's rewrite

$$\hat{H}_1(t) = \lambda \hat{\mathcal{H}}_1(t)$$ λ coupling constant

$$\left(\frac{-i}{\hbar}\right)^{n-1} \frac{1}{(n-1)!}\lambda^n = i\hbar\lambda\frac{\partial}{\partial\lambda}\left(\frac{-i}{\hbar}\right)^n \frac{1}{n!}\lambda^n$$

in order to get

$$\begin{aligned}
&\epsilon\sum_{n=1}^{\infty}\left(\frac{-i}{\hbar}\right)^{n-1}\frac{1}{(n-1)!}\int_{-\infty}^{0} dt_1 \ldots \int_{-\infty}^{0} dt_n \; e^{-\epsilon(|t_1|+\ldots+|t_n|)}\mathcal{T}\left[\hat{H}_1(t_1)\ldots\hat{H}_1(t_n)\right]|\Phi_0\rangle \\
&= i\hbar\lambda\epsilon\frac{\partial}{\partial\lambda}\left\{\sum_{n=0}^{\infty}\left(\frac{-i}{\hbar}\right)^{n}\frac{1}{n!}\int_{-\infty}^{0} dt_1 \ldots \int_{-\infty}^{0} dt_n \; e^{-\epsilon(|t_1|+\ldots+|t_n|)}\mathcal{T}\left[\hat{H}_1(t_1)\ldots\hat{H}_1(t_n)\right]|\Phi_0\rangle\right\} \\
&= i\hbar\lambda\epsilon\frac{\partial}{\partial\lambda}|\Psi_0^\epsilon\rangle \, .
\end{aligned}$$

As a consequence

$$\left(\hat{H}_0 - E_0\right)|\Psi_0^\epsilon\rangle = -\hat{H}_1|\Psi_0^\epsilon\rangle + i\hbar\lambda\epsilon\frac{\partial}{\partial\lambda}|\Psi_0^\epsilon\rangle$$

that it is equivalent to

$$\boxed{\left(\hat{H} - E_0\right)|\Psi_0^\epsilon\rangle = i\hbar\lambda\epsilon\frac{\partial}{\partial\lambda}|\Psi_0^\epsilon\rangle}$$

Too early for ε ⇒ 0

Gell Mann-Low theorem

Let's take into account the numerator and the denominator of the initial relation:

$$i\hbar\epsilon\lambda\frac{\partial}{\partial\lambda}\left\{\frac{|\Psi_0^\epsilon\rangle}{\langle\Phi_0|\Psi_0^\epsilon\rangle}\right\} = i\hbar\epsilon\lambda\frac{1}{\langle\Phi_0|\Psi_0^\epsilon\rangle}\frac{\partial}{\partial\lambda}|\Psi_0^\epsilon\rangle + i\hbar\epsilon\lambda|\Psi_0^\epsilon\rangle\left\{\frac{-1}{\langle\Phi_0|\Psi_0^\epsilon\rangle^2}\right\}\frac{\partial}{\partial\lambda}\langle\Phi_0|\Psi_0^\epsilon\rangle$$

$$= i\hbar\epsilon\lambda\frac{1}{\langle\Phi_0|\Psi_0^\epsilon\rangle}\frac{\partial}{\partial\lambda}|\Psi_0^\epsilon\rangle - i\hbar\epsilon\lambda\frac{|\Psi_0^\epsilon\rangle}{\langle\Phi_0|\Psi_0^\epsilon\rangle}\frac{\partial}{\partial\lambda}\ln\langle\Phi_0|\Psi_0^\epsilon\rangle$$

that can be written as follows

$$i\hbar\epsilon\lambda\frac{1}{\langle\Phi_0|\Psi_0^\epsilon\rangle}\frac{\partial}{\partial\lambda}|\Psi_0^\epsilon\rangle = i\hbar\epsilon\lambda\frac{\partial}{\partial\lambda}\left\{\frac{|\Psi_0^\epsilon\rangle}{\langle\Phi_0|\Psi_0^\epsilon\rangle}\right\} + i\hbar\epsilon\lambda\frac{|\Psi_0^\epsilon\rangle}{\langle\Phi_0|\Psi_0^\epsilon\rangle}\frac{\partial}{\partial\lambda}\ln\langle\Phi_0|\Psi_0^\epsilon\rangle$$

$$\boxed{\left(\hat{H} - E_0\right)|\Psi_0^\epsilon\rangle = i\hbar\lambda\epsilon\frac{\partial}{\partial\lambda}|\Psi_0^\epsilon\rangle}$$

divided by $\frac{1}{\langle\Phi_0|\Psi_0^\epsilon\rangle}$

$$\left(\hat{H} - E_0\right)\frac{|\Psi_0^\epsilon\rangle}{\langle\Phi_0|\Psi_0^\epsilon\rangle} = i\hbar\epsilon\lambda\frac{\partial}{\partial\lambda}\left\{\frac{|\Psi_0^\epsilon\rangle}{\langle\Phi_0|\Psi_0^\epsilon\rangle}\right\} + i\hbar\epsilon\lambda\frac{|\Psi_0^\epsilon\rangle}{\langle\Phi_0|\Psi_0^\epsilon\rangle}\frac{\partial}{\partial\lambda}\ln\langle\Phi_0|\Psi_0^\epsilon\rangle$$

Gell Mann-Low theorem

Multiply from the left-hand side by $\langle \Phi_0 | / \langle \Phi_0 | \psi_0^\epsilon \rangle$

$$\frac{\langle \Phi_0 | \hat{H}_1 | \Psi_0^\epsilon \rangle}{\langle \Phi_0 | \Psi_0^\epsilon \rangle} = \frac{i\hbar\epsilon\lambda}{\langle \Phi_0 | \Psi_0^\epsilon \rangle} \frac{\partial}{\partial \lambda} \langle \Phi_0 | \Psi_0^\epsilon \rangle = i\hbar\epsilon\lambda \frac{\partial}{\partial \lambda} \ln \langle \Phi_0 | \Psi_0^\epsilon \rangle$$

Using this relation we obtain

$$\left(\hat{H} - E_0 \right) \frac{|\Psi_0^\epsilon\rangle}{\langle \Phi_0 | \Psi_0^\epsilon \rangle} = i\hbar\epsilon\lambda \frac{\partial}{\partial \lambda} \left\{ \frac{|\Psi_0^\epsilon\rangle}{\langle \Phi_0 | \Psi_0^\epsilon \rangle} \right\} + \frac{|\Psi_0^\epsilon\rangle}{\langle \Phi_0 | \Psi_0^\epsilon \rangle} \frac{\langle \Phi_0 | \hat{H}_1 | \Psi_0^\epsilon \rangle}{\langle \Phi_0 | \Psi_0^\epsilon \rangle}$$

$$\Downarrow$$

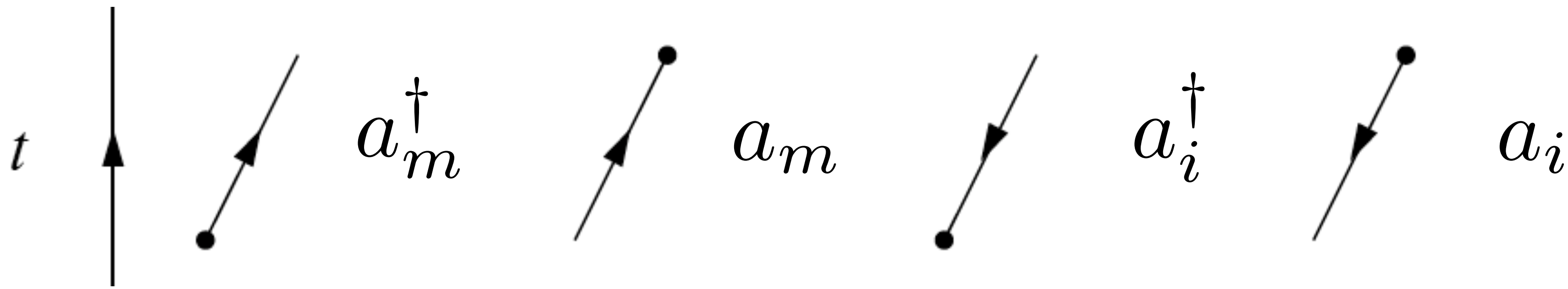
$$\left\{ \hat{H} - \frac{\langle \Phi_0 | \hat{H} | \Psi_0^\epsilon \rangle}{\langle \Phi_0 | \Psi_0^\epsilon \rangle} \right\} \frac{|\Psi_0^\epsilon\rangle}{\langle \Phi_0 | \Psi_0^\epsilon \rangle} = i\hbar\epsilon\lambda \frac{\partial}{\partial \lambda} \left\{ \frac{|\Psi_0^\epsilon\rangle}{\langle \Phi_0 | \Psi_0^\epsilon \rangle} \right\}$$

Limit for ε⇒0

$$\left\{ \hat{H} - \frac{\langle \Phi_0 | \hat{H} | \Psi_0 \rangle}{\langle \Phi_0 | \Psi_0 \rangle} \right\} \frac{|\Psi_0\rangle}{\langle \Phi_0 | \Psi_0 \rangle} = 0 \qquad \hat{H} \frac{|\Psi_0\rangle}{\langle \Phi_0 | \Psi_0 \rangle} = E \frac{|\Psi_0\rangle}{\langle \Phi_0 | \Psi_0 \rangle}$$

$$\boxed{E = \frac{\langle \Phi_0 | \hat{H} | \Psi_0 \rangle}{\langle \Phi_0 | \Psi_0 \rangle}}$$

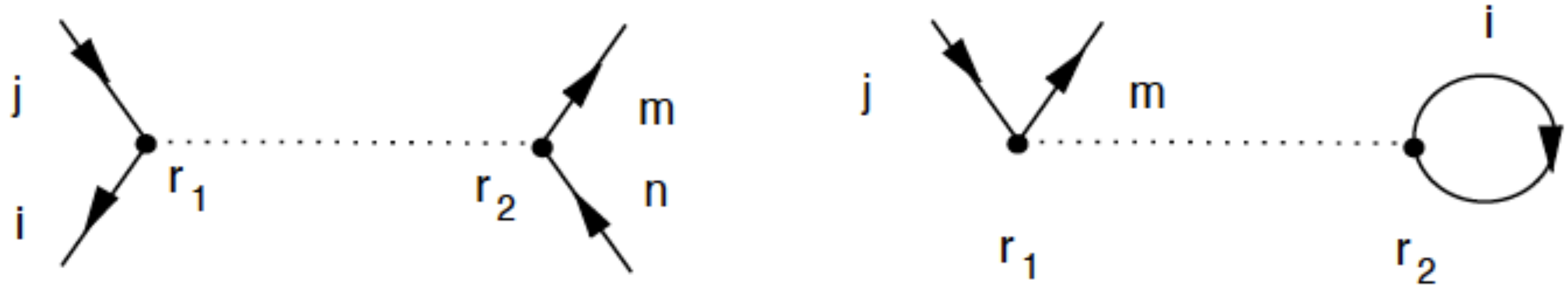
Goldstone theorem



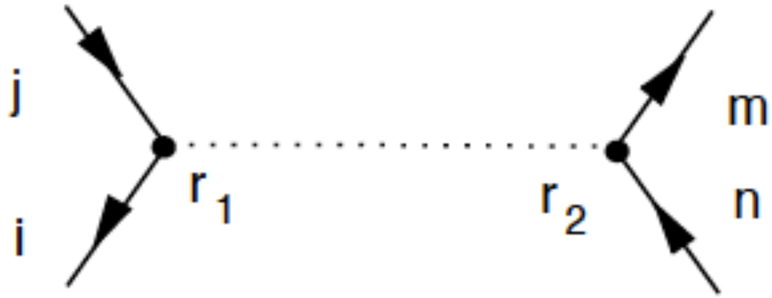
Particle creation *Particle destruction* *Hole creation* *Hole destruction*

Two-body potential - - - - - - - - - - - - - $V(\vec{x}, \vec{y})$

$$\langle \Psi_a | V(\mathbf{r}_1, \mathbf{r}_2) | \Psi_b \rangle$$

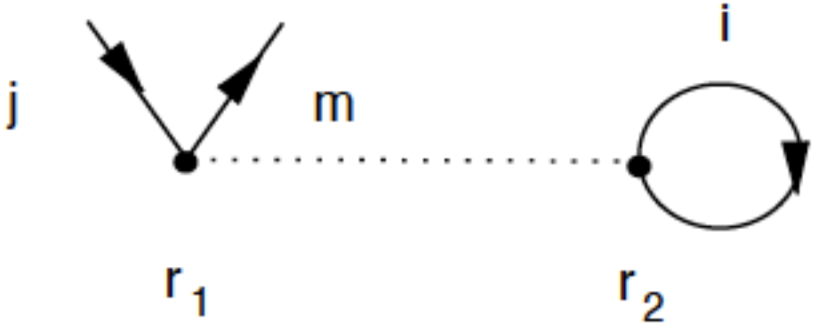


Goldstone theorem



$$\left.\begin{array}{l} |\Psi_b\rangle = a_m^+ a_j |\Phi_0\rangle \\ \\ \langle\Psi_a| = \langle\Phi_0| a_i^+ a_n \end{array}\right\} \langle\Phi_0|a_i^+ a_n V a_m^+ a_j|\Phi_0\rangle$$

$$V = \frac{1}{2}\sum_{\nu\nu'\mu\mu'} V_{\nu\mu\nu'\mu'} a_\nu^+ a_\mu^+ a_{\mu'} a_{\nu'}$$



$$\langle\Phi_0|a_i^+ a_j V a_m^+ a_i|\Phi_0\rangle$$

Initial and final hole states are the same state

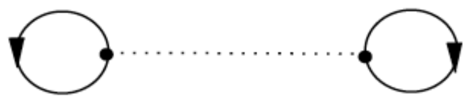
Goldstone theorem

The Goldstone theorem states that the difference between the energy of a system of **interacting** particles and the energy of a system of **non-interacting** particles can be written as follows

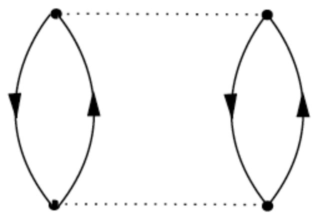
$$E - E_0 = \langle \Phi_0 | H_1 \sum_{n=0}^{\infty} \left(\frac{1}{E_0 - H_0} H_1 \right)^n | \Phi_0 \rangle_c$$

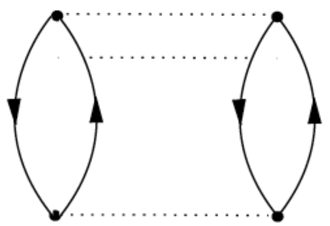
Goldstone theorem

explicitly



$$\begin{aligned} E - E_0 &= \langle \Phi_0 | H_1 | \Phi_0 \rangle \\ &+ \langle \Phi_0 | H_1 \frac{1}{E_0 - H_0} H_1 | \Phi_0 \rangle \\ &+ \langle \Phi_0 | H_1 \frac{1}{E_0 - H_0} H_1 \frac{1}{E_0 - H_0} H_1 | \Phi_0 \rangle \\ &+ \ \ldots \end{aligned}$$





$$E - E_0 = \langle \Phi_0 | H_1 | \Phi_0 \rangle + \sum_{n \neq 0} \frac{\langle \Phi_0 | H_1 | \Phi_n \rangle \langle \Phi_n | H_1 | \Phi_0 \rangle}{E_0 - E_n} + \ldots \qquad I = \sum_{n \neq 0} | \Phi_n \rangle \langle \Phi_n |$$

Goldstone theorem

Let's start from the Gell-Mann theorem

$$E - E_0 = \frac{\langle \Phi_0 | H_1 | \Psi_0 \rangle}{\langle \Phi_0 | \Psi_0 \rangle} = \frac{\langle \Phi_0 | H_1 U(0, -\infty) | \Phi_0 \rangle}{\langle \Phi_0 | U(0, -\infty) | \Phi_0 \rangle}$$

$$U(t, t_0) = \sum_{n=0}^{\infty} \left(\frac{-i}{\hbar} \right)^n \frac{1}{n!} \int_{t_0}^{t} dt_1 \ldots \int_{t_0}^{t} dt_n T \left[H_1, (t_1) \ldots H_1(t_n) \right]$$

using the definition of time evolution operator

$$\langle \Phi_0 | H_1 U(0, -\infty) | \Phi_0 \rangle = \sum_{\nu=0} \left(\frac{-i}{\hbar} \right)^{\nu} \frac{1}{\nu!} \int_{-\infty}^{0} dt_1 \ldots \int_{-\infty}^{0} dt_{\nu}$$
$$\langle \Phi_0 | T[H_1, H_1(t_1) \ldots, H_1(t_{\nu})] | \Phi_0 \rangle$$

Goldstone theorem

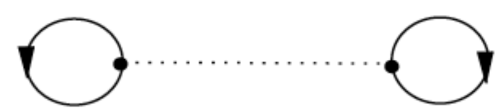
Example ν=1

$$\langle\Phi_0|H_1(0)H_1(t_1)|\Phi_0\rangle =$$
$$\langle\Phi_0|V_{\mu\nu\mu'\nu'}a^+_\mu(0)a^+_\nu(0)a_{\nu'}(0)a_{\mu'}(0)V_{\eta\xi\eta'\xi'}a^+_\eta(t_1)a^+_\xi(t_1)a_{\xi'}(t_1)a_{\eta'}(t_1)|\Phi_0\rangle$$

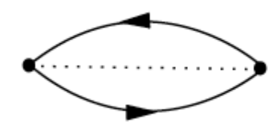
Non-connected contributions

$$\langle\Phi_0|V_{\mu\nu\mu'\nu'}a^+_\mu a^+_\nu a_{\nu'}a_{\mu'}|\Phi_0\rangle_{t=0}\langle\Phi_0|V_{\eta\xi\eta'\xi'}a^+_\eta a^+_\xi a_{\xi'}a_{\eta'}|\Phi_0\rangle_{t=t_1}$$

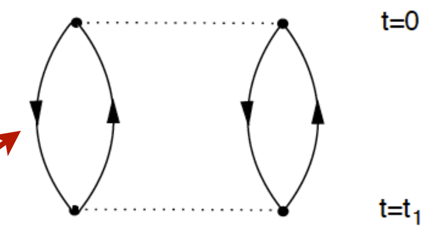
$$a^{+\alpha}_\mu(t)a^{+\beta}_\nu(t)a^\beta_{\nu'}(t)a^\alpha_{\mu'}(t) \rightarrow$$



$$a^{+\alpha}_\mu(t)a^{+\beta}_\nu(t)a^\alpha_{\nu'}(t)a^\beta_{\mu'}(t) \rightarrow$$



Connected contributions



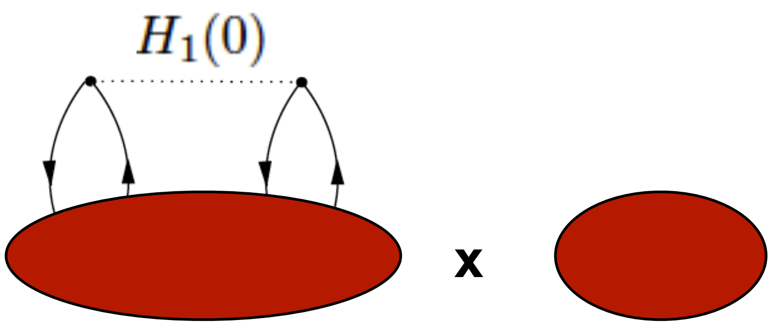
$$\langle\Phi_0|V_{\mu\nu\mu'\nu'}a^{+\alpha}_\mu(0)a^{+\beta}_\nu(0)a^\gamma_{\nu'}(0)a^\delta_{\mu'}(0)V_{\eta\xi\eta'\xi'}a^{+\gamma}_\eta(t)a^{+\delta}_\xi(t)a^\alpha_{\xi'}(t)a^\beta_\eta(t)|\Phi_0\rangle$$

Goldstone theorem

Case ν

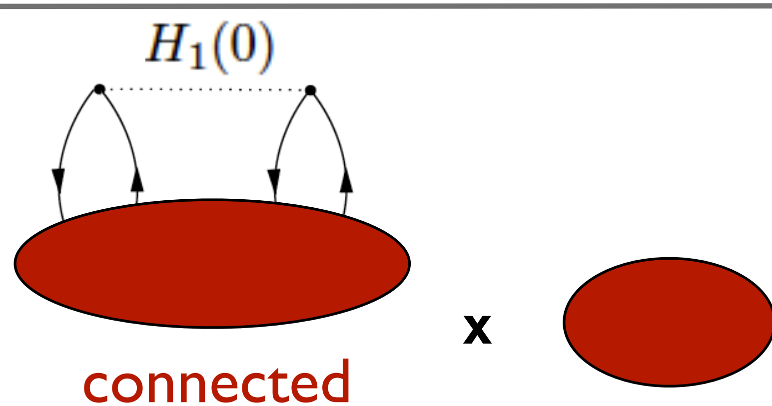
$$\langle\Phi_0|H_1 U(0,-\infty)|\Phi_0\rangle_\nu \quad = \quad \left(\frac{-i}{\hbar}\right)^\nu \frac{1}{\nu!}\int_{-\infty}^{0} dt_1 \quad \dots \quad dt_n \langle\Phi_0|T[H_1(0)\dots H_1(t_n)]|\Phi_0\rangle_c$$
$$\int_{-\infty}^{0} dt_{n+1} \quad \dots \quad \int_{-\infty}^{0} dt_{n+m}\langle\Phi_0|T[H_1(t_{n+1})\dots H_1(t_{n+m})]|\Phi_0\rangle_c$$

ν=n+m



connected not connected

Goldstone theorem



connected

not connected

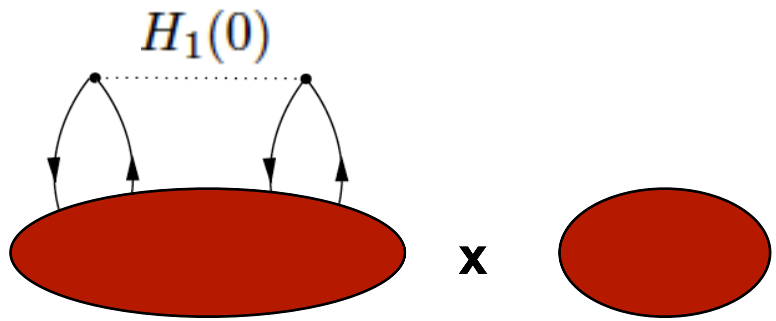
$$\sum_n \sum_m \left(\frac{-i}{\hbar}\right)^{n+m} \frac{\nu!}{n!m!}\frac{1}{\nu!}\int_{-\infty}^{0} dt_1 \quad \dots \quad \int_{-\infty}^{0} dt_n \langle\Phi_0|T[H_1, H_1(t_1)\dots H_n(t_n)]|\Phi_0\rangle_c$$
$$\int_{-\infty}^{0} dt_{n+1} \quad \dots \quad \int_{-\infty}^{0} dt_{n+m} \langle\Phi_0|T[H_1(t_{n+1})\dots H_1(t_{n+m})]|\Phi_0\rangle_c$$

any possible permutations

permutations without new diagrams

Exchanging between them 2 operators H_I defined at different times means that we exchange 4 creation and destruction operators, so the total phase generated by this exchange is always positive. The number of permutations is $\nu!$. On the other hand, the exchange of 2 operators H_I belonging to the same partition does not produce a new diagram.

Goldstone theorem



connected　　　not connected

$$\sum_n \sum_m \left(\frac{-i}{\hbar}\right)^{n+m} \frac{\nu!}{n!m!}\frac{1}{\nu!}\int_{-\infty}^{0} dt_1 \quad \dots \quad \int_{-\infty}^{0} dt_n \langle\Phi_0|T[H_1, H_1(t_1)\dots H_n(t_n)]|\Phi_0\rangle_c$$
$$\boxed{\int_{-\infty}^{0} dt_{n+1} \quad \dots \quad \int_{-\infty}^{0} dt_{n+m}\langle\Phi_0|T[H_1(t_{n+1})\dots H_1(t_{n+m})]|\Phi_0\rangle_c}$$

the only contribution in the denominator

Goldstone theorem

$$H_1(t) = e^{i\frac{H_0 t}{\hbar}} H_1 e^{-i\frac{H_0 t}{\hbar}}$$

nth order term

$$\begin{aligned}[E - E_0]_n \quad &= \quad \left(\frac{-i}{\hbar}\right)^n \int_{-\infty}^{0} dt_1 \ldots \int_{-\infty}^{t_1} dt_2 \ldots \int_{-\infty}^{t_{n-1}} dt_n e^{\epsilon(t_1+\ldots t_n)} \\ &\langle \Phi_0 | H_1 e^{i\frac{H_0 t_1}{\hbar}} H_1 e^{-i\frac{H_0 t_2}{\hbar}} H_1 \ldots e^{-i\frac{H_0}{\hbar} t_{n-1}} e^{i\frac{H_0}{\hbar} t_n} H_1 e^{-i\frac{H_0}{\hbar} t_n} | \Phi_0 \rangle_c\end{aligned}$$

we can eliminate T if we write explicitly the limits of integration

Goldstone theorem

Change of coordinates

$$x_1 = t_1 \quad , \quad x_2 = t_2 - t_1 \quad , \quad x_3 = t_3 - t_2 \quad , \quad \dots \quad , \quad x_n = t_n - t_{n-1}$$

$$t_1 = x_1 \quad , \quad t_2 = x_2 + x_1 \quad , \quad t_3 = x_3 + x_2 + x_1 \quad , \quad \dots \quad , \quad t_n = \sum_n x_n$$

$$[E - E_0]_n = \left(\frac{-i}{\hbar}\right)^n \langle\Phi_0|H_1 \int_{-\infty}^{0} dx_1 \int_{-\infty}^{0} dx_2 \dots \int_{-\infty}^{0} dx_n$$

$$e^{\epsilon(x_1+(x_2+x_1)+(x_3+x_2+x_1)+\dots(x_n+x_{n-1}\dots+x_2+x_1)}$$

$$e^{i\frac{H_0 x_1}{\hbar}} H_1 e^{i\frac{H_0 x_2}{\hbar}} H_1 \dots e^{i\frac{H_0 x_n}{\hbar}} H_1 e^{-i\frac{H_0 t_n}{\hbar}}|\Phi_0\rangle_c$$

$$e^{-i\frac{H_0 t_n}{\hbar}}|\Phi_0\rangle = e^{-i\frac{E_0}{\hbar}t_n}|\Phi_0\rangle = e^{-i\frac{E_0}{\hbar}(x_1+\dots x_n)}|\Phi_0\rangle_c$$

Goldstone theorem

$$[E - E_0]_n = \left(\frac{-i}{\hbar}\right)^n \langle \Phi_0 | H_1 \int_{-\infty}^{0} dx_1 e^{n\epsilon x_1} e^{i\frac{(H_0 - E_0)}{\hbar} x_1} \cdot H_1$$

$$\int_{-\infty}^{0} dx_2 e^{(n-1)\epsilon x_2} e^{i\frac{(H_0 - E_0)}{\hbar} x_2} \cdot H_1 \ldots \int_{-\infty}^{0} e^{\epsilon x_n} e^{i\frac{(H_0 - E_0)}{\hbar} x_n} H_1 | \Phi_0 \rangle_c$$

nth integrals

$$\int_{-\infty}^{0} dx_1 e^{\frac{i}{\hbar}(H_0 - E_0 - in\epsilon\hbar) x_1} = \frac{\hbar}{-i} \frac{1}{[E_0 - H_0 + in\varepsilon\hbar]}$$

nth contribution

$$[E - E_0]_n = \langle \Phi_0 | H_1 \frac{1}{E_0 - H_0 + i\epsilon n\hbar} H_1 \frac{1}{E_0 - H_0 + i\epsilon(n-1)\hbar} \ldots H_1 \frac{1}{E_0 - H_0 + i\epsilon\hbar} H_1 | \Phi_0 \rangle_c$$