One meson exchange potential

1π exchange potential

- in the momentum space

$$v_{12}^{\pi}(\boldsymbol{q}) = -\frac{f_{\pi NN}^2}{m_\pi^2}\frac{\boldsymbol{\sigma}_1\cdot\boldsymbol{q}\;\boldsymbol{\sigma}_2\cdot\boldsymbol{q}}{q^2+m_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$$

- in the coordinate space

$$v_{12}^{\pi}(\boldsymbol{r}) = -\frac{f_{\pi NN}^2}{4\pi}\frac{m_\pi}{3}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\left[T_\pi(r)S_{12}+Y_\pi(r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right]^{*}$$

- where the tensor operator is defined as follows

$$Y_\pi(r) = \frac{e^{-m_\pi r}}{m_\pi r}$$

$$T_\pi(r) = \left(1+\frac{3}{m_\pi r}+\frac{3}{m_\pi^2 r^2}\right)Y_\pi(r)$$

$$S_{12} = 3\boldsymbol{\sigma}_1\cdot\hat{\boldsymbol{r}}\;\boldsymbol{\sigma}_2\cdot\hat{\boldsymbol{r}}-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$$

*and divergent terms for *r = 0* absorbed into the hard-core

One meson exchange potential

- properties of the tensor operator

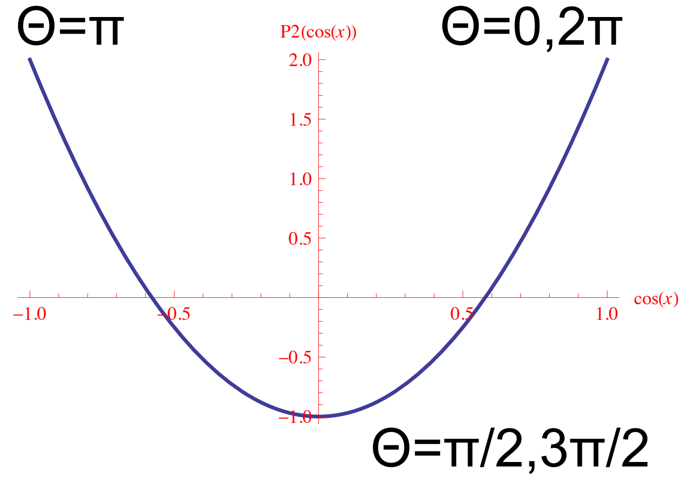
$$\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 S_{12} = S_{12} \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 = S_{12}$$
$$[S_{12}, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2] = 0$$

$$\begin{aligned} P_{S=0} &= \frac{1}{4}(1 - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \\ P_{S=1} &= \frac{1}{4}(3 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \end{aligned}$$

$$[P, S_{12}] = 0 \quad S_{12} P_0 = 0 \quad S_{12} P_1 = S_{12}$$

- The tensor force acts only in the triplet state *S=1*; it is zero in the singlet channel *S=0*. For M_S = *1 we have*

$$\langle \uparrow, \uparrow | S_{12} | \uparrow, \uparrow \rangle = 3\cos^2\theta - 1 = 2\, P_2(\cos\theta)$$



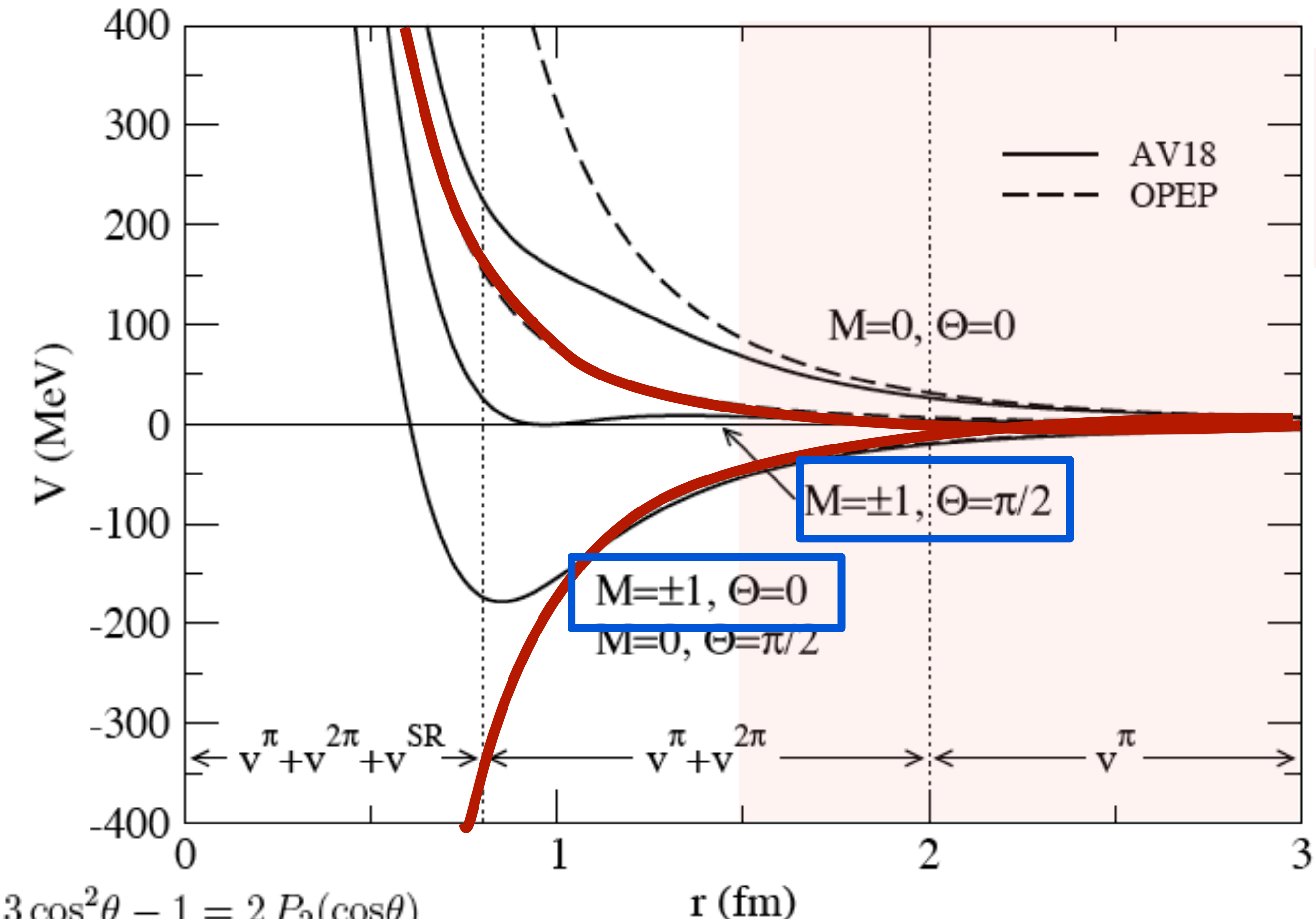
One meson exchange potential

1π exchange potential

Dominant term

$$\langle S=1, M=\pm1; \boldsymbol{r} | v^{\pi}_{12} | S=1, M=\pm1; \boldsymbol{r}\rangle = \frac{f_{\pi NN}}{4\pi}\frac{m_\pi}{3}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\left[2T_\pi(r)P_2(\cos\theta) + Y_\pi(r)\right]$$

T = 0
S = 1
$M_S = \pm1$



Long-range interaction is 1π exchange

Dominated by the tensor term, driven by a divergence *1/r³* for *r⇒0*.

$$\langle\uparrow,\uparrow|S_{12}|\uparrow,\uparrow\rangle = 3\cos^2\theta - 1 = 2\,P_2(\cos\theta)$$

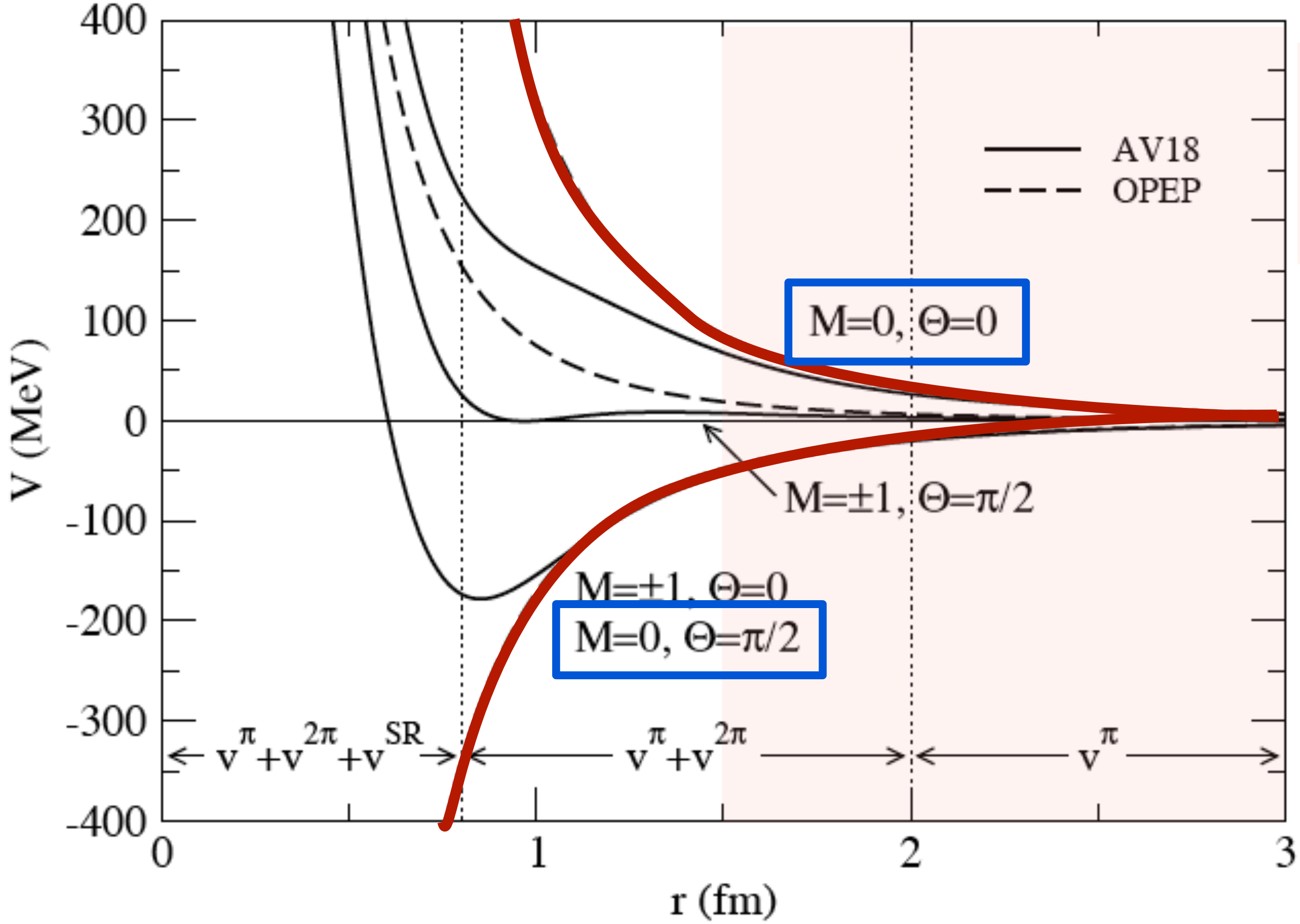
One meson exchange potential

1π exchange potential

Dominant term

$$\langle S=1, M=0; \boldsymbol{r}|v_{12}^{\pi}|S=1, M=0; \boldsymbol{r}\rangle = \frac{f_{\pi NN}}{4\pi}\frac{m_\pi}{3}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\left[-4T_\pi(r)P_2(\cos\theta)+Y_\pi(r)\right]$$

T = 0
S = 1
M_S = 0



Long-range interaction is 1π exchange

Dominated by the tensor term, driven by a divergence $1/r^3$ for $r \Rightarrow 0$.

One meson exchange potential

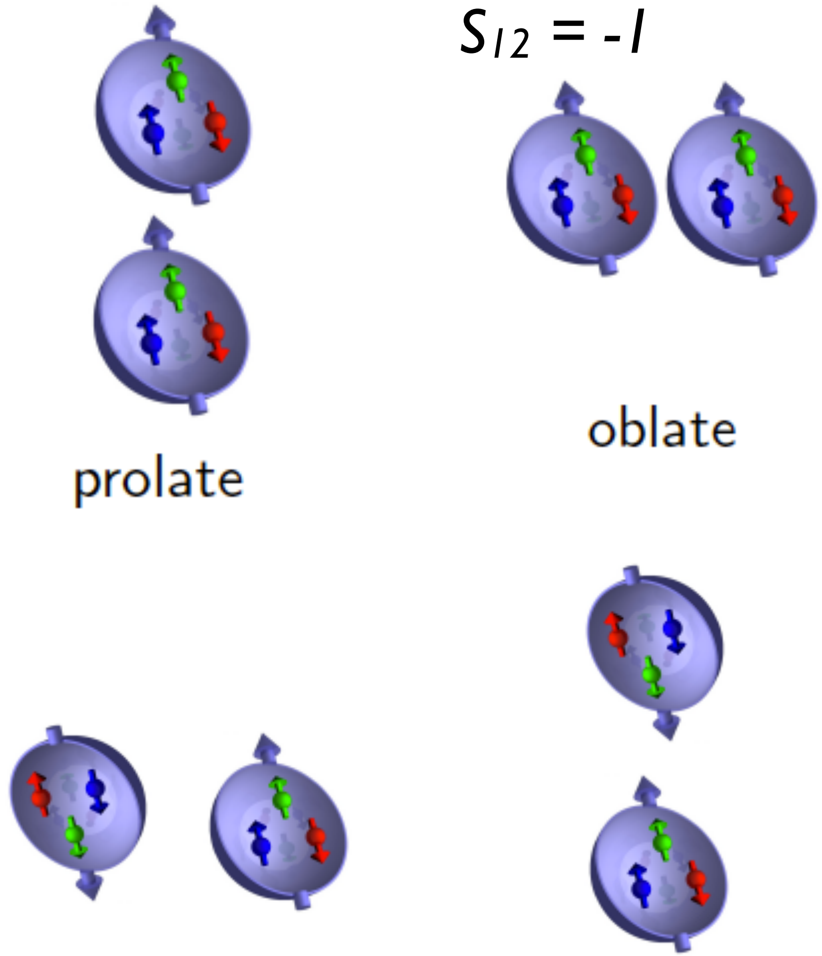
1π exchange potential

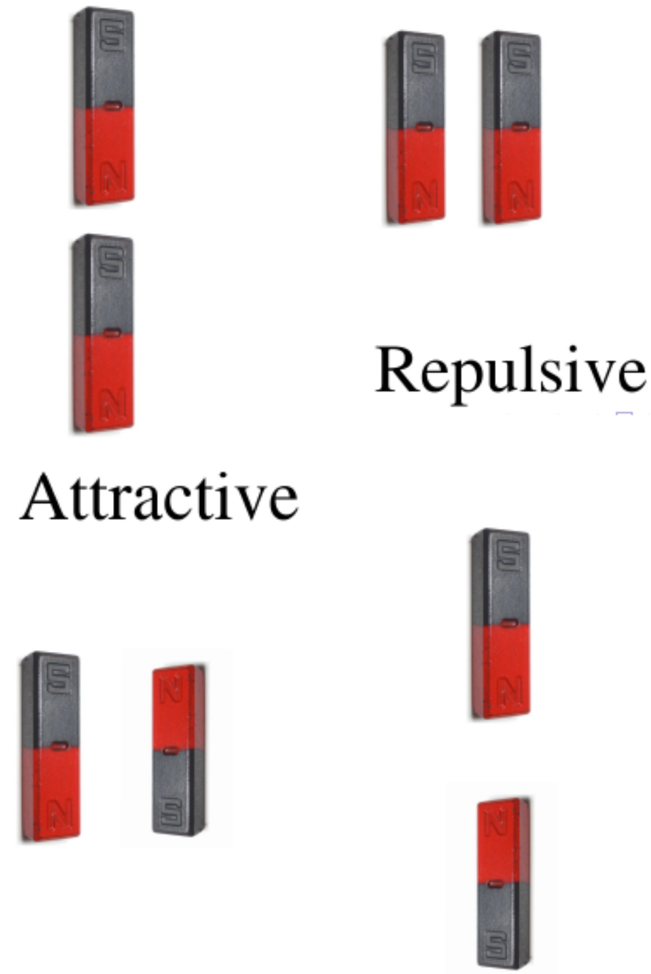
$S_{12} = +2$

$S_{12} = -1$

Caso M_S=1

Caso M_S=0





The S-D contribution is dominated by terms with $M_S = \pm 1$

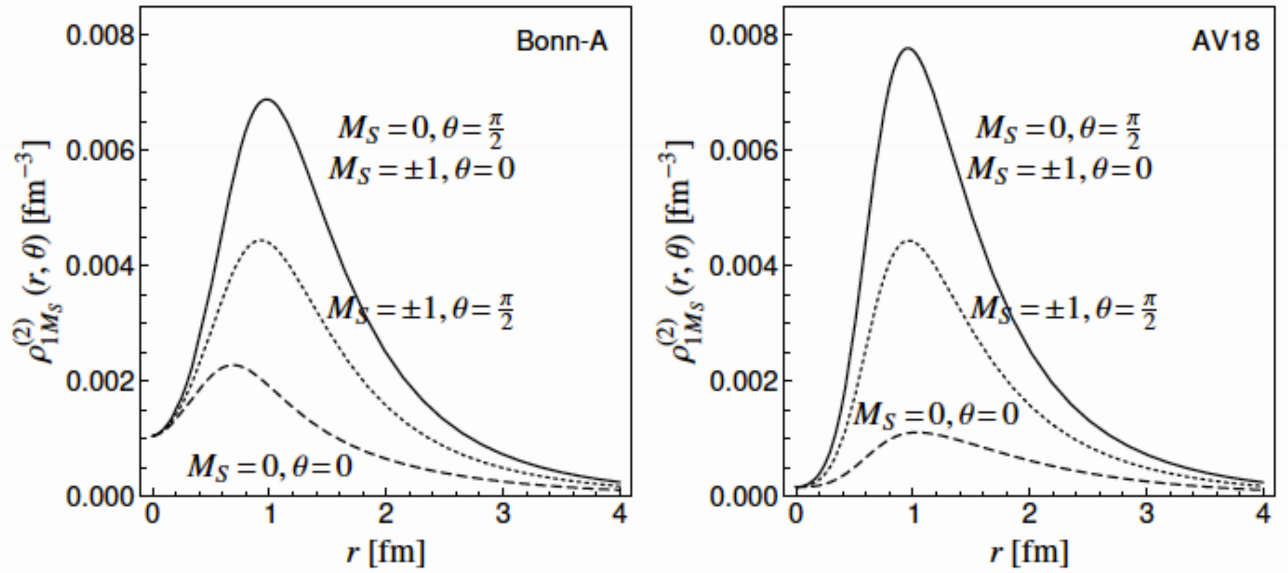
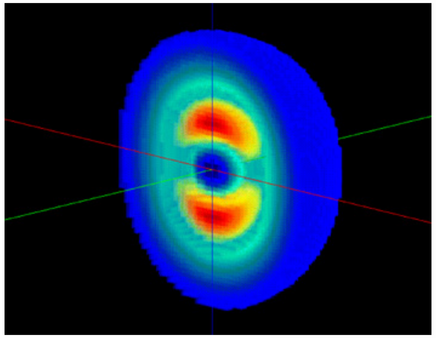
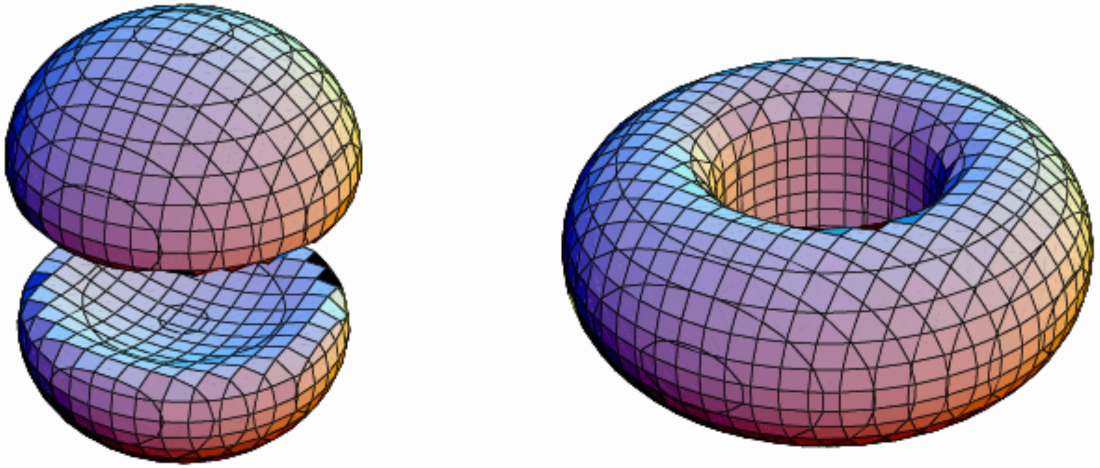


Figure 1.23: Deuteron two-body density as a function of the orientation in coordinate and spin-space for the Bonn-A interaction (left hand side) and the Argonne V18 interaction (right hand side).

prolate





oblate

Figure 1.24: Surfaces of constant density in the deuteron ($\rho^{(2)}_{1M_S} = 0.005\text{fm}^{-3}$) for $M_S = \pm1$ on the left and $M_S = 0$ on the right. The plots are done for the Argonne V18 interaction.

Tensor operator

(e) Matrix elements of the tensor operator S_{12}

We give here the matrix elements of the tensor S_{12} defined in eqn (A10.23)

$$\langle L1JM|\, S_{12}\, |L'1JM\rangle = \int \mathrm{d}\Omega\, \mathcal{Y}^*_{L1JM}(\hat{\mathbf{r}}) S_{12}(\mathbf{r}) \mathcal{Y}_{L'1JM}(\hat{\mathbf{r}}). \qquad \text{(A10.25)}$$

The coupled angular momentum eigenfunctions are

$$\mathcal{Y}_{LSJM}(\hat{\mathbf{r}}) = \sum_{M_L M_S} (LM_L SM_S \,|\, JM) Y_{LM_L}(\hat{\mathbf{r}}) \chi_{SM_S}. \qquad \text{(A10.26)}$$

The spin eigenfunctions χ_{SM_S} are

$$\chi_{SM_S} = \sum_{m_1 m_2} (\tfrac{1}{2}m_1\, \tfrac{1}{2}m_2 \,|\, SM_S) \chi_{\frac{1}{2}m_1} \chi_{\frac{1}{2}m_2} \qquad \text{(A10.27)}$$

Table A10.1. Values of $\langle L1J \,|\, S_{12} \,|\, L'1J\rangle$

L \ L'	$J+1$	J	$J-1$
$J+1$	$-\frac{2J(J+2)}{2J+1}$	0	$\frac{6(J(J+1))^{\frac{1}{2}}}{2J+1}$
J	0	2	0
$J-1$	$\frac{6(J(J+1))^{\frac{1}{2}}}{2J+1}$	0	$\frac{-2(J-1)}{2J+1}$

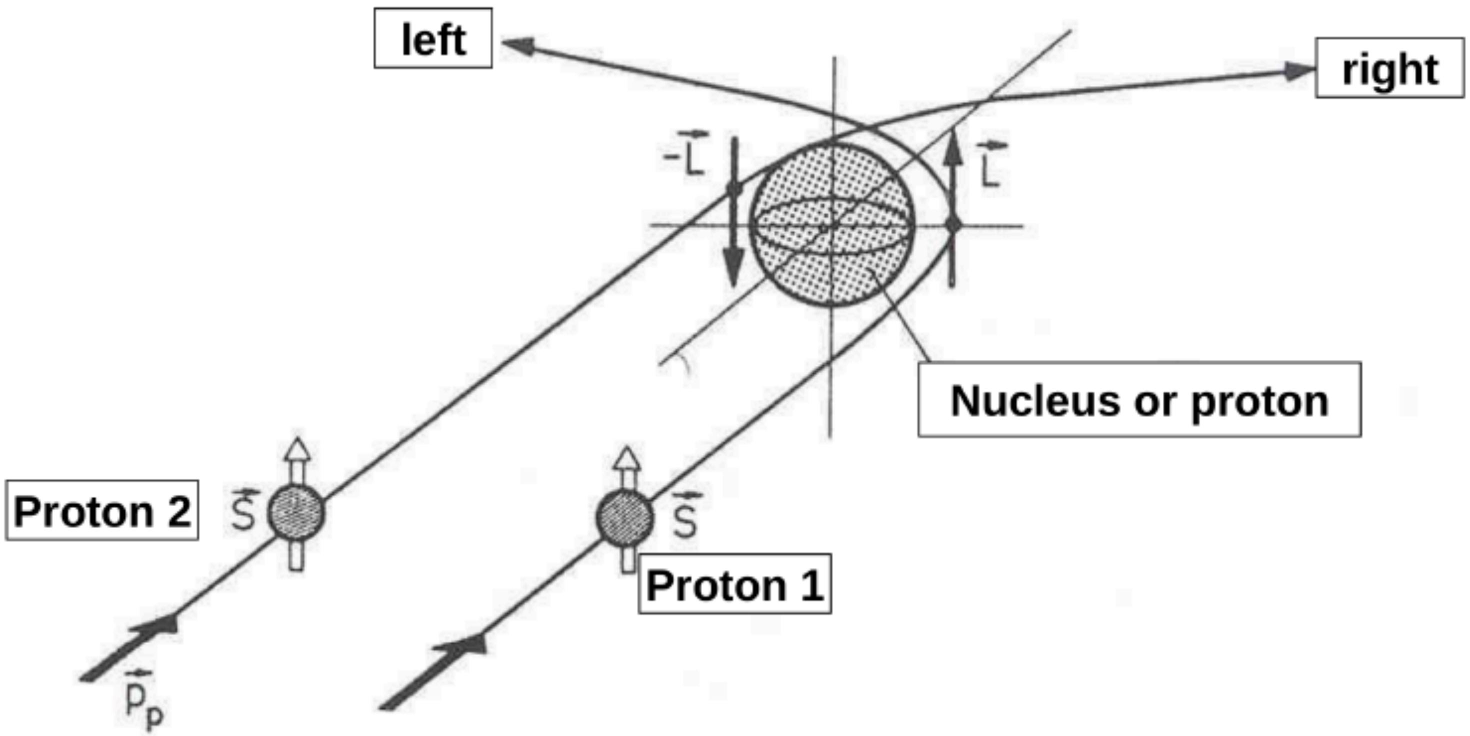
It couples an S state with a D state

OME potential, scalar field: σ meson (500 MeV)

$$V_\sigma(r) = -\frac{g_\sigma^2}{4\pi} m_\sigma \left[\underbrace{Y(m_\sigma r)}_{\text{CENTRAL}} + \underbrace{\frac{1}{2} Z_1(m_\sigma r) \boldsymbol{L} \cdot \boldsymbol{S}}_{\text{SPIN-ORBIT}} \right]$$

$$Y(x) = \frac{e^{-x}}{x}$$

$$Z(x) = \left(\frac{m_\sigma}{M}\right)^2 \left(1 + \frac{3}{x^2} + \frac{3}{x^2}\right) Y(x)$$

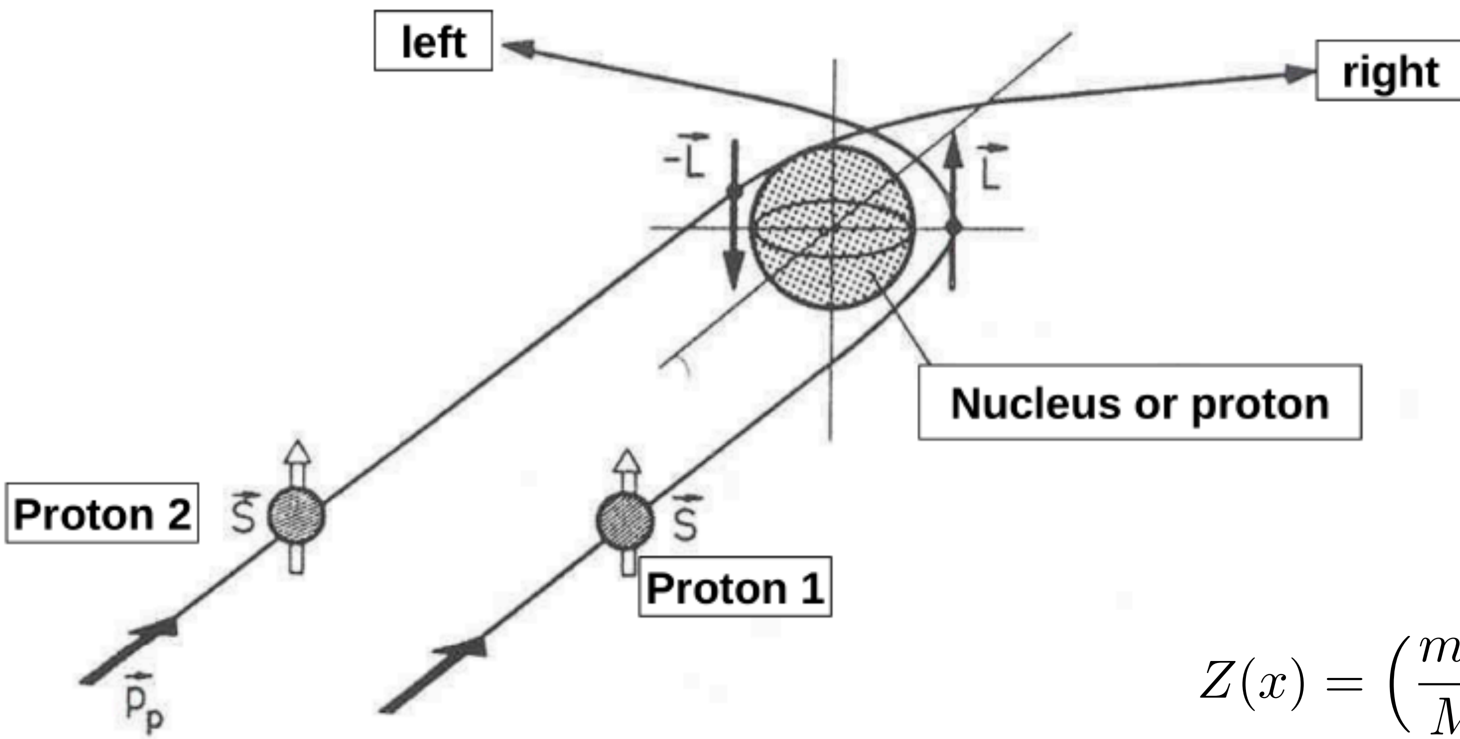


OME potential, vector field: ω meson (700 MeV)

CENTRAL

$$V_\omega(r) = \frac{g_\omega^2}{4\pi} m_\omega \left[Y(m_\omega r) + + \frac{1}{6}\left(\frac{m_\omega}{M}\right)^2 Y(m_\omega r) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 - \frac{3}{2} Z_1(m_\omega r) \boldsymbol{L}\cdot\boldsymbol{S} - \frac{1}{12} Z(m_\omega r) S_{12} \right]$$

SPIN-SPIN SPIN-ORBIT TENSOR



$$Y(x) = \frac{e^{-x}}{x}$$

$$Z(x) = \left(\frac{m_\omega}{M}\right)^2 \left(1 + \frac{3}{x^2} + \frac{3}{x^2}\right) Y(x)$$

Spin-orbit operator

The spin–orbit operator is

$$\mathbf{L}\cdot\mathbf{S} = \tfrac{1}{2}[J(J+1) - L(L+1) - S(S+1)]. \qquad \text{(A10.28)}$$

The quadratic spin–orbit operator Q_{12} of eqn (A.10.24) is

$$Q_{12} = 2(\mathbf{L}\cdot\mathbf{S})^2 - L(L+1) + \mathbf{L}\cdot\mathbf{S}. \qquad \text{(A10.29)}$$

The value of Q_{12} for a $J = L$ singlet state ($S = 0$) is $-L(L+1)$. Values for $S = 1$ are listed in Table A10.2.

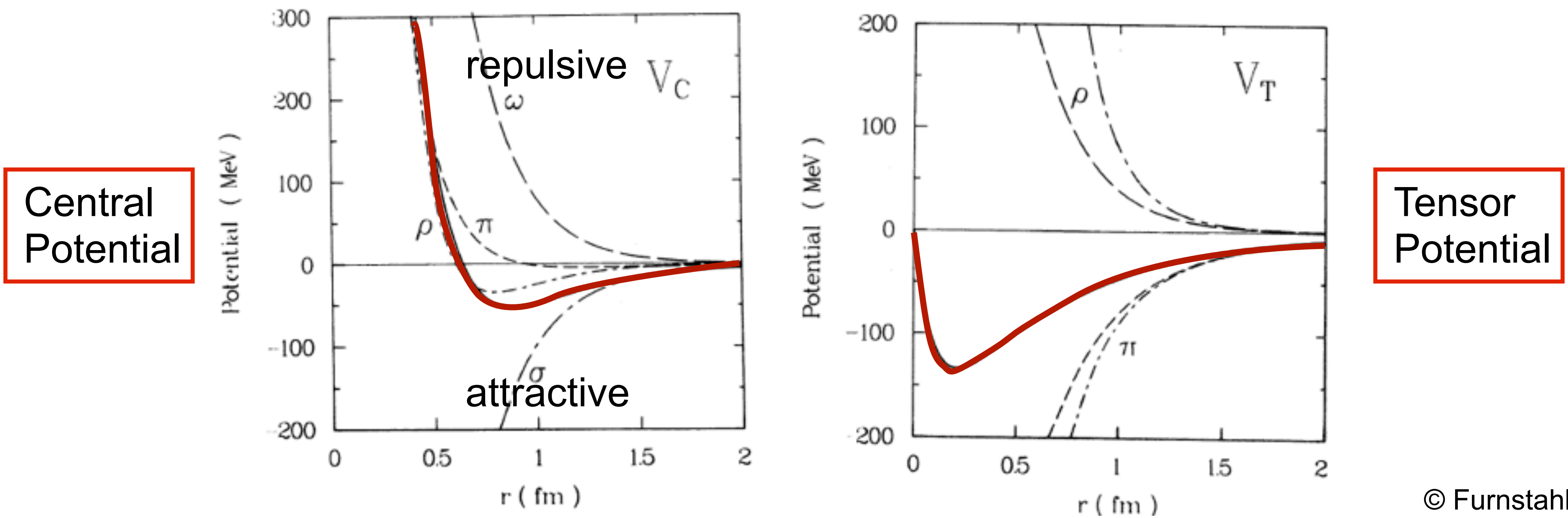
Table A10.2. Values of $\mathbf{L}\cdot\mathbf{S}$ and Q_{12} for $S = 1$ states

J	$L-1$	L	$L+1$
$\mathbf{L}\cdot\mathbf{S}$	$-(L+1)$	-1	L
Q_{12}	$(L+1)^2$	$-L(L+1)+1$	L^2

One meson exchange potential: review

One-Boson-Exchange Model

coupling	$T=0$ [1]	$T=1$ $[\tau_1 \cdot \tau_2]$	Central [1]	Spin-Spin $[\sigma_1 \cdot \sigma_2]$	Tensor $[S_{12}]$	Spin-Orbit $[\mathbf{L} \cdot \mathbf{S}]$
pseudoscalar	η	π	—	weak	strong	—
scalar	σ	δ	strong attractive	—	—	adds to vector
vector	ω	ρ	strong repulsive	weak	opposes ps	strong adds to s
tensor	ω	ρ	—	weak	opposes ps	—



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One meson exchange potential: review

One-Boson-Exchange Model

coupling	$T=0$ [1]	$T=1$ $[\tau_1 \cdot \tau_2]$	Central [1]	Spin-Spin $[\sigma_1 \cdot \sigma_2]$	Tensor $[S_{12}]$	Spin-Orbit $[\mathbf{L} \cdot \mathbf{S}]$
pseudoscalar	η	π	—	weak	strong	—
scalar	σ	δ	strong attractive	—	—	adds to vector
vector	ω	ρ	strong repulsive	weak	opposes ps	strong adds to s
tensor	ω	ρ	—	weak	opposes ps	—