Time-stepping and Krylov methods for large-scale instability problems

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Abstract ???

1 Introduction

2 Theoretical framework

Our attention is focused on the characterization of very high-dimensional nonlinear dynamical systems resulting from the spatial discretization of partial differential equations, e.g. the Navier-Stokes equations. Such dynamical systems can be written as

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = f(\mathbf{x}, \mu), \tag{1}$$

where $\mathbf{x}$ is the $n \times 1$ state vector of the system, t is time, μ is a control parameter and $f : \mathbb{R}^n \to \mathbb{R}^n$ is a nonlinear function. Alternatively, if temporal discretization is also accounted for, one can consider a discrete-time nonlinear dynamical system given by

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$$\mathbf{x}^{(n+1)} = \mathscr{F}(\mathbf{x}^{(n)}, \mu), \tag{2}$$

where $\mathscr{F} : \mathbb{R}^n \to \mathbb{R}^n$ is the discrete-time counterpart to f. Note that both continuous-time and discrete-time representation of the high-dimensional dynamical system considered will be used throughout this contribution, depending on which is the most convenient for the particular task considered.

As a prerequisite for §3, the reader will first be introduced to the concepts of fixed points and linear stability. Particular attention will be paid to *modal* and *non-modal stability*, two fundamental concepts that have become prevalent in fluid dynamics over the past two decades. Note that the concept of *nonlinear optimal perturbation* is beyond the scope of the present contribution. For interested readers, please refer to the recent work by [**?**] and references therein.

2.1 Equilibria and fixed points

Nonlinear dynamical systems described by Eq. (1) or Eq. (2) tend to admit a number of different equilibria forming the backbone of their phase space. These different equilibria can take the form of fixed points, periodic orbits or stange attractors for instance. In the rest of this work, our attention will be solely focused on fixed points.

For a continuous-time dynamical system described by Eq. (1), fixed points $\mathbf{x}^*$ are solution to

$$f(\mathbf{x}^*, \mu) = 0. \tag{3}$$

Conversely, fixed points $\mathbf{x}^*$ of a discrete-time dynamical system described by Eq. (2) are solution to

$$\mathscr{F}(\mathbf{x}^*, \mu) = \mathbf{x}^*. \tag{4}$$

It must be emphasized that both Eq. (3) and Eq. (4) may admit multiple solutions. Determining which of these fixed points is the most relevant one from a physical point of view is problem-dependent and left for the user to decide. Note however that computing these equilibrium points is a prerequisite to all of the analyses to be described in this chapter. Numerical methods to solve Eq. (3) or Eq. (4) are discussed in §3.1.

2.2 Modal stability analysis

2.3 Non-modal stability analysis

2.3.1 Optimal perturbation

Formulation using Rayleigh quotient

Formulation using Lagrange multipliers

2.3.2 Resolvent analysis

Formulation using Rayleigh quotient

Formulation using Lagrange multipliers

3 Numerical methods

3.1 Fixed points computation

3.1.1 Selective Frequency Damping

3.1.2 Newton-Krylov method

3.1.3 BoostConv

3.2 Modal stability analysis

3.2.1 Power Iteration method

3.2.2 Arnoldi decomposition

3.2.3 Krylov-Schur decomposition

3.3 Non-modal stability analysis

3.3.1 Optimal perturbation analysis

3.3.2 Resolvent analysis

4 Application to fluid dynamics

5 Conclusions and perspectives