Time-stepping and Krylov methods for large-scale instability problems

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Abstract ???

1 Introduction

2 Theoretical framework

Our attention is focused on the characterization of very high-dimensional nonlinear dynamical systems resulting from the spatial discretization of partial differential equations, e.g. the Navier-Stokes equations. Such dynamical systems can be written as

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \mathscr{F}(\mathbf{x}, \mu), \tag{1}$$

where $\mathbf{x}$ is the $n \times 1$ state vector of the system, t is time, μ is a control parameter and $\mathscr{F} : \mathbb{R}^n \to \mathbb{R}^n$ is the nonlinear dynamics. In this section, the reader will be introduced to the concepts of fixed points and linear stability. Particular attention will be paid to *modal* and *non-modal stability*, two fundamental concepts that have

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become prevalent in fluid dynamics over the past two decades. Note that the concept of *nonlinear optimal perturbation* is beyond the scope of the present contribution. For interested readers, please refer to the recent work by [?] and references therein.

2.1 Fixed points

2.2 Modal stability analysis

2.3 Non-modal stability analysis

2.3.1 Optimal perturbation

Formulation using Rayleigh quotient

Formulation using Lagrange multipliers

2.3.2 Resolvent analysis

Formulation using Rayleigh quotient

Formulation using Lagrange multipliers

3 Numerical methods

3.1 Fixed points computation

3.1.1 Selective Frequency Damping

3.1.2 Newton-Krylov method

3.1.3 BoostConv

3.2 Modal stability analysis

3.2.1 Power Iteration method

3.2.2 Arnoldi decomposition

3.2.3 Krylov-Schur decomposition

3.3 Non-modal stability analysis

3.3.1 Optimal perturbation analysis

3.3.2 Resolvent analysis

4 Application to fluid dynamics

5 Conclusions and perspectives