Research Statement

Francisco Blanco-Silva

Outline of Research

This is a description of my current research, both at the level of development of theoretical tools, as well as at the level of applications to real-life problems. I have thus divided the exposition in two parts.

In the first section, I describe my work in Approximation Theory derived from my dissertation, briefly explain the importance of these results to applications in Image Processing, and present some examples in which I am actively investigating.

In the second section, I describe some of the applied problems in which I have been involved:

- *Super-resolution applied to Electron Microscopy Imaging*, and
- *Modeling the impact of ebola and bushmeat hunting in Western Lowland gorillas.*

The former comes from a collaboration of the Nanotechnology Center of the University of South Carolina with the *Interdisciplinary Mathematics Institute*, for which I have been a post-doctoral researcher for the last three years. The latter is a pet-project that started as a collaboration among mathematicians, statisticians and forestry engineers, with a pasion for our environment.

1. Approximation Theory

My dissertation [**BS07**], "The Curvelet Transform—A Generalized Definition and Approximation Properties" was aimed to the construction of families of function spaces spanned by curvelet frames, as well as the computation of the associated approximation spaces. My work covered an improvement on the construction of curvelets defined by Candès and Donoho in [**CD05a**] and [**CD05b**], which offers the simple expression below for its corresponding transform, and allows the treatment of low-frequency functions with the same curvelet-like elements:

> *Curvelets* is an example of a family of square-integrable, complex-valued functions $\gamma_{\alpha\beta\theta}$ indexed by three parameters ($\alpha > 0$ for scale, $\beta \in \mathbb{R}^2$ for shift, and $\theta \in \mathbb{S}^1$ for angle) with the following fundamental property: For all functions $f, g \in L_2(\mathbb{R}^2)$,
>
> $$\langle f, g \rangle = \int_0^\infty \int_{\mathbb{S}^1} \int_{\mathbb{R}^2} \langle f, \gamma_{\alpha\beta\theta} \rangle \overline{\langle g, \gamma_{\alpha\beta\theta} \rangle} \, d\beta \, d\sigma(\theta) \, d\alpha,$$
>
> where $d\sigma(\theta)$ denotes the surface integral on the sphere.

One can choose a discrete set of this family to form a *frame* in $L_2(\mathbb{R}^2)$: If we denote $\phi_{nk\ell} = \gamma_{\alpha\beta\theta}$ for carefully chosen values $\alpha = \alpha_n$, $\theta = \theta_{nk}$ and $\beta = \beta_{nk\ell}$, then there exist constants $0 < A \leq B$ such that for all square-integrable functions $f \in L_2(\mathbb{R}^2)$,

$$A\|f\|^2_{L_2(\mathbb{R}^2)} \leq \sum_n \sum_k \sum_\ell \langle f, \phi_{nk\ell}\rangle^2 \leq B\|f\|^2_{L_2(\mathbb{R}^2)}$$

I presented an exhaustive analysis of my Curvelet Transform and show how to use it, for example, to study regularity of square-integrable complex-valued functions $f \in L_2(\mathbb{R}^2)$.

I also developed a discretization of this particular Curvelet Transform that allows the construction of tight frames of curvelets (frames where $A = B = 1$), and the posterior organization of these frames to build families of approximants (sequences of homogeneous spaces spanned by partial frames, whose union is dense in $L_2(\mathbb{R}^2)$).

These spaces, together with the regularity results produced above, are the building block for studyin the approximation properties of the corresponding frames, in the following sense: The regularity results are a previous step in the construction of spaces $Y \subset L_2(\mathbb{R}^2)$ for which Jackson and Bernstein inequalities are satisfied by the families of approximants spanned by curvelet frames. With both these inequalities, it is trivial to construct and compute the approximation spaces stated above.

The techniques I use are mostly based on the work of Ronald Devore ([**DeV98**] and [**DL93**]). My treatment of the Curvelet Transform pretends to mimic that of wavelets presented by Ingrid Daubechies in [**Dau92**].

The natural continuation of this work is to extend the same type of results to general frames with the *edge-detection property*: bandelets, chirplets, contourlets, dual-tree complex wavelets, noiselets, ridgelets, shearlets, wedgelets, wavelet packets, etc.

As a first step in this direction, I am currently working on the equivalence of description of smoothness spaces as decomposition spaces generated from frames of curvelets and shearlets (see section 1.1 below)

At the same time, I am exploring the computational advantages of using frames constructed from overcompleting well-known bases. The main example is given by the Dual-tree Complex Wavelet transform: a smart complexification of two different wavelet transforms related by a phase shift. Section 1.2 presents more information, as well as a good set of background citations.

The solution to the questions posed above serve as a starting point to developing new strategies to problems in Image Processing, where treatment of data with curved singularities is needed. This is one of the driving forces of my work.

1.1. Equivalence of Approximation Spaces generated from different frames. This is a natural problem when facing frames with a similar construction and structure. For example, in the search for frames with the edge-detection property, two stand out: the curvelets of Donoho and Candés defined above, and the shearlets of Guo, Kutyniok, Labate and Weiss (see e.g. [**GKL06**]). At implementation time (for signal processing purposes), shearlets have a great advantage over curvelets: not only their elements are easily represented in the lattice in Fourier domain, but also the shearlets construction is based on multiresolution analysis, which makes design of algorithms very convenient and fast. On the other hand, the

polar nature of elements from curvelet frames makes them a more desirable pick when proving Theorems. Yet, frames in both elements share very similar geometric properties; their support in frequency domain is compactly supported in slits of growing annulae (be it circular or square), and they are both indexed by a triple, that indicates frequency, location and some sense of directionality in the plane.

It is thus conjectured that both frames have the same approximation properties: If this is true, a researcher may regard them as the same object (only for approximation purposes) and make use of one or another depending on the situation at hand.

The generalization of this idea to general frames is an ambitious long-term project, in which I mean to devote many of my efforts. My personal take for a successful resolution of this problem requires a treatment of function spaces similar to that of computation of invariants of manifolds in the scope of Algebraic Topology. I aim to describe invariants of function spaces by means of (properly defined) homology groups associated to these spaces, in a similar fashion as in the theory of Loop spaces as complex manifolds.

Baby steps in this direction start by defining this machinery for smoothness spaces defined from frames by decomposition methods (see e.g. [**FG85**] or [**BN07**]). They are a natural first step prior to attacking the general problem stated above, from which one will certain learn valuable techniques to deal with approximation spaces in the future.

1.2. The Dual-Tree Complex Wavelet Transform. Although the discrete wavelet transform (DWT) is a powerful signal-processing tool, it has three disadvantages that undermine its usage in many applications. First, it is shift sensitive because input-signal shifts generate unpredictable changes in DWT coefficients. Second, the DWT suffers from poor directionality because DWT coefficients reveal only three spatial orientations. Third, DWT analysis of real signals lacks the phase information that accurately describes non-stationary signal behavior. To overcome these problems, Kingsbury [**Kin01**] created the dual-tree complex wavelet transform ($\mathbb{C}$WT), a redundant, complex wavelet transform with excellent directionality, reduced shift sensitivity and explicit phase information. Because of these advantages, the $\mathbb{C}$WT yields excellent results in applications where redundancy is acceptable.

The wavelets associated with the filter banks are an almost-Hilbert pair (see [**YCW07**]). This property is critical since it provides the advantages of reduced shift sensitivity, improved directionality and explicit phase information. However, the design of Kingsbury's $\mathbb{C}$WT filters is very complicated because it requires an iterative optimization over the space of "perfect-reconstruction" filter banks. To simplify and generalize the $\mathbb{C}$WT design procedure, Selesnick [**Sel01**] proposed a new method to obtain filters that may be used instead.

Design of similar frames that overcome these computational difficulties is part of my research. We mainly aim to reduce the redundancy and further simplify the design procedure: since transform coefficients require more storage space than the input signals, the Dual-tree complex wavelet transform cannot be used in applications such as image/video compression where thriftier signal representations are crucial.

I am also involved in researching the advantages of using these transforms in applications to lower-level analysis in Image Processing: edge-detection, horizon-detection, and computation of smoothness are an example of the tasks I have analyzed in my work.

2. Applications

2.1. Super-resolution applied to Electron Microscopic Imaging. This work is a collaboration among mathematicians, material and electrical engineers, chemists and electron microscopists. It is an ongoing effort to solve an ambitious question in Nanotechnology: extracting structural information (at the atomic level) from micrographs of molecules of different materials. A successful outcome will provide scientist with precise information on how the structural defects of a material provide its chemical properties, thus leading to the possibility of nano-manufacturing a similar (cheaper) product with the same properties.

Modern electron microscopic imaging has reached resolutions significantly better than 100pm, which allows for unprecedented measurements of the composition and structure of materials. However, one faces several severe obstacles to fully exploiting the information provided by aberration-corrected instruments. On the one hand, one needs to constantly remediate and reduce environmental perturbations such as air flow, acoustic noise, floor vibrations, AC and DC magnetic fields, and temperature fluctuations. On the other hand, high resolution and a good signal to noise ratio requires a high density of electrons per square nanometer. Unfortunately, soft materials are very susceptible to beam damage, and can only be visualized with low dose beams, resulting in poor resolution and a prohibitively low signal to noise ratio. Imaging matter using electron microscopes, in particular STEM (scanning transmission electron microscopes), will become increasingly important in the near future, especially in Biology.

By using a STEM and a high-angle annular detector, it is possible to obtain atomic resolution images where the contrast is directly related to the atomic number. As for beam sensitivity, a critical issue in electron microscopy is the amount of dose needed to produce an image. Higher dose scans can damage the specimen while lower dose scans result in very low signal to noise ratio. In STEM mode, the electron dose onto the sample can be controlled in a variety of ways: The number of electrons per unit time can be varied by changing the demagnification of the electron source through the strength of the first condenser lens. The dwell time of the probe is typically varied between $7\mu s$ and $64\mu s$ per pixel in practice, although a much larger range is possible. The size of the image can be varied from a very small number of pixels in a frame (256×256) to over 64 million pixels per image (8192×8192). Finally, the magnification of the image sets the area of the specimen exposed to the electrons and thereby affects the dose per unit area onto the specimen.

This leads us to the concept of *super-resolution*. We aim to recover the level of resolution set by the microscope, but by using a *time series* of low resolution—viz. low dose—images.

The standard way of producing high resolution reconstructions from a series of low resolution/noisy images is classically formulated as a global model (with local noise $\boldsymbol{n}$) of the form

$$\boldsymbol{y}_t = (D \cdot B_t \cdot M_t)\boldsymbol{x} + \boldsymbol{n}_t, \tag{1}$$

where $\boldsymbol{x}$ is the desired high-resolution image which is assumed constant during the acquisition of the multiple micrographs, except for any motion and degradation allowed by the model. Therefore, the observed low-resolution images result from warping (M_t), blurring (B_t), and sub-sampling (D) operators performed on $\boldsymbol{x}$. It is also assumed that each micrograph is corrupted by additive noise $\boldsymbol{n}_t$ only.

Unfortunately, this paradigm is not applicable in this form: Tracking and estimating the motion by a sufficiently accurate model M_t in (1) based on low resolution possibly very noisy data, is not feasible. Thus, standard super-resolution concepts as described above that are based on registration and motion tracking are not applicable.

In [**BBSB^{+}09**] we propose an alternative strategy that is motivated by the above observations and can be summarized as follows:

Step #1. *Time series*: As before, a high resolution image is to be recovered from a timeseries of HAADF STEM micrographs $\boldsymbol{y}_t$ of the same object, where the *time t* is the frame index and runs through a finite set.

Step #2. *The key tool*: Denoising based on time series is based on averaging the same specimen portion appearing in different frames. As explained above it is impossible to identify such portions from the low resolution frames. It is therefore crucial to employ a technique that avoids an explicit registration and motion tracking. The concept of *nonlocal means*, developed by Buades, Coll and Morel in [**BCM05**] as a denoising algorithm, offers exactly this property. The key point is to average image portions whose intensity distributions are close to each other. This can be done as follows:

With every pixel position p we associate a neighborhood N_p centered at p in as well as a (small) patch $R(p,t)$ in the frame $\boldsymbol{y}_t$ also centered at p. We wish to produce an updated (target) value $z(p,t)$ at position p in the frame at t from source values $y(p',t')$ at positions p' in a (usually different) frame at t' by computing

$$z(p,t) = \frac{\sum_{p' \in N_p, t' \in N_t} w(p,p',t,t')y(p',t')}{\sum_{p' \in N_p, t' \in N_t} w(p,p',t,t')} \tag{2}$$

where N_t denotes a *time neighborhood* of t, i.e. a collection of timewise neighboring frames that are to be taken into account for the averaging process. Here the *weights* $w(p,p',t,t')$ have the form

$$w(p,p',t,t') := \exp\left\{-\frac{\mathrm{dist}\left(R(p,t),R(p',t')\right)}{2h^2}\right\}, \tag{3}$$

where h is a filtering parameter that is thought to be solely related to the variance of the noise corrupting the data, and the *distance* dist$\left(R(p,t),R(p',t')\right)$ between two patches is to quantify the *similarity* between those patches. The distance notion is a crucial parameter of such a scheme which, in particular, allows us to incorporate knowledge about data acquisition and special artifacts.

Step #3. *A two-stage approach*: Nonlocal means alone does not yet provide super-resolution. On the one hand, it should be complemented by a deblurring step. On the other hand, in order to exploit the full potential of nonlocal means we propose an *iterated averaging procedure*. This procedure may be viewed as gradually refining the image formation in HAADF STEM and modeling the distortions encountered during the imaging process.

2.2. Modeling the impact of ebola and bushmeat hunting on Western Lowland Gorillas. The 2003 outbreak of ebola in the Republic of Congo killed 114 people and up to 800 western lowland gorillas. This outbreak and all outbreaks between 2001–2003 began with human handling of infected animal carcasses. Ebola has since spread, putting the entire gorilla population at risk.

We presented an epidemiological model in [**RBSG07**] to describe the combined effects of ebola and hunting on persistence of gorillas. This model is initially based on dynamical systems, more concretely on the so-called **SIR** models (**S**usceptible-**I**nfected-**R**ecovered). For the numerical solution of the system, we employed solving techniques that account for environmental stochasticity (in the form of future ebola outbreaks).

References

[BBSB+09] P. Binev, F. J. Blanco-Silva, D. Blom, W. Dahmen, and R. Sharpley, *Super-resolution image reconstruction by nonlocal means applied to high-angle annular darkfield scanning transmission electron microscopy*, Preprint (2009), http://imi.cas.sc.edu/IMI/resources/technical–reports/2009/reports/0906.pdf.

[BCM05]   A. Buades, B. Coll, and J. M. Morel, *A review of image denoising algorithms with a new one*, Multiscale Modeling and Simulation **4** (2005), 490–530.

[BDK02]   P. E. Batson, N. Dellby, and O. L. Krivanek, *Sub-angstrom resolution using aberration corrected electron optics*, Nature **418** (2002), no. 6898, 617–620.

[BN07]   Lasse Borup and Morten Nielsen, *Frame decomposition of decomposition spaces*, J. Fourier Anal. Appl. **13** (2007), no. 1, 39–70. MR MR2296727

[BS07]   Francisco Blanco-Silva, *The curvelet transform. a generalized definition and approximate properties*, Ph.D. thesis, Purdue University, 2007.

[CD04]   Emmanuel J. Candès and David L. Donoho, *New tight frames of curvelets and optimal representations of objects with piecewise C^2 singularities*, Comm. Pure Appl. Math. **57** (2004), no. 2, 219–266. MR MR2012649 (2004k:42052)

[CD05a]   _____, *Continuous curvelet transform. I. Resolution of the wavefront set*, Appl. Comput. Harmon. Anal. **19** (2005), no. 2, 162–197. MR MR2163077 (2006d:42058a)

[CD05b]   _____, *Continuous curvelet transform. II. Discretization and frames*, Appl. Comput. Harmon. Anal. **19** (2005), no. 2, 198–222. MR MR2163078 (2006d:42058b)

[CDLL98]   Antonin Chambolle, Ronald A. DeVore, Nam-yong Lee, and Bradley J. Lucier, *Nonlinear wavelet image processing: variational problems, compression, and noise removal through wavelet shrinkage*, IEEE Trans. Image Process. **7** (1998), no. 3, 319–335. MR MR1669536 (2001i:94006)

[Chr03]   Ole Christensen, *An introduction to frames and Riesz bases*, Applied and Numerical Harmonic Analysis, Birkhäuser Boston Inc., Boston, MA, 2003. MR MR1946982 (2003k:42001)

[Dau92]   Ingrid Daubechies, *Ten lectures on wavelets*, CBMS-NSF Regional Conference Series in Applied Mathematics, vol. 61, Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 1992. MR MR1162107 (93e:42045)

[dC76]   Manfredo P. do Carmo, *Differential geometry of curves and surfaces*, Prentice-Hall Inc., Englewood Cliffs, N.J., 1976, Translated from the Portuguese. MR MR0394451 (52 #15253)

[DeV98]   Ronald A. DeVore, *Nonlinear approximation*, Acta numerica, 1998, Acta Numer., vol. 7, Cambridge Univ. Press, Cambridge, 1998, pp. 51–150. MR MR1689432 (2001a:41034)

[DL93]   Ronald A. DeVore and George G. Lorentz, *Constructive approximation*, Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], vol. 303, Springer-Verlag, Berlin, 1993. MR MR1261635 (95f:41001)

[ELL08]   Glenn Easley, Demetrio Labate, and Wang-Q Lim, *Sparse directional image representations using the discrete shearlet transform*, Appl. Comput. Harmon. Anal. **25** (2008), no. 1, 25–46. MR MR2419703

[FG85]   Hans G. Feichtinger and Peter Gröbner, *Banach spaces of distributions defined by decomposition methods. I*, Math. Nachr. **123** (1985), 97–120. MR MR809337 (87b:46020)

[GKL06]   Kanghui Guo, Gitta Kutyniok, and Demetrio Labate, *Sparse multidimensional representations using anisotropic dilation and shear operators*, Wavelets and splines: Athens 2005, Mod. Methods Math., Nashboro Press, Brentwood, TN, 2006, pp. 189–201. MR MR2233452 (2007c:42050)

[GL07]   Kanghui Guo and Demetrio Labate, *Optimally sparse multidimensional representation using shearlets*, SIAM J. Math. Anal. **39** (2007), no. 1, 298–318. MR MR2318387 (2008k:42097)

[GZL06]   Shi Long Gao, Jie Zhou, and Wang Yong Lü, *A feature extraction based on dual-tree complex wavelet*, Sichuan Daxue Xuebao **43** (2006), no. 6, 1183–1186. MR MR2360164 (2008i:94008)

[Kin01]   Nick Kingsbury, *Complex wavelets for shift invariant analysis and filtering of signals*, Appl. Comput. Harmon. Anal. **10** (2001), no. 3, 234–253. MR MR1829802 (2002g:94016)

[NS05]   Julia Neumann and Gabriele Steidl, *Dual-tree complex wavelet transform in the frequency domain and an application to signal classification*, Int. J. Wavelets Multiresolut. Inf. Process. **3** (2005), no. 1, 43–65. MR MR2131368 (2006b:94008)

[PETM09]   M Protter, M Elad, H Takeda, and P Milanfar, *Generalizing the nonlocal-means to super-resolution reconstruction*, Image Processing, IEEE Transactions on **18** (2009), no. 1, 36–51.

[RBSG07]   C. Rizkalla, F. J. Blanco-Silva, and S. Gruver, *Modeling the impact of ebola and bushmeat hunting on western lowland gorillas*, EcoHealth **4** (2007), no. 2, 151–155.

[Sel01]   Ivan W. Selesnick, *Smooth wavelet tight frames with zero moments*, Appl. Comput. Harmon. Anal. **10** (2001), no. 2, 163–181. MR MR1815726 (2001m:42061)

[WCC06]   Hong-Xia Wang, Bo Chen, and Li-Zhi Cheng, *A new construction method for the dual tree complex wavelet based on direction sensitivity*, Acta Automat. Sinica **32** (2006), no. 1, 47–53. MR MR2229652

[YCW07]   Fengxia Yan, Lizhi Cheng, and Hongxia Wang, *The design of Hilbert transform pairs in dual-tree complex wavelet transform*, Wavelet analysis and applications, Appl. Numer. Harmon. Anal., Birkhäuser, Basel, 2007, pp. 431–441. MR MR2297932 (2007m:42030)

230 CORLEY WOODS DRIVE, LEXINGTON, SC, 29072

E-mail address: `francisco.blanco.silva@gmail.com`

Research Projects

Francisco Blanco-Silva

Additional Research Projects

An outline of my research can be read in a different attached document, titled "Research Statement." On it, I summarize the main lines of investigation in which I have been professionally involved in the last four years. The decision of focusing on those topics was always heavily influenced by the group of scientists with whom I teamed up, but mostly by the leader of said team. The same figures that inspired and guided me as a scientist, did also spark the pleasure of passing this knowledge toward those students following our steps. I had the privilege to learn a great deal of people's management from talented and brilliant mathematicians, chemists, physicists, computer scientists and engineers, and I aspire to bring that experience into an undergraduate research group of my own in a near future.

In that sense, and following the ideas behind my main research line, I started the planning of several possible ways to interact and engage undergraduate talent. The following is a brief summary of such a project:

Art Authentication. Vincent Van Gogh died in 1889 poor and unknown. He was an avid painter but lacked of organizational skills and any cataloging interest. He would take any opportunity to sketch and paint, but after he was somewhat satisfied with the result, he would store his production and forget it in the mess that was his studio. Often, most of his sketches will end up in the property of friends and acquaintances, that would keep them as a memento of their friendship, or as a token of appreciation for a service to the painter. Van Gogh's immediate family inherited the majority of the master's production after his death, and these few hundred pieces went unnoticed for several years until a group of very enthusiastic French painters "discovered" the genius behind the style of the Dutch master. In the following five to ten years, his originals started being acquired by art dealers and museums, and the momentum was so great that thirty years after his death, in the 1920's, his paintings were already among the most expensive in the World.

Many art dealers, second rate painters and museum owners saw the opportunity to profit from the lack of a catalog of Vincent's opus, and very many *new* Van Gogh's arose. An army of art experts would face the problem of establishing the authenticity of these new pieces, but often the financial interest of both experts and museums would cloud the truth and admit as authentic a fake.

In the late 1990's the Project Van Gogh was born, as the first exponent of the so-called "Scientific Art Authentication": a set of mathematical and computational

techniques destined to explore the **dating**, **authorship** and **description of features** of any given painting. This is an example of multi-modal research where Chemistry, Mathematics, Computer Science and other material sciences interact with Statistics to answer any and all of the three questions posed above.

One of the most important aspects in art authentication starts from the extraction and study of the brush-stroke technique used on a given picture. And here is where the field of Approximation Theory (and especially the use of the Curvelet or Shearlet frames described in my early work) has its impact. We know that any given image, once digitally rendered, can be represented in very many different ways: as a collection of pixels, or as a collection of coefficients in some other smart basis or frame (wavelets, Fourier, Gabor, curvelets, etc). The advantage of representation in a frame like the curvelets is in the fact that each coefficient offers certain information that can be exploited to better understand the characteristics of the image at a given location in the canvas, in a given direction, and at a certain scale—a brush-stroke! This allows quick computer codes to decide statistical properties of the artistic technique of a given painter, for example, or the possibility of comparison of the technique on two different paintings.

While most of the theory behind Curvelets is only accesible to experienced mathematicians, the implementation of the decomposition of a given image by curvelets coefficients is easy to code for anybody with enough knowledge of computer programming and familiar with the techniques of Vector Calculus. A group of students with the right background would have surely no trouble to implement, for example, a set of applications for a device with camera (such as an iPhone, iPad or an Android-based device). Such a program captures a painting with the camera, or from a sufficiently good image online, and would be able to perform basic brush-stroke analysis of the object in a matter of seconds.

A group of students would need no more than the necessary funds to code, test and publish these applications, reporting the scientific knowledge acquired to a scientific document or a poster, and funding to travel to meetings to present their research.

230 CORLEY WOODS DRIVE, LEXINGTON, SC, 29072
E-mail address: `francisco.blanco.silva@gmail.com`