STATEMENT OF TEACHING INTEREST

FRANCISCO BLANCO SILVA

This document summarizes my progression as an instructor, teaching philosophy and goals. It contains four parts: in the first one, I enumerate the different assignments in which I have participated at the University level, with a brief description of my duties. Where available and relevant, I also include short statistics of the results of my teaching evaluations. I summarize my views on instruction and education on a second paragraph. In a third section, I include a description of some of the courses and topics for which I am passionate, and for which I can make a strong contribution as an educator. I ended with practical examples of my previous statements, including one of my latest syllabi, teaching evaluations, a lesson plan of one of my more advanced classes, and interaction with student through my professional blog. I believe they strongly support my pursue of excellence in teaching.

Teaching Assignments

(1) Grader and recitation instructor for all 100 and 200 levels of Algebra/Trigonometry (pre-Calculus), Calculus (including Business and honor Calculus), 300 level Geometry, and Differential Equations at Purdue University. Average recitation evaluation near 5/5.
(2) Main lecturer for almost all 100 and 200 levels of Mathematics at Purdue University, including all courses in the Algebra/Trigonometry series (pre-Calculus), and all Calculus courses. Average instructor evaluation over 4/5.
(3) Main lecturer for all 100 and 200 levels of Calculus (including Business Calculus and Elementary Differential Equations) at the University of South Carolina. Average instructor evaluation over 4/5.
(4) Main lecturer for 300 level courses in Geometry and Differential Equations at Purdue University. Average instructor evaluation over 4/5.
(5) Recitation instructor for graduate-level courses on Abstract Algebra and Partial Differential Equations at the University of Minnesota.
(6) Main lecturer for a graduate-level summer course on Measure Theory at Purdue University. No instructor evaluation offered.
(7) Main lecturer for a reading course on Mathematical Imaging at the University of South Carolina. No instructor evaluation offered.

(8) Programmer for the SAGE project (**S**tudent **A**ssignments **G**raded **E**lectronically) at Purdue University, helping develop a computer-based homework system. This appointment required not only strong coding abilities, but also a deep familiarity with all courses offered in the department of Mathematics, and in particular all minute details of each homework assignment. It required working closely with instructors, course coordinators, and students simultaneously.

Teaching Philosophy

I base my teaching philosophy on two concepts: feedback and motivation. My approach to lectures, is classical, in the sense that I feel more comfortable interacting with my students in front of either a blackboard, or an interactive screen, going over theory and posterior problem solving. The structure of each class allows me to obtain as much feedback from my students as possible, and this accomplishes two important goals:

- The speed and order in which new material is covered depends mostly on the background of the students, and not solely based on a rigorous syllabus.
- In case of a student's failure in the course, I have enough information to provide very accurate and personalized advise.

Motivation plays a big role in my teaching principles. I use it, for example, to raise awareness of unsolved questions, or current applications to real-life situations that are close to their interests. I can push this a little further and, with the right guidance, offer all my students the opportunity to research at their level on a topic for which they are passionate.

In the stage where motivation comes into play, I deviate from traditional methods in favor of new technologies. For example, my students benefit from the interaction with my professional blog, where they can obtain course-related material and interact with their classmates. I have attached an example to this document. I am extremely proficient on the use of Mathematical engines (Maple, MATLAB, Mathematica, sage, etc.) and scientific typesetting with LaTeX. I use these skills to create visual examples, or generate enough different problems with certain characteristics. I have also included another example to support this claim, from one 400-level Geometry class I taught at Purdue University.

A good example of the benefits of this methodology can be readily observed on some of my students' research projects every semester: study of the behavior of the oil slick in the Gulf of Mexico, applications to financial mathematics, dynamics of networks of enzymes, improvement of metal structures subject to vibrations, study of tension on strings, artificial intelligence, etc. Some of these projects have even received substantial coverage in the news. I have included some of these projects in their corresponding course-pages online for future reference, and I encourage the readers to take a minute to examine them.

In summary: My recipe for success in education starts by supporting students' passion through motivation. It is accomplished by helping them achieve their goals by putting emphasis on their stronger skills. It is improved by identifying shortcomings in their scientific background and helping them overcome those issues.

Future Goals

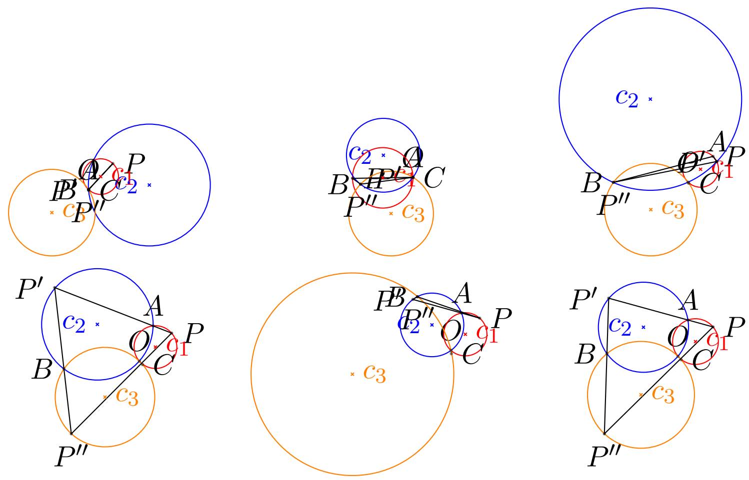
Among all the possible undergraduate-level courses available, I am especially interested in those where several different mathematical techniques come together: differential equations, linear algebra, discrete mathematics, pattern recognition or data mining, to name a few. It is not only the beauty of the material that drives my interest, but the countless possibilities to guide students in the application of these topics to interesting research projects.

I am also capable of teaching both at the advanced-undergraduate and graduate levels. In the past ten years I have been working on my particular take of some topics for which I am extremely passionate: In the theoretical arena, the fields of algebraic topology, theory of distributions and differential geometry; in the field of practical mathematics, applications of functional analysis, the theory of distributions, variational methods, partial differential equations, approximation theory and numerical methods to mathematical imaging. I have been typing extensive notes and posting online examples of lectures and assignments, again through my professional blog. Once more I point the reader to that resource for a more detailed review of my potential as instructor.

Example of Use of Technology in the Classroom

An IGSE (Interactive Geometry Software Environment) is a computer program that helps create dynamic diagrams in different Geometries (planar or 3D Euclidean, or even spherical or hyperbolic). It allows the user to drop points on screen, or select two points to construct the segment through them, or the circle centered in the first that goes through the second, etc. Once done with a geometric construction, it is possible to click and hold on any of its elements (let it be a point, a segment, a circle, etc.) and while dragging them all over the screen, change the shape of the construction. It is a very valuable tool that allows students to gain a huge insight and intuition on geometric structures, and has helped many a mathematician to pose and solve interesting problems. The best exponent of these IGSE is probably `GeoGebra`, with which I am very proficient, and have used it in countless occasions to carry my lectures.

The usual way to include diagrams from an IGSE in a document is by asking the software to produce a proper image, or by grabbing screen shots. But with `tikz` and a few of its libraries, one can mass produce these diagrams within LaTeX code, without the need of a IGSE. The figure below was generated as a tabular with three centered columns, and on each cell of the table a function holding a tikzpicture environment was included. The function produces a random example of the Theorem below.

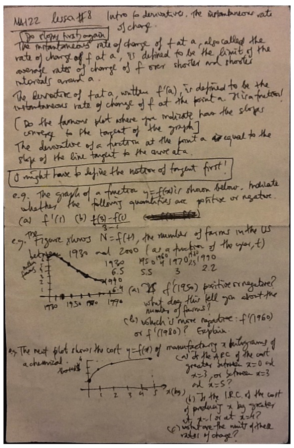


Consider three circles c_1, c_2, c_3 that intersect in a common point O. (see figure above) Circles c_1 and c_2 intersect also at point A. Circles c_2 and c_3 intersect also at point B. Circles c_1 and c_3 intersect also at a point C. Consider any point $P \in c_1$, and trace a line through P and A. This line intersects circle c_2 at a second point P'. Trace a line through P' and B. This line intersects circle c_3 at a second point P''. We want to prove that the points P, P'' and C are collinear.

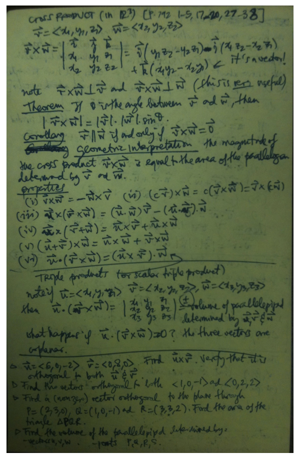
Example of Lesson Plans and Handouts

I keep all lesson plans in `Evernote` and `Dropbox` for future reference, as well as resources for my students. The following are examples from some of the most relevant lessons of some of my classes at USC.

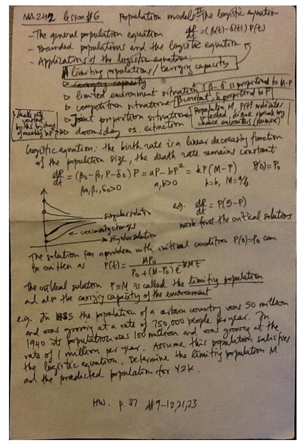
Figure 1. Handwritten notes/exercises



MA122

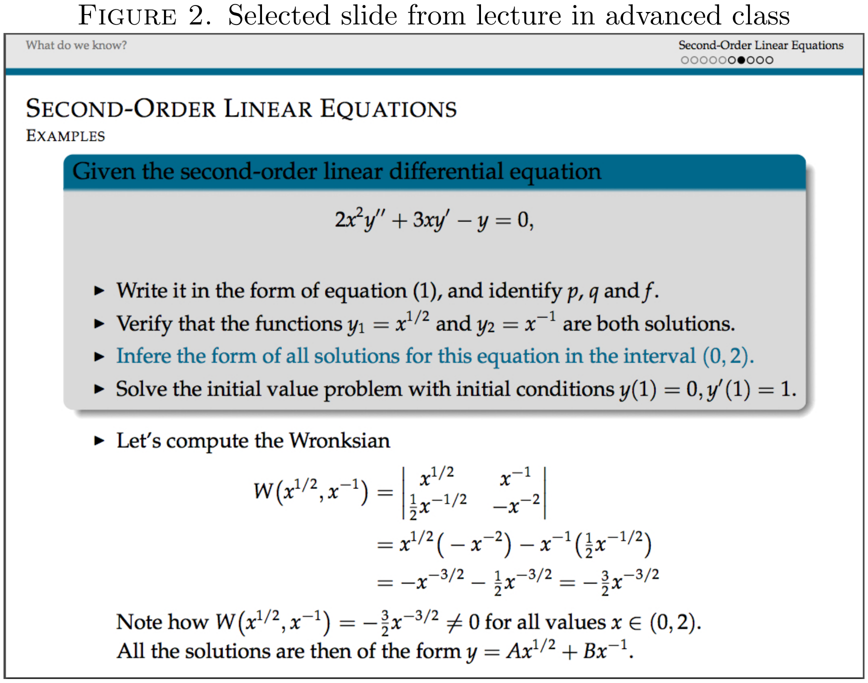


MA241



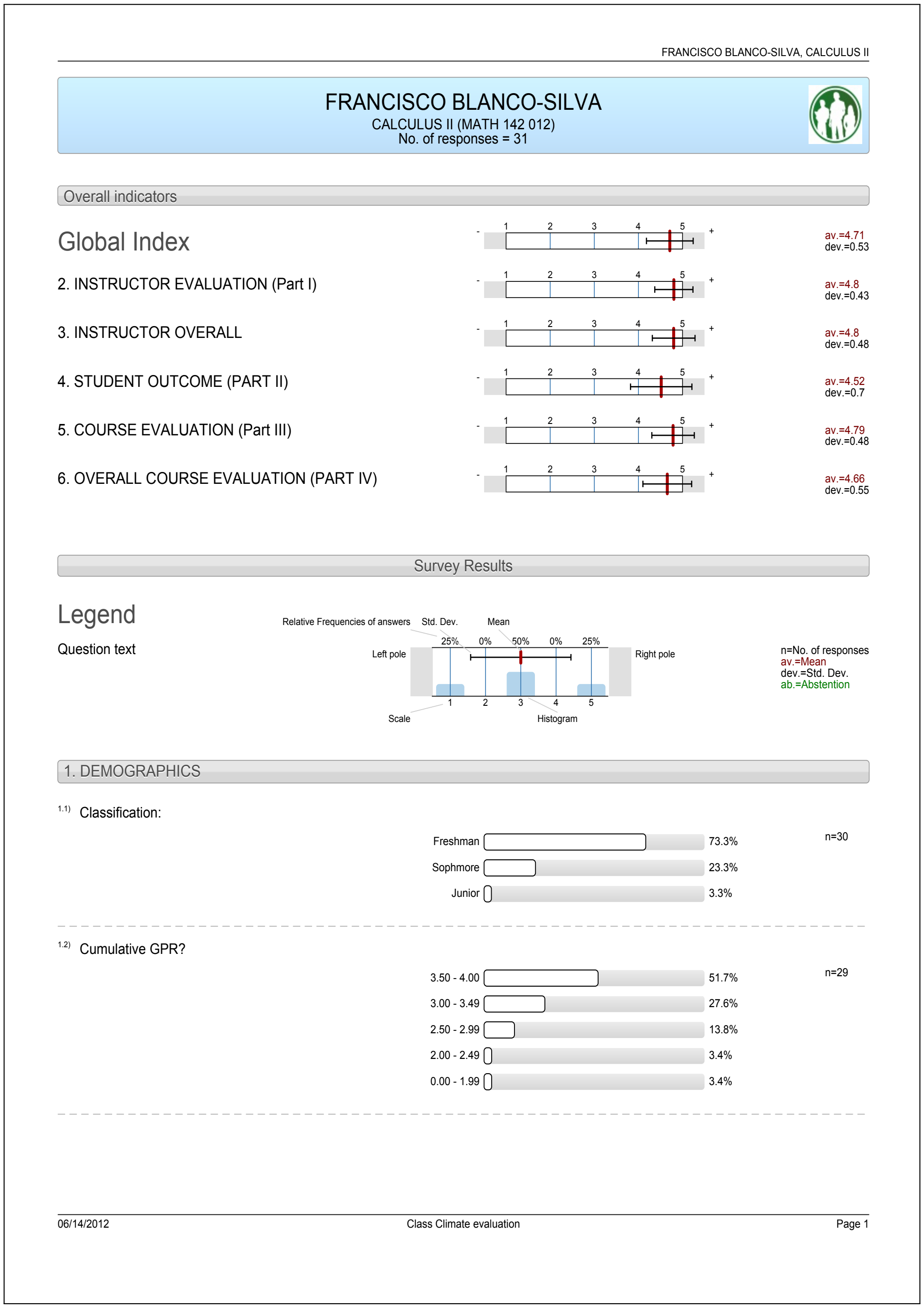
MA242

Figure 2. Selected slide from lecture in advanced class



Example of Teaching Evaluations

The following is the first page of one of the most recent "climate class surveys" administered in one of my classes of Calculus II. It indicates the students' appraisal of my general performance, as well as a course evaluation.



FRANCISCO BLANCO-SILVA, CALCULUS II

FRANCISCO BLANCO-SILVA
CALCULUS II (MATH 142 012)
No. of responses = 31

Overall indicators

Global Index		av.=4.71 dev.=0.53
2. INSTRUCTOR EVALUATION (Part I)		av.=4.8 dev.=0.43
3. INSTRUCTOR OVERALL		av.=4.8 dev.=0.48
4. STUDENT OUTCOME (PART II)		av.=4.52 dev.=0.7
5. COURSE EVALUATION (Part III)		av.=4.79 dev.=0.48
6. OVERALL COURSE EVALUATION (PART IV)		av.=4.66 dev.=0.55

Survey Results

Legend

Question text

Relative Frequencies of answers Std. Dev. Mean
25% 0% 50% 0% 25%
Left pole Right pole
1 2 3 4 5
Scale Histogram

n=No. of responses
av.=Mean
dev.=Std. Dev.
ab.=Abstention

1. DEMOGRAPHICS

1.1) Classification:

Freshman	73.3%	n=30
Sophmore	23.3%	
Junior	3.3%	

1.2) Cumulative GPR?

3.50 - 4.00	51.7%	n=29
3.00 - 3.49	27.6%	
2.50 - 2.99	13.8%	
2.00 - 2.49	3.4%	
0.00 - 1.99	3.4%	

06/14/2012 Class Climate evaluation Page 1

Example of Recent Syllabus

These are the first four pages of one of the syllabi of my classes on Business Calculus (MA122). The complete document and other resources associated to this class can be accessed online through the corresponding course webpage.

Type text to search here...

MA122—Section 22

Edit

Instructor

Francisco Blanco-Silva

e-mail: blanco at math dot sc dot edu
office: LeConte 307

Meeting Times

Lectures: **TTh** 12:30 PM – 1:45 PM Davis 209
Office Hours: **TTh** 2.00 PM — 5.00 PM LeConte 307

Prerequisites

Qualifications through algebra placement or a grade of **C** or better in MATH 111/111L. The deadline to drop/add is Friday, January 13th. The first day in which a "W" grade is assigned is therefore Saturday, January 14th. The last day to obtain a "W" grade or to elect a pass/fail grade is Monday, February 27th. The first day in which a "WF" grade is assigned is therefore Tuesday, February 28th.

Text

Applied Calculus by Hughes-Hallett, Gleason, Lock, Flath et al. **Wiley** 2009 (fourth edition)

Applied Calculus

Calculator

A graphic calculator is required for this course. Either the TI-83 or TI-84 is preferred, and as a matter of fact, highly recommended. A TI-89 or a similar calculator with a computer algebra system is not allowed on examinations and quizzes.

Texas Instruments TI-83 Plus Graphing Calculator Texas Instruments TI-84 Plus Graphing Calculator

Course Structure and Grading Policies

Homework problems will be assigned at the end of each lecture. We will be using WileyPLUS in the course, and the homework will be assigned and completed online. In order to register for WileyPLUS, you need to have a registration code, which should be included with your textbook. If you do not have a registration code, you will need to either return your book and purchase a package that includes a registration code, or you may purchase a registration code separately online at wileyplus.com

The Registration code includes access to the entire contents of the textbook online, so you may opt to purchase only the registration code and then use the online transcription of the textbook to study. In order to sign up for our section of the course on WileyPLUS, visit

http://edugen.wileyplus.com/edugen/class/cls251270/

There you will be able to enter the registration code from your textbook and enroll in our section of the course online. Once you have successfully enrolled, use wileyplus.com to login to your account and complete homework assignments.

The result of your HW assignment has no impact whatsoever over your final grade, but it serves as a guideline to understand the type of problems that you will encounter on your quizzes and tests. Your final score for the course will be computed as follows:

- Quizzes: 15% of the course grade. **Only the 10 best scores are counted.** A ten-minute quiz will be given during selected sessions or through wileyplus.com. There will be no make-up quizzes, since only the best 10 grades count towards the course grade.
- Midterms: each test counts 15%, for a total of 45% of the course grade. There will be three in-class midterm exams tentatively scheduled as follows:

Test #	Date
1	Tue, Jan 31
2	Thu, Feb 23
3	Tue, Mar 27

No make-up tests will be given. Only medical, death in the family, religious or official USC business reasons are valid excuses for missing a test and must be verified by letter from a doctor, guardian or

supervisor to the instructor.

- Final exam: 40% of the course grade. The final exam is scheduled on Tuesday, May 1st, at 2:00 PM.

The course grade will be determined as follows:

GRADE	RANGE
A	90%-100%
B+	85%-89%
B	80%-84%
C+	75%-79%
C	70%-74%
D+	65%-69%
D	60%-64%
F	below 60%

ATTENDANCE POLICY: Attendance is mandatory. Penalties to your final grade apply as follows:

- Students missing four sessions without a valid excuse will have their final grade lowered by half a letter grade (e.g. from C to D+).
- Students missing six sessions without a valid excuse will have their final grade lowered by a full letter grade (e.g. from B to C)
- Students missing eight sessions without a valid excuse will have their final grade lowered by a letter-and-a-half (e.g. from A to C+)

Further Information

Some useful information:

- Remember to change your e-mail address on Blackboard if necessary [blackboard.sc.edu]
- **ADA**: If you have special needs as addressed by the *Americans with Dissabilities Act* and need any assistance, please notify the instructor immediately.
- The Math Tutoring Center is a free tutoring service for MATH 111, 115, 122, 141, 142, 170, 221, 222, and 241. The center also maintains a list of private tutors for math and statistics. The center is located in LeConte, room 105, and the schedule is available at the Department of Mathematics website (www.math.sc.edu). No appointment is necessary.
- The **Student Success Center** and one of four **Academic Centers for Excellence (ACE)** are on the mezzanine level of the Thomas Cooper Library and can be reached by phone at (803) 777-0684 or by going online at www.sc.edu/academicsuccess Other ACE locations around campus make access to these resources easy (Sims Hall, Bates House, Columbia Hall). The centers are at the crossroads of services and information about many special resources for stucents, including advice on developing successful study habits, time management, and effective learning strategies as well as availability of tutoring.
- The Supplemental Instructor for this course is Candler Paige. Candler will be holding three sessions a week in the Humanities Building (HU). Her schedule is as follows:

Mon 7.00 PM HU 317
Tue 5.00 PM HU 405
Wed 8.00 PM HU 317

Learning Outcomes

A student who successfully completes Applied Calculus (MATH 122) will master concepts based on derivatives and integrals of elementary algebraic, exponential and logarithmic functions. Students will be able to solve applications involving maxima, minima, rates of change, motion, work, area under a curve, and volume.

HW Assignments, Quizzes, Exams

- **Tue Jan 10**: 1.1 & 1.2. Introduction to functions. Linear functions. [p.5 #2,3,4,7,8,10,11,12a,13,14,16,23,24a; p.12 #5,6,7,8,14,15]
- **Thu Jan 12**: 1.2 & 1.3. More linear functions. Average Rate of Change. [p.12 #1,2,3,4,12,25; p.22 #12,13,15,16,20,27]
- **Tue Jan 17**: 1.3 & 1.4. Relative change. Applications of functions to Economics. [p.22 #42--46; p.35 #4,6,8,9,10,11,19,20,22,23] [Quiz #1]
- **Thu Jan 19**: 1.5. Exponential functions [p.43 #2,4,6--12,19] [Quiz #2]
- **Tue Jan 24**: 1.6. The natural logarithm [p.50 #1--17,21,27--29]
- **Thu Jan 26**: 1.7. Exponential growth and decay [p.56 #1,3--5,8,10--12,16]
- **Tue Jan 31**: First Midterm. Sections 1.1, 1.2, 1.3, 1.4, 1.5 and 1.6.
- **Thu Feb 02**: 1.8 & 1.9. Power functions, polynomials. Compositions, shifts, stretches. [p.62 #1--9,32--41; p.67 #1--12] [Quiz #3]
- **Tue Feb 07**: 2.1-2.3. Intro to derivatives: instantaneous rate of change [Quiz #4]
- **Thu Feb 09**: 2.3 & 2.4. Notation and interpretation of the derivative. The second derivative and interpretation in terms of concavity [p.106 #1--4,7,10,16; p.113 #3--8,16,17] [Quiz #5]
- **Tue Feb 14**: 3.1 & 3.2. Derivative rules [p.139 #1-36, 40,41,45,50; p.144 #1--28,33,34,40] [Quiz #6]
- **Thu Feb 16**: 3.4. The product and quotient rules. Applications: Marginal analysis [p.119 #9--11,13; p.154 #3,4,7--14,16,20,21,23--28,35,36,41,42; p.140 #59] [Quiz #7]
- **Tue Feb 21**: 3.3. The chain rule.
- **Thu Feb 23**: Second Midterm. Sections 1.7, 1.8, 1.9, 2.1-2.5, 3.1, 3.2 and 3.4
- **Tue Feb 28**: 4.1 & 4.2. Local maxima and minima. Inflection points. [Quiz #8]
- **Thu Mar 01**: 4.3 & 4.4. Global maxima and minima. Applications to Finance [online version of test#2 posted. due Sat March 3 at 11pm]
- **Tue Mar 13**: 7.1. Intro to antiderivatives and integration. [Quiz #9]
- **Thu Mar 15**: 7.2. Integration by substitution [Quiz #10]
- **Tue Mar 20**: 7.4. Integration by parts [Quiz #11]
- **Thu Mar 22**: 5.3 & 7.3. The Fundamental Theorem of Calculus. The

Example of Student Interaction on a Practice Exam Online

For each midterm offered in class, I prepare a practice exam online. I never include an answer key, because the intention is precisely to foster interaction among students. I included below a printout with the forum started on one of my Vector Calculus midterms. I urge the reader to follow the entire discussion, as well as other similar ones, on their corresponding course pages online.

Type text to search here...

Second Midterm-Practice test

Edit Leave a comment Go to comments

You know the drill: work these problems and let me know if you have any trouble. I will try to answer *some* questions until Monday evening (Oct 10). This is a good opportunity to compare notes and work with other students. Enjoy!

1. Find three positive numbers whose sum is 12 and the sum of whose squares is as small as possible.
2. The base of an aquarium with volume 9,000cm^3 is made of slate and the sides are made of glass. If slate costs five times as much as glass (per square centimeter), find the dimensions of the aquarium that minimizes the cost of the materials.
3. If $u = f(x, y)$, where $x = e^s \cos t$ and $e^s \sin t$, show that
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = e^{-2s}\Big(\frac{\partial^2 u}{\partial s^2} + \frac{\partial^2 u}{\partial t^2}\Big)$$
4. If $z = f(x, y)$, where $x = s + t$ and $y = s - t$, show that
$$\Big(\frac{\partial z}{\partial x}\Big)^2 - \Big(\frac{\partial z}{\partial y}\Big)^2 = \frac{\partial z}{\partial s}\frac{\partial z}{\partial t}$$
5. Suppose that over a certain region of space the electrical potential V is given by $V(x, y, z) = 5x^2 - 3xy + xyz$.
   - Find the rate of change of the potential at the point $P = (3, 4, 5)$ in the direction of the vector $\boldsymbol{v} = \boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$.
   - In which direction does V change most rapidly at P?
   - What is the maximum rate of change at P?
6. Find the directional derivative of $f(x, y) = \sqrt{xy}$ at $P = (2, 8)$ in the direction of $Q = (5, 4)$.
7. Find the local maximum and minimum values and saddle point of the function $f(x, y) = x^2 y e^{-x^2-y^2}$.
8. Find the local maximum and minimum values and saddle point of the function $f(x, y) = \sin x \sin y$ in the square $-\pi \le x \le \pi, -\pi \le y \le \pi$.
9. Find the absolute maximum and minimum values of $f(x, y) = xy^2$ in the set $D = \big\{(x, y) : x \ge 0, y \ge 0, x^2 + y^2 \le 3\big\}$.
10. Find and sketch the domain of the following functions
$$f(x, y) = \ln(9 - x^2 - 9y^2), f(x, y) = \sqrt{y} + \sqrt{25 - x^2 - y^2}, f(x, y) = \frac{\sqrt{y - x^2}}{1 - x^2}$$
11. Find the following limits if they exist, or show that they do not exist:
$$\lim_{(x,y)\to(0,0)} \frac{x^2 y e^y}{x^4 + 4y^2}, \lim_{(x,y)\to(0,0)} \frac{6x^3 y}{2x^4 + y^4}.$$
12. Find all the second partial derivatives of $v = \dfrac{xy}{x - y}$.
13. Verify that the function $u = (x^2 + y^2 + z^2)^{-1/2}$ is a solution of the three-dimensional Laplace equation
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

14. Find an equation of the tangent plane to the surface $z = y\cos(x - y)$ at the point $(2, 2, 2)$.
15. Use differentials to estimate the amount of tin in a closed tin can with diameter 8cm and height 12cm if the tin is 0.04cm thick.
16. if $z = x^2 - xy + 3y^2$ and (x, y) changes from $(-3, -1)$ to $(2.96, -0.95)$, compare the values of Δz and dz.
17. Four positive numbers, each less than 50, are rounded to the first decimal place and then multiplied together. Use differentials to estimate the maximum possible error in the computed product that might result from the rounding.

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Comments (41) Leave a comment

1. Nate Fuller
   October 7, 2011 at 11:52 am | #1
   Reply | Quote | Edit

   I need help with number 3 and number 4. How do you get the second partials with the s and t variables?

   - Francisco Blanco-Silva
     October 7, 2011 at 1:18 pm | #2
     Reply | Quote | Edit

     Look at example 7 in page 905

   - Lauretta G.
     October 8, 2011 at 6:16 pm | #3
     Reply | Quote | Edit

     Hey, Nate. For number 4 this is what did:

     The partial derivative of z with respect to s= the partial derivative of z with respect to x times the partial derivative of x with repsect to s plus the partial derviative of z with respect to y times the partial derivative of y with respect to s. The partial derivative of x with respect to s = 1, and the partial derivative of y with respect to s =1. Therefore, the partial derivative of z with repsect to s = the partial dervative of z with respect to x + the partial derivative of z with respect to y. Next, the partial derivative of z with respect to t =the partial derivative of z with repsect to x time the partial derivative of x with repsect to t plus the partial derivative of z with respect to y times the partial derivative of y with respect to t. The partial derivative of x with repsect to t equals 1 and the partial derivative of y with repsect to t= -1. Thus, the partial derivative of z with respect to t = the partial derivative of z with respect to x – the partial derivitive of z with repsect to y. Therefore the partial derivative of z with respect to s times the partial derivative of

     t with respect to t= (the partial derivative of z with respect to x + the partial derivative of z with repsect to y) times (the partial derivative of z with respect to x – the partial derivative of z with repsect to y). When you foil it, you get (the partial derivative of z with respect to x)^2 – (the partial derivative of z with respect to y)^2. Hence, you have shown what the question wants you to show.

     p.s. sorry i had to write everything out in words, but i have no clue how to type it to make it look nice.

     - Francisco Blanco-Silva
       October 10, 2011 at 10:14 am | #4
       Reply | Quote | Edit

       Lauretta, thanks a lot for the help. Something that might make solutions easier to read is using for example Maple's notation instead of words. Something like "diff(f,x,x)" is easier to interpret, for example.

       Y'all are doing a good job, keep it up!

2. Lauretta G.
   October 8, 2011 at 5:09 pm | #5
   Reply | Quote | Edit

   Hello. For number one I got x=4, y=4, and z=4, which I think is correct. Is that correct? Also, I used the Lagrange method. How would you go about solving the problem if it said that the sum of the squares was a large as possible?

   - Francisco Blanco-Silva
     October 9, 2011 at 6:28 am | #6
     Reply | Quote | Edit

     Did you take into account all the different constraints?
     $x + y + z = 12$,
     $x, y, z \ge 0$.

     With only the first constraint you may apply the method of Lagrange multipliers. Note how it gave you only one solution for this particular problem. This is very significative, right? It means that, with only one of the constraints of this problem, there is only one critical point that satisfies the solution. This point might be either a max or a min—did you check that it is a min? (it is)

     It also means that (with only one constraint) there is no max: you can find three numbers that add up to 12 and the sum of whose squares is as big as you want.

     We need to use the other constraints of the problem: if all of the variables are to be non-negative, then the maximum is attained at $(12, 0, 0)$, $(0, 12, 0)$ or $(0, 0, 12)$.

     We did in class a problem where the method of Lagrange multipliers gave us four critical points: two were min and two were max. If the method gives you only one, then you only have one kind.

     Good question!

3. Lauretta G.
   October 8, 2011 at 5:12 pm | #7
   Reply | Quote | Edit

   Also, in number two we are assuming the aquarium is in the shape of a rectangular prism, correct?

   - Francisco Blanco-Silva
     October 9, 2011 at 6:10 am | #8
     Reply | Quote | Edit

     Yes

4. Lauretta G.
   October 8, 2011 at 5:40 pm | #9
   Reply | Quote | Edit

   For #2, if x and y represent the dimension of the base of the aquarium and z represents the height of the aquarium, I got that x= (3600)^(1/3), y=(3600)^(1/3), and z= (5/2)(3600)^(1/3). Are those the correct dimensions?

   - Bobby F
     October 9, 2011 at 5:11 pm | #10
     Reply | Quote | Edit

     For #2 I'm having problems with solving the system of equations. My constraint function is g(x,y,z) = xyz – 9000 and my function to minimize is f(x,y,z) = 2xz + 2yz + 5xy

     Are these correct functions to start with? The reason I ask is that I get some crazy system of equations that is really difficult to solve. For example, my first equation is yz / (2z + 5y) = lamda. The other three look the same. I'm having trouble solving for x, y, and z with these equations.

     - Francisco Blanco-Silva
       October 9, 2011 at 5:38 pm | #11
       Reply | Quote | Edit

It looks good, except that it should be, for example the first equation, $2z + 5y = yz\lambda$. (it looks like you placed the lambda in the wrong side of the equation). If you consider now the first two equations and equal the lambdas, you should get (after simplifying) something like $x = y$. Go from there.

- Bobby F
  October 9, 2011 at 6:08 pm | #12
  Reply | Quote | Edit

  Aaah, you are right. I switched my two gradient vectors. Thanks!

- rachelodom
  October 10, 2011 at 12:01 pm | #13
  Reply | Quote | Edit

  I got the exact same answer…

5. Lauretta G.
October 8, 2011 at 6:48 pm | #14
Reply | Quote | Edit

For #5, I got
1) 32/(3^(1/3))
2)
3) 1624^(1/2)

Are those answers correct?

- Bobby F
  October 9, 2011 at 5:50 pm | #15
  Reply | Quote | Edit

  For this one I actually got
  1) 50 / 3^(1/2)
  2)
  3) 2164^(1/2)

- Bobby F
  October 9, 2011 at 5:51 pm | #16
  Reply | Quote | Edit

  Hah, I think putting vectors in angle brackets is making the blog think that we're using html

  tags… Anyway, for part 2 my vector is
  (38, 24, 12)

- J
  October 10, 2011 at 9:57 am | #17
  Reply | Quote | Edit

  I got

  1) 32/(3^(1/2))
  2) <38,6,12
  3) 1624^(1/2)

6. Lauretta G.
October 8, 2011 at 6:49 pm | #18
Reply | Quote | Edit

Also, is there an answer key anywhere so I don't have to keep asking you if my answers are right?

- Francisco Blanco-Silva
  October 9, 2011 at 6:10 am | #19
  Reply | Quote | Edit

  Nope. I want y'all to obtain the correct answers through "networking." It is part of the learning process. I will only offer hints in the right direction.

7. Lauretta G.
October 8, 2011 at 6:59 pm | #20
Reply | Quote | Edit

Is the answer to #6 2/5 ?

- J
  October 10, 2011 at 10:05 am | #21
  Reply | Quote | Edit

  That's what I got.

8. Lauretta G.
October 8, 2011 at 7:49 pm | #22

Reply | Quote | Edit

When you are finding local maxima and minima and saddle points, what do you do if the second partial derivative of the function with respect to x twice of a critical point = 0?

- Francisco Blanco-Silva
  October 9, 2011 at 6:07 am | #23
  Reply | Quote | Edit

  That's a good question. If $\frac{\partial^2 z}{\partial x^2}(a,b) = 0$, then most probably your Hessian $D(a,b)$ is negative (substitute in the formula to realize this fact). This means that you have a saddle point.

  The other possibility is that the Hessian is zero, and you have no means to decide the kind of critical point with this method. We use then other methods.

9. Lauretta G.
October 9, 2011 at 12:13 pm | #24
Reply | Quote | Edit

For number 8 I found 11 critical points. Am I on the right track?

- Francisco Blanco-Silva
  October 9, 2011 at 5:33 pm | #25
  Reply | Quote | Edit

  There are many critical points, yes. Which ones did you find?

  - Bobby F
    October 9, 2011 at 6:07 pm | #26
    Reply | Quote | Edit

    Yeah, I actually found 13 critical points. I got
    (pi/2, pi/2) (pi/2, -pi/2)
    (-pi/2, pi/2) (-pi/2, -pi/2)
    (pi, pi) (pi, -pi) (pi, 0)
    (-pi, pi) (-pi, -pi) (-pi, 0)
    (0, pi) (0, -pi) (0, 0)

10. Joe H
October 9, 2011 at 7:47 pm | #27
Reply | Quote | Edit

for number 5, what exactly is it asking in part 2? im kind of confused

11. Nate Fuller
October 9, 2011 at 9:47 pm | #28
Reply | Quote | Edit

Guys…i have a bunch of problems…i cannot figure this out…the second partials on numbers 3 and 4…i also dont understand d^2x/dxdy or d^2y/dydx…i need some major help

12. Mitch G.
October 10, 2011 at 9:44 am | #29
Reply | Quote | Edit

On number 9, how do we find the boundary given by x^2+ y^2 ≤ 3? I got two of the boundaries since we can say that the max y value will happen when x is 0 and the max x value will happen when y is 0, so our two boundaries are (0,0) to (root(3), 0) and (0,0) to (0, root(3)), however since the last boundary occurs on a curve and not a straight line, how do we account for this?

Also, when we find critical points where all points of the form (0, a) are critical points and the Hessian comes out to 0 such as in number 7, how do we conclude this? If you look at the function for number 7 you can see that the points (0, a) are minima, but they all occur in a straight line, so are they still local minima or are they not critical points at all?

- Francisco Blanco-Silva
  October 10, 2011 at 9:48 am | #30
  Reply | Quote | Edit

  Good questions too!

  About the first one: That part of the boundary is a piece of a circle, so you can parametrize it as $h(\theta) = \big(\cos\theta, \sin\theta\big)$. You just have to find what values of θ give you that piece of the circle. Go from there.

  About the second part: good job at realizing that the points of the form $(0, a)$ are indeed critical points. Since the second derivative test doesn't give you information, we need to do something different. This one is specially tricky. Some of the points are going to be maxima, and some others are going to be minima! Graph the function (with WolframAlpha, or Maple or whatever) to realize it, and see how you can prove it algebraically.

13. J
October 10, 2011 at 10:13 am | #31
Reply | Quote | Edit

For #7 I keep getting that y(x^2) = 0. Wouldn't this yield an infinite number of critical points when either x, y, or both are 0?

- Francisco Blanco-Silva
  October 10, 2011 at 10:19 am | #32
  Reply | Quote | Edit

  Remember to combine the "zeros" with both equations. The first equation will give you three possibilities (something like $x = 0, y = 0$ or $x = \pm 1.$). The second equation will give you some other possibilities… but then you have to combine them all: (1) the first possibility from the first equation, with the first possibility of the second equation; (2) the first from the first, with the second of the second; (3) first from first, third from second; etc.

  Note that at most, you will have to make eight different combinations, since the "degrees" of your expressions are 4 and 4.

14. J
    October 10, 2011 at 10:36 am | #33
    Reply | Quote | Edit

    Are both of the limits in #11 DNE?

    - Bobby F
      October 10, 2011 at 4:05 pm | #34
      Reply | Quote | Edit

      I got zero for the first limit and DNE for the second limit.

15. J
    October 10, 2011 at 12:42 pm | #35
    Reply | Quote | Edit

    Would the last problem be equal to 50^4 – 49.95^4 ?

16. Mitch G.
    October 10, 2011 at 1:32 pm | #36
    Reply | Quote | Edit

    In number 17, I'm not sure how to set up the problem. Since there are constraints, it seems like it should be a LaGrange Multipliers problem but I don't see anyway to relate all of the variables in a single equation. Also, for the equation that we need to maximize, I set it up as

    F(w,x,y,z) = (w+0.05)(x+0.05)(y+0.05)(z+0.05) – wxyz

    I did this because 0.05 is the maximum error per number that could be caused by rounding to the first decimal place but I'm not sure if this is correct.

    - Bobby F
      October 10, 2011 at 5:03 pm | #37
      Reply | Quote | Edit

      You're going about number 17 the wrong way. You use differentials to solve this one.

      Your equation is f(w,x,y,z) = wxyz So the first thing to do is calculate the differential. The differential is
      df = xyz(dw) + wyz(dx) + wxz(dy) + wxy(dz)
      When you're using differentials to estimate df you simply use dx = delta(x), etc…

      The tricky part to this problem is what to do with the actually parameters he gave you. You're definitely right about the maximum error. It would be 0.05. But what about the "less than 50″ part? Well… the question just asks what the maximum error could be…

      Think about it this way… Four numbers multiplied together… would your error be very big if, say, all the numbers were really small (like less than 1)? No, you'd have a really small error. The difference between multiplying 0.90^4 and 0.95^4 is not that big… So… what about if the four numbers were really big? The difference between 49.90^4 and 49.95^4 is pretty significant. So if we want to find the MAXIMUM POSSIBLE ERROR, then we should consider a number as close to 50 as we can get.

      So then you just take your differential equation (above) and substitute in w = x = y = z = 50 and dw = dx = dy = dz = 0.05 .

      The answer I got was 25000 as the maximum possible error.

      - Mitch G.
        October 10, 2011 at 6:12 pm | #38
        Reply | Quote | Edit

        Thanks, that makes a lot more sense! I got the same answer once I worked it out.

        Also, for number 11 you said that you got 0 for the first limit and DNE for the second limit.
        I got DNE for both of them. I plugged in the polar coordinates and got 0/[4*sin^2(theta)]. So in most cases the limit is 0, however in the direction of theta = 0, pi, -pi and so forth wouldn't you get 0/0? Maybe I interpreted this wrong but I just wanted to see what your thoughts were on it.

        - Bobby F
          October 10, 2011 at 8:36 pm | #39
          Reply | Quote | Edit

          Yeah, you're exactly right. I totally forgot to take into account when sin^2theta was zero. Good catch.

17. Nate Fuller
    October 10, 2011 at 2:58 pm | #40
    Reply | Quote | Edit

    Can anyone help me with number 13?

    - Mitch G.
      October 10, 2011 at 5:58 pm | #41
      Reply | Quote | Edit

      Hey Nate, I figured this one out earlier after I left the tutoring center. The best way to work this one out is to make sure you always keep the (x^2 + y^2 + z^2)^(-1/2) and anything similar to it as just a negative exponent and not try to move it around.

      For starters, just look at the partial of u for x. You should get something like this:

      (-1/2) * (x^2 + y^2 + z^2)^(-3/2) * (2x)

      Now take the second partial of u for x of this using the product rule. Leave the (x^2 + y^2 + z^2) stuff alone for the moment but simplify the rest of it and you should get something like this:

      3x^2 * (x^2 + y^2 + z^2)^(-5/2) – (x^2 + y^2 + z^2)^(-3/2)

      Notice at this point that all of the second partials will look exactly like this. The only difference is that the second differential for y will have 3y^2 at the beginning instead of 3x^2 and the z will have 3z^2. Put all three of these second partials together and you will notice that each of the parts of this whole equation has (x^2 + y^2 + z^2) to some sort of fractional exponent. Since the base is the same for all of them, you can go ahead and pull out (x^2 + y^2 + z^2)^(-5/2). What this means for the ones that are raised to the (-3/2) is that they are now raised to the 1st power since if you were to multiply them out and add exponents then (-5/2) + 1 = (-3/2). So now you have two sets of equations multiplied by each other and only one of them needs to equal 0, so you can ignore (x^2 + y^2 + z^2)^(-5/2). Now if you combine everything together in the other equation and simplify as much as possible you should be able to cancel out the rest of the equation.

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