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Performance of the Shiryaev-Roberts-type scheme in comparison to the CUSUM and EWMA schemes in monitoring weibull scale parameter based on Type I censored data

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Abstract

This paper introduces a process monitoring scheme based on the Shiryaev-Roberts (SR) procedure for Type I right-censored Weibull lifetime data for detecting shifts in the scale parameter for a fixed value of the shape parameter. The performance of the new charting scheme is evaluated under different censoring rates and sample sizes via Monte Carlo simulation. Comparison with two existing schemes, namely, the exponentially weighted moving average (EWMA) and cumulative sum (CUSUM) schemes, is performed in terms of average run length (ARL). Both zero-state and steady-state shifts are considered in this study for impartial comparisons. The numerical results based on simulation are highly encouraging in favour of the SR procedure. Our findings show that the new scheme outperforms the other two schemes in some cases. An illustrative example is given to illustrate the application of the proposed scheme.

KEYWORDS

censoring, Shiryaev-Roberts, statistical process monitoring, weibull distribution

1 | INTRODUCTION

Statistical process monitoring (SPM) and control schemes play a vital role in business and industry, healthcare, environment and network surveillance. Graphical devices, such as control charts, are practical tools for SPM to ensure the stability of a process. They can be broadly classified into two categories, namely, memoryless schemes and memory-type schemes. A Shewhart-type scheme introduced originally by Shewhart[1] nearly a century ago is memoryless because it uses only current information to detect process shifts. Page[2] and Roberts[3] proposed the cumulative sum (CUSUM) and the exponentially weighted moving average (EWMA) procedures, respectively, popularly known as the memory-type schemes. These schemes use past and current information in tandem, making them sensitive to small to moderate and persistent shifts.

In the emerging competitive market with the rapid growth of manufacturing and service industries, it is essential to maintain a high quality of products and services for sustainable business development. Industries need to ensure a robust product quality guarantee. To this end, one may note that the product lifetime or its endurance is a crucial indicator of product quality. Currently, most competitive manufacturing companies are implementing SPM schemes in various applications. Therefore, a quick and effective method to detect deviations in product lifetime is vital. Two aspects specific to lifetime data make the development of sophisticated SPM schemes more challenging. First, the data tend to be skewed and non-normally distributed. Second, the data are typically censored. For example, Weibull distribution is widely used in modelling the lifetime of a product. Weibull[4] used his proposed distribution, which was later attributed to his name, to describe material breaking strengths. This distribution has flexibility in modelling various types of failure mechanisms and is very popular in reliability and survival analysis.

Due to time and cost limitations, the collected data are often censored in natural processes. Type I censoring is common and occurs when a product has not failed by the end of the monitoring period. There are many research results in this field. Steiner and Mackay[5] proposed SPM schemes based on the simple idea of replacing each censored observation with its conditional expected value (CEV) and detected changes in the mean from censored lifetime data in Steiner and Mackay.[6] Zhang and Chen[7] constructed a lower one-sided and an upper one-sided EWMA CEV chart to detect decreases and increases in the mean for processes characterised by Type I right-censored Weibull lifetimes. CUSUM schemes for monitoring censored Weibull lifetimes were proposed by Dickinson et al.[8] Choi and Lee[9] proposed a binomial CUSUM scheme for monitoring Type I right-censored Weibull lifetimes, and Xu and Jeske[10] proposed a Shewhart-type SPM scheme based on the likelihood ratio test (LRT) statistic. Wang[11] considered the MaxEWMA control chart for a Weibull process with individual measurements. Khan et al.[12] discussed a new variable control chart under failurecensored reliability tests for Weibull distribution. Ho et al.[13] used an improving Shewhart control chart performance for monitoring the Weibull mean. Gong and Mukherjee[14] studied and compared some Shewhart-type schemes for simultaneous monitoring of Weibull parameters, and Huwang et al.[15] introduced adaptive EWMA variable sampling intervals charts for simultaneous monitoring of Weibull parameters.

The Shiryaev-Roberts (SR) procedure, proposed by Shiryaev[16] for the Brownian motion case, is based on a conditional average delay time criterion. In addition, Pollak and Siegmud[17] compared the performance of the CUSUM and SR for detecting shift in the mean of the Gaussian process when the in-control (IC) value of the mean is unknown. Srivastava and Wu[18] compared EWMA, CUSUM and SR for detecting mean shifts. Pollak and Rskm[19] proposed the SR scheme for surveillance of a non-homogeneous Poisson distribution. Moustakides et al.[20] considered integral equations for performance metrics and numerical approximations to evaluate the SR procedures efficiency. Zhang et al.[21] proposed an SR procedure for detecting the process mean and variability simultaneously. Zhang et al.[22,23] discussed adaptive-type SR procedures for monitoring the process mean and variance over a range of shift sizes, respectively. Ottenstreuer and Sebastian[24] discussed the SR control chart for Markovian count time series.

This paper applies the SR method to the censored Weibull distribution noting its flexibility and practical relevance in reliability engineering. The proposed approach is compared with the traditional EWMA CEV and CUSUM schemes using the average run length (ARL) as the performance metric. However, our proposed methodology could be extended to any other suitable lifetime distributions. According to the analysis of the results, the SR procedure is better than the corresponding CUSUM scheme for small shifts, and the SR procedure has certain advantages over the EWMA CEV scheme when the shape parameter is less than 1.

The rest of this paper is organised as follows: we review the necessary background information about the censored Weibull distribution and related work, such as EWMA CEV and CUSUM charts, in Section 2. Section 3 and Section 4 provide details about the proposed SR methods for monitoring censored Weibull lifetimes. The comparison with the existing EWMA CEV and CUSUM methods is given in Section 5. A real example is discussed in Section 6 to illustrate the implementation of the SR chart. Finally, conclusions are presented in Section 7.

2 | PRELIMINARIES AND EXISTING WORK

This section reviews the related properties of the Weibull distribution and the two existing SPM schemes, namely, EWMA CEV and CUSUM schemes.

2.1 | Censoring and the Weibull Distribution

Let $T_{i1}, T_{i2},..., T_{in}$ be the i^{th} random sample of size n from a Weibull distribution with shape parameter β and scale parameter η for any batch $i = 1, 2, ...$ The probability density function (pdf) of the failure time T is

$$f\left(T_{ij} \mid \beta, \eta\right) = \frac{\beta}{\eta}\left(\frac{T_{ij}}{\eta}\right)^{\beta-1} \exp\left[-\left(\frac{T_{ij}}{\eta}\right)^{\beta}\right], T_{ij} > 0, \eta > 0, \beta > 0, \tag{1}$$

and the cumulative distribution function (cdf) is

$$F\left(T_{ij} \mid \beta, \eta\right) = 1 - \exp\left[-\left(\frac{T_{ij}}{\eta}\right)^{\beta}\right]. \tag{2}$$

The mean and variance of T are

$$E[T] = \eta\Gamma\left(1 + \frac{1}{\beta}\right),$$

$$Var[T] = \eta^2\Gamma\left(1 + \frac{2}{\beta}\right) - \left(\eta\Gamma\left(1 + \frac{1}{\beta}\right)\right)^2,$$

respectively, where $\Gamma(\cdot)$ is the gamma function. The mean and variance are both functions of the Weibull shape and scale parameters. In industry, more attention is given to the average life span of products. Since $E[T] = \eta\Gamma(1 + \frac{1}{\beta})$, detecting the mean lifetime is equivalent to determining η when β is fixed. Let C denote the censoring time; then, the censoring rate P_c, which is the probability of censoring, is given by

$$P_c = P(T \geq C) = \exp\left[-\left(\frac{C}{\eta}\right)^{\beta}\right]. \tag{3}$$

2.2 | EWMA CEV chart proposed by Zhang and Chen[7]

We assume that $T_{i1}, T_{i2}, \dots, T_{in}$ represent the observed lifetimes of the i^{th} batch; however, it is unknown whether these data are censored. The EWMA CEV scheme aims to detect a decrease in the scale parameter η when the shape parameter β is constant. The observed lifetimes can be transformed into $(\frac{T_{ij}}{\eta})^{\beta}$. Based on Steiner,[5] the censoring lifetimes can be replaced by the IC CEV, namely,

$$CEV = E\left[\left(\frac{T_{ij}}{\eta}\right)^{\beta} \mid \left(\frac{T_{ij}}{\eta}\right)^{\beta} > \left(\frac{c}{\eta}\right)^{\beta}\right] = \left(\frac{c}{\eta}\right)^{\beta} + 1 = -\log\left(P_c\right) + 1. \tag{4}$$

Therefore, the transformed variables can be defined as

$$X_{ij} = \begin{cases} \left(\frac{T_{ij}}{\eta}\right)^{\beta} & T_{ij} \leq C, \\ -log(P_c) + 1 & T_{ij} > C. \end{cases} \tag{5}$$

Consequently, the plotting statistic of the lower-sided EWMA CEV scheme proposed by Zhang[7] is defined as

$$Q_i^- = \min\left((1 - \lambda)Q_{i-1}^- + \lambda\bar{X}_i, w_0\right), Q_0^- = w_0,$$

and the lower control limit (LCL) for a stable process is

$$LCL = K_L w_0,$$

where $\bar{X}_i = \frac{(X_{i1}+X_{i2}+\ldots X_{in})}{n}$, $\lambda \in (0, 1]$ is the smoothing parameter, $K_L \in (0, 1)$ is the control limit coefficient and $w_0 = 1$ is a reflecting barrier. The concept of a reflecting barrier for the EWMA procedure was first proposed by Gan[25] to increase the sensitivity of the SPM schemes. As Zhang and Chen[7] stated, the mean of transformed lifetimes can act as the reflecting barrier. The scheme raises an alarm if $Q_i^- < LCL$.

Similarly, the plotting statistic of the upper-sided EWMA CEV scheme is defined as

$$Q_i^+ = \max\left((1-\lambda)Q_{i-1}^+ + \lambda\bar{X}_i, w_0\right), Q_0^+ = w_0.$$

The upper control limit (UCL) for a stable process is

$$UCL = K_U w_0.$$

The scheme raises an alarm if $Q_i^+ > h_Q^+$. Here h_Q^+ is the UCL determined by equating the IC ARL (ARL_0) of the charting scheme with a pre-specified value, such as 370 or 500.

2.3 | CUSUM Chart proposed by Dickinson et al.[8]

Dickinson et al.[8] considered a likelihood ratio-based CUSUM scheme considering the right-censored samples of size $n \geq 1$ for monitoring the scale parameter η of a Weibull process. According to the authors, the parameter η reflects the characteristic life. They assumed that the shape parameter β is fixed and known. In general, the likelihood function for any distribution, including right-censored data proposed by Meeker and Escobar,[26] is

$$L(\beta, \eta; T) = \prod_{j=1}^{n} f\left(T_{ij} \mid \beta, \eta\right)^{\delta_{ij}} \left[1 - F\left(T_{ij} \mid \beta, \eta\right)\right]^{1-\delta_{ij}}, \tag{6}$$

where $\delta_{ij} = 1$ if δ_{ij} is an exact failure time and $\delta_{ij} = 0$ if δ_{ij} is censored. Here, f and F are the pdf and cdf of the underlying distribution, respectively. Then, according to Dickinson et al.,[8] the lower-sided likelihood ratio-based CUSUM chart based on this hypothesis is given by

$$D_i^- = \max\left(0, D_{i-1}^- + Z_i\right), D_0^- = 0, i = 1, 2, \ldots,$$

where Z_i is given by

$$\begin{aligned} Z_i &= \log\left(\frac{L(T_i \mid \beta, \eta_1)}{L(T_i \mid \beta, \eta_0)}\right) \\ &= \sum_{j=1}^{n} \delta_{ij} \log\left[\frac{f_1(T_{ij})}{f_0(T_{ij})}\right] + \sum_{j=1}^{n}(1-\delta_{ij}) \log\left[\frac{1-F_1(T_{ij})}{1-F_0(T_{ij})}\right]. \end{aligned} \tag{7}$$

In Equation (7), η_0 is the standard value corresponding to the IC situation, in which T, the lifetime random variable, follows a Weibull (β, η_0) distribution with density function $f_0(T)$, and η_1 is the value corresponding to the out-of-control (OOC) situation, in which T follows a Weibull (β, η_1) distribution with density function $f_1(T)$. When the random variable T follows a Weibull(β, η), then the random variable $X = (T/\eta)^\beta$ follows an exponential distribution with a mean of 1. If $\eta = \eta_0$, the random variable $X = (T/\eta_0)^\beta$ follows a standard exponential distribution. In an OOC situation with $\eta = \eta_1$, $X = (T/\eta_0)^\beta$ follows an exponential distribution with mean $(\frac{\eta_1}{\eta_0})^\beta$.

The authors considered the problem of detecting a change in η of known magnitude. Suppose that in this case, $\eta_1 = (1 - d_P)\eta_0$, where the reference value of d_P is pre-specified and known before monitoring. We will discuss how to deal with unknown d_P in a practical situation in Section 4. We introduced the parameter d_P by denoting the anticipated shift size in the process parameter η. The shift size $d_P * 100\%$ represents the percent change in the IC parameter η_0, which is desired to be detected quickly. The values $d_P = 0, d_P < 0$, or $0 < d_P < 1$ indicate that the process mean is IC, increased, or decreased, respectively. We defer the discussion on the effect of d_P on the CUSUM chart and on the SR type charts introduced in the subsequent Section till Section 4.

According to Dickinson et al.,[8] for a given sample $x_i = (x_{i1}, x_{i2}, \dots, x_{in})$, from Equation (7), we have

$$\begin{aligned} Z_i &= \delta_i \beta \log\left(\frac{\eta_0}{\eta_1}\right) - \left[\left(\frac{\eta_0}{\eta_1}\right)^{\beta} - 1\right] \sum_{j=1}^{n} x_{ij} \\ &= \delta_i \beta \log\left(\frac{1}{1 - d_P}\right) - \left[\left(\frac{1}{1 - d_P}\right)^{\beta} - 1\right] \sum_{j=1}^{n} x_{ij}, \end{aligned} \tag{8}$$

where δ_i is the number of uncensored observations in the i-th batch ($i.e.$, $\delta_i = \sum_{j=1}^{n} \delta_{ij}$). Equivalently, the chart is based on

$$C_i^- = \min\left(0, C_{i-1}^- + \sum_{j=1}^{n} x_{ij} - k_i\right), C_0^- = 0, i = 1, 2, \dots,$$

where $k_i = \frac{\delta_i \beta \log(1-d_P)}{1-(1-d_P)^{-\beta}}$. The chart produces a signal if $C_i^- < h_C^-$, where h_C^- is the LCL. The value of h_C^- is so determined that the charting scheme attains the pre-specified target ARL_0.

3 | THE SR CONTROL CHART

This section introduces an SPM scheme for the scale parameter of the Weibull processes based on type-I censored samples using the SR procedure. To the best of our knowledge, the SR procedure for the Weibull processes under type-I censoring is not discussed in the literature. Recall that $T_{i1}, T_{i2}, \dots, T_{in}$ be independent random variables of Weibull (β, η); C and P_c are the censoring time and censoring rate mentioned in Equation (3), respectively. When the process is IC, T follows Weibull (β, η_0) with density function $f_0(T_{ij})$. In this paper, we consider the Phase II monitoring with standards case in which the initial conditions η_0 and β are assumed to be known, i.e., it is assumed that the IC dataset used in Phase I is sufficient to estimate the parameters well. In order to be consistent with Dickinson et al.[8] and Zhang and Chen,[7] we set $\eta_0 = 1$.

According to Moustakides et al.,[20] for $i \geq 1$, define $\Lambda_i = \frac{f_1(T_{ij})}{f_0(T_{ij})} = \exp(Z_i)$ to be the "instantaneous" likelihood ratio between the post-change and pre-change hypotheses with pdf $f_1(T_{ij})$ and $f_0(T_{ij})$, respectively. Then, the SR-type plotting statistic is defined as

$$R_i = \sum_{k=1}^{i} \prod_{j=k}^{i} \Lambda_j. \tag{9}$$

The scheme produces an OOC signal if $R_i > h$, where h is determined by setting the ARL_0 equal to the desired level. It can be easily verified that the SR statistic allows the following recursive representation:

$$R_i = (1 + R_{i-1})\Lambda_i, \tag{10}$$

where $R_0 = 0$. According to Equation (7), Λ_i can be converted into:

$$\begin{aligned}\Lambda_i &= \exp(Z_i)\\ &= \exp\left\{\sum_{j=1}^{n} log\left[\frac{f_1(T_{ij})}{f_0(T_{ij})}\right] + \sum_{j=1}^{n}(1-\delta_{ij})log\left[\frac{1-F_1(T_{ij})}{1-F_0(T_{ij})}\right]\right\}\\ &= \exp\left\{\delta_i \beta log\left(\frac{\eta_0}{\eta_1}\right) - \left[\left(\frac{\eta_0}{\eta_1}\right)^{\beta} - 1\right]\sum_{j=1}^{n} x_{ij}\right\}\\ &= \exp\left\{\delta_i \beta log\left(\frac{1}{1-d_P}\right) - \left[\left(\frac{1}{1-d_P}\right)^{\beta} - 1\right]\sum_{j=1}^{n} x_{ij}\right\}.\end{aligned} \tag{11}$$

A common practice of evaluating the performance of a charting scheme is studying its run length characteristics, particularly the ARL. The ARL represents the average number of samples plotted on a control chart before obtaining an OOC signal. The ARL_0 should be sufficiently large for an IC process to avoid false alarms. For an OOC process, the OOC ARL (ARL_1) should be sufficiently small to rapidly detect all shifts in the process. In this paper, the run length properties of the charting scheme are computed using extensive Monte Carlo simulations through an algorithm developed in the R language. The simulations assume that the IC value of the shape parameter β is fixed and known.

Algorithm: Monte Carlo simulation steps of the SR control chart for computing the control limits is as follows:

Step 1. Determine the values of the parameters, including the sample size n, the censoring rate P_c, the pre-specified percent change parameter d_P and the shape parameter β, in the IC process. The number of replications is set to 50,000. Select the initial value UCL denote as h.

Step 2. Generate a sample of size n from the Weibull distribution with the corresponding parameters.

Step 3. The plotting statistic R_i is calculated. If $R_i \leq h$, turn to Step 2. If $R_i > h$, record the corresponding sample number as the one that produces the first OOC signal and denote the number as RL.

Step 4. Repeat Step 2 - Step 3 50,000 times; and compute the average of RLs as the ARL_0 corresponding to h. Then adjust the value of h until the specified ARL_0 is reached. Finally set the UCL equal to h.

When the process undergoes a shift, the OOC performance of the chart is evaluated and compared via ARL_1. We assume that the occurrence of an OOC condition shifts the IC η_0 to $\eta = (1 - d_T)\eta_0$, where the value of d_T is the true shift size, i.e., the percent change in the IC parameter η_0. Note that d_T is different from d_P. d_P is pre-specified and known, while d_T is not fixed and unknown before monitoring. The following algorithm can be used in R to find ARL_1:

Step 1. Specify the values of n, β, P_c, d_P, and obtain the values of the control limits;

Step 2. Generate a sample of size n with shift size d_T. Then the plot statistic R_t is calculated by Equations (9) and (11);

Step 3. If the control chart signal, RL is recorded; if not, turn to Step 2;

Step 4. After 50,000 replications of Step 2 -Step 3, ARL_1 is calculated by the average of RLs.

4 SIMULATION STUDIES

In this section, we investigate the influence of two types of data transformation, subsequently discuss the impact of d_P, consider the performance of the charting schemes in terms of ARL, and study the effect of censoring rate P_c and sample sizes n in the SR control chart.

It is recommended to evaluate the control chart's performance using zero-state and steady-state ARL values. The zero-state ARL (ZS-ARL) are computed assuming that a sustained shift in the parameter occurs at the beginning of the Phase-II monitoring. On the other hand, a steady-state ARL (SS-ARL) is based on a delayed shift. In this case, the Phase-II monitoring begins with a prevailing IC state, and the process parameter shifts at a later stage, often at an unknown time point. It is often more reasonable to assume that a period of IC behaviour exists before an OOC shift occurs in applications. As Zwetsloot and Woodall[27] pointed out: "...we prefer steady-state performance metrics is because some methods have an implicit headstart that results which introduced in relatively good zero-state performance, but relatively poor steady-state performance".

To provide a fair comparison, each of the charts was designed to have IC zero-state ARL (ARL_0) values of 370 under different pre-specified values of d_P. We obtain 50,000 replicates of run lengths via the above algorithms to estimate ARL values for each charting scheme and various parameter combinations. Using the control limit of the SR chart, an adequate steady-state condition was simulated by running the process in the IC state for 50 samples before the shift in the parameter occurred. If a signal occurs before the first 50 IC samples, that particular run was discarded, and a new set of 50 samples was generated.

4.1 | Performance comparison of the SR Chart with two types of data transformation

Most of the literature suggests a special treatment in monitoring processes with censored data. Steiner and Mackay[5] proposed applying the CEV method for high censoring rates in monitoring problems, especially when the censoring proportion is greater than 50%. In this subsection, we compare the performance of the SR charts with two types of data transformation which can be defined as follows:

Type-I data transformation:

$$X_{ij} = \begin{cases} \left(\frac{T_{ij}}{\eta_0}\right)^{\beta} & T_{ij} \leq C, \\ \left(\frac{C}{\eta_0}\right)^{\beta} & T_{ij} > C. \end{cases} \tag{12}$$

Type-II data transformation:

$$X_{ij} = \begin{cases} \left(\frac{T_{ij}}{\eta_0}\right)^{\beta} & T_{ij} \leq C, \\ -\log(P_c) + 1 & T_{ij} > C. \end{cases} \tag{13}$$

In this comparison, we take different values of β_0 and P_c for illustration, and the SS-ARL comparison results are summarised in Figure 1. From Figure 1, we can see that the SR chart using the Type-I data transformation performs much better than that of using Type-II data transformation under the same shift size in most cases, especially for detecting small to moderate shifts. Therefore, we adopt Type-I data transformation to construct the SR chart in the remainder of this paper.

4.2 | The impact of the pre-specified value of d_P

In this subsection, the performance of the SR chart with different pre-specified values of d_P are evaluated. We only take $\beta_0 = 3$, $n = 5$, $P_c = 0.15$, 0.5, and 0.7 for illustration. The SS-ARL values of SR charts for detecting the scale parameter shift in the range of $[0.05, 0.5]$ are presented in Table 1. It should be noted that the performance of the SR charts heavily depends on the assumption that the shift magnitude, say d_P, is known. In other words, optimal selection of d_P relies on the target shift size. We rarely know the exact shift size before it is detected in practice. Consequently, it may be more important to look at a range of known or unknown mean shifts. Clearly, the SR chart with a pre-specified d_P usually can not have optimal or nearly optimal performance for both small and large shifts. It is, therefore, desirable that a control chart performs well over a range of mean shift sizes. To this end, we use the relative mean index (RMI) employed by Han and Tsung[28] to evaluate the overall performance. A control chart with a smaller RMI is considered to have better overall performance. The RMI can be computed by the following formula:

$$RMI = \frac{1}{N} \sum_{\tau=1}^{N} \frac{ARL_{d_{T_\tau}} - ARL_{ds_{T_\tau}}}{ARL_{ds_{T_\tau}}}, \tag{14}$$

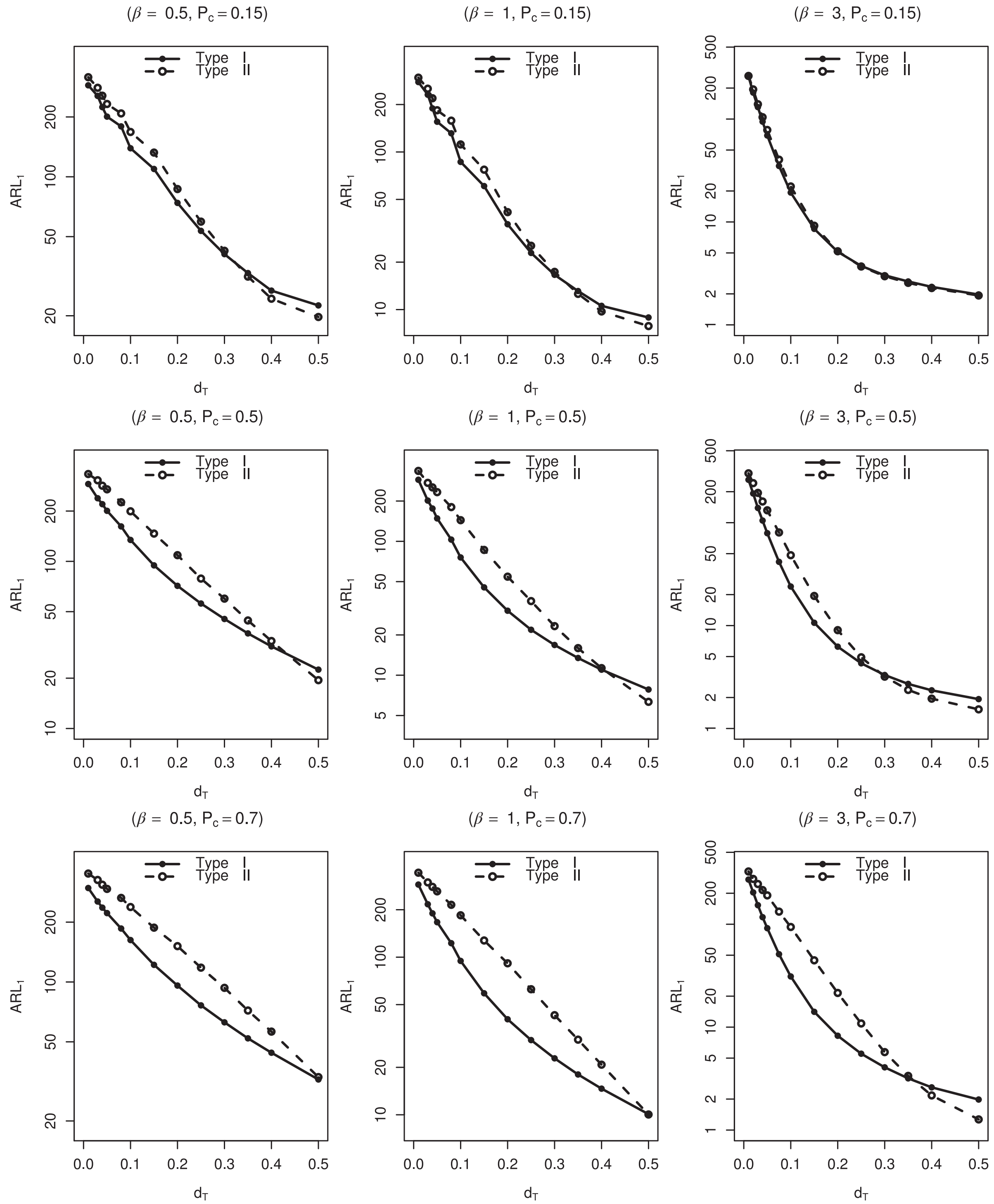

FIGURE 1 SS-ARL comparison between Type-I and Type-II data transformation ($d_P = 0.2$)

where N is the number of shifts utilized. $\mathrm{ARL}_{d_{T_\tau}}$ is the ARL_1 value when the true shift is $d_{T_\tau}, \tau = 1, 2, \ldots, N$ and ARL_{ds_T} is the smallest ARL among all the charts when the true shift is d_T. In this paper, the RMI values are evaluated with N = 9. The RMI values and the corresponding control limits are also presented in the second row of Table 1. From Table 1, we can see that when $\beta = 3$ and $n = 5$, the SR charts with small pre-specified d_P values can effectively detect small to moderate shifts, while control charts with large pre-specified d_P values are effective for detecting large shifts. This finding is consistent with the CUSUM charts. In the other hand, the simulation results also indicate that the SR chart with a pre-specified value of $d_P = 0.2$ has robust overall performance. In this case, the RMI

TABLE 1 SS-ARL and control limits of the SR control chart (ARL$_0$ = 370, $\beta_0 = 3$, $n = 5$)

$P_c = 0.15$							
d_T	d_P=0.10	d_P=0.15	d_P=0.20	d_P=0.25	d_P=0.30	d_P=0.35	d_P=0.40
	h=269.06	h=226.56	h=187.11	h=157.51	h=127.36	h=107.23	h=100.84
-	367.89	370.03	372.73	369.65	370.96	371.37	370.81
0.05	41.44	53.34	69.77	88.96	108.00	130.28	157.80
0.10	14.54	15.76	19.50	25.42	34.08	46.12	64.60
0.15	8.45	7.98	8.49	9.93	12.76	17.42	26.09
0.20	6.12	5.36	5.15	5.37	6.15	7.80	11.19
0.25	4.91	4.13	3.76	3.63	3.76	4.24	5.50
0.30	4.19	3.45	3.04	2.80	2.72	2.82	3.23
0.35	3.75	3.04	2.64	2.33	2.21	2.15	2.23
0.40	3.45	2.76	2.35	2.06	1.95	1.79	1.73
0.50	3.09	2.50	1.97	1.87	1.74	1.31	1.19
RMI	49.88%	32.64%	**27.66%**	33.99%	48.96%	70.65%	115.98%
$P_c = 0.50$							
d_T	d_P=0.10	d_P=0.15	d_P=0.20	d_P=0.25	d_P=0.30	d_P=0.35	d_P=0.40
	h=278.50	h=235.00	h=196.29	h=161.00	h=130.22	h=111.47	h=104.20
-	371.60	371.49	368.96	368.13	371.00	369.25	372.57
0.05	52.75	64.61	79.37	95.77	114.39	135.58	161.09
0.10	19.06	20.25	24.03	29.75	38.08	49.88	67.38
0.15	10.67	10.17	10.61	12.16	14.89	19.89	27.95
0.20	7.21	6.45	6.24	6.48	7.24	8.93	12.18
0.25	5.45	4.70	4.31	4.18	4.31	4.85	6.05
0.30	4.40	3.71	3.31	3.07	3.00	3.13	3.50
0.35	3.72	3.10	2.72	2.47	2.34	2.28	2.37
0.40	3.31	2.74	2.34	2.11	1.98	1.84	1.77
0.50	2.86	2.37	1.94	1.84	1.69	1.31	1.18
RMI	43.30%	28.83%	**23.32%**	27.40%	37.72%	54.66%	88.24%
$P_c = 0.70$							
d_T	d_P=0.10	d_P=0.15	d_P=0.20	d_P=0.25	d_P=0.30	d_P=0.35	d_P=0.40
	h=288.21	h=247.50	h=207.50	h=175.80	h=146.57	h=122.50	h= 98.43
-	368.39	369.24	372.06	370.26	369.22	370.88	369.88
0.05	65.80	77.26	91.83	107.10	124.61	143.20	161.95
0.10	25.45	26.88	30.45	36.91	45.30	56.17	69.97
0.15	14.18	13.63	14.23	15.93	18.85	23.80	30.82
0.20	9.36	8.47	8.24	8.46	9.30	11.24	14.09
0.25	6.79	5.94	5.54	5.39	5.55	6.09	7.29
0.30	5.22	4.50	4.05	3.82	3.74	3.86	4.22
0.35	4.20	3.59	3.18	2.92	2.77	2.70	2.79
0.40	3.51	2.97	2.60	2.37	2.20	2.06	2.01
0.50	2.74	2.30	1.98	1.84	1.67	1.36	1.27
RMI	36.54%	24.16%	**19.71%**	22.25%	29.44%	41.69%	63.27%

values are 27.66%, 23.32% and 19.71%, respectively, P_c = 0.15, 0.5, and 0.7, which are much smaller than the other charts.

One may alternatively consider the SR procedure for monitoring the Weibull distribution with adaptive design to deal with both smaller and larger shifts simultaneously. We leave this for feature research. For more work on the adaptive design, readers are referred to Capizzi, G. and Masarotto, G.,[29] Sparks,[30] Zhang et al.[22,23] and the references therein.

TABLE 2 SS-ARL, SDRL and run length quantile values ($P_c = 0.5$, $n = 5$, $d_P = 0.35$)

$\beta = 0.5$								$\beta = 3$							
d_P	ARL	SDRL	Q_5	Q_{25}	Q_{50}	Q_{75}	Q_{95}	d_P	ARL	SDRL	Q_5	Q_{25}	Q_{50}	Q_{75}	Q_{95}
-	369.81	339.71	49	132	267	498	1,055	-	369.25	368.26	22	107	255	515	1,106
0.05	216.78	209.84	15	67	154	299	642	0.05	135.58	132.42	8	39	94	189	400
0.10	140.21	129.18	13	48	101	192	401	0.10	49.88	49.48	4	15	35	69	147
0.15	96.74	84.98	11	36	72	132	263	0.15	19.89	18.13	2	7	14	27	56
0.20	70.45	58.34	10	30	54	94	186	0.20	8.93	7.57	2	4	7	12	24
0.25	52.06	40.00	8	24	42	70	131	0.25	4.85	3.30	2	3	4	6	11
0.30	40.00	28.90	7	20	33	53	96	0.30	3.13	1.67	1	2	3	4	6
0.35	31.61	21.26	7	17	27	41	73	0.35	2.28	0.96	1	2	2	3	4
0.40	25.43	16.15	6	14	22	33	56	0.40	1.84	0.64	1	1	2	2	3
0.45	20.93	12.23	5	12	19	27	44	0.45	1.55	0.53	1	1	2	2	2
0.50	17.44	9.55	5	11	16	23	35	0.50	1.31	0.47	1	1	1	2	2
0.55	14.70	7.51	4	10	14	19	29	0.55	1.12	0.32	1	1	1	1	2
0.60	12.53	6.06	4	8	12	16	24	0.60	1.01	0.12	1	1	1	1	1
0.65	10.74	4.87	4	7	10	14	19	0.65	1.00	0.03	1	1	1	1	1
0.70	9.22	3.96	3	6	9	12	16	0.70	1.00	-	1	1	1	1	1

4.3 | Run length performance of the SR control chart

In this section, we present several run length characteristics as the statistical performance indicators of the SR control chart, such as ARL, the standard deviation of the run length (SDRL), and the few quantiles of the run-length distribution. The SDRL usually gives us an idea about the possible error one can commit in estimating the ARL values. The smaller the values of the SDRL are, the better the performance of a control chart. We consider $\beta_0 = 0.5$, 3, $\mathrm{P}_c = 0.5$, $d_P = 0.35$ and $n = 5$ for illustration. The simulation results are presented in Table 2. From this Table, we can see that the SDRLs are slightly smaller than the corresponding ARLs, indicating that the run length calculated by the SR method is relatively stable. This difference is more significant as the shift size increases. In the case of small shifts, the median run length, tjat is, Q_{50}, is much less than the ARL, which indicates that the run lengths are skewed to the right. For example, when the true shift size $d_T = 0.1$, the ARL and Q_{50} are 140.21 and 101, respectively. In the case of large shifts, the median run length is slightly smaller than the ARL. For other choices of parameters, the corresponding results are available from the authors upon request.

4.4 | The impact of different censoring rates and sample sizes

From Tables 1 and 2 and Figure 1 presented above, we can see that the shape parameter β, the sample size of a life test n, and the censoring rate P_c are all factors that impact the OOC performance of the SR chart. As Dickinson et al.[8] pointed out, a practitioner cannot control the size of the shape parameter; however, they can control the sample size and censoring rate. So these two factors are analysed in this comparison. Figure 2 shows the SS-ARL values with different shifts for censoring rates $P_c = 0.15$, 0.5, and 0.7, respectively, for different choices of β, n and d_P. We see that as the censoring rate increases, the detection ability of the charting scheme decreases. The phenomenon is natural because having a large proportion of censored observations induces low accuracy. Consequently, more samples are needed to obtain the desired information.

Figure 3 shows the ARL_1 with different shifts for sample sizes $n = 3$, 5, and 10 when β, P_c and d_P are constant. As the number of samples in each group increases, the detection capability of the control chart continues to improve. Intuitively, more samples in each group can provide more information, leading to a rapid response of the control chart.

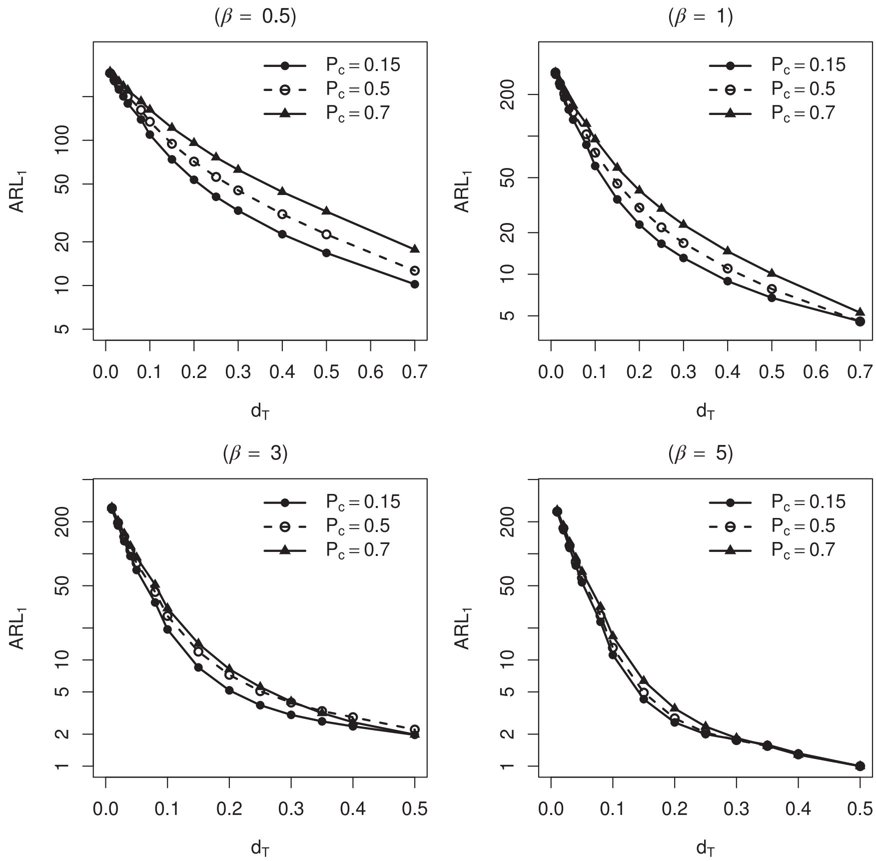

FIGURE 2 SS-ARL comparison of the SR chart with different censoring rates ($n = 5, d_P = 0.2$)

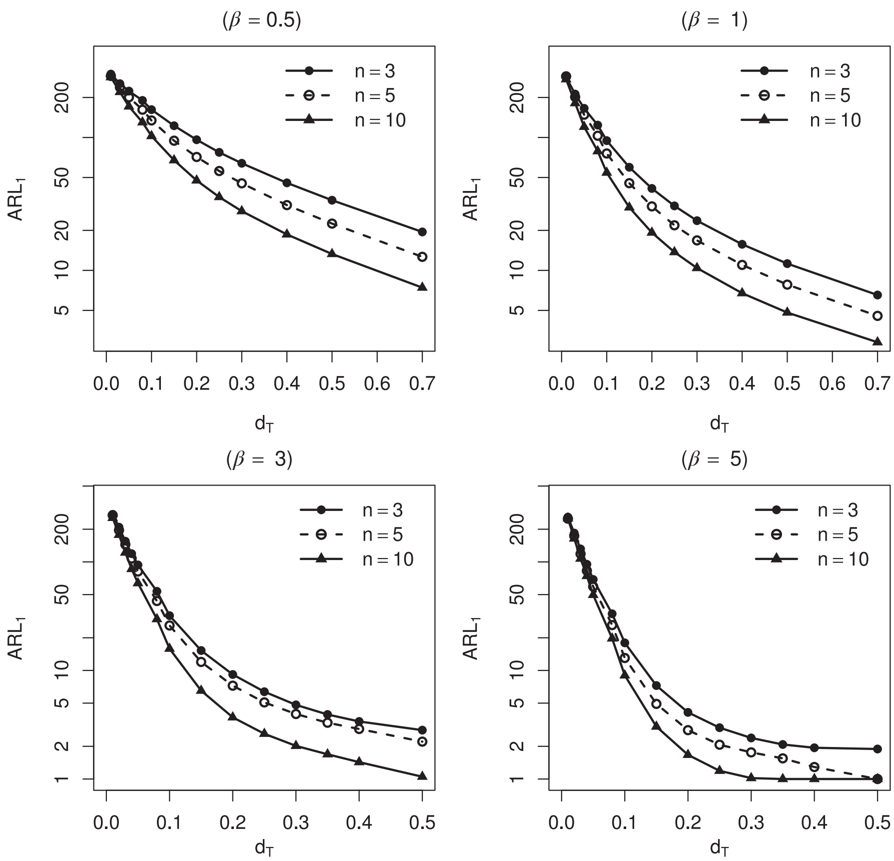

FIGURE 3 SS-ARL comparison of the SR chart with different sample sizes ($P_c = 0.5, d_P = 0.2$)

TABLE 3 ZS-ARL comparison for $\eta_0 = 1$, $n = 5$, $P_c = 0.15$, $\mathrm{ARL}_0 = 370$

$\beta = 0.5$											
d_T	d_P=0.10		d_P=0.20		d_P=0.30		d_P=0.40		λ=0.05	λ=0.1	λ=0.2
	SR	CUSUM	SR	CUSUM	SR	CUSUM	SR	CUSUM	EWMA	EWMA	EWMA
	h=351.41	h=-26.50	h=330.47	h=-19.88	h=306.25	h=-15.39	h=283.89	h=-12.02	h=0.846	h=0.767	h=0.659
-	371.33	368.62	369.70	368.40	367.62	368.96	370.46	370.08	368.90	370.82	370.88
0.05	231.91	206.89	214.49	215.91	217.80	222.65	228.56	239.88	230.61	247.72	262.90
0.10	166.05	129.05	137.59	134.78	138.53	145.47	146.32	157.55	146.94	168.31	188.48
0.15	127.87	89.71	99.23	90.09	93.75	94.96	97.76	104.46	99.33	114.06	135.54
0.20	104.45	65.89	76.01	63.63	68.12	65.37	68.86	71.65	67.47	78.40	96.94
0.25	87.64	51.85	60.78	47.92	52.10	48.48	50.19	50.83	50.26	56.76	71.21
0.30	75.60	41.75	50.40	37.53	41.51	36.22	38.72	38.10	37.55	41.05	50.76
0.35	66.28	34.60	42.80	30.43	34.04	28.94	30.40	28.70	29.16	30.76	37.49
0.40	58.80	29.22	37.09	25.26	28.94	23.34	25.08	22.55	23.40	23.73	28.07
0.50	47.64	21.95	28.94	18.33	21.62	16.36	17.82	15.06	16.12	15.16	16.44
0.60	39.54	16.98	23.33	14.02	16.88	12.14	13.54	10.83	11.85	10.62	10.41
0.70	33.17	13.45	19.13	10.97	13.56	9.31	10.63	8.14	9.07	7.75	7.03
$\beta = 1$											
d_T	d_P=0.10		d_P=0.20		d_P=0.30		d_P=0.40		λ=0.05	λ=0.1	λ=0.2
	SR	CUSUM	SR	CUSUM	SR	CUSUM	SR	CUSUM	EWMA	EWMA	EWMA
	h=332.81	h=-20.50	h=294.77	h=-13.29	h=258.91	h=-9.15	h=221.72	h=-6.43	h=0.846	h=0.767	h=0.659
-	368.45	369.06	369.48	369.46	369.89	369.07	372.61	369.73	371.05	367.94	368.79
0.05	142.47	136.80	146.43	157.74	165.44	173.89	185.70	198.56	155.40	171.75	196.94
0.10	81.18	68.22	73.11	75.18	80.94	88.07	97.44	105.90	75.16	86.21	105.90
0.15	55.93	42.07	43.66	42.24	45.89	48.11	54.37	59.85	42.63	47.19	60.28
0.20	42.89	29.56	30.42	27.34	28.78	29.14	32.49	35.04	28.00	29.35	35.62
0.25	34.91	22.69	23.03	19.64	20.34	19.82	21.23	21.84	20.32	19.96	22.73
0.30	29.39	18.18	18.61	15.17	15.30	14.15	14.76	15.11	15.60	14.75	15.50
0.35	25.42	15.15	15.58	12.14	12.17	10.86	11.24	10.83	12.71	11.47	11.51
0.40	22.42	12.94	13.37	10.12	10.11	8.83	8.89	8.28	10.65	9.30	8.75
0.50	18.16	9.98	10.43	7.57	7.57	6.25	6.17	5.57	8.01	6.69	5.87
0.60	15.36	8.09	8.58	6.01	6.04	4.83	4.73	4.12	6.44	5.26	4.39
0.70	13.36	6.87	7.34	5.04	5.07	3.98	3.86	3.30	5.38	4.36	3.52
$\beta = 3$											
d_T	d_P=0.10		d_P=0.20		d_P=0.30		d_P=0.40		λ=0.05	λ=0.1	λ=0.2
	SR	CUSUM	SR	CUSUM	SR	CUSUM	SR	CUSUM	EWMA	EWMA	EWMA
	h=269.06	h=-10.16	h=187.11	h=-4.69	h=127.36	h=-2.37	h= 100.84	h=-1.25	h=0.846	h=0.767	h=0.659
-	368.29	370.84	366.15	369.50	367.29	369.47	368.26	369.28	368.60	372.16	372.85
0.01	220.82	230.74	257.08	264.20	291.40	292.58	311.80	318.32	210.44	234.07	249.19
0.03	92.45	98.50	132.00	141.89	177.16	183.91	223.99	223.98	87.35	102.32	124.79
0.05	47.57	49.38	70.52	79.12	109.39	114.98	157.63	163.76	45.38	51.85	65.57
0.08	26.93	26.29	35.79	39.44	60.45	66.52	102.27	107.46	26.10	27.16	32.65
0.10	18.29	16.84	20.59	22.33	34.58	38.31	64.87	68.52	17.81	17.35	19.48
0.15	11.30	9.59	9.35	9.52	13.08	14.18	26.27	28.54	11.16	9.84	9.44
0.20	8.44	6.77	5.81	5.60	6.46	6.75	11.17	12.49	8.27	6.97	6.16
0.30	5.97	4.56	3.50	3.21	2.89	2.86	3.28	3.37	5.77	4.72	3.84
0.40	5.01	3.78	2.80	2.37	2.08	2.02	1.77	1.77	4.84	3.95	3.08
0.50	4.29	3.07	2.17	2.01	1.93	1.74	1.21	1.18	4.06	3.30	2.99
0.70	4.00	3.00	2.00	2.00	1.03	1.00	1.00	1.00	4.00	3.00	2.01

(Continues)

TABLE 3 (Continued)

$\beta = 5$											
d_T	d_P=0.10		d_P=0.20		d_P=0.30		d_P=0.40		λ=0.05	λ=0.1	λ=0.2
	SR	CUSUM	SR	CUSUM	SR	CUSUM	SR	CUSUM	EWMA	EWMA	EWMA
	h=215.94	h=-6.21	h=120.71	h=-2.19	h=86.95	h=-0.89	h= 5.66	h=-0.15	h=0.846	h=0.767	h=0.659
-	371.39	369.40	368.12	369.58	368.83	369.17	371.89	370.47	371.81	369.13	368.09
0.01	188.55	198.52	244.23	251.50	289.69	295.65	295.39	296.26	157.88	170.44	191.72
0.03	60.28	67.33	112.47	119.55	181.63	182.88	194.13	193.35	45.92	51.69	64.53
0.05	25.90	27.44	53.97	57.67	110.89	113.85	128.59	126.67	23.16	23.79	27.93
0.08	12.97	13.04	23.46	25.63	59.00	60.60	74.03	75.93	14.17	12.97	13.51
0.10	8.40	7.95	11.58	12.63	30.76	32.51	44.08	44.71	10.35	8.97	8.39
0.15	5.16	4.61	4.48	4.63	9.20	9.89	15.81	16.21	7.01	5.81	4.96
0.20	3.97	3.45	2.72	2.69	3.61	3.74	6.09	6.12	5.63	4.56	3.73
0.30	3.03	2.69	1.93	1.80	1.41	1.41	1.51	1.52	4.46	3.70	3.00
0.40	3.00	2.02	1.45	1.19	1.01	1.02	1.01	1.01	4.00	3.00	2.84
0.50	3.00	2.00	1.01	1.00	1.00	1.00	1.00	1.00	4.00	3.00	2.02
0.70	3.00	2.00	1.00	1.00	1.00	1.00	1.00	1.00	4.00	3.00	2.00

5 | COMPARISON WITH EXISTING CHARTS

This section compares the SR chart with two existing memory-type control charts: the CUSUM and EWMA CEV charts. To facilitate the comparison, all the control charts are calibrated such that they have ARL_0 values of approximately 370 and sample size $n = 5$. The SS-ARL values with a warm-up period equal to 50 were obtained via Monte Carlo simulation with 50,000 iterations. The control charts are obtained for the 12 combinations of shape parameters β ($\beta = 0.5, 1, 3, 5$), the censoring rates P_c ($P_c = 0.15, 0.5, 0.7$), and the pre-specified values of d_P ($d_P = 0.1, 0.2, 0.3, 0.4$). For the purpose of comparison, the SR and CUSUM charts are constructed with the same values of d_P. In each Table, for the EWMA CEV chart, three choices of smoothing parameter λ ($\lambda = 0.05, 0.1, 0.2$) are considered.

5.1 | SR Chart versus CUSUM

In most circumstances, we may not have the insight into what size shift to expect. It is often of interest to evaluate the performance of the CUSUM and SR charts designed for a certain pre-specified shift while the true shift size is of a different magnitude. Tables 3-5 display, for ($P_c = 0.15, 0.5, 0.7$), respectively, the ZS-ARL values for the SR and CUSUM charts when $n = 5$ and ($\beta = 0.5, 1, 3, 5$). Four different pre-specified designed shift size of d_P are considered, i.e., $d_P = 0.1, 0.2, 0.3, 0.4$.

The results presented in Table 3 are for $P_c = 0.15$. The CUSUM method performs slightly better than the SR method for all four pre-specified shifts in an OOC situation. For instance, when $P_c = 0.15$, $\beta = 0.5$, $d_P = 0.4$ and the true shift size is $d_T = 0.40$, the corresponding ZS-ARL values are 25.08 and 22.55 for the SR and CUSUM charts, respectively. As the scale parameter β increases, this difference is negligible. When $\beta \leq 1$ and $d_P \leq 0.2$, the CUSUM chart always performs better than the SR chart, while when $d_P > 0.2$, the SR chart outperforms the CUSUM chart. When $\beta > 1$, i.e., $\beta = 3, 5$, the SR chart does better than the CUSUM chart when $d_T < d_P$, while the CUSUM chart does better for large shifts. With the increase of β and d_P, there is no clear winner between the CUSUM and SR charts.

The results presented in Tables 4 and 5 are for $P_c = 0.5$ and 0.7, respectively. Except for small values of d_P and β ($\beta < 1, d_P \leq 0.2$), the CUSUM method performs slightly better than the SR method in detecting small to moderate shifts. Concerning the steady-state case, the SR method performs better than the CUSUM method with the same pre-specified values of d_P in most cases considered in Tables 6-8, except for the case of $d_P = 0.1$ and $\beta = 0.5$. In addition, when the actual process shift size is smaller than d_P, i.e., the shift size $d_T < d_P$, the SR chart also performs better than the CUSUM chart, and the difference is significant, especially for small shifts. For instance, in Table 6, when $\beta = 0.5$, $d_P = 0.40$ and $P_c = 0.15$, the SS-ARL values are 213.06, 133.61, 87.08, 59.68, 42.41, 31.61, 24.02, and 19.17 when the shift size $d_T = 0.05$, 0.1, 0.2, 0.25, 0.3, 0.35, and 0.4, respectively. The corresponding SS-ARL values for the CUSUM chart are 226.94, 149.2, 97.98,

TABLE 4 ZS-ARL comparison for $\eta_0 = 1$, $n = 5$, $P_c = 0.50$, $\text{ARL}_0 = 370$

$\beta = 0.5$											
d_T	d_P=0.10		d_P=0.20		d_P=0.30		d_P=0.40		λ=0.05	λ=0.1	λ=0.2
	SR	CUSUM	SR	CUSUM	SR	CUSUM	SR	CUSUM	EWMA	EWMA	EWMA
	h=355.00	h=-21.8	h=334.53	h=-17.09	h=312.50	h=-13.57	h=289.80	h=-10.83	h=0.877	h=0.809	h=0.710
-	372.14	370.49	371.70	371.13	368.18	368.34	367.96	371.71	367.49	370.52	370.76
0.05	264.57	229.88	242.24	233.52	238.38	245.29	243.21	255.71	255.57	272.62	283.71
0.10	201.61	155.70	170.17	159.96	163.15	168.73	166.54	174.84	174.11	193.72	214.69
0.15	162.65	111.21	127.63	111.04	117.27	117.18	118.47	124.38	124.92	140.86	164.20
0.20	135.21	83.98	99.84	80.90	89.05	85.60	87.85	88.24	90.71	103.61	125.81
0.25	115.48	65.55	81.76	62.66	69.32	62.53	65.86	65.54	66.20	76.65	93.53
0.30	100.36	53.18	68.28	49.42	56.14	48.17	51.00	48.51	49.68	56.64	70.58
0.35	88.75	43.31	58.13	39.75	46.40	38.31	41.19	38.05	38.57	42.86	53.37
0.40	78.68	36.62	50.27	32.75	39.22	30.78	33.93	29.68	30.46	32.59	40.02
0.50	62.97	26.53	38.63	23.14	29.08	21.08	24.00	19.74	20.21	20.11	23.12
0.60	50.99	19.81	30.47	17.02	22.26	15.05	17.89	13.78	13.90	13.10	13.66
0.70	41.40	14.96	24.13	12.69	17.25	11.00	13.50	9.78	9.92	8.95	8.55
$\beta = 1$											
d_T	d_P=0.10		d_P=0.20		d_P=0.30		d_P=0.40		λ=0.05	λ=0.1	λ=0.2
	SR	CUSUM	SR	CUSUM	SR	CUSUM	SR	CUSUM	EWMA	EWMA	EWMA
	h=336.25	h=-17.47	h=300.16	h=-11.83	h=264.06	h=-8.40	h=227.85	h=-6.03	h=0.877	h=0.809	h=0.710
-	371.93	371.89	368.30	367.88	371.73	370.48	369.24	369.93	368.05	372.74	371.95
0.05	174.01	162.74	167.28	174.41	180.93	188.88	201.43	210.83	179.65	198.48	220.91
0.10	107.03	87.80	91.69	91.05	96.67	103.54	111.28	119.44	97.00	111.17	133.15
0.15	75.35	55.05	58.65	54.88	58.27	60.59	65.56	72.15	56.99	65.34	79.59
0.20	58.27	38.39	41.11	35.99	38.18	37.87	41.38	43.67	36.95	41.22	50.73
0.25	47.11	29.04	31.28	25.38	27.20	25.68	27.63	28.35	25.63	26.99	32.70
0.30	39.27	22.87	25.07	19.44	20.54	18.46	19.29	19.51	19.29	19.31	21.93
0.35	33.44	18.52	20.62	15.15	16.10	14.01	14.61	14.04	15.06	14.34	15.43
0.40	28.94	15.40	17.28	12.37	13.23	10.93	11.43	10.68	12.12	11.23	11.47
0.50	22.35	11.14	12.90	8.66	9.40	7.42	7.65	6.71	8.43	7.35	6.86
0.60	17.81	8.39	10.03	6.45	7.09	5.36	5.54	4.65	6.23	5.25	4.62
0.70	14.53	6.58	8.01	4.98	5.55	4.06	4.22	3.46	4.80	3.97	3.36
$\beta = 3$											
d_T	d_P=0.10		d_P=0.20		d_P=0.30		d_P=0.40		λ=0.05	λ=0.1	λ=0.2
	SR	CUSUM	SR	CUSUM	SR	CUSUM	SR	CUSUM	EWMA	EWMA	EWMA
	h=278.50	h=-9.24	h=196.29	h=-4.48	h=130.22	h=-2.31	h=104.20	h=-1.25	h=0.877	h=0.809	h=0.710
-	372.78	368.36	370.75	367.45	369.80	367.95	372.09	369.53	369.05	371.40	368.55
0.01	238.31	241.23	263.32	276.25	287.63	294.72	313.08	319.99	242.83	256.75	268.80
0.03	110.61	117.02	142.06	153.27	180.97	188.02	225.67	232.88	112.85	129.36	150.02
0.05	61.59	61.81	81.15	90.88	114.70	121.06	161.89	168.86	60.98	69.16	84.39
0.08	36.19	34.56	41.71	48.11	64.84	72.42	103.22	111.82	34.55	37.76	46.67
0.10	24.64	21.95	24.07	27.71	38.20	42.84	67.47	73.01	22.72	23.41	26.99
0.15	14.89	12.00	10.65	12.04	14.92	16.95	28.13	31.19	12.98	11.88	12.39
0.20	10.59	8.09	6.26	6.85	7.16	8.06	12.06	13.64	8.81	7.71	7.19
0.30	6.73	4.78	3.32	3.49	2.99	3.17	3.51	3.74	5.35	4.45	3.81
0.40	5.18	3.51	2.34	2.39	1.98	2.07	1.78	1.80	4.07	3.27	2.67
0.50	4.36	3.02	1.94	2.02	1.69	1.66	1.19	1.19	3.35	3.00	2.08
0.70	4.00	3.00	1.83	2.00	1.01	1.00	1.00	1.00	3.00	3.00	2.00

(Continues)

TABLE 4 (Continued)

$\beta = 5$											
d_T	d_P=0.10		d_P=0.20		d_P=0.30		d_P=0.40		λ=0.05	λ=0.1	λ=0.2
	SR	CUSUM	SR	CUSUM	SR	CUSUM	SR	CUSUM	EWMA	EWMA	EWMA
	h=222.19	h=-5.83	h=125.00	h=-2.16	h=86.79	h=-0.89	h=5.75	h=-0.15	h=0.877	h=0.809	h=0.710
-	367.73	368.29	370.48	371.42	369.27	370.91	372.25	371.29	371.44	369.80	371.22
0.01	202.38	211.88	253.75	259.60	286.21	293.84	309.88	301.93	185.02	203.49	214.18
0.03	72.35	78.94	121.29	129.41	181.56	185.06	197.44	201.52	62.41	71.85	85.15
0.05	33.15	35.01	59.41	64.89	110.02	113.90	127.75	129.32	30.55	32.31	39.01
0.08	16.83	16.67	26.73	29.46	59.20	63.35	75.06	76.55	17.11	16.82	18.24
0.10	10.91	9.99	13.41	14.87	31.24	32.95	44.47	44.46	11.69	10.73	10.61
0.15	6.20	5.32	5.21	5.35	9.71	10.56	16.04	16.17	7.07	6.06	5.36
0.20	4.43	3.68	3.04	2.97	3.83	4.03	6.07	6.31	5.15	4.29	3.63
0.30	3.08	2.42	1.96	1.77	1.42	1.42	1.52	1.54	3.77	3.05	2.31
0.40	3.00	2.00	1.49	1.16	1.01	1.01	1.01	1.01	3.01	3.00	2.00
0.50	3.00	2.00	1.01	1.00	1.00	1.00	1.00	1.00	3.00	3.00	2.00
0.70	3.00	2.00	1.00	1.00	1.00	1.00	1.00	1.00	3.00	3.00	2.00

67.21, 46.90, 33.50, 25.44, and 19.92. When $\beta = 3, 5$ and $d_P = 0.40$, the SR chart and CUSUM chart performs similarly. (Table 7)

Note that a control chart with better SS-ARL performance is more meaningful because, in most cases, the process is initially IC but shifts to an OOC state at the change point. The overall conclusion is that when the value of the fixed shape parameter is small, a large shift will need to be specified to achieve more acceptable OOC ARL values for both the SR and the CUSUM charts.

5.2 | SR Chart versus EWMA CEV

Dickinson et al.[8] only compared the SS-ARL performance of the CUSUM and EWMA CEV charts, in this section, we compare the performance of the SR and EWMA CEV charts in terms of ZS-ARL and SS-ARL, respectively. For the EWMA CEV charts, three values of λ are considered, that is, $\lambda = 0.05$, 0.1, and 0.2.

TABLE 5 ZS-ARL comparison for $\eta_0 = 1$, $n = 5$, $P_c = 0.70$, $ARL_0 = 370$

$\beta = 0.5$											
d_T	d_P=0.10		d_P=0.20		d_P=0.30		d_P=0.40		λ=0.05	λ=0.1	λ=0.2
	SR	CUSUM	SR	CUSUM	SR	CUSUM	SR	CUSUM	EWMA	EWMA	EWMA
	h=355.47	h=-17.80	h=339.45	h=-14.45	h=321.88	h=-11.88	h=303.13	h=-9.63	h=0.902	h=0.847	h=0.761
-	371.34	367.39	371.90	370.02	369.23	370.75	369.47	369.85	371.06	368.34	370.71
0.05	290.70	251.07	266.97	259.41	259.01	264.57	262.43	260.97	275.62	292.55	297.58
0.10	236.06	184.10	200.04	186.31	190.02	192.02	191.73	194.64	204.90	224.19	242.82
0.15	199.88	136.43	159.26	135.39	144.59	140.28	143.73	144.37	152.86	172.81	194.52
0.20	170.83	104.02	129.54	103.46	113.89	105.21	109.52	109.89	113.83	133.07	154.50
0.25	148.94	83.41	107.87	79.60	92.10	81.38	86.21	82.46	87.48	101.40	121.27
0.30	131.38	67.34	91.73	64.29	75.83	63.25	68.78	63.42	68.24	77.99	95.16
0.35	116.55	55.48	78.88	51.85	63.70	49.88	56.09	49.69	52.39	60.81	73.41
0.40	104.58	46.54	68.80	42.91	54.35	41.17	47.08	39.84	42.08	46.32	57.25
0.50	84.21	33.81	53.39	30.21	40.56	28.14	33.70	26.4	26.68	28.07	33.47
0.60	68.30	24.73	41.92	22.05	31.05	19.79	24.78	18.45	17.99	17.67	20.19
0.70	55.01	18.14	32.81	15.90	23.67	14.27	18.58	12.84	12.29	11.42	11.89

(Continues)

TABLE 5 (Continued)

$\beta = 1$											
d_T	d_P=0.10		d_P=0.20		d_P=0.30		d_P=0.40		λ=0.05	λ=0.1	λ=0.2
	SR	CUSUM	SR	CUSUM	SR	CUSUM	SR	CUSUM	EWMA	EWMA	EWMA
	h=340.63	h=-14.75	h=306.72	h=-10.50	h=277.50	h=-7.66	h=240.98	h=-5.59	h=0.902	h=0.847	h=0.761
-	369.91	368.82	368.83	370.03	368.45	370.53	368.91	370.31	371.08	371.39	371.41
0.05	206.00	188.46	190.76	199.69	200.95	211.58	214.56	220.39	209.65	222.99	245.68
0.10	136.39	107.42	115.10	116.56	118.86	124.80	129.47	136.04	122.51	137.07	163.01
0.15	101.19	71.00	77.51	70.73	74.65	77.40	80.40	86.61	76.80	88.52	105.65
0.20	79.19	50.60	56.08	47.96	50.80	49.73	52.93	55.94	50.62	56.41	69.35
0.25	64.47	37.76	43.45	34.42	37.36	34.91	36.77	37.34	35.05	37.91	46.08
0.30	54.00	29.64	34.73	26.24	28.46	25.17	26.44	25.92	25.48	26.85	32.15
0.35	45.97	23.78	28.58	20.53	22.51	18.86	20.19	19.25	19.59	19.54	22.47
0.40	39.51	19.49	24.10	16.61	18.34	14.80	15.83	14.38	15.45	14.75	16.03
0.50	29.95	13.67	17.53	11.24	12.89	9.71	10.48	8.87	10.15	9.23	9.03
0.60	23.10	9.80	13.11	7.92	9.36	6.75	7.34	5.90	7.06	6.16	5.61
0.70	17.87	7.17	9.90	5.73	6.92	4.80	5.28	4.11	5.01	4.28	3.69

$\beta = 3$											
d_T	d_P=0.10		d_P=0.20		d_P=0.30		d_P=0.40		λ=0.05	λ=0.1	λ=0.2
	SR	CUSUM	SR	CUSUM	SR	CUSUM	SR	CUSUM	EWMA	EWMA	EWMA
	h=288.21	h=-8.36	h=207.50	h=-4.20	h=146.57	h=-2.29	h= 98.43	h=-1.20	h=0.902	h=0.847	h=0.761
-	369.97	371.09	367.17	370.42	372.45	370.97	371.47	369.57	368.30	370.02	368.10
0.01	253.62	261.10	279.17	283.00	294.86	304.83	312.35	312.75	265.02	280.00	288.29
0.03	132.59	139.01	158.43	170.92	191.21	204.92	224.04	232.17	137.50	154.83	177.10
0.05	79.82	80.82	96.45	104.15	125.10	135.53	162.54	167.88	80.75	93.32	111.39
0.08	48.83	46.20	54.45	58.89	76.48	83.19	106.85	109.68	46.76	52.57	64.56
0.10	34.42	30.28	33.70	36.00	46.22	51.45	70.70	75.17	30.52	32.25	39.57
0.15	20.70	16.33	16.36	15.98	19.52	21.59	30.55	33.23	16.47	16.07	17.58
0.20	14.43	10.60	9.92	9.04	10.02	10.53	14.47	15.59	10.68	9.64	9.63
0.30	8.59	5.76	5.08	4.32	4.14	3.97	4.34	4.46	5.74	4.93	4.38
0.40	5.98	3.82	3.31	2.74	2.48	2.27	2.08	2.07	3.82	3.23	2.72
0.50	4.73	3.05	2.47	2.08	2.00	1.65	1.31	1.28	3.06	2.42	2.08
0.70	4.00	2.95	2.00	2.00	1.18	1.00	1.00	1.00	3.00	2.00	2.00

$\beta = 5$											
d_T	d_P=0.10		d_P=0.20		d_P=0.30		d_P=0.40		λ=0.05	λ=0.1	λ=0.2
	SR	CUSUM	SR	CUSUM	SR	CUSUM	SR	CUSUM	EWMA	EWMA	EWMA
	h=238.37	h=-5.43	h=140.72	h=-2.14	h=71.25	h=-0.84	h=8.59	h=-0.18	h=0.902	h=0.847	h=0.761
-	371.84	368.25	367.30	369.03	369.45	370.83	369.97	369.49	370.60	370.52	371.04
0.01	218.61	227.86	259.31	263.30	287.16	288.86	304.46	311.11	209.74	224.39	245.60
0.03	88.30	94.06	129.89	141.82	177.92	182.18	203.21	208.39	81.45	94.21	114.42
0.05	43.49	45.41	67.70	75.68	110.03	114.58	136.18	137.78	41.50	45.87	57.52
0.08	23.39	22.49	32.61	35.47	60.02	63.71	83.23	85.36	22.69	23.27	27.37
0.10	14.98	13.58	17.30	18.55	32.94	35.31	50.19	51.39	14.68	13.92	15.41
0.15	8.33	6.96	6.87	7.02	10.79	11.59	18.64	19.07	8.27	7.24	6.87
0.20	5.63	4.48	3.81	3.66	4.45	4.66	7.31	7.36	5.47	4.69	4.16
0.30	3.44	2.59	2.10	1.83	1.59	1.58	1.71	1.75	3.31	2.79	2.32
0.40	3.00	2.01	1.63	1.14	1.04	1.03	1.02	1.02	3.00	2.06	2.00
0.50	3.00	2.00	1.03	1.00	1.00	1.00	1.00	1.00	3.00	2.00	2.00
0.70	3.00	2.00	1.00	1.00	1.00	1.00	1.00	1.00	3.00	2.00	2.00

TABLE 6 SS-ARL comparison for $\eta_0 = 1$, $n = 5$, $P_c = 0.15$, $\text{ARL}_0 = 370$

$\beta = 0.5$											
d_T	**d_P=0.10**		**d_P=0.20**		**d_P=0.30**		**d_P=0.40**		**λ=0.05**	**λ=0.1**	**λ=0.2**
	SR	**CUSUM**	**SR**	**CUSUM**	**SR**	**CUSUM**	**SR**	**CUSUM**	**EWMA**	**EWMA**	**EWMA**
	h=351.41	**h=-26.50**	**h=330.47**	**h=-19.88**	**h=306.25**	**h=-15.39**	**h=283.89**	**h=-12.02**	**h=0.846**	**h=0.767**	**h=0.659**
-	370.99	371.52	370.48	368.19	368.94	369.73	367.88	370.71	370.29	368.81	368.82
0.05	188.52	183.94	179.27	192.74	197.25	211.74	213.06	226.94	216.43	239.05	259.26
0.10	125.69	109.17	109.47	117.47	118.63	130.01	133.61	149.20	137.68	159.35	188.54
0.15	92.66	73.95	73.91	78.48	77.05	87.16	87.08	97.98	91.54	106.36	133.98
0.20	71.98	54.28	53.46	54.04	54.28	58.53	59.68	67.21	61.26	73.95	96.48
0.25	58.22	40.80	40.83	39.85	39.61	42.44	42.41	46.90	43.79	51.67	68.15
0.30	48.62	32.35	32.79	31.86	30.63	31.53	31.61	33.50	31.90	37.53	49.65
0.35	41.10	26.66	26.76	24.65	24.15	24.32	24.02	25.44	24.58	27.39	35.52
0.40	35.50	22.35	22.55	20.16	19.94	19.54	19.17	19.32	19.36	20.90	26.44
0.50	27.25	16.46	16.76	14.44	14.01	13.52	13.20	13.11	13.01	12.90	14.73
0.60	21.63	12.85	12.82	11.00	10.65	9.87	9.54	9.26	9.39	8.81	9.15
0.70	17.43	10.17	10.20	8.53	8.23	7.58	7.26	6.81	7.00	6.32	6.05
$\beta = 1$											
d_T	**d_P=0.10**		**d_P=0.20**		**d_P=0.30**		**d_P=0.40**		**λ=0.05**	**λ=0.1**	**λ=0.2**
	SR	**CUSUM**	**SR**	**CUSUM**	**SR**	**CUSUM**	**SR**	**CUSUM**	**EWMA**	**EWMA**	**EWMA**
	h=332.81	**h=-20.50**	**h=294.77**	**h=-13.29**	**h=258.91**	**h=-9.15**	**h=221.72**	**h=-6.43**	**h=0.846**	**h=0.767**	**h=0.659**
-	371.17	372.24	369.63	372.87	367.98	368.20	370.85	367.59	371.88	372.27	367.29
0.05	110.82	122.15	131.39	145.58	156.44	168.63	181.31	195.10	142.32	163.52	193.25
0.10	57.37	58.78	60.77	68.88	74.99	82.40	94.72	102.18	65.83	82.31	100.27
0.15	36.64	34.71	34.80	37.45	40.33	44.37	51.00	57.68	36.56	43.53	56.25
0.20	26.56	23.73	22.88	23.25	24.72	27.06	29.94	33.90	23.26	26.07	33.35
0.25	20.56	17.88	16.62	16.64	16.64	17.63	19.03	20.93	16.72	17.36	21.05
0.30	16.78	14.25	13.11	12.72	12.28	12.44	13.06	13.99	12.66	12.45	14.04
0.35	14.01	11.80	10.56	10.12	9.62	9.60	9.59	10.05	10.06	9.71	10.12
0.40	12.06	10.03	8.91	8.52	7.81	7.68	7.48	7.63	8.32	7.75	7.78
0.50	9.45	7.69	6.75	6.29	5.65	5.48	5.09	5.05	6.24	5.44	5.01
0.60	7.72	6.24	5.41	5.01	4.43	4.22	3.84	3.72	4.94	4.24	3.70
0.70	6.59	5.32	4.56	4.18	3.68	3.46	3.11	2.98	4.16	3.52	2.94
$\beta = 3$											
d_T	**d_P=0.10**		**d_P=0.20**		**d_P=0.30**		**d_P=0.40**		**λ=0.05**	**λ=0.1**	**λ=0.2**
	SR	**CUSUM**	**SR**	**CUSUM**	**SR**	**CUSUM**	**SR**	**CUSUM**	**EWMA**	**EWMA**	**EWMA**
	h=269.06	**h=-10.16**	**h=187.11**	**h=-4.69**	**h=127.36**	**h=-2.37**	**h= 100.84**	**h=-1.25**	**h=0.846**	**h=0.767**	**h=0.659**
-	367.89	368.51	372.73	369.40	370.96	368.70	370.81	368.14	369.88	371.41	372.80
0.01	210.06	219.15	260.44	264.49	295.72	287.36	311.44	319.96	201.54	229.99	253.88
0.03	83.95	93.44	131.25	138.98	175.57	182.02	220.63	227.42	78.05	96.31	122.81
0.05	41.44	46.45	69.77	77.35	108.00	112.40	157.80	166.65	39.37	47.67	62.52
0.08	22.35	23.77	34.77	38.51	61.08	64.44	102.11	108.83	22.15	24.65	31.05
0.10	14.54	14.85	19.50	21.09	34.08	36.43	64.60	69.46	14.59	15.18	17.57
0.15	8.45	8.27	8.49	8.98	12.76	14.09	26.09	28.76	8.83	8.18	8.26
0.20	6.12	5.82	5.15	5.21	6.15	6.48	11.19	12.26	6.39	5.70	5.26
0.30	4.19	3.92	3.04	2.96	2.72	2.73	3.23	3.35	4.47	3.80	3.24
0.40	3.45	3.21	2.35	2.22	1.95	1.93	1.73	1.74	3.70	3.12	2.56
0.50	3.09	2.76	1.97	1.92	1.74	1.64	1.19	1.18	3.27	2.70	2.32
0.70	2.79	2.61	1.87	1.85	1.02	1.00	1.00	1.00	3.03	2.49	1.88

(Continues)

TABLE 6 (Continued)

$\beta = 5$											
d_T	d_P=0.10		d_P=0.20		d_P=0.30		d_P=0.40		λ=0.05	λ=0.1	λ=0.2
	SR	CUSUM	SR	CUSUM	SR	CUSUM	SR	CUSUM	EWMA	EWMA	EWMA
	h=215.94	h=-6.21	h=120.71	h=-2.19	h=86.95	h=-0.89	h= 5.66	h=-0.15	h=0.846	h=0.767	h=0.659
-	368.58	369.10	368.70	369.74	370.33	368.49	372.33	367.74	372.28	368.64	370.31
0.01	186.81	196.35	245.18	250.33	287.17	293.36	296.19	297.82	145.45	165.60	195.68
0.03	57.79	65.16	113.20	119.83	177.45	182.12	194.15	194.90	40.16	47.57	62.41
0.05	23.92	26.33	53.79	58.26	109.45	111.74	126.37	127.53	19.23	20.92	26.38
0.08	11.42	11.90	22.86	25.20	57.90	60.44	75.21	75.71	11.31	11.02	12.00
0.10	7.21	7.29	11.14	12.00	30.40	32.54	44.36	44.08	8.08	7.32	7.37
0.15	4.29	4.17	4.27	4.43	9.08	9.79	15.97	15.98	5.41	4.73	4.17
0.20	3.25	3.11	2.58	2.61	3.59	3.74	6.10	6.20	4.33	3.68	3.12
0.30	2.53	2.39	1.78	1.72	1.40	1.40	1.52	1.53	3.46	2.90	2.43
0.40	2.26	1.94	1.32	1.17	1.01	1.01	1.01	1.01	3.14	2.58	2.17
0.50	2.07	1.92	1.00	1.00	1.00	1.00	1.00	1.00	3.03	2.49	1.89
0.70	1.94	1.91	1.00	1.00	1.00	1.00	1.00	1.00	2.96	2.43	1.87

Concerning the ZS-ARL, from Tables 3–5, we choose $d_P = 0.3$ and $\lambda = 0.1$ for illustration. It can be seen that when $\beta \leq 1$, the performance of SR chart is superior to EWMA CEV chart for small shifts. When $\beta > 1$, the EWMA CEV chart does better for small shifts, while the SR chart outperforms the EWMA CEV chart for $d_T > d_P$. For instance, when $P_c = 0.15$, $\beta = 0.5$, $d_P = 0.30$ and the true shift size $d_T = 0.10$, the ARL_1 values of the SR and EWMA CEV charts are 138.53 and 168.31, respectively. However, when $\beta = 3$, the corresponding ARL_1 values of the SR and EWMA CEV charts are 34.58 and 17.35, respectively.

Concerning the SS-ARL, from Tables 6-8, we can get the same conclusion as the zero-state that when $\beta \leq 1$ the performance of SR is superior to the EWMA CEV for small shifts; however, when $\beta > 1$ EWMA CEV is better. For instance, in Table 6, when $P_c = 0.15$, $\beta = 0.5$, $d_P = 0.2$, and $\lambda = 0.05$, the ARL values are 179.27, 109.47, 73.91, 53.46, and 40.83, with the true shift size $d_T = 0.05, 0.1, 0.15, 0.2$, and 0.25, respectively. The corresponding ARL values for the EMWA CEV chart are 216.43, 137.68, 91.54, 61.26 and 43.79. The SR chart performs slightly better than the EWMA CEV chart for $\beta > 1$ and $d_T > d_P$. For instance, from Table 6 for $P_c = 0.15$, $\beta = 3$, $d_P = 0.20$ and the shift size $d_T > d_P$, the ARL values are 5.17, 3.04, 2.35, 1.97 and 1.87, which are smaller than the those of the EWMA CEV.

TABLE 7 SS-ARL comparison for $\eta_0 = 1$, $n = 5$, $P_c = 0.50$, $\mathrm{ARL}_0 = 370$

$\beta = 0.5$											
d_T	d_P=0.10		d_P=0.20		d_P=0.30		d_P=0.40		λ=0.05	λ=0.1	λ=0.2
	SR	CUSUM	SR	CUSUM	SR	CUSUM	SR	CUSUM	EWMA	EWMA	EWMA
	h=355.00	h=-21.8	h=334.53	h=-17.09	h=312.50	h=-13.57	h=289.80	h=-10.83	h=0.877	h=0.809	h=0.710
-	372.47	370.36	369.26	371.20	370.77	369.49	369.65	367.54	370.30	368.51	372.98
0.05	216.57	207.45	200.63	214.14	207.18	231.15	225.98	240.30	236.70	257.73	282.70
0.10	158.75	135.53	134.31	142.38	138.31	152.55	149.32	162.92	167.18	183.33	213.34
0.15	123.01	94.56	94.61	98.62	94.99	104.26	102.55	116.31	118.73	135.65	159.26
0.20	98.28	71.03	71.37	70.60	67.74	75.33	73.43	80.68	82.33	98.73	123.34
0.25	80.72	53.01	55.94	52.18	51.13	54.67	53.97	58.88	59.35	71.45	92.03
0.30	67.76	41.80	45.21	41.61	39.84	41.25	41.02	43.49	43.97	52.83	68.17
0.35	57.94	34.31	37.06	32.23	32.03	31.94	31.81	33.57	33.57	39.55	51.48
0.40	49.86	28.50	30.99	26.42	25.99	25.72	25.37	25.06	26.33	29.92	38.24
0.50	37.73	20.38	22.50	18.51	18.28	17.29	17.04	16.85	16.78	17.67	21.49
0.60	29.19	15.09	16.78	13.41	13.34	12.25	12.01	11.63	11.38	11.33	12.52
0.70	22.49	11.24	12.65	9.95	9.84	8.90	8.70	8.25	8.00	7.43	7.64

(Continues)

TABLE 7 (Continued)

$\beta = 1$											
d_T	d_P=0.10		d_P=0.20		d_P=0.30		d_P=0.40		λ=0.05	λ=0.1	λ=0.2
	SR	CUSUM	SR	CUSUM	SR	CUSUM	SR	CUSUM	EWMA	EWMA	EWMA
	h=336.25	h=-17.47	h=300.16	h=-11.83	h=264.06	h=-8.40	h=227.85	h=-6.03	h=0.877	h=0.809	h=0.710
-	368.12	367.40	371.72	368.49	370.75	372.17	372.06	370.17	372.32	371.59	369.28
0.05	136.15	145.82	148.38	161.93	170.75	188.94	191.62	209.84	171.82	194.37	215.31
0.10	76.92	74.33	75.80	83.69	88.25	98.65	106.51	116.11	88.92	105.17	128.06
0.15	50.74	45.04	45.31	48.61	50.47	56.43	60.80	68.63	51.23	60.62	77.91
0.20	36.58	31.28	30.32	31.15	32.02	34.95	36.98	41.93	32.81	37.28	47.73
0.25	28.20	22.93	21.85	21.87	21.79	22.97	24.24	27.06	22.29	24.50	30.81
0.30	22.54	17.85	16.82	16.41	15.90	16.26	16.76	17.72	16.16	16.75	20.16
0.35	18.68	14.37	13.45	12.76	12.30	12.37	12.48	12.83	12.52	12.48	14.05
0.40	15.52	11.91	11.00	10.27	9.85	9.73	9.46	9.67	9.89	9.62	10.13
0.50	11.41	8.55	7.82	7.15	6.69	6.39	6.19	6.03	6.75	6.14	6.05
0.60	8.70	6.46	5.86	5.32	4.85	4.62	4.31	4.21	4.89	4.34	3.95
0.70	6.88	5.06	4.55	4.13	3.77	3.52	3.27	3.10	3.77	3.29	2.84

$\beta = 3$											
d_T	d_P=0.10		d_P=0.20		d_P=0.30		d_P=0.40		λ=0.05	λ=0.1	λ=0.2
	SR	CUSUM	SR	CUSUM	SR	CUSUM	SR	CUSUM	EWMA	EWMA	EWMA
	h=278.50	h=-9.24	h=196.29	h=-4.48	h=130.22	h=-2.31	h=104.20	h=-1.25	h=0.877	h=0.809	h=0.710
-	371.60	368.52	368.96	371.95	371.00	369.54	372.57	372.65	372.28	368.21	369.60
0.01	218.08	239.64	269.22	272.30	289.17	294.18	316.97	318.90	226.61	246.04	268.41
0.03	98.46	109.62	144.59	151.14	183.72	189.02	225.99	233.19	105.45	121.57	151.66
0.05	52.75	58.40	79.37	87.73	114.39	122.34	161.09	167.30	54.82	66.66	85.19
0.08	28.92	31.09	43.65	47.28	66.27	70.19	104.83	111.53	30.27	33.91	44.88
0.10	19.06	19.59	24.03	27.12	38.08	41.99	67.38	72.25	19.06	20.49	25.70
0.15	10.67	10.44	10.61	11.38	14.89	16.52	27.95	31.17	10.49	10.19	11.20
0.20	7.21	6.89	6.24	6.38	7.24	7.71	12.18	13.15	6.97	6.39	6.39
0.30	4.40	4.09	3.31	3.23	3.00	3.02	3.50	3.67	4.21	3.65	3.25
0.40	3.31	3.04	2.34	2.23	1.98	1.96	1.77	1.79	3.16	2.70	2.29
0.50	2.86	2.64	1.94	1.91	1.69	1.56	1.18	1.17	2.70	2.38	1.90
0.70	2.55	2.46	2.00	1.83	1.05	1.00	1.00	1.00	2.49	2.15	1.81

$\beta = 5$											
d_T	d_P=0.10		d_P=0.20		d_P=0.30		d_P=0.40		λ=0.05	λ=0.1	λ=0.2
	SR	CUSUM	SR	CUSUM	SR	CUSUM	SR	CUSUM	EWMA	EWMA	EWMA
	h=222.19	h=-5.83	h=125.00	h=-2.16	h=86.79	h=-0.89	h=5.75	h=-0.15	h=0.877	h=0.809	h=0.710
-	369.62	370.35	370.45	369.44	370.93	368.36	368.28	370.28	367.51	369.56	371.61
0.01	196.24	206.42	250.10	255.84	288.12	290.08	301.95	292.14	173.08	196.57	225.52
0.03	68.07	78.28	118.62	125.84	182.39	183.61	193.83	195.79	55.91	65.73	84.32
0.05	29.91	33.19	59.21	64.06	113.37	114.12	127.41	127.62	26.14	29.17	38.22
0.08	14.60	15.54	26.19	29.35	59.29	60.34	74.30	73.59	14.20	14.51	16.98
0.10	9.07	9.25	13.03	14.49	31.65	33.42	43.32	44.61	9.35	9.08	9.68
0.15	4.92	4.81	4.92	5.19	9.82	10.44	16.05	16.84	5.58	4.98	4.70
0.20	3.43	3.32	2.82	2.85	3.75	3.99	5.99	6.29	4.07	3.52	3.11
0.30	2.42	2.20	1.76	1.69	1.40	1.42	1.51	1.53	2.92	2.50	2.04
0.40	2.09	1.91	1.29	1.13	1.01	1.01	1.01	1.01	2.57	2.26	1.84
0.50	1.93	1.89	1.00	1.00	1.00	1.00	1.00	1.00	2.49	2.16	1.81
0.70	1.87	1.87	1.00	1.00	1.00	1.00	1.00	1.00	2.46	2.11	1.79

TABLE 8 SS-ARL comparison for $\eta_0 = 1$, $n = 5$, $P_c = 0.70$, $\mathrm{ARL}_0 = 370$

$\beta = 0.5$											
d_T	**d_P=0.10**		**d_P=0.20**		**d_P=0.30**		**d_P=0.40**		**λ=0.05**	**λ=0.1**	**λ=0.2**
	SR	**CUSUM**	**SR**	**CUSUM**	**SR**	**CUSUM**	**SR**	**CUSUM**	**EWMA**	**EWMA**	**EWMA**
	h=355.47	**h=-17.80**	**h=339.45**	**h=-14.45**	**h=321.88**	**h=-11.88**	**h=303.13**	**h=-11.88**	**h=0.902**	**h=0.847**	**h=0.761**
-	369.05	368.41	368.23	367.78	370.67	369.47	367.50	371.40	371.67	372.90	368.22
0.05	240.06	223.68	222.09	233.96	223.92	241.75	234.45	261.34	267.61	281.37	296.72
0.10	191.04	156.39	162.51	162.50	158.26	176.39	166.69	187.89	196.13	217.57	235.78
0.15	155.73	115.28	121.96	118.88	115.59	123.90	120.61	137.53	143.85	169.88	188.88
0.20	129.57	87.62	95.93	86.83	87.11	93.27	89.24	101.39	109.69	129.56	146.72
0.25	109.53	68.08	76.29	67.93	68.26	71.80	68.34	76.31	80.84	99.55	118.60
0.30	94.36	56.04	62.68	55.40	53.49	54.55	52.48	57.37	61.66	74.12	91.26
0.35	81.44	44.38	52.06	42.31	43.88	43.21	41.67	44.46	47.56	57.14	71.10
0.40	71.24	36.68	44.08	34.45	35.99	34.43	33.69	34.39	36.83	43.55	56.36
0.50	54.14	26.00	32.33	24.24	25.22	22.88	22.83	23.08	23.02	25.80	32.13
0.60	41.88	19.00	23.77	17.19	18.23	16.17	16.10	15.48	15.11	15.71	18.31
0.70	31.90	13.71	17.67	12.42	13.26	11.47	11.43	10.73	10.07	9.89	10.84
$\beta = 1$											
d_T	**d_P=0.10**		**d_P=0.20**		**d_P=0.30**		**d_P=0.40**		**λ=0.05**	**λ=0.1**	**λ=0.2**
	SR	**CUSUM**	**SR**	**CUSUM**	**SR**	**CUSUM**	**SR**	**CUSUM**	**EWMA**	**EWMA**	**EWMA**
	h=340.63	**h=-14.75**	**h=306.72**	**h=-10.50**	**h=277.50**	**h=-7.66**	**h=240.98**	**h=-5.59**	**h=0.902**	**h=0.847**	**h=0.761**
-	369.39	370.92	370.92	372.00	372.79	369.88	371.21	371.31	369.88	369.84	370.31
0.05	164.00	166.86	166.68	186.69	183.51	198.02	205.78	220.67	197.33	218.23	240.98
0.10	101.42	94.63	94.67	102.53	104.37	118.65	119.17	134.65	111.87	131.54	156.29
0.15	70.30	59.67	58.99	63.21	63.83	70.30	74.02	81.46	69.43	81.65	105.16
0.20	52.23	40.98	40.38	40.59	41.81	45.63	47.33	52.92	45.06	53.03	68.45
0.25	40.37	30.46	29.78	29.53	29.36	30.32	31.92	35.34	30.07	35.34	44.12
0.30	32.32	23.54	22.81	22.78	21.48	22.41	22.32	24.20	22.02	24.06	30.39
0.35	26.56	18.68	18.02	16.93	16.55	16.79	16.49	17.49	16.62	17.41	21.30
0.40	22.11	15.22	14.65	13.51	12.96	12.89	12.67	12.77	12.78	12.88	15.00
0.50	15.76	10.68	10.09	9.23	8.67	8.33	8.06	7.93	8.24	7.88	8.22
0.60	11.47	7.60	7.28	6.42	6.08	5.68	5.43	5.26	5.63	5.15	4.95
0.70	8.49	5.54	5.27	4.63	4.37	4.08	3.83	3.67	3.98	3.55	3.20
$\beta = 3$											
d_T	**d_P=0.10**		**d_P=0.20**		**d_P=0.30**		**d_P=0.40**		**λ=0.05**	**λ=0.1**	**λ=0.2**
	SR	**CUSUM**	**SR**	**CUSUM**	**SR**	**CUSUM**	**SR**	**CUSUM**	**EWMA**	**EWMA**	**EWMA**
	h=288.21	**h=-8.36**	**h=207.50**	**h=-4.20**	**h=146.57**	**h=-2.29**	**h= 98.43**	**h=-1.20**	**h=0.902**	**h=0.847**	**h=0.761**
-	368.39	367.98	372.06	372.00	369.22	370.58	369.88	368.77	371.76	369.53	370.93
0.01	235.59	246.63	273.44	287.24	289.35	304.37	295.42	310.82	251.39	269.04	281.02
0.03	115.07	127.99	154.71	164.98	189.74	197.31	227.47	231.35	133.38	147.06	178.36
0.05	65.80	73.38	91.83	102.81	124.61	135.15	161.95	163.36	74.28	88.46	110.30
0.08	38.35	40.68	51.15	56.30	72.25	81.86	106.37	111.85	41.88	49.22	63.16
0.10	25.45	26.42	30.45	34.15	45.30	51.13	69.97	74.19	26.58	29.94	37.94
0.15	14.18	14.05	14.23	14.94	18.85	20.83	30.82	32.41	13.79	14.20	16.42
0.20	9.36	8.92	8.24	8.43	9.30	10.09	14.09	15.35	8.72	8.43	8.76
0.30	5.22	4.83	4.05	4.00	3.74	3.78	4.22	4.39	4.62	4.12	3.86
0.40	3.51	3.24	2.60	2.49	2.20	2.17	2.01	2.01	3.06	2.67	2.36
0.50	2.74	2.56	1.98	1.93	1.67	1.53	1.27	1.26	2.41	2.04	1.82
0.70	2.36	2.20	1.78	1.76	1.02	1.00	1.00	1.00	2.09	1.80	1.64

(Continues)

TABLE 8 (Continued)

$\beta = 5$											
d_T	d_P=0.10		d_P=0.20		d_P=0.30		d_P=0.40		λ=0.05	λ=0.1	λ=0.2
	SR	CUSUM	SR	CUSUM	SR	CUSUM	SR	CUSUM	EWMA	EWMA	EWMA
	h=238.37	h=-5.43	h=140.72	h=-2.14	h=71.25	h=-0.84	h=8.59	h=-0.18	h=0.902	h=0.847	h=0.761
-	368.67	371.79	368.42	370.47	370.80	372.98	371.47	370.50	371.50	371.75	372.65
0.01	207.16	224.94	256.76	270.44	292.65	297.74	305.27	306.05	199.54	221.62	246.80
0.03	81.54	91.71	128.88	142.37	180.86	185.80	202.31	203.93	75.84	89.22	113.25
0.05	38.88	43.44	67.90	73.43	111.49	115.52	137.75	138.85	36.74	42.12	55.25
0.08	19.37	21.01	31.64	35.30	59.91	63.72	82.61	82.88	19.15	20.52	26.01
0.10	12.10	12.43	16.74	18.15	32.57	35.98	51.00	51.66	12.22	12.34	14.13
0.15	6.39	6.20	6.35	6.75	10.63	11.67	18.64	19.42	6.62	6.10	6.10
0.20	4.13	3.97	3.49	3.51	4.37	4.61	7.22	7.48	4.36	3.91	3.60
0.30	2.49	2.31	1.84	1.76	1.54	1.57	1.72	1.73	2.70	2.31	2.02
0.40	1.96	1.87	1.28	1.12	1.03	1.03	1.02	1.02	2.23	1.85	1.70
0.50	1.85	1.85	1.00	1.00	1.00	1.00	1.00	1.00	2.09	1.80	1.65
0.70	1.83	1.83	1.00	1.00	1.00	1.00	1.00	1.00	2.05	1.77	1.60

6 | EXAMPLE

To illustrate the application of the SR chart, we re-examine the rust test example provided by Steiner and Mackay[6]where they monitored the rust-resistant capabilities of the painting process in the manufacture of outdoor electrical boxes. This example were also used by Zhang and Chen[7] to illustrate the EWMA CEV chart and Dickinson et al.[8] to illustrate the CUSUM chart. In the example, a life test consisted of placing $n = 3$ cut panels from a test box, scratched in a prescribed manner, into a salt spray chamber maintained at 30°C. The test units in the chamber were checked daily for rust, and remained in the chamber until rust appeared or until 20 days passed. The fixed time was set to limit the cost of testing and to allow for quicker feedback. For panels that showed rust before or on the 20th day, an exact failure time (in days) is known, but exact failure times are unknown for the cut panels that did not show rust on the 20th day; these observations are censored at 20 days.

A good lifetime monitoring process should be able to quickly detect deteriorations in the rust-resistant capabilities of the paint, even with some censored observations. To accomplish this the first step is to estimate the IC process performance to provide a benchmark for comparison. In this initial step, called the implementation phase, the process parameters are estimated from a fairly large initial sample of the process output, a control chart is built. With an initial censored Weibull sample, we may derive maximum likelihood estimates (MLEs) for the underlying Weibull parameters using the likelihood function given by Lawless.[31] In this example, the rust-resistant data under current investigation consist of 100 subgroups of $n = 3$. Of the 300 data points, 230 are recorded as censored. According to Lawless,[31] Zhang and Chen[7] and Dickinson et al.,[8] the IC observations are assumed to follow the Weibull distribution, and the IC parameters are estimated to be $\eta_0 = 48.04$, $\beta = 1.51$ and $P_c = 0.767$.

To compare the SR method with the other two methods, we consider the threshold determined via simulation to detect a 33% decrease in the scale parameter η. Zhang and Chen[7] designed their EWMA CEV chart using $\lambda = 0.1$. For an $ARL_0 = 370$, the control limits of the EWMA CEV, CUSUM and SR charts are 0.819, -3.930 and 244, respectively. To study the OOC performance, Zhang and Chen[7] simulated additional points from a Weibull distribution with $\eta = 32$, an approximately 33% decrease in the scale parameter. This simulated group has 50 OOC points. Table 9 shows the simulated additional points from this OOC Weibull distribution and the monitoring statistics of three control charts. The plots of the EWMA CEV, CUSUM and SR charts are presented in Figure 4. The EWMA CEV chart signals at the 24th sample, the CUSUM chart signals at the 23th sample, as noted in the previous papers. The SR chart signals at the 23th simulated sample, which is the same as the CUSUM chart. Clearly, the SR chart is an effective alternative method to detect the decrease in the scale parameter of a Weibull distribution with a high censoring rate.

TABLE 9 Subgroup data from the rust test with monitoring statistics

Subgroup	Observation Data			CUSUM	EWMA	SR
1	25.4997	57.9954	39.2197	0.0000	1.0000	2.2599
2	23.3581	70.2749	20.7643	0.0000	1.0000	1.5720
3	38.1081	25.6001	28.9188	0.0000	1.0000	1.2403
4	47.9104	39.0709	28.1769	0.0000	1.0000	1.0804
5	79.8087	23.3544	25.8403	0.0000	1.0000	1.0032
6	13.2431	21.6178	71.0747	−0.0364	0.9891	2.0711
7	64.2551	67.4670	47.2051	0.0000	1.0000	1.4810
8	34.6664	4.6946	42.0510	−0.1494	0.9853	2.8450
9	56.2218	18.3263	4.1000	−1.0460	0.9376	8.7449
10	47.1059	17.2177	23.2392	−1.0128	0.9352	9.4534
11	60.6636	56.1943	14.1106	−1.0348	0.9313	10.6661
12	11.7594	6.8154	57.0731	−2.0173	0.8861	28.7053
13	49.6321	32.3780	19.6625	−1.9370	0.8905	27.5980
14	30.6804	10.2627	21.8167	−2.0190	0.8890	30.8303
15	12.5557	30.9334	14.6037	−2.8758	0.8522	69.8020
16	40.3100	2.6253	25.0658	−3.0427	0.8518	82.4984
17	13.1955	21.6034	20.8153	−3.0798	0.8557	86.3884
18	45.7540	32.7602	8.8958	−3.1807	0.8571	95.8534
19	44.7734	20.4689	29.0185	−2.3849	0.8979	46.7059
20	47.2498	17.1957	8.4363	−3.2549	0.8597	105.8868
21	10.7058	23.7613	28.7098	−3.3305	0.8616	114.5551
22	26.6099	31.7590	9.4222	−3.4242	0.8626	125.9258
23	29.6613	10.0579	13.5682	−4.3360	0.8266	292.7127
24	25.6839	14.8015	13.2469	−5.1783	0.7965	635.5881
25	35.2412	30.8197	47.1052	−4.3825	0.8434	306.9839
26	34.2396	26.6027	6.3945	−4.5141	0.8450	347.4753
27	40.4076	74.5548	34.5544	−3.7183	0.8870	168.0464
28	3.4324	60.1728	30.8787	−3.8790	0.8833	195.8580
29	36.4370	34.9054	9.2534	−3.9750	0.8821	214.9783
30	4.1869	9.9357	27.3789	−5.0116	0.8400	558.4516
31	26.0625	30.4383	18.4261	−4.9556	0.8482	531.4424
32	7.3139	37.9039	1.2431	−6.0476	0.8076	1448.4144
33	24.4593	1.8968	73.7558	−6.2192	0.8115	1696.3214
34	16.5039	11.9385	30.1998	−7.0521	0.7832	3641.3708
35	25.9842	63.5993	1.2358	−7.2273	0.7894	4277.0064
36	9.4711	53.4755	25.9835	−7.3204	0.7976	4659.0798
37	62.6317	42.7677	30.5907	−6.5246	0.8444	2247.2449
38	30.8937	25.5906	10.6351	−6.6013	0.8477	2411.8200
39	31.6307	1.0165	17.1628	−7.5412	0.8123	5710.1164
40	45.5791	49.8509	15.6476	−7.5367	0.8215	5687.1668
41	50.5929	36.2086	35.6295	−6.7408	0.8659	2743.0225
42	25.3759	20.5546	36.9077	−5.9450	0.9058	1323.2585
43	31.1133	9.7119	9.2147	−6.9272	0.8632	3257.6168
44	14.1045	50.7164	8.9467	−7.8453	0.8269	7558.7687
45	25.6885	13.3298	66.3770	−7.8803	0.8334	7805.7362
46	41.9506	14.8362	14.7175	−8.6974	0.8035	16507.5202
47	14.5249	29.9232	30.6256	−8.7123	0.8129	16736.4341
48	27.3264	10.6609	27.9668	−8.7886	0.8194	17949.0026
49	25.1678	9.3687	31.8351	−8.8831	0.8247	19574.0632
50	63.7633	3.7819	25.2610	−9.0408	0.8273	22618.8898

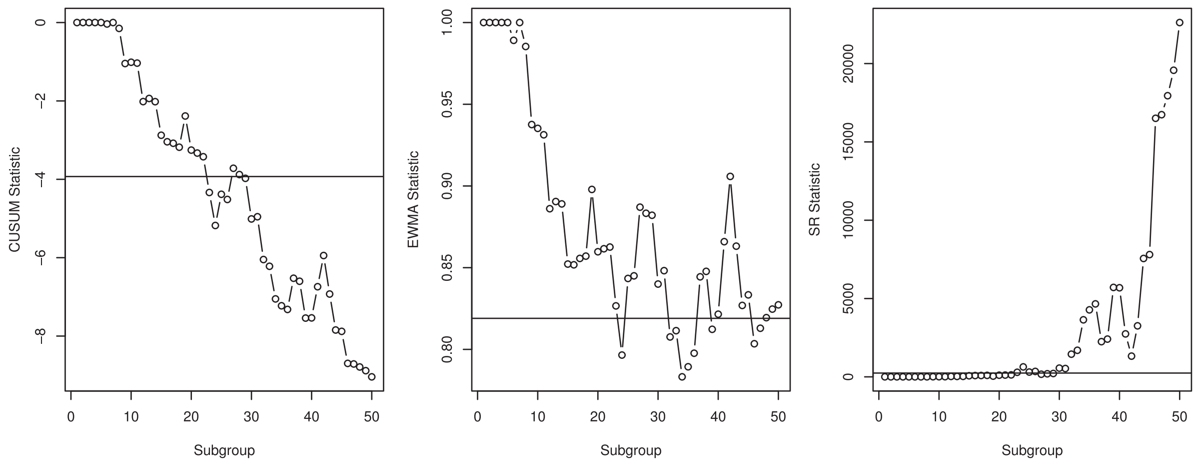

FIGURE 4 Lower-sided CUSUM, EWMA CEV and SR charts designed to detect a 33% decrease in the scale parameter η

7 | CONCLUSION

This paper applies the SR method to the censored Weibull distribution process. As the scale parameter is the characteristic life of the Weibull processes, detecting a change in the mean lifetime is often done via monitoring the scale parameter and assuming that the shape parameter is given and fixed. Under such a setup, the problem of detecting a decrease in the service life of a product boils down to detecting a downward shift in the scale parameter. Furthermore, the proposed method can help detect upward shifts in the scale parameter when a technical improvement needs to be evaluated. One can also consider a two-sided scheme for detecting both upward and downward shifts. For different values of β, the performance of the SR, CUSUM, and EWMA CEV charts is different.

We observe that when $\beta \leq 1$, the proposed SR type procedure is superior to the CUSUM and EWMA schemes in detecting small to moderate shifts. When $\beta > 1$, the SR scheme is better than the CUSUM and EWMA schemes for specific shifts. For any β, under the same adjustable parameters d_P, the SR procedure is better than the CUSUM scheme for small to moderate shifts. This paper only considers the desired $\text{ARL}_0 = 370$. Other values should also be considered. We also notice that both the censoring rate P_c and the sample size n impact the detection ability of the scheme. The parameters can be set according to the actual situation in practical applications. The SR method could be extended to monitor the shape parameter with fixed scale parameters or simultaneously monitor the scale and shape parameters, which warrants future research. Another important future direction of research is to extend the proposed procedure when standards are unknown and estimated from a reference sample. Finally, the charting performance of the SR method heavily depends on the assumption that the shift magnitude is known. In other words, the selection of the reference value relies on the target shift. However, the magnitudes of future shifts are seldom known in practical applications. To this end, the adaptive design of the proposed scheme for both smaller and larger shift sizes are worth pursuing.

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DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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