8.1.

Set the concentration of enzymes E, substrate S, intermediate species ES, product P are [E], [S], [ES], and [P] respectively

As we already know k1, k2, k3 are the first-order kinetic rate constant

The rate of changes = Generation – Consumption

Hence,

$$\frac{d[E]}{dt} = k_2[ES] + k_3[ES] - k_1[E][S]$$

$$\frac{d[S]}{dt} = k_2[ES] - k_1[E][S]$$

$$\frac{d[ES]}{dt} = k_1[E][S] - k_2[ES] - k_3[ES]$$

$$\frac{d[P]}{dt} = k_3[ES]$$

8.2.

The **MATLAB code** is shown below, I also provide the original code in the uploaded folder. Please uncomment the equation 1 to 4 and run them separately.

```
clear all
clc

h = 0.01; % Steps
x = 0:h:1; % Time
k1 = 100; % Initial forward rate
k2 = 600; % Initial reverse rate
k3 = 150; % Initial forward rate
E = 1; % Initial value
S = 10; % Initial value
ES = 0; % Initial value
P = 0; % Initial value

% Equation 1
% y(1) = E; % Derivative of variable E with respect to time dE/dx
% F_xy = @(x,y) k2*ES+k3*ES-k1*y*S; % Equation 1 for rate of changes

% Equation 2
% y(1) = S; % Derivative of variable S with respect to time dS/dx
% F_xy = @(x,y) k2*ES-k1*E*y; % Equation 2 for rate of changes

% Equation 3
% y(1) = ES; % Derivative of variable ES with respect to time dES/dx
% F_xy = @(x,y) k1*E*S-k2*y-k3*y; % Equation 3 for rate of changes

% Equation 4
y(1) = P; % Derivative of variable P with respect to time dP/dx
F_xy = @(x,y) k3*ES; % Equation 4 for rate of changes
```

```
for i = 1:(length(x)-1)
   k_1 = F_xy(x(i),y(i)); % Slope at the beginning of the time period
   k_2 = F_xy(x(i)+0.5*h,y(i)+0.5*h*k_1); % Slope at the middle time
   k_3 = F_xy(x(i)+0.5*h,y(i)+0.5*h*k_2); % Slope at the midpoint
   k_4 = F_xy(x(i)+h,y(i)+k_3*h); % Slope at the end of the time period
   y(i+1) = y(i)+(1/6)*(k_1+2*k_2+2*k_3+k_4)*h; % The slope of the midpoint has a greater weight
end
```

8.3.

We have equation (1) from Question 2.1:

$$\frac{d[ES]}{dt} = k_1[E][S] - k_2[ES] - k_3[ES] \tag{1}$$

At the large concentrations of S, the velocity v saturates to a maximum value. The rate of change of the product ES is 0:

$$\frac{d[ES]}{dt} = 0 \tag{2}$$

$$k_1[E][S] - k_2[ES] - k_3[ES] = 0 \tag{3}$$

$$k_1[E][S] = (k_2 + k_3)[ES] \tag{4}$$

To define:

$$[E_t] = Total\ [E] \tag{5}$$

$$[E_t] = [E] + [ES] \tag{6}$$

According to equation (4) and (6), we can obtain:

$$k_1([E_t] - [ES])[S] = (k_2 + k_3)[ES] \tag{7}$$

$$[ES] = \frac{[E_t][S]}{\frac{k_2 + k_3}{k_1} + [S]} \tag{8}$$

To define:

$$k_m = \frac{k_2 + k_3}{k_1} \tag{9}$$

Then, we have

$$[ES] = \frac{[E_t][S]}{k_m + [S]} \tag{10}$$

From Question 2.1, we know the velocity of change of product P is:

$$v = k_3[ES] \tag{11}$$

According to equation (10) and (11), we can obtain:

$$v = k_3 \frac{[E_t][S]}{k_m + [S]} \tag{12}$$

To define:

$$V_m = k_3[E_t] \tag{13}$$

where V_m is the velocity v that reaches the maximum value, then we have:

$$v = \frac{V_m[S]}{k_m + [S]} \tag{14}$$

To set:

$$v = \frac{V_m}{2} \tag{15}$$

Then, we have:

$$\frac{V_m}{2} = \frac{V_m[S]}{k_m + [S]} \tag{16}$$

$$k_m = [S] \tag{17}$$

Eventually, we prove that k_m equals the concentration S when the initial rate is half its maximum value. We also plot the equation (14) in Figure 1 and label the V_m.

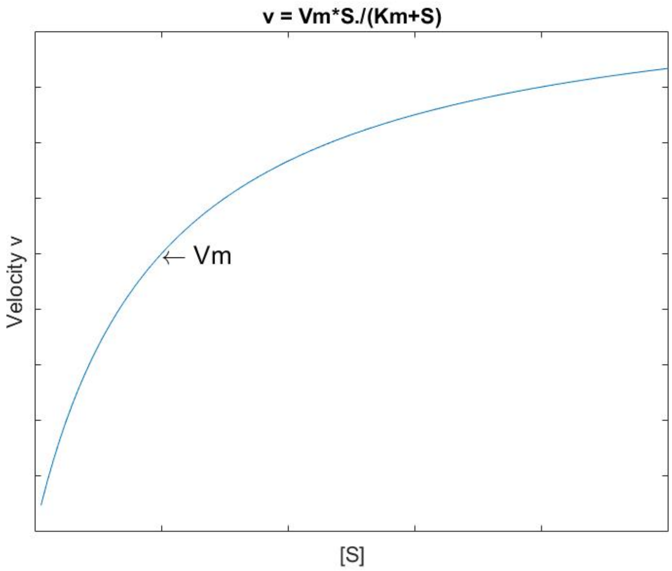

Figure 1 The velocity V as a function of the concentration of the substrate S