

1. Bubble sort is $O(n^2)$. Looking at the algorithm can you explain to your group why that is?

- Outer Loop: The outer loop runs n times, where n is the size of the array. This loop ensures that each element is bubbled to its correct position.
- Inner Loop: For each iteration of the outer loop, the inner loop runs up to $n-1-i$ times, where i is the current iteration of the outer loop. This loop compares adjacent elements and swaps them if they are in the wrong order.
- Number of Comparisons: In the worst-case scenario, the inner loop performs approximately $n(n-1)/2$ comparisons.
- Swaps: In the worst-case scenario, the inner loop performs approximately $n(n-1)/2$ comparisons.

Comparisons (worst case)

0 : $n-1$

1 : $n-2$

2 : $n-3$

\vdots

$n-1$: 1

$$\therefore S(n) = (n-1) + (n-2) + (n-3) + \dots + 1$$

$$= \frac{(n-1)(n-1+1)}{2}$$

$$= \frac{n(n-1)}{2}$$

Swaps (worst case)

0 : $n-1$

1 : $n-2$

2 : $n-3$

\vdots

$n-1$: 1

$$S_n = \frac{(n-1) + (n-2) + (n-3) + \dots + 1}{2}$$

$$= \frac{n(n-1)}{2}$$

So: for the worst case, the overall time complexity is:

$$\frac{n(n-1)}{2} + \frac{n(n-1)}{2} = n(n-1) = n^2 - n$$

which is $O(n^2)$

2. Can you come up with an array that will generate the "worst case"?

worst case: [5, 4, 3, 2, 1]

3. Can you come up with an array that will generate the "best case"?

best case:[1, 2, 3, 4, 5]

* By default, the best case is the same, but can you optimize it?

Yes

* Given the example above, optimize it so it stops running after the array is sorted?

We can add a flag(in my code, I used is_swapped) to detect whether any swaps were made.

At the start of each outer loop iterate, I set is_swapped to 0.

Then, as j iterates through the inner loop, if any swap occurs, I set is_swapped to 1.

After completing the inner loop, check the is_swapped flag. If it remains 0, it means the array is already sorted, and we can break out of the loop early.

* The optimized version is more common, and gives us the "best case".

best case:[1, 2, 3, 4, 5]

After the first pass, the is_swapped flag remains 0 because the array is sorted and no swaps are needed. The algorithm terminates early, and it performs $O(n)$ comparisons.