

Example Sheet 08

Def.(Syntax of predicate logic)

- 1 There are infinitely many individual constants.
- 2 There are infinitely many individual variables.
- 3 Every individual constant and every individual variable is a term.
- 4 For every natural number n there are infinitely many n -place predicates.
- 5 If P is an n -place predicate and t_1, \dots, t_n are terms, then $P(t_1, \dots, t_n)$ is an atomic formula.
- 6 If t_1 and t_2 are terms, $t_1 = t_2$ is an atomic formula.
- 7 Every atomic formula is a formula.
- 8 If φ and ψ are formulas, then $\neg\varphi$, $\varphi \wedge \psi$, $\varphi \vee \psi$, $\varphi \rightarrow \psi$ and $\varphi \leftrightarrow \psi$ are also formulas.
- 9 If v is a variable and φ a formula, then $\forall v(\varphi)$ and $\exists v(\varphi)$ are also formulas.

Exercise 1: Translating from natural language into predicate logic

Determine the right number of places of a predicate.

- (1) John studies at University of Tübingen.

Translating 'everyone', 'nobody' and 'at least one person'

- (2) Everyone likes Mary.
(3) Nobody likes John.
(4) At least one person knows John.

'the good couples'

- (5) All humans are Mortal.
(6) Some Greeks are philosophers.

'himself'

- (7) Every man cheats himself.

Hints:

Domain + Translation keys + Predicate logic FMLs

Domain: domain of the variables.

Translation keys:

-**Predicates**(e.g. properties, verbs...):

0-place verbs (like to rain) are translated as 0-place predicates

intransitive verbs and predicateive adjectives(describing a property) are translated as 1-place predicates

transitive verbs are translated as 2-place predicates

ditransitive verbs are translated as 3-place predicates

-Constants(identity):

proper nouns and definite descriptions are translated as individual constants

** constant or predicate? John has a Tesla.*

Constructing FMLs:

Hard to translate? => paraphrase first!

-variables:

*pronouns are translated as variables in predicate logic.
Sometimes come with quantifiers.(everyone, nobody...)*

-quantifiers:

Universal quantifier and existential quantifier

In most cases:

Restriction of the universal quantifier is translated using the implication

$\forall x(\text{Human}(x) \rightarrow \text{Mortal}(x))$

Restriction of the existential quantifier is translated using conjunction

$\exists x(\text{Greek}(x) \wedge \text{Philosopher}(x))$

To sum this => Rule of thumb:

given: English sentence S that needs a quantifier to be translated paraphrase S in such a way that it starts with for all P it holds that ... or there is a P such that ... (where "P" is a noun)

translate as

$\forall x(P(x) \rightarrow \dots)$ or $\exists x(P(x) \wedge \dots)$

("P" is the translation of the noun in question) translate the rest of the sentence

All humans are Mortal.

(For each object it holds: if it is human, it is mortal.) $\forall x(\text{Human}(x) \rightarrow \text{Mortal}(x))$

Some Greeks are philosophers.

(There is an object such that it is a Greek and a philosopher.) $\exists y(\text{Greek}(y) \wedge \text{Philosopher}(y))$

Every man cheats himself.

(For every man it holds that he cheats himself.) $\forall x(\text{Man}(x) \rightarrow \text{Cheat}(x, x))$

One last example about quantifier:

Dogs are intelligent. => For every dog it holds that it is intelligent. $\forall x(\text{Dog}(x) \rightarrow \text{Intelligent}(x))$

** universal quantifiers can be used even if words s.t. 'all' are not directly found.*

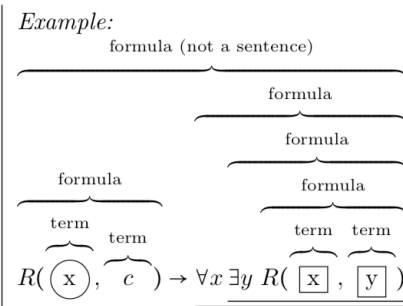
Exercise 2: Translating from predicate logic into natural language

$$\forall y(\text{Lion}(y) \rightarrow \exists w(\text{Mane}(w) \wedge \text{Has}(y, w)))$$

=> For every lion it holds that there is a mane such that it has it.

=> Lions have a mane.

Exercise 3: Syntax of predicate logic



- (a) well-formed? => Syntax of predicate logic
- (b) variable or constant ?
- (c) terms and sub-formulas?

Every individual constant and every individual variable is a term.

“If P is an n -place predicate and t_1, \dots, t_n terms, then $P(t_1, \dots, t_n)$ is an atomic formula.”

- (d) Scope.

The formula within the bracket pair after a quantifier is called the scope of the quantifier

#A quantifier Q binds a variable occurrence v

iff

v occurs in the scope of Q ,

and between Q and v there is no intervening co-indexed quantifier Q' such that v is in the scope of Q' (and that would therefore bind v)#

- (e) Sentence?

FML with no free-variables.

#Definition (Free and bound variable occurrences)

All variable occurrence in an atomic formula φ are free in φ .

Every free occurrence of a variable v in φ is also free in $\neg\varphi$.

Every free occurrence of a variable v in φ and ψ is also free in $\varphi \wedge \psi$, $\varphi \vee \psi$, $\varphi \rightarrow \psi$ and $\varphi \leftrightarrow \psi$.

Every free occurrence of a variable v in φ is also free in $\forall w(\varphi)$ and $\exists w(\varphi)$, if $v \neq w$.

Every free occurrence of a variable v in φ is bound in $\forall v(\varphi)$ by $\forall v$, and bound in $\exists v(\varphi)$ by $\exists v$.

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If a variable occurrence v is bound in φ , it is also bound in every formula that contains φ as a sub-formula.