Thursday, 12. Dec. 2019

Example Sheet 08

Def.(Syntax of predicate logic)
1 There are infinitely many individual constants.
2 There are infinitely many individual variables.
3 Every individual constant and every individual variable is a term.
4 For every natural number n there are infinitely many n-place predicates.
5 If P is an n-place predicate and t1, . . . , tn are terms, then P(t1, . . . , tn) is an atomic formula.
6 If t1 and t2 are terms, t1 = t2 is an atomic formula.
7 Every atomic formula is a formula.
8 If φ and ψ are formulas, then $\neg\varphi$, $\varphi \wedge \psi$, $\varphi \vee \psi$, $\varphi \rightarrow \psi$ and $\varphi \leftrightarrow \psi$ are also formulas.
9 If v is a variable and φ a formula, then $\forall v(\varphi)$ and $\exists v(\varphi)$ are also formulas.

Exercise 1: Translating from natural language into predicate logic

<u>Determine the right number of places of a predicate.</u>
(1) John studies at University of Tübingen.

<u>Translating ‘everyone’, ‘nobody’ and ‘at least one person’</u>
(2) Everyone likes Mary.
(3) Nobody likes John.
(4) At least one person knows John.

<u>‘the good couples’</u>
(5) All humans are Mortal.
(6) Some Greeks are philosophers.

<u>‘himself’</u>
(7) Every man cheats himself.

Hints:

Domain + Translation keys + Predicate logic FMLs

Domain: domain of the variables.

Translation keys:

-Predicates(e.g. properties, verbs…):

0-place verbs (like to rain) are translated as 0-place predicates
intransitive verbs and predicateive adjectives(describing a property) are translated as 1-place predicates
transitive verbs are translated as 2-place predicates
ditransitive verbs are translated as 3-place predicates

-Constants(identity):

proper nouns and definite descriptions are translated as individual constants

** constant or predicate? John has a Tesla.*

Constructing FMLs:

Hard to translate? => paraphrase first!

-variables:

pronouns are translated as variables in predicate logic.
Sometimes come with quantifiers.(everyone, nobody…)

-quantifiers:

Universal quantifier and existential quantifier

In most cases:

Restriction of the universal quantifier is translated using the implication
$\forall x(Human(x) \rightarrow Mortal(x)$
Restriction of the existential quantifier is translated using conjunction
$\exists x(Greek(x) \wedge Philosopher(x))$

To sum this => Rule of thumb:

given: English sentence S that needs a quantifier to be translated paraphrase S in such a way that it starts with for all P it holds that ... or there is a P such that ... (where "P" is a noun)
translate as
$\forall x(P(x) \rightarrow ...)$ or $\exists x(P(x) \wedge ...)$
("P" is the translation of the noun in question) translate the rest of the sentence

All humans are Mortal.
(For each object it holds: if it is human, it is mortal.)$\forall x(Human(x) \rightarrow Mortal(x))$
Some Greeks are philosophers.
(There is an object such that it is a Greek and a philosopher.)$\exists y(Greek(y) \wedge Philosopher(y))$
Every man cheats himself.
(For every man it holds that he cheats himself.)$\forall x(Man(x) \rightarrow Cheat(x, x))$

One last example about quantifier:
Dogs are intelligent. => For every dog it holds that it is intelligent. $\forall x(Dog(x) \rightarrow Intelligent(x))$
** universal quantifiers can be used even if words s.t. 'all' are not directly found.*

Exercise 2: Translating from predicate logic into natural language

∀y(Lion(y) → ∃w(Mane(w) ∧ Has(y, w)))

=> For every lion it holds that there is a mane such that it has it.
=> Lions have a mane.

Exercise 3: Syntax of predicate logic

Example:

$$\overbrace{\overbrace{R(\overbrace{x}^{\text{term}}, \overbrace{c}^{\text{term}})}^{\text{formula}} \to \overbrace{\forall x\, \overbrace{\exists y\, \overbrace{R(\overbrace{\boxed{x}}^{\text{term}}, \overbrace{\boxed{y}}^{\text{term}})}^{\text{formula}}}^{\text{formula}}}^{\text{formula}}}^{\text{formula (not a sentence)}}$$

(a) well-formed? => Syntax of predicate logic
(b) variable or constant ?
(c) terms and sub-formulas?

Every individual constant and every individual variable is a term.

“If P is an n-place predicate and t1, . . . , tn terms, then P(t1, . . . , tn) is an atomic formula.”

(d) Scope.
The formula within the bracket pair after a quantifier is called the scope of the quantifier

#A quantifier Q binds a variable occurrence v
iff
v occurs in the scope of Q,
and between Q and v there is no intervening co-indexed quantifier Q′ such that v is in the scope of Q′ (and that would therefore bind v)#

(e) Sentence?

FML with no free-variables.

#Definition (Free and bound variable occurrences)
All variable occurrence in an atomic formula φ are free in φ.
Every free occurrence of a variable in v in φ is also free in ¬φ.
Every free occurrence of a variable v in φ and ψ is also free in in φ ∧ ψ, φ ∨ ψ, φ → ψ and φ ↔ ψ.
Every free occurrence of a variable v in φ is also free in ∀w(φ) and ∃w(φ), if v 6= w.
Every free occurrence of a variable v in φ is bound in ∀v(φ) by ∀v, and bound in ∃v(φ) by ∃v.
#
If a variable occurrence v is bound in φ, it is also bound in every formula that contains φ as a sub-formula.