Thursday, 24. Oct

Examples for Assignment 01

Ex.1: Sentences and Statements

For each of the following expressions, determine whether it is a statement in the sense treated by statement logic, and if it is, give its truth value in the real world.

(1) Whoops.
(2) Did you wake early this morning to go to the tutorial?
(3) I will never wake up early to go to a tutorial at 8am unless I am the tutor.
(4) Jupiter is the fifth planet from the Sun and the largest in the Solar System.

Hints:

-Statements are sentences that are, in principle, either true or false.
-Statement logic (and predicate logic) only deal with statements the truth of which do not depend on the situation in which they are uttered.

-> A statement in the sense treated by statement logic has to meet the above 2 rules.

Ex. 2: Syntax of statement logic

For each of the following expressions, determine if it is a well-formed formula of statement logic. If it is, atomic? Also find out the principle connective

(1) $(p \rightarrow (p \rightarrow q) \rightarrow q)$
(2) $p \vee (q)$
(3) $(\neg p \vee \neg\neg p)$

Hints:

-Atomic statements ("statement variables"): statements that do not consist of statements themselves.

-Syntax of statement logic:

Let A be a set of atomic statements.
1 Every statement in A is a formula in L(A).
2 If ψ is a formula in L(A), then $\neg\psi$ is also a formula in L(A).
3 If ϕ and ψ are formulas in L(A),then$(\phi \wedge \psi)$,$(\phi \vee \psi)$,$(\phi \rightarrow \psi)$ and $(\phi \leftrightarrow \psi)$ are also formulas in L(A).

4 There are no other formulas in L(A).

Ex. 3: Semantics of statement logic by truth table

Draw a truth table.

$((p \rightarrow q) \vee (q \rightarrow p))$

Hints:

to compute the truth conditions of a complex formula ϕ,

-it is not necessary to consider all conceivable valuation functions,
but
-only all possible combinations of truth values of the atomic statements that occur in ϕ,
i.e.
-2n different combinations of truth values, for n atomic statements.

Ex. 4: Semantics of statement logic not by truth table

Given the following valuation function V :

$V(p) = 0, V(q) = 1, V(r) = 0$

determine stepwise the truth values of the following formulas under V according to the arithmetic definition (without using a truth table).

$(\neg q \vee r) \rightarrow \neg p$

Hints:

valuation function V : Function that assigns each formula of a language of propositional logic a truth value
admissible valuation functions must agree with the interpretation of the logical connectives:

Definition
A function V from the formulas of a language of statement logic L(A) into the set of truth values {0, 1} is a valuation function iff it holds for all formula ϕ and ψ:
1 $V(\neg\phi) = 1 - V(\phi)$
2 $V(\phi \wedge \psi) = V(\phi) \times V(\psi)$
3 $V(\phi \vee \psi) = V(\phi) + V(\psi) - V(\phi) \times V(\psi)$
4 $V(\phi \rightarrow \psi) = 1 - V(\phi) \times (1 - V(\psi))$
5 $V(\phi \leftrightarrow \psi) = 1 - (V(\phi) - V(\psi))2$

Summary:

-Syntax of statement logic: determine if a formula if well formed or not

-Semantics of statement logic: compute the truth condition of a well-formed formula(draw truth table or not)