Thursday, 30. Oct

Q&A sheet again:

Tricky “implication”
Annoying brackets
Bonus 5 about modal logic

A correction of last tutorial:

¬The moon is made out of cheese → The moon is made out of chocolate
Is not a statement since no natural language is allowed in statement logic formulas.

Something was not made quite clear last week:

Colourless green ideas sleep furiously (Chomsky 57) => semantically incorrect
The king of France is bald.(Russel) => pragmatically incorrect

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Key definitions and theorems

Definition (Tautology)
A formula of statement logic ϕ is a tautology of statement logic, formally written as

$$\Rightarrow \phi$$

if and only if it holds for all valuations V :

$$V(\phi) = 1$$

Definition (Contradiction)
A formula ϕ is a contradiction of statement logic if and only if it holds for all valuation functions V :

$$V(\phi) = 0$$

Theorems
If ϕ is a tautology, then ¬ϕ is a contradiction.
If ϕ is a contradiction, then ¬ϕ is a tautology.

Definition (Logical equivalence)
Two formulas ϕ and ψ are logically equivalent, formally written as

$$\phi \Leftrightarrow \psi$$

if and only if for all valuation functions V it holds that:

$$V(\phi) = V(\psi)$$

Note: “Logical equivalence” is a meta-linguistic notion, while “equivalence” in the sense of ↔ is an operator of the object language.

Logical equivalence can be decided with the help of truth tables as well.

Theorem
ϕ and ψ are logically equivalent if and only if ϕ ↔ ψ is a tautology.