Thursday, 19. Dec

Example Sheet 09

Definition (Semantics of predicate logic)
Let M = hE, Fi be a model and g an assignment function for M.
1 [c]M g = F(c), if c is an individual constant.
2 [v]M g = g(v), if v is an individual variable.
3 [P(t1, . . . , tn)]M g = 1 iff h[t1]M g , . . . , [tn]M g i ∈ F(P)
~~4 [t1 = t2]M g iff [t1]M g = [t2]M g~~
5 [¬φ]M g = 1 − [φ]M g
6 [φ ∧ ψ]M g = min([φ]M g , [ψ]M g)
7 [φ ∨ ψ]M g = max([φ]M g , [ψ]M g)
8 [φ → ψ]M g = max(1 − [φ]M g , [ψ]M g)
9 [φ ↔ ψ]M g = 1 − ([φ]M g − [ψ]M g) 2
10 [∀v(φ)]M g = min({[φ]M g[a/v] |a ∈ E})
11 [∃v(φ)]M g = max({[φ]M g[a/v] |a ∈ E})

Ex. 01 Translation and interpretation of expressions

a) Translation

⟦constant⟧M,g => element
⟦variable⟧M,g => the individual domain
⟦Predicate(term1, term2, …, termn)⟧M,g => relation of elements(or individual domains)

Hints => first determine the right interpretation for terms then plug them in the predicates
(1)⟦know(p, j)⟧M,g
John knows Peter.
(2)⟦know(y, z)⟧M,g
She loves him.

(3)⟦∃xknow(j, x)⟧M,g
There is someone Peter knows

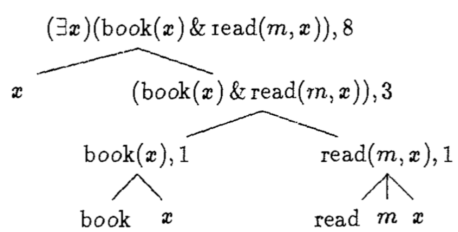
b) Interpretation: A stepwise (!) compositional interpretation of the expression.

[P(c1, . . . , cn)]M = 1 iff <F(c1), . . . , F(cn)> ∈ F(P)

Example 1: bottom-up tree notation(in book chapter 13)

Mary is reading a book.

(13–11)



Example 2: top-down construction

$[\![P(x) \wedge Q(m,p)]\!]^{M,g} = 1$
iff $[\![P(x)]\!]^{M,g} = 1$ and $[\![Q(m,p)]\!]^{M,g} = 1$ rule (3)
iff $\langle [\![x]\!]^{M,g} \rangle \in [\![P]\!]^{M,g}$ and $\langle [\![m]\!]^{M,g}, [\![p]\!]^{M,g} \rangle \in [\![Q]\!]^{M,g}$ rule (1)
iff $\langle g(x) \rangle \in F(P)$ and $\langle F(m), F(p) \rangle \in F(Q)$ rule (0)
iff John $\in$ {John, Peter} and $\langle$Mary, Peter$\rangle \in \{\langle$Peter, Mary$\rangle\}$

= 0

"See also the tupperware model examples Natalie uploaded on Moodle; the calculations there are a little more detailed. This is roughly what it should look like, but any presentation that contains all the necessary steps is fine."

Exercise 2: Understanding models and interpretations

For reference:

Semantics model M, consists of individual domain E and interpretation function F
Interpretation function maps individual constants to elements of E,
and n-place predicates to n-place relations over E.

extension to formulas: [P(c1, . . . , cn)]M = 1 iff <F(c1), . . . , F(cn)> ∈ F(P)

Definition (assignment function)
An assignment function g for a model M = <E, F> is a function from the set of variables into the individual domain E.

Quantifiers interpretation:
Let M = <E, F> be a model.

[∀v(φ)]M g = 1 if and only if [φ] M g[a/v] = 1 for all a ∈ E

[∃v(φ)]M g = 1 if and only if there is an object a ∈ E such that [φ] M g[a/v] = 1

a) ellipse diagram(seen in Function): Everyone loves John.
b) Understanding model.
c) Understanding assignment function.

Merry Christmas!