Example exercise 3(Updated)

*updated parts are marked in green.

Exercise 1: Logical inference:

Decide which of the following logical inferences hold by means of your choice (truth table, arguing about valuation functions), using each of the two methods at least once.

$p, q \wedge r \Rightarrow q$

Hints:

Think of Def. Logical inference.(on board, also found in slides)

truth tables: look into the rows where your premises all have truth value one and check if the conclusion also have truth value one in every such row.(demo on board)
or
valuation functions: See if the valuation functions that make your premises true also make the conclusion true. Analyse by breaking complex premises into smaller ones with the help of the functions equations learnt in lecture one.(demo on board)

Exercise 2: Truth Trees

Start a truth tree for the following formulas and state whether or not they are valid. If the truth tree remains open, give the counter valuation function associated with each open branch.

Example: $\vdash_{TT} \neg(p \rightarrow q) \leftrightarrow p \wedge q$

Hints:

How to draw truth trees?

-Rules of Branching:

Double negation, conjunction, disjunction, implication, equivalence, negation + conjunction, Negation + disjunction,…

-Rules of Thumb:

Try to use the non-branching rules first.
When applying a branching rule, try to do it so that one of the two branches can be closed soon.
The double negation of an atomic statement is usually good for nothing; therefore develop doubly negated atoms only if it is necessary for closing a branch.

Step 1: Negate the given formula and make it the root node(assumption) of your tree.

Step 2: Branching from the root node.
Follow rules of branching when you branch, and always keep rules of thumb in mind when branching.

What are your trees supposed to look like?

each line consists of
1. *line number*
2. *formula that is assumed to be true*
3. *number of the line from which the current line is derived (the first line is called "assumption" (A))*

If a branch
contains the formula ϕ which is dominated by ¬ϕ, or contains the formula ¬ϕ which is dominated by ϕ,
then this branch is marked as contradictory with an "x".
A truth tree is closed if all branches are contradictory, i.e. all leaves are marked with "x".

What if not closed?

give the counter valuation function associated with each open branch.

Exercise 4: Object and meta language

For each of the following expressions, state whether it is an object-language expression, a meta-language expression or syntactical nonsense.
Note: We write "→"/"↔" to abbreviate a meta-linguistic "if"/"if and only if", respectively

$0 \rightarrow 1 = 0$	non-sense
$p \vee q$	object-language expression
$V(p \leftrightarrow \neg p) = 0$	meta-language expression

Hints:

object-language expression: basically well-formed SL formula.

meta-language expression:

Think about meta data, which is the information(data) used to describe other information(data).

MLEs do not have to be well-formed formulas, they allow natural language, valuation functions, logical inference, and meta-linguistic (e.g. "if"/"if and only if")they describe certain properties or facts(e.g. validity, tautologicity, inference, derivability) mostly in the form of statement.

syntactical nonsense:
Syntactically incorrectness(not syntactical instead of not semantical)!

e.g. $\neg(\Rightarrow \neg q \leftrightarrow r)$ non-sense!

Why? can't apply object lang. symbol ¬ to meta lang. symbol ⇒

Further disambiguation(clarification/correction):

Forgot about what I said about “make sense, nonsense”, “sense is about semantics”, sorry :P
Syntactical nonsense here is just UNSYNTACTICAL!!!

nonsense is nonsense, when it is neither ole nor mle.