Thursday, 30. Oct

Examples for Assignment 02

Ex. 1: Translation

Translate the following statements in natural languages into statement logic formulas.

(1) Lily and Lucy are friends.
(2) John unloads the truck.
(3) Neither John nor Peter are in Tübingen.

Hints:

“A statement A is an adequate translation of a statement A′ if and only if A and A′ have the same truth conditions.”

Expected answer: **translation key + FML**

Translation key: atomic statements, how to decide? -> refer to rules of thumb:

rule of thumb: If an English statement that contains “not” (or “n’t”) can be paraphrased without problems by a formulation using “it is not the case that”, then A can be translated into a negated formula.

rule of thumb: If a statement A that contains “and” can be paraphrased by a sentence where “and” connects two clauses, then A can be translated as a conjunction.

rule of thumb: Suppose A is an English statement which might possibly be translated as an implication $\phi \rightarrow \psi$. To test the adequacy of this translation, it is important to understand under what conditions A is false. If the translation is correct, then under these very conditions, ϕ must be true and ψ false.

FML: find the right connectives(negation, conjunction, disjunction, implication, equivalence). How? -> take a reference from the examples presented in slides when it is hard to decide.

Some possible indications of certain connectives:

“nobody”, “nothing”, “neither…nor…”, “unreasonable”,… -> negation
“and”, “not…but…”, “even… though…”, “both… and…”,… -> conjunction
“or”, … -> disjunction
There is no real counterpart to implication in English.
Some grammatical constructions can approximately be translated by implications.
“if…then…”, “…only if…”, “…is a necessary condition that…”, “…is a sufficient condition that…”, “…implies…”… -> implication
“…if and only if…”, “…just in case…”, … -> equivalence

This can only serve as an inspiration, it is not definitely the case. Some exceptional situations are shown above in this example exercise.
ACHTUNG: Always be careful of such situations where you can easily make mistakes.

What if the statement in NL is not very intuitive to be translated? -> Paraphrase

Ex. 2: Properties of formulas

Draw truth tables for the following formulas and state for each formula whether it is a tautology, contingency or contradiction and if it is satisfiable.

$((p \rightarrow (q \wedge \neg r))$

Hints:

Whether or not a formula is logically true(tautology) can be decided with the help of truth tables.
Whether or not a formula is logically false(contradiction) can also be determined by using truth tables.
A formula is satisfiable (German: erfüllbar) iff it is true under at least one valuation, i.e. iff it is not a contradiction.
A proposition that is neither a tautology nor a contradiction is called a contingency.
For example $(q \vee p)$ is a contingency. This can also be decided by drawing truth tables.

=> Draw truth tables.(find tips in example sheet 01)

Ex. 3: Relations between formulas

Find out the equivalent pairs of formulas among the following given ones.

$((p \rightarrow (q \wedge \neg r)) \wedge (p \rightarrow (q \vee \neg r)))$

Hint:

Draw truth tables again lol
Compare the truth tables and find out the identical truth tables of equivalent formulas.

Ex. 4: Tautology and Contradiction

Prove the following statements by arguing about valuation functions.

ϕ and ψ are logically equivalent if and only if $\phi \leftrightarrow \psi$ is a tautology.

Hints:

Refer to the proof of theorems presented in slides.
Prove towards both directions individually when you have to("if and only if").
Note by valuation function.

Sample solution(from slides 29-30):

Forward direction: Suppose $\phi \Leftrightarrow \psi$. Let V be an arbitrary valuation function. By assumption, it holds that $V(\phi) = V(\psi)$. Hence either $V(\phi) = V(\psi) = 0$ or $V(\phi) = V(\psi) = 1$. In either case, it follows from the semantics of the equivalence that $V(\phi \leftrightarrow \psi) = 1$.

Backward direction: Suppose $\phi \leftrightarrow \psi$ is a tautology. Let V be an arbitrary valuation function. We have to distinguish two cases:
$V(\phi) = 1$. It follows from the semantics of equivalence that $V(\psi) = 1$.
$V(\phi) = 0$. It follows from the semantics of equivalence that $V(\psi) = 0$.
In both cases it holds that $V(\phi) = V(\psi)$. Hence ϕ and ψ are logically equivalent.