Thursday , 21. November

Example Sheet 05

A sum-up/recapitulation of (classical)statement logic

Exercise 1: Formally proving arguments

a) formalize the argument (the premises and the conclusion) into statement logic with translation keys for the atomic statements,

=> Translation: NL into SL
(something we have done in Assignment 02, hints can be found in Example Sheet 02)

b) state whether the argument is valid or invalid,

=> Argument valid? Logical inference
(some similar thing we did in Assignment 03, hints can be found in Example Sheet 03)

c) if the argument is valid, prove it by presenting a natural deduction derivation with the formulas you obtained in a);

=> Natural Deduction

if the argument is invalid, refute it by presenting one counter valuation function along with an informal description of a matching real life situation under which all of the premises are true but the conclusion is false.

=> A counter valuation function for an invalid formula is a valuation function under which the formula does not hold.

"-it is not possible to prove via natural deduction that a formula is not derivable from a given set of premises
-if you suspect that the conclusion doesn't follow from the premises, it is safer to work with truth trees"

you can try coming up with the counter valuation function by truth tree method, but this is not the only way.

=> In truth tree method: To determine the counter valuation associated with an open branch, walk through the branch and assign all atomic propositional variables the truth value 1 and all negated propositional variables the truth value 0.

Truth tree method is simply an idea here to help you find a counter valuation function to refute invalid arguments, as mentioned, it is not the only way, but it is safe. And in this exercise, a truth tree is not required to be presented as part of the solution.

=> Refer to Assignment 03 & 04

An example can be found on Assignment Sheet 05.

Natural Deduction

What do we already know about natural deduction before this week?

Fist impression: last week

=> What & How? in Example Sheet 04.

Further tips added last Thursday after tutorial on our website.

What's been added this week?

New tips introduced that will make your life easier.

Tip 1:

always keep track which sub-goal you are currently proving.

What to do towards each sub-goal.

Sub-goals:

**additional assumption is the auxiliary assumption we talked about last week*

if the current sub-goal is $\phi \wedge \psi$:
first prove ϕ then prove ψ then apply $\wedge$I

if the current sub-goal is $\neg\phi$:
start a sub-proof with ϕ as additional assumption
for some convenient formula ψ: prove both ψ and $\neg\psi$ and conclude $\perp$
finish the sub-proof with $\neg$I

if the current sub-goal is $\phi \rightarrow \psi$(implication construction):
start a new sub-proof with ϕ as additional assumption
try to prove ψ
if successful: finish the sub-proof with $\rightarrow$ I

if the current sub-goal is $\phi \vee \psi$:
prove ϕ or prove ψ
if successful, introduce $\phi \vee \psi$ via $\vee$I, 1(2)

otherwise: if there is an accessible formula $\xi \vee \zeta$ combine $\vee$E and $\vee$I:
start a sub-proof with the assumption ξ and prove ϕ (or ψ) derive $\phi \vee \psi$ using $\vee$I and finish sub-proof
start a second sub-proof and prove ψ (ϕ)
from this, derive $\phi \vee \psi$ via $\vee$I and finish sub-proof via $\vee$E, derive $\phi \vee \psi$

if the current sub-goal is $\phi \leftrightarrow \psi$(implication both directions):

start sub-proof with the additional assumption ϕ
prove ψ
finish sub-proof and start new sub-proof withe the assumption ψ prove ϕ
finish the second sub-proof and apply $\leftrightarrow$ I

further rules of thumb:

apply $\wedge$E, → E and ↔ E whenever possible

also, apply ¬I as soon as possible; if the current line in the proof is the negation of an undischarged assumption, apply ⊥I, followed by ¬I

if none of these rules of thumb is applicable:

Tip 2:

indirect proof

suppose you want to prove ϕ
start your sub-proof with the assumption $\neg\phi$
try to derive a contradiction
i.e.: try to derive both ψ and $\neg\psi$ for some formula ψ
if successful: end the current sub-proof with ⊥I followed by ¬I result is $\neg\neg\phi$
applying ¬E leads to ϕ, as desired

Sum: when and how to plug in aux assumptions?

2 typical cases:

Conditional construction and indirect proof.

Exercise 2: Understanding inferences and derivations

What about ex02?

Hints:

Thinking page of the unfinished last Example sheet.

Recapitulation(better understanding) of logical inference.

Soundness and completeness.

Contraposition.

Bonus 3: Soundness and Completeness

Soundness of ND = "If $M \vdash_{ND} \phi$, then $M \Rightarrow \varphi$"
(= If there is an ND derivation for an inference, then the inference is semantically valid)

Completeness of ND = "If $M \Rightarrow \varphi$, then $M \vdash_{ND} \phi$"
(= If some inference holds, then we can find an ND derivation for it)

Together, this combines to "$M \vdash_{ND} \phi$ if and only if $M \Rightarrow \varphi$"
(**soundness and completeness**, also sometimes just called completeness for short).

Define sound and complete sets of rules for exclusive disjunction.

a) in the calculus of truth trees
b) in the calculus of natural deduction.

=> Develop the rules for this logical operator(exclusive or) for truth tree method and natural deduction respectively.