

## Example exercise 3

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### Exercise 1: Logical inference:

Decide which of the following logical inferences hold by means of your choice (truth table, arguing about valuation functions), using each of the two methods at least once.

$$p, q \wedge r \Rightarrow q$$

Hints:

Think of Def. Logical inference.

**truth tables:** look into the rows where your premises all have truth value one and check if the conclusion also have truth value one in every such row.

or

**valuation functions:** See if the valuation functions that make your premises true also make your conclusion true.

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### Exercise 2: Truth Trees

Start a truth tree for the following formulas and state whether or not they are valid. If the truth tree remains open, give the counter valuation function associated with each open branch.

$$\text{Example: } \vdash \text{TT } \neg(p \rightarrow q) \Leftrightarrow p \wedge q$$

Hints:

How to draw truth trees?

-Rules of Branching:

Double negation, conjunction, disjunction, implication, equivalence, negation + conjunction, Negation + disjunction,...

-Rules of Thumb:

Try to use the non-branching rules first.

When applying a branching rule, try to do it so that one of the two branches can be closed soon. The double negation of an atomic statement is usually good for nothing; therefore develop doubly negated atoms only if it is necessary to close a branch.

Step 1: Negate the given formula and make it the root node(assumption) of your tree.

Step 2: Branching from the root node.

Follow rules of branching when you branch, and always keep rules of thumb in mind when branching.

What are your trees supposed to look like?

*each line consists of*

- 1 line number*
- 2 formula that is assumed to be true*
- 3 number of the line from which the current line is derived (the first line is called "assumption" (A))*

*If a branch*

*contains the formula  $\phi$  which is dominated by  $\neg\phi$ , or contains the formula  $\neg\phi$  which is dominated by  $\phi$ ,*

*then this branch is marked as contradictory with an "x".*

*A truth tree is closed if all branches are contradictory, i.e. all leaves are marked with "x".*

What if not closed?

give the counter valuation function associated with each open branch.

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## Exercise 4: Object and meta language

For each of the following expressions, state whether it is an object-language expression, a meta-language expression or syntactical nonsense.

Note: We write " $\rightarrow$ " / " $\leftrightarrow$ " to abbreviate a meta-linguistic "if" / "if and only if", respectively

$0 \rightarrow 1 = 0$	non-sense
$p \vee q$	object-language expression
$V(p \leftrightarrow \neg p) = 0$	meta-language expression

Hints:

**object-language expression:** basically well-formed SL formula.

**meta-language expression:**

Think about meta data, which is the information(data) used to describe other information(data).

MLEs do not have to be well-formed formulas, they allow natural language, valuation functions, logical inference, and meta-linguistic (e.g. "if" / "if and only if") they describe certain properties or facts (e.g. validity, tautologicity, inference, derivability) mostly in the form of statement.

**syntactical nonsense:** nonsense is nonsense, when it is neither ole nor mle.