Thursday, 28 November

Example Sheet 06

What is set?

Georg Cantor (1845-1918)
“A set is a collection into whole of definite, distinct objects of our intuition or our thought. The objects are called the elements of the set.”

* Sets can be finite or infinite.
* Every well-defined object can be member/element of a set

Exercise 1: Writing down sets

=> four ways to describe sets:

1-list notation

only applicable to finite sets
A = {the Volga, Emmanuel Macron, 16}

2- separation notation

{ variable $\in$ domain | sentence that contains the variable }
or
{ variable $\in$ domain : sentence that contains the variable }

e.g. => $\{x \in N | x \text{ is even}\}$

3-recursive definition

consists of three components:
1 a finite list of objects that definitely belong to the set to be defined
2 rules that allow to generate new elements from existing elements
3 statement, that all elements of the set in question can be generated via finitely many application of the rule from (2) to the objects from (1)

=> e.g.

1 $4 \in E$
2 If $x \in E$, then $x+2 \in E$.
3 Nothing else is in E.

Recursive definition of set of natural numbers?

4-set theoretic operations later in ex 03

5-Venn diagram

See examples on board.

Exercise 2: Elements and subsets: member, subset, proper subset

. Given the following sets:

$$\begin{array}{lllllll} A & = & \{a,b,c,2,3,4\} & \quad E & = & \{a,b,\{c\}\} \\ B & = & \{a,b\} & \quad F & = & \emptyset \\ C & = & \{c,2\} & \quad G & = & \{\{a,b\},\{c,2\}\} \\ D & = & \{b,c\} & & & \end{array}$$

classify each of the following statements as true or false

(a)	$c \in A$	**(g)**	$D \subset A$	**(m)**	$B \subseteq G$
(b)	$c \in F$	**(h)**	$A \subseteq C$	**(n)**	$\{B\} \subseteq G$
(c)	$c \in E$	**(i)**	$D \subseteq E$	**(o)**	$D \subseteq G$
(d)	$\{c\} \in E$	**(j)**	$F \subseteq A$	**(p)**	$\{D\} \subseteq G$
(e)	$\{c\} \in C$	**(k)**	$E \subseteq F$	**(q)**	$G \subseteq A$
(f)	$B \subseteq A$	**(l)**	$B \in G$	**(r)**	$\{\{c\}\} \subseteq E$

. For any arbitrary set S,

(a) is S a member of $\{S\}$?

(b) is $\{S\}$ a member of $\{S\}$?

(c) is $\{S\}$ a subset of $\{S\}$?

(d) what is the set whose only member is $\{S\}$?

Exercise 3: Set Operations

Given the sets $A, \ldots, G$ as in Exercise 1, list the members of each of the following:

(a)	$B \cup C$	**(g)**	$A \cap E$	**(m)**	$B - A$
(b)	$A \cup B$	**(h)**	$C \cap D$	**(n)**	$C - D$
(c)	$D \cup E$	**(i)**	$B \cap F$	**(o)**	$E - F$
(d)	$B \cup G$	**(j)**	$C \cap E$	**(p)**	$F - A$
(e)	$D \cup F$	**(k)**	$B \cap G$	**(q)**	$G - B$
(f)	$A \cap B$	**(l)**	$A - B$		

-Power set:
set of all subsets A.

If A is finite and has n elements, then ℘(A) always has 2n elements.

For all sets A: ∅∈℘(A) and A∈℘(A).

-Cardinality of sets:
|A| is the number of elements of A

-Identity of sets
Two sets are identical if and only if they have the same elements.

Set theoretic laws

1. idempotence laws:
   1. $A \cup A = A$
   2. $A \cap A = A$
2. commutativity laws:
   1. $A \cup B = B \cup A$
   2. $A \cap B = B \cap A$
3. associativity laws:
   1. $(A \cup B) \cup C = A \cup (B \cup C)$
   2. $(A \cap B) \cap C = A \cap (B \cap C)$
4. distributivity laws:
   1. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   2. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

1. identity laws:
   1. $A \cup \emptyset = A$
   2. $A \cup U = U$
   3. $A \cap \emptyset = \emptyset$
   4. $A \cap U = A$
2. complement laws:
   1. $A \cup A' = U$
   2. $(A')' = A$
   3. $A \cap A' = \emptyset$
   4. $A - B = A \cap B'$
3. De Morgan's laws:
   1. $(A \cup B)' = A' \cap B'$
   2. $(A \cap B)' = A' \cup B'$
4. consistency principle:
   1. $A \subseteq B$ iff $A \cup B = B$
   2. $A \subseteq B$ iff $A \cap B = A$