

Candidates must complete this page and then give this cover and their final version of the extended essay to their supervisor.

Candidate session number

Candidate name

School number

School name

Examination session (May or November)

May

Year

2013

Diploma Programme subject in which this extended essay is registered: Physics

(For an extended essay in the area of languages, state the language and whether it is group 1 or group 2.)

Title of the extended essay: How do the properties of a catenary affect the build process of a suspension bridge?

Candidate's declaration

This declaration must be signed by the candidate; otherwise a grade may not be issued.

The extended essay I am submitting is my own work (apart from guidance allowed by the International Baccalaureate).

I have acknowledged each use of the words, graphics or ideas of another person, whether written, oral or visual.

I am aware that the word limit for all extended essays is 4000 words and that examiners are not required to read beyond this limit.

This is the final version of my extended essay.

Candidate's signature:

Date:

Supervisor's report and declaration

The supervisor must complete this report, sign the declaration and then give the final version of the extended essay, with this cover attached, to the Diploma Programme coordinator.

Name of supervisor (CAPITAL letters)

Please comment, as appropriate, on the candidate's performance, the context in which the candidate undertook the research for the extended essay, any difficulties encountered and how these were overcome (see page 13 of the extended essay guide). The concluding interview (viva voce) may provide useful information. These comments can help the examiner award a level for criterion K (holistic judgment). Do not comment on any adverse personal circumstances that may have affected the candidate. If the amount of time spent with the candidate was zero, you must explain this, in particular how it was then possible to authenticate the essay as the candidate's own work. You may attach an additional sheet if there is insufficient space here.

K: Holistic judgement:

_____ wants to be an engineer and decided to choose a topic related to that. The physics behind the properties of a catenary is quite hard and involves math beyond the requirements, differential equations especially. The initial difficulty for _____ was to identify what property to study and how to go about it without facing mathematical problems that were too hard. Over the summer he independently came up with the idea of having an experimental approach and to build a catenary model himself. On his own he contacted a university to get sensor equipment as he did not want to wait for the new term to start. The model was crafted with the variables tested in mind.

_____ has demonstrated the equipment for me and shown how the placing of the weights affect the strain and shape of the catenary.

In his viva voce _____ explained that through this project he has started to fully appreciate the hard work needed to get a full understanding of the physics behind engineering, and he still wants to pursue this as a carrier. Of course I cannot guarantee that _____ understands all of the physics that is referred to in his bibliography, but I can clearly say that he has acquired the necessary knowledge in this literature to understand and analyse his investigation.

_____ has followed the internal deadlines, which has allowed me to steadily monitor the progress of the investigation and writing. I can confidently say that this is his work.

This declaration must be signed by the supervisor; otherwise a grade may not be issued.

I have read the final version of the extended essay that will be submitted to the examiner.

To the best of my knowledge, the extended essay is the authentic work of the candidate.

I spent 5 hours with the candidate discussing the progress of the extended essay.

Supervisor's signature

Date

Assessment form (for examiner use only)

Criteria	Achievement level					
	Examiner 1	maximum	Examiner 2	maximum	Examiner 3	
A research question	<input type="text" value="2"/>	2	<input type="text"/>	2	<input type="text"/>	
B introduction	<input type="text" value="0"/>	2	<input type="text"/>	2	<input type="text"/>	
C investigation	<input type="text" value="3"/>	4	<input type="text"/>	4	<input type="text"/>	
D knowledge and understanding	<input type="text" value="2"/>	4	<input type="text"/>	4	<input type="text"/>	
E reasoned argument	<input type="text" value="3"/>	4	<input type="text"/>	4	<input type="text"/>	
F analysis and evaluation	<input type="text" value="2"/>	4	<input type="text"/>	4	<input type="text"/>	
G use of subject language	<input type="text" value="2"/>	4	<input type="text"/>	4	<input type="text"/>	
H conclusion	<input type="text" value="2"/>	2	<input type="text"/>	2	<input type="text"/>	
I formal presentation	<input type="text" value="2"/>	4	<input type="text"/>	4	<input type="text"/>	
J abstract	<input type="text" value="1"/>	2	<input type="text"/>	2	<input type="text"/>	
K holistic judgment	<input type="text" value="2"/>	4	<input type="text"/>	4	<input type="text"/>	
Total out of 36	<input type="text" value="21"/>		<input type="text"/>		<input type="text"/>	

Extended Essay
Session: May 2013
Word count: 3743

Catenary

How do the properties of a catenary affect the build
process of a suspension bridge?



Abstract

A catenary is the shape that a flexible wire of uniform density takes when it is hanging freely between two points. Whilst the shape is simple, the mathematics underlying the catenary is complex – particularly when weight is applied. This essay examines the mathematics and investigates the way in which the properties of a catenary affect the design and engineering of a suspension bridge. The changes to the catenary under different weights are plotted using both mathematics and experimentation and then related back to the build process of a suspension bridge.

The first experiment checks the calculations associated with catenaries and demonstrates that the experimental set-up is valid. In this experiment, catenaries were created and measurements taken which, when calculated, produced a scalar “a”. This worked in the standard parametric equation to generate a mirror image of the curve in a computer graphing application.

The second experiment looks at the effects of applying weight to the catenary. It is designed to replicate, on a small scale, the catenary’s impact on the build process of a suspension bridge and the design and engineering issues this raises. In this experiment both the changes to the shape of the catenary and the tension at its top ends were measured. The results showed the centre of the catenary rising as weight was applied and demonstrated the importance of weight being applied to the catenary in balanced pairs. This is then correlated to images of partially constructed suspension bridges. The measurements of tension in the different catenaries generated results which relate directly to bridge design showing that shallower catenaries are more suited to long suspension bridges, whereas deeper catenaries should be chosen for shorter bridges.

The design of the investigation was relatively simple but generated results relevant to issues faced by engineers building suspension bridges.

Word count: 299

Table of Contents

Abstract	1
1. Introduction	4
1.1. Background.....	5
1.2. Mathematical Concepts	6
2. Setup	7
2.1. Materials	7
2.2. Build	7
3. Experiment 1	9
3.1. Question.....	9
3.2. Method.....	9
3.3. Fair test	10
3.3.1. Independent variables.....	10
3.3.2. Dependent variables	10
3.3.3. Constant variables	10
3.4. Results:	10
3.5. Visual Observation	11
3.6. Discussion.....	11
3.7. Conclusion	12
4. Experiment 2	13
4.1. Question.....	13
4.2. Method.....	13
4.3. Fair test	14
4.3.1. Independent variables.....	14
4.3.2. Dependent variables	14
4.3.3. Constant variables	14
4.4. Results	14
4.4.1. Test 1	15
4.4.2. Test 2	15
4.5. Discussion.....	16
4.6. Conclusion	17
5. Summary	20
6. Appendix.....	22
6.1. Experiment 1.....	22
6.1.1. Test 2:.....	22
6.1.2. Test 3:.....	22

6.1.3. Test 4:.....	22
6.2. Experiment 2.....	23
6.2.1. Test 1	23
6.2.2. Test 2	23
6.3. Bridge examples	25
6.3.1. Forth Road Bridge	25
6.3.2. Manhattan Bridge.....	25
6.3.3. Bronx-Whitestone Bridge	25
6.3.4. Golden Gate Bridge.....	26
7. Bibliography.....	27

1. Introduction

People have been building suspension bridges, in one form or another, for thousands of years¹. The earliest examples were simple rope bridges slung between two points. Gradually, the bridges became more sophisticated, for example adding wooden walkways. In more modern times, suspension bridges have become longer and need to be more stable. Engineers have invested significant time in determining the best way to design and build stable, flat, suspended structures. The lynchpin to all this is being able to understand the properties of the catenary.

A catenary is the shape that a flexible wire of uniform density takes when it is hanging freely². They are everywhere in the modern world; from suspension bridges, to overhead power lines, to the simple ropes between poles used to mark out where to queue at the airport. Although the shape is simple, the mathematics underlying the catenary is complex – particularly when weight is applied. When a person walks along a simple rope bridge, the shape of the catenary changes according to their position on the bridge.

Understanding the mathematics, and using it to predict how a catenary will respond to the application of a given weight or weights, is of key importance when designing a structure such as a suspension bridge. The aim of this essay is to examine the effect that the properties of the catenary have on the build process of suspension bridges. To do this I will investigate the effect that the application of altering weights has on a catenary and how this then impacts the way in which suspension bridges need to be planned and constructed. I will plot a number of catenaries using both mathematics and experimentation to show how the catenary changes shape and tension under the influence of weights. From this I will draw conclusions as to how the bridge should be built.

¹ Tilden, Eric. The History of the Suspension Bridge. *ehow*. [Online] Demand Media, Inc, date not given. [Cited: july 13, 2012.] http://www.ehow.com/facts_5184027_history-suspension-bridge.html.

² Collins, Harper;. catenary. *Collins English Dictionary*. s.l. : HarperCollins Publishers, HarperCollins Publishers.

1.1. Background

The catenary was first examined and investigated by the mathematician Galileo Galilei. Although he did not find the formula to describe a catenary, he did note that it is a close relative to a parabola, as shown in figure 1.1. The word “catenary”, which is derived from the Latin word for chain, “catena”, was first used by Christiaan Huygens in the context of the challenge set by Jakob Bernoulli, a Swiss

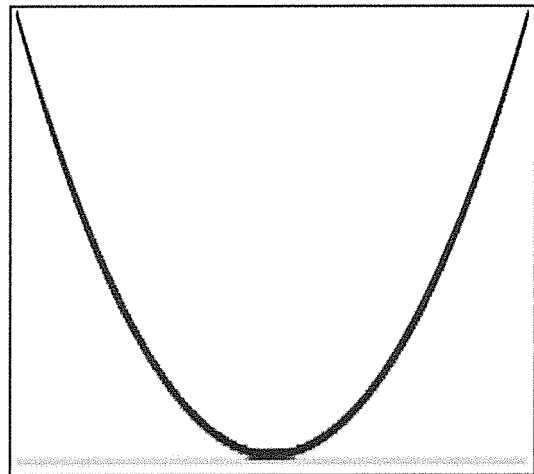


Figure 1.1
Catenary – black dotted line
Parabola – red solid line (5)

mathematician, in the May 1690 edition of *Acta Eruditorum*³. This challenge, to find the equation of the “chain curve”, was answered by three mathematicians, Huygens, Leibniz and Johann Bernoulli, in three different ways⁴. As a result there are still several different notations for the formula of a catenary. However, the most frequently used notation is the parametric equation which is easy to graph using modern calculators. This is the notation I use in this essay.

There is a significant body of research and academic articles on catenaries and suspension bridges. However, these all examine the mathematics which proves that whilst the bridge’s suspension wires are a catenary at the beginning of the build process, they become a parabola once the roadway is complete and all the weight is applied⁵⁶.

³ . **anonymous**. The Gateway Arch - A Trigonometric delight. *Maths in the city*. [Online] University of Oxford , 2011. [Cited: 28 october 2012.] <http://www.mathsinthecity.com/sites/gateway-arch-trigonometric-delight>.

⁴ **Weisstein, Eric W**. Catenary. *wolfram mathworld*. [Online] Wolfram Research, Inc, no date given. [Cited: july 9, 2012.] <http://mathworld.wolfram.com/Catenary.html>

⁵ *A General Form of the Suspension Bridge Catenary*. **Freeman, Ira**. 8, s.l. : Bulletin of the American Mathematical Society, 1925, Vol. 31. 1088-9485.

⁶ *A classroom investigation into the catenary*. **Staples, Ed**. 2, s.l. : Australian Senior Mathematics Journal, 2011, Vol. 25, pp. 43-54. 08194564.

Since my experiments focus only on the point before the roadway is completed, these articles are not of direct relevance.

1.2. Mathematical Concepts

The parametric equation I use to describe a catenary comes in the form of a hyperbolic function using a scalar “a”, as follows.

$$y = \frac{1}{a} \cosh(ax) = \left[\frac{e^{ax} + e^{-ax}}{2a} \right] \text{ Hyperbolic function}^{78} \text{ of a catenary}$$

Where $a = \frac{T_0}{\lambda g}$

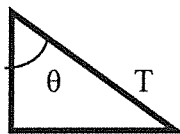
λ = weight of cable per unit length

g = the earth's gravity

T_0 = tension along the x axis where T is the tension of the rope

$T_0 = T \sin \theta$ (relation between the two tensions described above)

$$a = \frac{T \sin \theta}{\lambda g}$$



91011

⁷ Durell, C V and Robson, A. *Advanced Trigonometry*. London : G. Bell and Sons, LTD, 1967. p. 104.

⁸ *A classroom investigation into the catenary*. Staples, Ed. 2, s.l. : Australian Senior Mathematics Journal, 2011, Vol. 25, pp. 43-54. 08194564

⁹ Sokolnikoff, I S and Redheffer, R M. *mathematics of physics and modern engineering*. New York; Toronto; London : McCraw-Hill book company, INC.; Kogakusha company, LTD., 1958. pp. 40-42. 57-12914.;

¹⁰ Cmglee. catenaries. *wikipedia*. [Online] Wikimedia Foundation, Inc., 19 January 2012. [Cited: 4 jly 2012.] http://en.wikipedia.org/w/index.php?title=File:Comparison_catenary_parabola.svg.

¹¹ Weisstein, Eric W. Catenary. *wolfram mathworld*. [Online] Wolfram Research, Inc, no date given. [Cited: july 9, 2012.] <http://mathworld.wolfram.com/Catenary.html>.

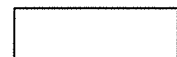
2. Setup

2.1. Materials

- a) 9 weights of 100 grams each with wire hanging attachment
- b) 150 cm heavy cord
- c) A strain gauge (a luggage scale)
- d) Building materials (Meccano)
 - A base or stand
 - Two uprights to support the structure
 - Two pulleys
 - A small, strong clamp
 - A 60 cm x 49 cm backboard made of thick plastic hobby board
 - A frame to hold the strain gauge in place.
- e) 9 wires to hang the weights from with washers on one end
- f) Squared paper on which to make markings and measurements
- g) Tape
- h) Several coloured pens to plot measurements with

2.2. Build

- a) Build two upright supports approximately 60 cm tall, ensuring that they are mirror images of each other.
- b) Attach both of these to a base.
- c) Attach pulleys close to the top of the front side of each stand. These can be level or at different heights since it won't affect the curve.



d) Attach the clamp that holds the cord in place to one upright and attach the strain gauge to the other side directly below the pulley.

e) Attach the backboard between the two stands (Figure 2.1).

f) Tape the squared paper to backboard. (Figure 2.2)

g) Once the basic build is complete, attach one end of the cord to the strain gauge, drape the cord over the two pulleys and thread the other end through the clamp.

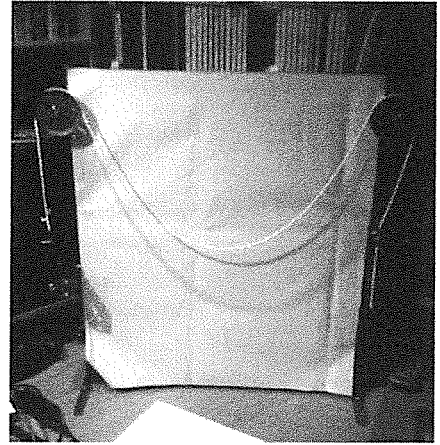


Figure 2.2

3. Experiment 1

3.1. Question

Is it possible to check the theoretical mathematics of catenaries using my equipment without any weights attached, in particular to calculate a value for the scalar “a”, in order to provide a control for later tests using weights?

3.2. Method

- a) Take the cord off the test rig. Weigh it and measure its length to find its weight per unit length
- b) Reattach one end of the cord to the clamp and allow the cord to form a balanced catenary between the two pulleys.
- c) Measure the amount of excess cord hanging down over the pulley once the system is balanced and use the measurement to calculate its weight. This will allow the net tension acting on the cord on the other side of the pulley to be measured (n.b. this step is only necessary if the strain gauge being used is not able to measure the tension of the catenary).
- d) Measure the angle of the catenary coming off the pulley above the strain gauge and use Pythagoras’ theorem to calculate the tension of the cord along the x-axis using the net tension obtained in step c). ($T_0 = T \sin \theta$, see math section above).
- e) Take the value of T_0 obtained in this way and place it into the formula for finding the scalar “a”

$$a = \frac{T \sin \theta}{\lambda g}$$

- f) Repeat the experiment 3 times with different depths of catenary.
- g) Record the results in a table.
- h) Graph the results observed using an online application and compare the graphs to actual observations.

3.3. Fair test

3.3.1. Independent variables

The tension held by the cord; the value of scalar a .

3.3.2. Dependent variables

The depth of the catenary (vertical distance between centre of the pulleys and the lowest point of the catenary); the length of the cord.

3.3.3. Constant variables

Distance between the two pulleys; the weight per unit length,

3.4. Results:¹²

All errors were created with the help of an error calculator

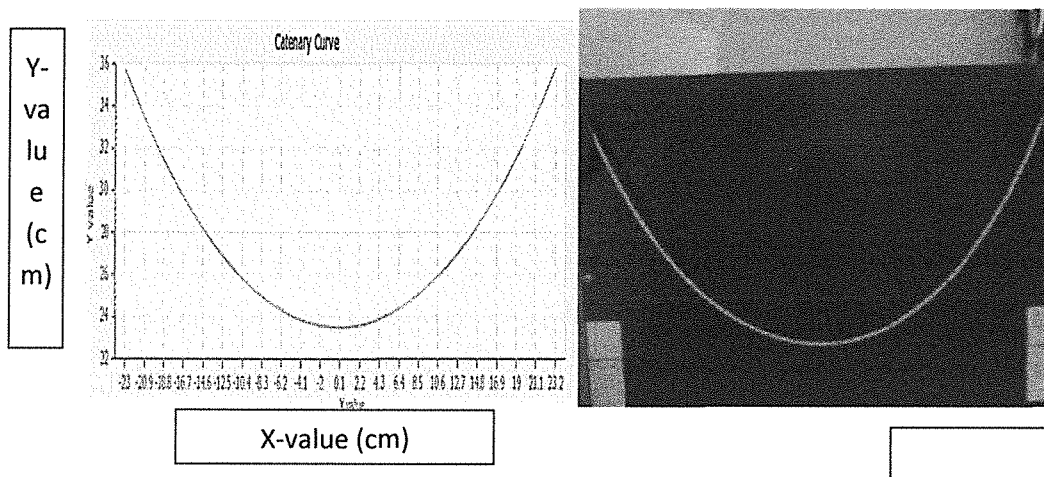
	Test 1	Test 2	Test 3	Test 4
Distance (L)(cm)	46±0.2	46±0.2	46±0.2	46±0.2
Depth (H)(cm)	23,5±0.2	28,5±0.2	19,5±0.2	14,5±0.2
Length (s)(cm)	70,5±0.3	78,5±0.3	64,5±0.3	58,5±0.3
Weight per unit length (λ)(g/cm)	0,7±0.0067	0,7±0.0067	0,7±0.0067	0,7±0.0067
overhang (cm)	36,5±0.2	39,5±0.2	35±0.2	30,5±0.2
Tension (y-axis)(g)	25,5±0.28	27,6±0.3	24,5±0.27	21,3±0.25
Angle (ϕ)(degree°)	65±1	70±1	60±1	48±1
Tension (x-axis)(g)	23,1±0.5	25,9±0.55	21,2±0.48	15,8±0.41
Scalar (a)	3,36±0.08	3,77±0.088	3.09±0.076	2.3±0.064

Figure 3.1

¹² Bertrand, Gary L. Uncertainty Calculator. *Welcome to Chez Bertrand*. [Online] Missouri University of Science and Technology, No Date of Creation. [Cited: 11 november 2012.] <http://web.mst.edu/~gbert/java/uncertainty.html>

3.5. Visual Observation

Test 1¹³



The visual comparisons of the other tests are contained in the appendix.

3.6. Discussion

During the set-up, it became clear that the strain gauge I intended to use to measure the tension in the cord wasn't actually sensitive enough for this experiment. I therefore decided to use gravity to calculate the tensions instead. To do this, I set up the experiment so that the loose end of the cord would balance out the gravitational pull on the catenary. This allowed me to calculate manually the tension of the cord. Calculating the tension in this way is relatively accurate because the balanced catenary requires a specific tension to ensure that the neither the weight of the excess cord pulls the catenary straight over the pulleys nor that the catenary itself becomes deeper. However, it is necessary to consider whether the pulleys ran freely or whether they caused some frictional resistance which made it easier to balance out the system.

In order to take the measurements used in the tables and graphs, I used basic equipment such as rulers and relied on my own visual observations. This may have

¹³ **Anonymous.** <http://easycalculation.com/graphs/catenary.php>. *easycalculation.com*. [Online] No Date of Creation. [Cited: 20 June 2012.] <http://easycalculation.com/graphs/catenary.php>.

caused some of the more noticeable errors in the raw data (Table 3.1). However, despite this, once I had used the raw data in the equations to get the scalar “a” in each test, the errors were reduced to a relatively low level. This is apparent in both the error bands in the table and in the similarity between the actual catenary used for taking the measurements and the one produced by graphing (see figure 3.2)

Although it might appear that the graphs produced in tests 3 and 4 (see appendix) do not match those of their real life counter parts, this is because of peculiarities in the graphing application which meant that the graphs could only be drawn over a fixed width which did not match that of the real life tests. This is more evident in tests 3 and 4 than in the first two. In order to correct this peculiarity the graphs had to be squeezed along the x-axis to make them match the actual width. No adjustment was made to the y-axis or to the photographs of the tests. However, in spite of these errors, the visual similarity remains evident.

3.7. Conclusion

The objective of this test was to check that my test rig allowed me to set up true catenaries as a control for my later tests using weights. In this regard, the experiment was a success. The graphs generated with the raw data collected from the catenaries created in my experiments matched closely the real catenaries, as evidenced in the photographs.

As a result, I was reassured that the design of my second experiment was valid and would produce relevant data with respect to the way in which catenaries react when weights are applied.

4. Experiment 2

4.1. Question

How does distributing weight along a catenary affect the shape of a catenary? What strain does the tension from these loads cause in the pylons holding the catenary up? How does adding weight to the catenary affect how a bridge might be built?

4.2. Method

- a) Attach the loose end of the cord to the stress gauge. Set up a catenary.
- b) Measure the distance between the two pulleys. Divide this distance by 10 to obtain the spacing between the nine wires and draw vertical lines on the backing paper at these points.

- c) Attach wires to the cord level with each line.
- d) Attach a washer to the end of each wire and record the tension using the strain gauge. Use the tension recorded and the angle of the catenary to calculate the forces that would be acting on the pylon on both the x- and the y-axes. (Figure 4.1)

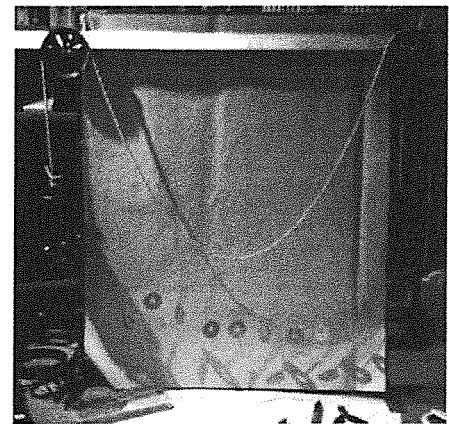
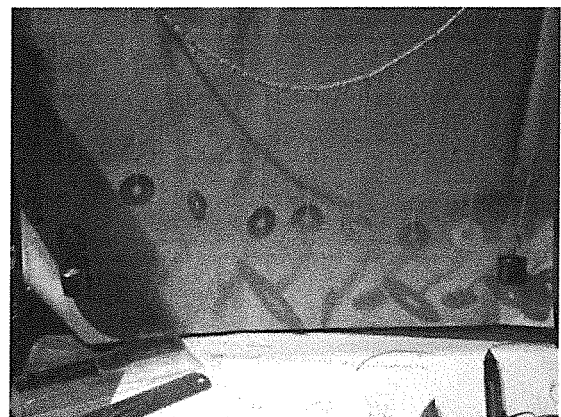


Figure 4.1

- e) Mark a point on each wire on a level horizontal line as reference points for the displacement graphs. (Figure 4.2)
- f) Mirror these points onto the squared paper on the backboard.
- g) Attach a weight to the washers on the two outermost wires.



- h) Transfer the new positions of the points on the wires to the squared paper.
- i) Record the new tension in the cord.

- j) Working inwards, attach weights to the next two wires.
- k) Repeat steps h) – j) until all 9 wires have a weight hanging from them.
- l) Reset the catenary to a new depth. Repeat steps c) – k).

4.3. Fair test

4.3.1. *Independent variables*

The load held by the cord; the distribution of said load.

4.3.2. *Dependent variables*

Shape of the catenary; effects of the cord's tension on the pylons.

4.3.3. *Constant variables*

The distance between the two pulleys; the number of wires and washers attached to the cord; the weight per unit length of the cord.

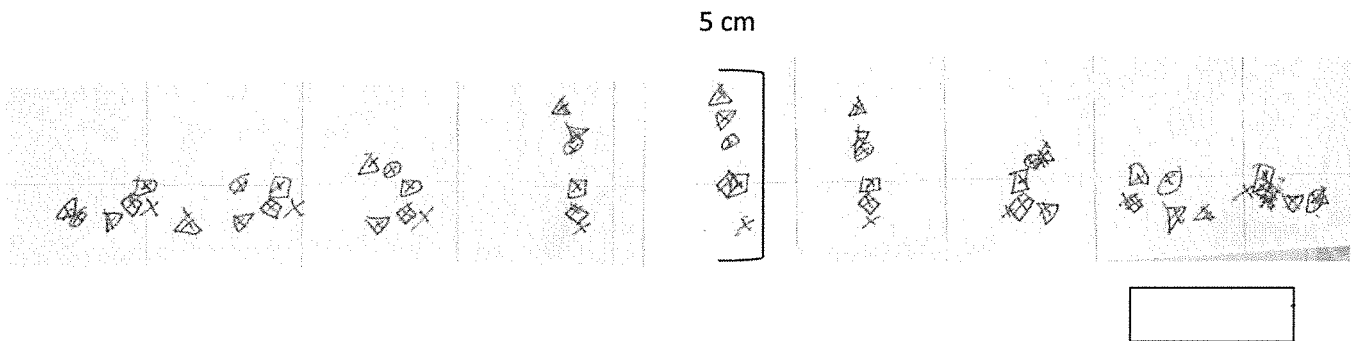
4.4. Results

The results obtained on the backing paper were differentiated as follows :

- Squares = no weights,
- Circle = 2 weights,
- Triangle = 4 weights,
- Upside-down triangle = 6 weights,
- Diamond = 8 weights,
- Green cross = 9 weights.

Figures 4.3 and 4.5 show the raw data of the displacement of the catenary during the experiment. This raw data is shown in graphic form in the appendix.

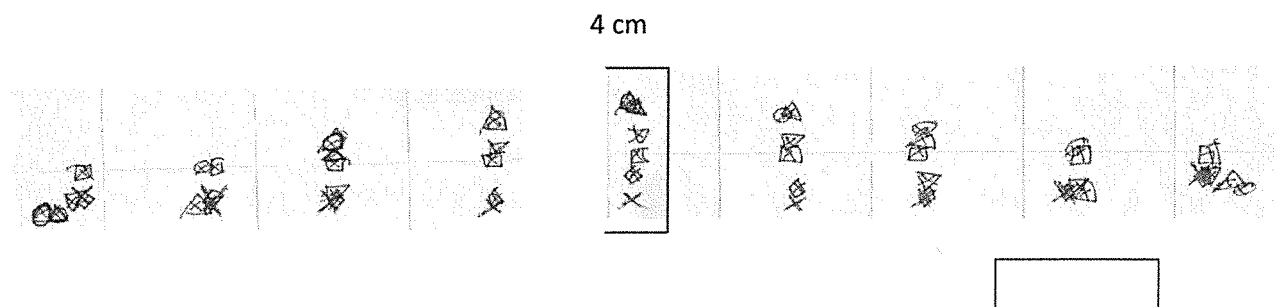
4.4.1. Test 1



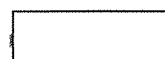
	Tension (net total)(grams)	Angle (degree°)	Tension (x-axis)(grams)	Tension (y-axis)(grams)
No weights	40±1	45±1	28,3±0.98	28,3±0.71
2 weights	140±1	30±1	70±2.4	121,1±0.87
4 weights	260±1	35±1	149±4.9	212,9±0.82
6 weights	400±1	35±1	230±6.8	327,6±0.83
8 weights	540±1	40±1	347±9.2	413,6±0.78
9 weights	600±1	40±1	390±10	456,6±0.79

4.4.2. Test 2

In the second test all variables were set up the same except the depth of the catenary was much shallower. This was done so the two experiments could represent different types of bridges; the first being a short bridge with relatively high pylons, the second a much longer bridge with lower pylons.

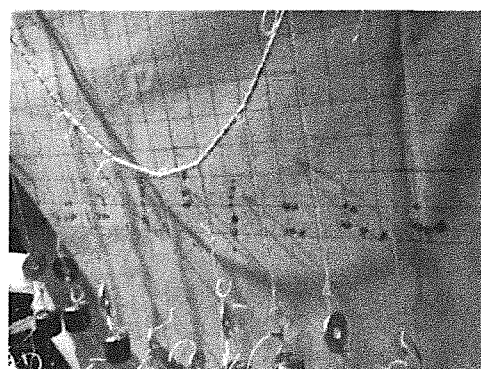


	Tension (y-axis)(grams)	Angle (Tension (x-axis)(grams)	Tension (y-axis)(grams)
No weights	Too small	15 ± 1	Can't calculate	Can't calculate
2 weights	120 ± 1	5 ± 1	10 ± 2	$119,5 \pm 0.1$
4 weights	220 ± 1	10 ± 1	38 ± 3.7	$216,5 \pm 0.99$
6 weights	320 ± 1	10 ± 1	56 ± 5.4	$314,9 \pm 0.99$
8 weights	420 ± 1	15 ± 1	109 ± 7.1	$405,3 \pm 0.97$
9 weights	480 ± 1	15 ± 1	124 ± 8.2	$463,2 \pm 0.98$



4.5. Discussion

I had to set up this experiment twice because the first time the results were distorted. The distortion was caused by the initial decision to use twine to hang the weights on. When I attached the weight, the twine either stretched or unwound slightly. As a result, when all nine weights were attached, the line formed by the marked points



was not even remotely horizontal (Figure 4.7). I resolved this issue by substituting the twine with wire. This had the added advantage of being easier to work with.

I considered whether the cord itself could have stretched and damaged the results. However, I discounted this problem because I had measured the length of the catenary before starting the experiment and had, therefore, reference points showing where the catenary met the pulleys. These allowed me to double check the final length of the cord and determine whether it had stretched or not. I concluded from this that any stretch could only have been a few millimetres which would have had negligible effects on the catenary.

Another issue I encountered came from the fact that I mapped the points from the wires to the paper by eye, which is not very accurate. Therefore, I used crosses to indicate a margin of error in mapping the position of the points to the paper. The shapes over the crosses indicate which set of weights each point belongs to. Unfortunately, this makes them quite difficult to read (Figures 4.3 and 4.5). I resolved this by graphing each component of the experiment separately. (Appendix 6.2.1 and 6.2.2)

In addition, I made my own weights for these experiments. Although they were all made to the same measurements there could be some slight variation due to weight lost during the sanding process. This could mean that the load on the catenary were not evenly dispersed. However checks showed that any differences were so minor that they should not cause errors in the data.

The placement of the clamp may have had an impact on my results because the total tensions recorded were slightly higher than expected. This is discussed in further detail in the conclusion.

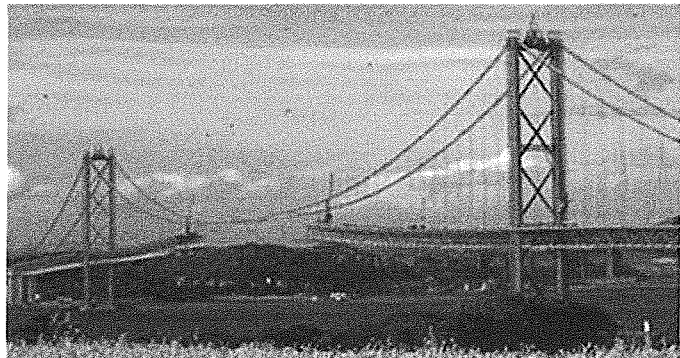
The error margin for the tension measurement is quite large. The strain gauge I used was set to measure in kilos which, when combined with the low weights used, lead to quite large errors. The size of the errors therefore dictated the choice of grams as the unit of measurement rather than Newtons. Translating my results into Newtons would simply have magnified the errors.

4.6. Conclusion

It would have been expected that the strain gauge indicate a tension of approximately 450g since the catenary was loaded with 900g of weight. However, this was not the case. It is probable that this is due to the set-up of the experiment. The clamp may have been sited too high on the test-rig, causing the pulley with the stress gauge to take more of the total tension, particularly on the shallower catenary.

However, in spite of this unexpected result, the data obtained from the second experiment is of interest from a structural and construction point of view. It shows that when weights are added to the outermost points of the cord, the points on the cord between those weights move upwards. This upward movement continues until the third set of weight is added. From this stage, the points on the cord start to drop until they make a flat line again.

This is a physical ¹⁴ demonstration of a crucial aspect of suspension bridge construction. When suspension bridges are being constructed, the platform is built out from each end simultaneously. Furthermore, the engineers have to ensure that the platform is also built



upwards at a slight angle so that it sits slightly above its final position until the full weight is applied. Construction needs to proceed from both ends of the bridge at the same pace to prevent the platform on one side of the bridge from rising into the air and damaging a large section of the bridge. This construction process is very obvious in photographs of partially built suspension bridges (Figure 4.8 and appendix).

The movements on the catenaries used in the experiments were only small but had they been scaled up to the size of even a small bridge, they would represent huge changes to the shape of the supporting catenaries.

Observation of the asymmetrical aspects of the experiments performed reinforced the fact that the build must occur from both ends of the bridge simultaneously. When the first

¹⁴ Figure 4.8 : Cornwell, Tim (not photographer). Firth of Forth Suspension Bridge under construction, Edinburgh to Fife, Scotland. [Photo] Edinburgh : Flickr: Totally random scanned 35mm slides, 1963

weight in a pair was added, the catenary was significantly lower on that side than on the unweighted side. This is a direct example of the fact that if one side of a suspension bridge is being built faster than the other then the platform on that side will droop while the other side rises. The stresses on the platform would be detrimental to the construction. On the contrary, the experiments show that if both sides are weighted at an even pace, the vertical displacement changes very little. In reality, photographs show that it is preferable to build with a slight upwards curve on the platform so that added compression makes the bridge stronger and sturdier when the two halves meet.

The tension measurements obtained from the experiment, give an insight into the kind of forces imposed upon the pylons. In the very deep catenary tested it would appear that the inward force caused by the tension in the cord is minimal compared to the downward force. However, in the second test, with a shallower catenary, the downward and inward forces are almost equal. In this case, the pylon not only has to brace against compression but also has to be very stiff and well anchored.

This would indicate that when building long suspension bridges a shallow catenary would be the best choice since there is less movement vertically and it allows easier calculation of the upward bow needed for stability. Whereas, when building a shorter suspension bridge, it may be better to choose a deeper catenary in order to minimize the amount of anchoring needed for the bridge.

5. Summary

Both of the experiments I ran for this essay enhanced my appreciation of the complexities of building a suspension bridge and the role of the catenary in this process.

My objective in the first experiment was to check both the mathematics associated with calculating a catenary and the reliability of my experimental set-up. The results showed that the test rig I designed and built worked correctly. I was able to use it to create catenaries and take measurements from them which, when calculated, produced a scalar “a” which worked in the equations to generate a mirror image of the curve in a computer graphing application.

The second experiment was somewhat more complex in that it looked at the effects of the application of weight to the catenary. The objective in this case was to replicate, on a small scale, the impact of the construction process of a suspension bridge on the catenary. In this experiment I measured changes to the shape of the catenary as weight was applied and the variation in tension at the top ends of the catenary. The reason for concentrating on these aspects was their direct relationship to issues faced by engineers planning and building suspension bridges. The results of this experiment demonstrated clearly the fact that, as weight is applied, the centre of the catenary rises. I also observed that in order to preserve the integrity of the catenary, it was necessary to apply the weights to the catenary in pairs at equal distance from the ends. I was able to relate this directly to images of partially constructed suspension bridges which showed both the platform rising towards the centre and the fact that construction proceeds from both end simultaneously.

In addition, the measurements I took of the tension in different catenaries gave me some insight into bridge design. The figures I generated showed that when the catenary was shallow, the requirement to brace against downward tension was less, although the catenary itself required more anchoring. This replicates the requirements for a longer suspension bridge with lower pylons. On the contrary, when the catenary was deeper, there was less

inward tension compared to the downward tension. This equates to a shorter suspension bridge span with higher pylons but less requirement to anchor.

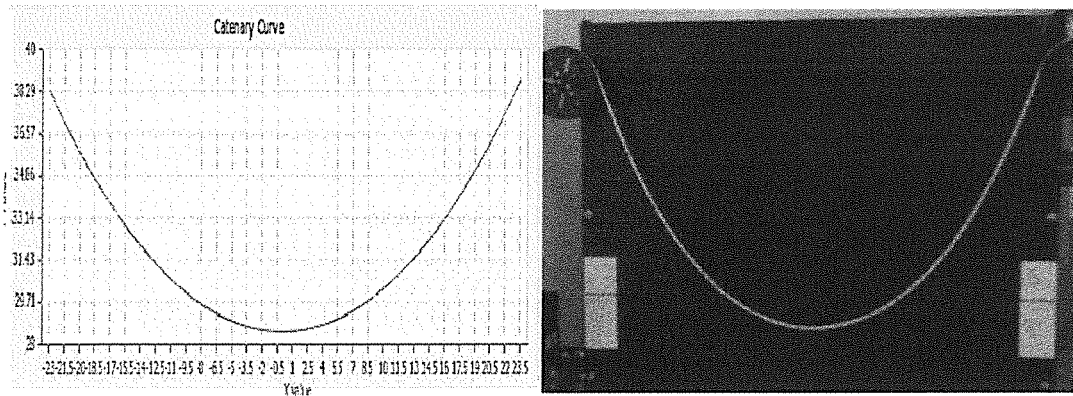
Although the design of my investigation was relatively simple, it generated highly relevant results which related directly to issues faced by engineers designing and constructing suspension bridges.

6. Appendix

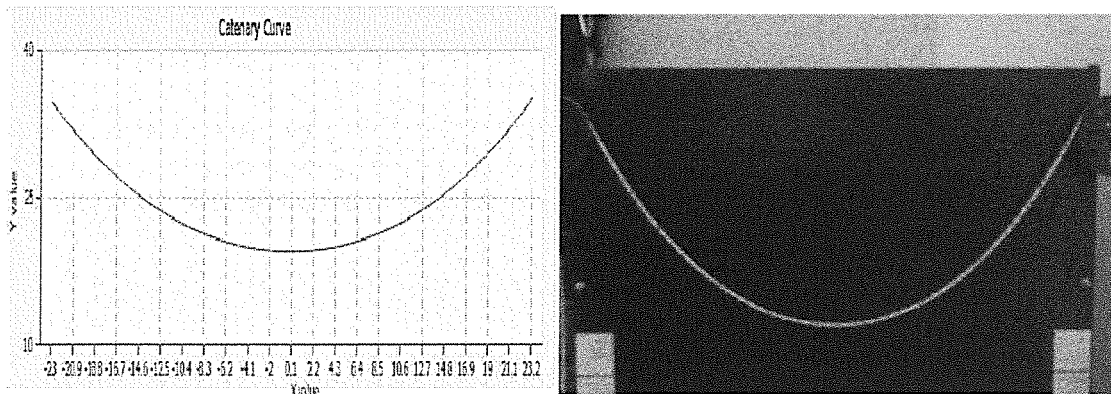
6.1. Experiment 1

The following are visual checks of the calculations obtained from tests 2, 3 and 4 of the first experiment.

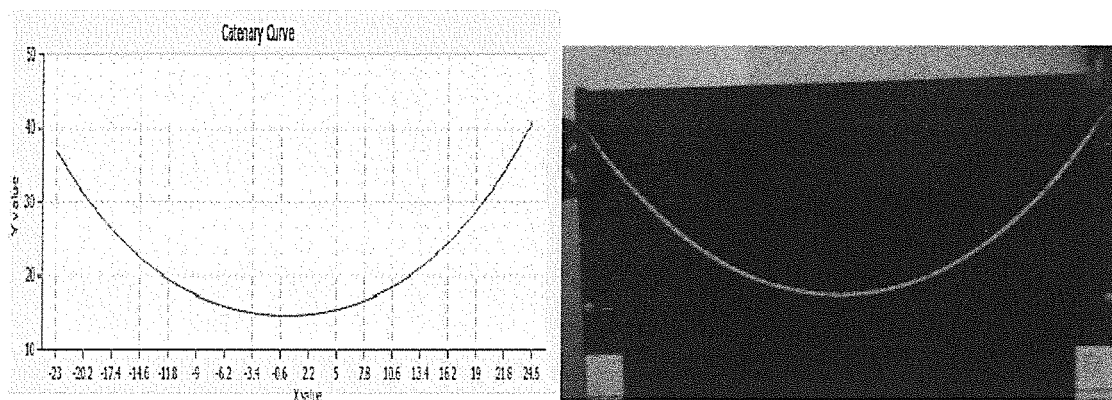
6.1.1. Test 2:



6.1.2. Test 3:



6.1.3. Test 4:

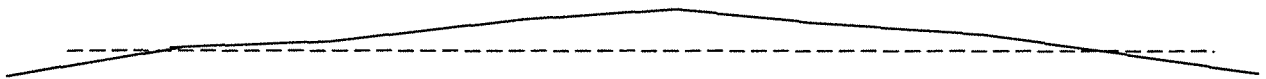


6.2. Experiment 2

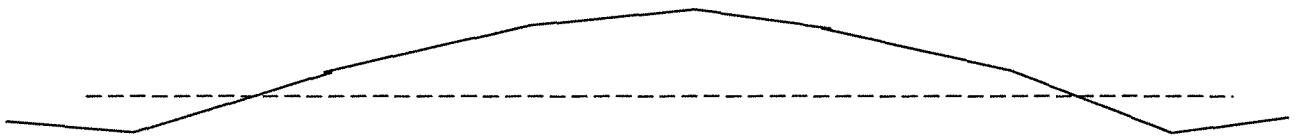
The visual displacements shown below are made in an attempt to convert the information contained in Figures 4.1 and 4.3 into easily understood images. Each one of the images is a scale representation and contains two elements: a base line, the dashed line, which represents the unaltered shape of the catenary; and the solid line which stands for the displacement of the catenary as additional weights are added. The baseline is flat is because the displacement was measured using wires strung along the catenary. Since their net weight was less than that of the catenary, the experiment was unaffected.

6.2.1. *Test 1*

2 weights:



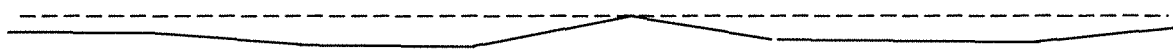
4 weights:



6 weights:



8 weights:

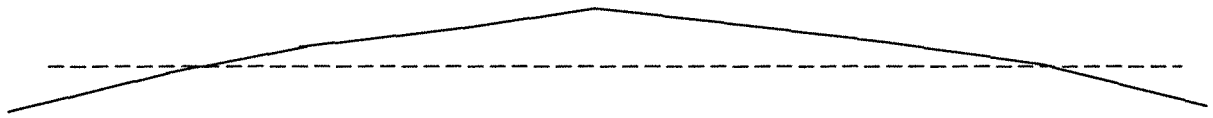


9 weights:

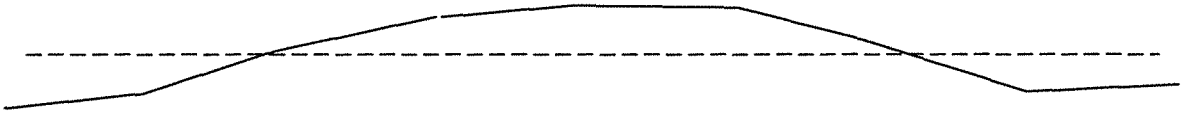


6.2.2. *Test 2*

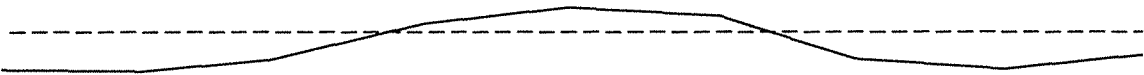
2 weights:



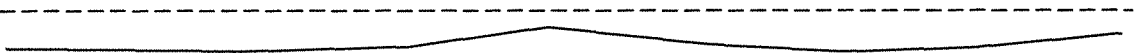
4 weights:



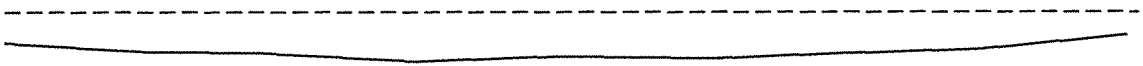
6 weights:



8 weights:



9 weights:

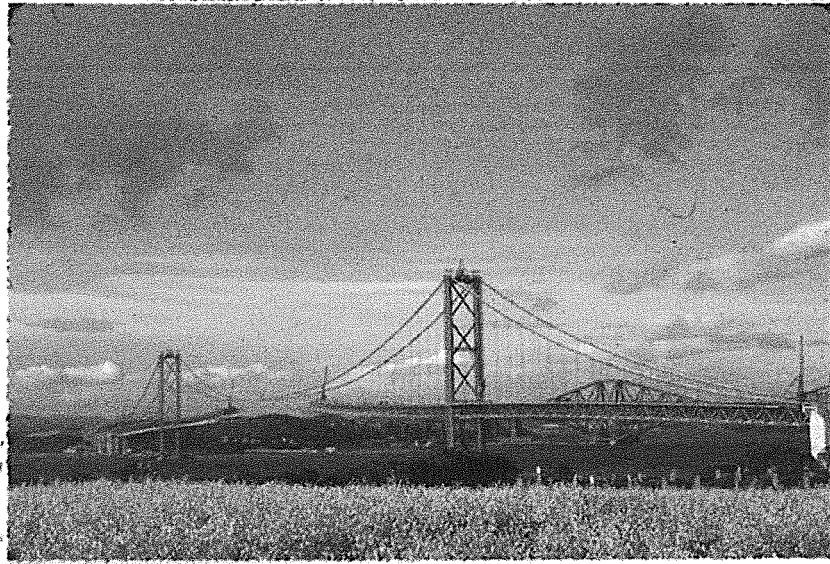


6.3. Bridge examples

The pictures below show a number of famous suspension bridges under construction.

The effect of hanging weight from the main suspension cables during the build process is clearly visible in each picture as is the symmetrical nature of construction.

6.3.1. *Forth Road Bridge*¹⁵



6.3.2. *Manhattan Bridge*¹⁶



6.3.3. *Bronx-Whitestone Bridge*¹⁷

¹⁵ Cornwell, Tim (not photographer). *Firth of Forth Suspension Bridge under construction, Edinburgh to Fife, Scotland*. [Photo] Edinburgh : Flickr: Totally random scanned 35mm slides, 1963

¹⁶ Underhill , Irving. [Photo] New York : s.n., 1909



6.3.4. *Golden Gate Bridge*¹⁸



¹⁷ Chan, Sewell (not photographer);. *70th Birthday of the Bronx-Whitestone Bridge*. [Photo] New York : The New YorkTimes, 1930s

¹⁸ Anonymous. *From The Chronicle Archives: Golden Gate Bridge*. [Photo] San Francisco : San Francisco Chronicle, 1935

7. Bibliography

1. **Tilden, Eric.** The History of the Suspension Bridge. *ehow*. [Online] Demand Media, Inc, date not given. [Cited: 13 july 2012.] http://www.ehow.com/facts_5184027_history-suspension-bridge.html.
2. **Collins, Harper;.** catenary. *Collins English Dictionary*. s.l. : HarperCollins Publishers, HarperCollins Publishers.
3. **Weisstein, Eric W.** Catenary. *wolfram mathword*. [Online] Wolfram Research, Inc, no date given. [Cited: 9 july 2012.] <http://mathworld.wolfram.com/Catenary.html>.
4. **Sokolnikoff, I S and Redheffer, R M.** *mathematics of physics and modern engineering*. New York; Toronto; London : McCraw-Hil book company, INC.; Kogakusha company, LTD., 1958. pp. 40-42. 57-12914.
5. **Cmglee.** catenaries. *wikipedia*. [Online] Wikimedia Foundation, Inc., 19 January 2012. [Cited: 4 jly 2012.] http://en.wikipedia.org/w/index.php?title=File:Comparison_catenary_parabola.svg.
6. **Bertrand, Gary L.** Uncertainty Calculator. *Welcome to Chez Bertrand*. [Online] Missouri University of Science and Technology, No Date of Creation. [Cited: 11 november 2012.] <http://web.mst.edu/~gbert/java/uncertainty.html>.
7. **Durell, C V and Robson, A.** *Advanced Trigonometry*. London : G. Bell and Sons, LTD, 1967. p. 104.
8. **Anonymous.** <http://easycalculation.com/graphs/catenary.php>. *easycalculation.com*. [Online] No Date of Creation. [Cited: 20 June 2012.] <http://easycalculation.com/graphs/catenary.php>.
9. *A classroom investigation into the catenary*. **Staples, Ed. 2**, s.l. : Australian Senior Mathematics Journal, 2011, Vol. 25, pp. 43-54. 08194564.
10. **Anonymous.** *From The Chronicle Archives: Golden Gate Bridge*. [Photo] San Francisco : San Francisco Chronicle, 1935.
11. **Underhill , Irving.** [Photo] New York : s.n., 1909.
12. **Chan, Sewell (not photographer);.** *70th Birthday of the Bronx-Whitestone Bridge*. [Photo] New York : The New YorkTimes, 1930s.
13. **Cornwell, Tim (not photographer).** *Firth of Forth Suspension Bridge under construction, Edinburgh to Fife, Scotland*. [Photo] Edinburgh : Flickr: Totally random scanned 35mm slides, 1963.
14. **anonymous.** The Gateway Arch - A Trigonometric delight. *Maths in the city*. [Online] University of Oxford , 2011. [Cited: 28 october 2012.] <http://www.mathsinthecity.com/sites/gateway-arch-trigonometric-delight>.
15. *A General Form of the Suspension Bridge Catenary*. **Freeman, Ira. 8**, s.l. : Bulletin of the American Mathematical Society, 1925, Vol. 31. 1088-9485.