

Solve 2B and 2C

Basis

a basis of V is a list of vectors in V that is linearly independent and spans V

- * a list v_1, v_2, \dots, v_n of vectors in V is a basis of V if and only if every $v \in V$ can be uniquely written in the form
$$v = a_1 v_1 + \dots + a_n v_n$$
where $a_1, \dots, a_n \in F$

- * every spanning list in vector space can be reduced to basis of vector space
- * every finite dimensional space has a basis
- * every linearly independent list extends to a basis

Dimension

A reasonable dimension should force dimension of F^n to equal n .

- * any 2 basis of a finite dimensional space have same length

V is finite, B_1 and B_2 are bases of V .

B_1 is linearly independent in V .

B_2 spans V .

So B_1 is at most the length of B_2 and vice versa.

\therefore We can formally define -
dimension of a finite ^{dimensional} vector space is the length of any basis of vector space
(length is the count of basis)