What is a vector space?

A set of elements that can do addition and scalar multiplication and follow a set of rules.

1. associative
2. commutative
3. additive identity, (there exists $0 \in V$ such that $v + 0 = v$ for all $v \in V$)
4. additive inverse (for every $v \in V$, there is $w \in V$ where $v + w = 0$)
5. mul identity
6. distributive

Q. vector space has unique identity

$v \in V$, $v + 0 = v \rightarrow$ additive identity.

proof by contradiction, if we have 2 identity 0 and $0'$.

$v + 0 = v \qquad v + 0' = v$

$v + 0 = v + 0'$

if $v = 0'$

~~$0 + 0 = 0 + 0'$~~ (since 0 and $0'$ are additive identity

~~$= 0 = 0'$~~

$0' + 0 = 0'$ (0 is identity)

also $0' + 0 = 0$ ($0'$ is identity)

$\therefore 0 = 0'$

Q every element ~~in vector space~~ has unique additive inverse.

$v \in V$ $\quad w$ and w' are inverses

$v + w = 0$ (w is inverse) $\rightarrow$ (1)

$v + w' = 0$ (w' is inverse)

$w = w + 0$ (identity)

from (1)

$w = w + 0$

$= w + (v + w')$

$w = (v + w) + w'$

w is inverse, so

$w = 0 + w'$

$w = w'$

Exercise 1B

1. prove that $-(-v) = v$ for every $v \in V$

$v + (-v) = 0 \longrightarrow$ additive inverse property

$-(-v) = v$ (means v is inverse of $-v$)

~~$v + (-v) = 0$~~

$-v - (-v) = 0$

so take

$-v - (-v) = 0$

$-v + v = 0$

we know for every v, there is a unique inverse.

$\therefore v = -(-v)$

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2 suppose $a \in F$, $v \in V$ and $av = 0$, prove that $a = 0$ or $v = 0$.

$av = 0$, so either a or v must be 0.

Lol, doesnt work.

assume $a \neq 0$

$a \in F$, so a has an inverse

$$\frac{1}{a}(av) = 0 \times a$$
$$v = 0$$

4. empty set is not a vector space, why?

no additive identity.

5. show for a vector space, additive inverse can be replaced with

$0v = 0 \quad \forall v \in V$

$v + w = 0 \longrightarrow$ inverse additive

$v + w = 0v$

we know for inverse $w = -v$

$$v + (-v) = 0v$$
$$= 0 = 0$$