(3) if you take a scalar like $\frac{3}{2}$

$\frac{3}{2}(u,v) = (1.5, 1.5) \notin R^2$

not R^2

$\therefore$ false

Chapter - 2

Finite Dimensional Vector Spaces

Linear combination

The sum of scalar multiples of vectors in a list

a linear combination of a list $v_1, v_2 \ldots v_m$ of vectors in V is of form

$a_1 v_1 + a_2 v_2 \text{ --- } a_n v_n$

$a_1, a_2 \ldots \in F$

Span

The set of all linear combinations of list of a vector $v_1, \ldots v_m$ in V is called the span denoted by span$(v_1, v_2 \ldots v_m)$

$\text{span}(v_1, v_2 \ldots v_m) = \{a_1 v_1 + a_2 v_2 \ldots a_n v_n : a_1, a_2 \in F\}$

span of empty list ( ) is $\{0\}$.

Linearly independent

a list $v_1, \dots v_m$ of vectors in V is linearly independent if the only choice of $a_1 \dots a_m \in$ that makes

$a_1v_1 + \dots a_mv_m = 0 \rightarrow (1)$

is $a_1 = a_2 = \dots a_m = 0$

empty list ( ) is linearly independent.

(No vector can be made by combining each other)

the list of vectors are linearly independent if the only way to combine them to get zero vector is by using zero co-efficients

if there are non-zero scalars $a_1, a_2 \dots a_m$ that satisfies (1) they are dependent, meaning atleast one vector is a combination of another.

eg:

$v_1 = (1,0)$

$v_2 = (0,1)$

$a_1(v_1) + a_2(v_2) = (0,0)$

$a_1(1,0) + a_2(0,1) = (0,0)$

$(a_1, a_2) = (0,0)$

$\therefore a_1 = 0 \quad a_2 = 0$

$\therefore$ independent

$v_1 = (1,0)$

$v_2 = (2,0)$?

$a_1(1,0) + a_2(2,0) = (0,0)$

$(a_1 + 2a_2) = (0,0)$

$a_1 + 2a_2 = 0$

$a_1 = -2a_2$

if $a_2 = 1$

$a_1 = -2$

$-2(1,0) + 1(2,0)$

$= (-2+2, 0)$

$= (0,0)$

$\therefore$ dependent.

* a list of length one in vector space is independent if and only if the vector is not 0

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linear dependence lemma

Given a linearly dependent list of vectors, one of the vectors is in the span of the previous ones. Also, we can throw out that vector without changing the span of original list.

$c_1, c_2, c_3 \ldots c_n$ all are NOT zero, such that
$c_1 v_1 + c_2 v_2 \ldots c_n v_n = 0$

suppose a scalar $c_k \neq 0$, then v_k :

$c_k v_k = -c_1 v_1 - c_2 v_2 \ldots$

$v_k = \frac{-c_1}{c_k} v_1 - \frac{c_2}{c_k} v_2$

v_k is a linear combination of vectors,
v_k is in the span of $\{v_1, v_2 \ldots v_k\}$

Part 2 of the statement

say vector v_k can be written as -

$v_k = b_1 v_1 + b_2 v_2 \ldots b_{k-1} v_{k-1} \ldots b_n v_2$

(v_k is already in the span of other vectors
$v_1, v_2, v_{k-1}, v_{k+1} \ldots v_n$)

if we remove v_k -

a vector w in the span can be written

$w = a_1 v_1 + a_2 v_2 + a_k v_k + \ldots a_n v_n$

we know $v_k = b_1 v_1 + b_2 v_2 + b_{k-1} v_{k-1} + b_{k+1} v_{k+1} \dots b_n v_n$

substitute to (1)

$w = a_1 v_1 + a_2 v_2 + \dots a_k (b_1 v_1 + b_2 v_2 + \dots) + \dots a_n v_n$

$w = v_1 (a_1 + a_k b_1) + v_2 (a_2 + a_k b_2) \dots v_n (a_n + a_k b_n)$

$\therefore$ w is a linear combi of $\{v_1, v_2 \dots v_n\}$

$\therefore$ w is in span of $\{v_1, v_2, \dots v_n\}$.

- in a finite dimensional vector space, the length of every linearly independent list of vectors is less than or equal to the span of vectors.

(1) no list of length 4 is linearly independent in R^3

(2) no list of length 3 spans R^4.

Exercise 2A

2. prove or disprove if v_1, v_2, v_3, v_4 spans V, then list $v_1 - v_2, v_2 - v_3, v_3 - v_4, v_4$ also spans V.

$\text{span}(v_1, v_2, v_3, v_4) = \text{span}(v_1 - v_2, v_2 - v_3, v_3 - v_4, v_4)$

$V = a_1 v_1 + a_2 v_2 + a_3 v_3 + a_4 v_4 = a_1(v_1 - v_2) + a_2(v_2 - v_3) + a_3(v_3 - v_4) + a_4 v_4$

$= a_1 v_1 - a_1 v_2 + a_2 v_2 - a_2 v_3 + a_3 v_3 - a_3 v_4 + a_4 v_4$

$= a_1 v_1 + (a_2 - a_1) v_2 + (a_3 - a_2) v_3 + (a_4 - a_3) v_4$

TO - DO