Question 1

(1) Timo is a cow. (2) No (3) No (4) Zero

Question 2

(1) $\{ML\} \sqsubseteq AI\_course \sqcap \exists teach^{-}\{ZZH\}$, $\{ZZH\} \sqsubseteq professor \sqcap \exists work\_at\{NJU\}$.

(2) $\{NJU\} \sqsubseteq university \sqcap \exists members\_are\{school\} \sqcup \{department\}$

(3) $\{NJU\} \sqsubseteq (\geqslant 30000\ have.\ students)$

(4) $\exists member\_of\ AI\_school \sqsubseteq \{undergraduates\} \sqcup \{graduates\} \sqcup \{teachers\}$.

(5) $\exists citizen\_of.\top \sqsubseteq countries$

FoL: (4) $\forall x(\exists member\_of\ AI\_school \sqsubseteq \{ug\} \sqcup \{g\} \sqcup \{t\}) =$

$\forall x(member\_of(x.AI\_school) \rightarrow \{ug\}(x) \vee \{g\}(x) \vee \{t\}(x))$

(5) $\forall x(\exists y.\ citizen\ of(x,y) \wedge \top \rightarrow Countries(x)) =$

$\forall x(\exists y.\ citizen\ of(x,y) \rightarrow countries(x))$.

Question 3

(1) True. Consider this example: The ontology describes a world with only three objects and no relations. It has a finite number of models, each one corresponding to a different assignment of properties to the objects. So it's possible for an ontology by focusing on a limited, specific domain or scenario.

(2) False. According to (1), this statement is obvious wrong.

(3) True. By definition, a class is satisfiable if and only if there exists an interpretation under which the class is true. So a satisfiable class must always have at least one interpretation, hence a non-empty interpretation.

(4) False. By definition, an unsatisfiable class is one that cannot be true under any interpretation. So it cannot have a non-empty interpretation in any model.

(5) True. In the context of DL, an unsatisfiable class is indeed a subclass of any other class. This is because the set of instances that satisfy an unsatisfiable class is empty, and it is a subset of any set.

Question 4

(1) d、f (2) e、d (3) h、f、i (4) g、h、i、f、d (5) e、g、f、h、i.

Question 5

(1) $\emptyset$

$\{d, b, c, e\}$

$\{b, c, e\}$

$\{b, d, e\}$

$\emptyset$

(2) True:

- $\mathcal{I} \models A \equiv \exists r.B$
- $\mathcal{I} \models A \sqcap B \sqsubseteq \top$
- $\mathcal{I} \models \exists r.A \sqsubseteq A \sqcap B$

Question 6

1. It holds. If $C \sqsubseteq D$ holds, then $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$. Consider $\exists r.C = \{d \in \Delta_C^{\mathcal{I}} \mid$ there is $e \in \Delta_C^{\mathcal{I}}$ $(d, e) \in r^{\mathcal{I}}$ and $e \in C^{\mathcal{I}}\}$

$\exists r.D = \{d \in \Delta_D^{\mathcal{I}} \mid$ there is $e \in \Delta_D^{\mathcal{I}} = (d, e) \in r^{\mathcal{I}}$ and $e \in D^{\mathcal{I}}\}$.

$d \in \Delta_C^{\mathcal{I}} \Rightarrow d \in \Delta_D^{\mathcal{I}}$; $e \in \Delta_C^{\mathcal{I}} \Rightarrow e \in \Delta_D^{\mathcal{I}}$; $e \in C^{\mathcal{I}} \Rightarrow e \in D^{\mathcal{I}}$ $\therefore$ It holds.

2. It doesn't hold. $\leq 1 r.\top = \{d \in \Delta^{\mathcal{I}} \mid card(\{e \mid (d, e) \in r^{\mathcal{I}}\}) \leq 1\}$. Just need to make $\Delta^{\mathcal{I}} = \{a, b\}$

$C^{\mathcal{I}} = \{a\}$, $r^{\mathcal{I}} = \{(a, b)\}$, then $\exists r.C = \emptyset$. $\leq 1 r.\top = \{a, b\}$. So it doesn't hold.

3. It holds. Consider it: $\leq 0r.\top = \{d\in\Delta^I \mid card(\{e \mid (d,e)\in r^I \wedge e\in\Delta^I\}) \leq 0\}$, it means e doesn't exist.

$\forall r.\bot = \{d\in\Delta^I \mid$ for all $e\in\Delta^I = (d,e)\in r^I$ implies $e\in\bot\}$ also means e doesn't exist. Same, so it holds.

4. It doesn't hold. Let $\Delta^I=\{a,b,c\}$. $r^I=\{(c,a),(c,b)\}$. $A^I=\{a\}$ $B^I=\{b\}$. So $\forall r.(A\sqcup B)$ is $\{a,b,c\}$.

and $(\forall r.A)\sqcup(\forall r.B)$ is $\{a,b\}$. So it doesn't hold.

5. It holds. $\exists r.(A\sqcup B)=\{d\in\Delta^I \mid$ there is $e\in\Delta^I = (d,e)\in r^I$ and $e\in(A\sqcup B)^I\}$.

Similarly. $\exists r.A=\{d\in\Delta^I \mid$ there is $e\in\Delta^I = (d,e)\in r^I$ and $e\in A^I\}$.

$\exists r.B=\{d\in\Delta^I \mid$ there is $e\in\Delta^I = (d,e)\in r^I$ and $e\in B^I\}$.

So $(\exists r.A)\sqcup(\exists r.B)=\{d\in\Delta^I \mid$ there is $e\in\Delta^I = (d,e)\in r^I$ and $e\in A^I\cup B^I\}$ and $A^I\cup B^I=(A\sqcup B)^I$. So it holds.

Question 7

Let $\Delta^I=\{a,b,c,d,e,f\}$. and a~f are all person and a~d are parents. and a has child b, c has child d and c is d's mother. All the relations can be shown in a graph below:

$$\begin{array}{cccc} \text{parent} & \text{parent mother} & & \\ a & c & e & f \\ \downarrow & \downarrow & & \\ b & d & & \end{array}$$

Thus. $I\models T$ means $(Parent)^I\subseteq(\exists\, has\,Child.\,Person)^I$, $\{a,c\}\subseteq\{a,c\}$. holds.

$(Mother)^I=\{c\}\subseteq\{a,c\}=(Parent)^I$. so $I\models T$. $(Parent)^I=\{a,c\}\nsubseteq\{c\}=(Mother)^I$

So $I\not\models Parent\sqsubseteq Mother$.

Question 8.

1. From $X\sqsubseteq_T Y$ we get $(X)^I\subseteq(Y)^I$, where I is a model of T. So $(X)^I\cap\neg(Y)^I=\emptyset$, then $X\sqcap\neg Y$ is not satisfiable; if $X\sqcap\neg Y$ is not satisfiable, so there is no model I such that $(X)^I\cap(\neg Y)^I\neq\emptyset$. So we know $(X)^I$ and Δ^I/Y^I are disjoint sets, so $(X)^T\subseteq(Y)^T$. then $X\sqsubseteq_T Y$. Proved.

2. From 1. let $Y \equiv \bot$ then $X \sqsubseteq_T \bot$ iff X is not satisfiable. So X is satisfiable iff $X \not\sqsubseteq_T \bot$. Proved.

Question 9

1. 801 and 322. Because some axioms are not logical, such as some declaration axioms.

2. (1) Country Equivalent To Domain Thing and ({America, England, France, Germany, Italy}).

(2) Vegetarian Pizza Equivalent To Pizza and (not (has Topping some Seafood Topping))
and (not (has Topping some Meat Topping))

Nonvegetarian Pizza Equivalent To Pizza and (not (Vegetarian Pizza)).

(3) hasBase is SubProperty has ingredient
hasTopping is SubProperty has ingredient
is Base of is SubProperty is Ingredient of
is Topping of is SubProperty is Ingredient of

(4) has Ingredient inverse of is Ingredient of
has Base inverse of is Base of
has Topping inverse of is Topping of

3. Because Ice Cream is empty, it is equivalent to Nothing. Thus the reasoner cannot work correctly.

4. Spicy Topping

Cheesy Pizza, Interesting Pizza, Meety Pizza, Spicy Pizza and Spicy Pizza Equivalent.

5. The result changes to an internal reasoner error.