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Ray-Sphere Intersection

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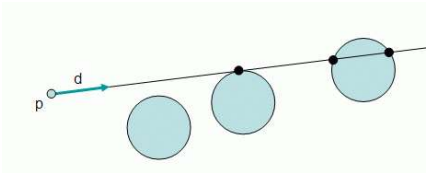
Given a ray, defined by a point and a unit vector, and a sphere do they intersect? If so where? Do they intersect in a single point, or in two points? In the figure below we can see several possibilities.

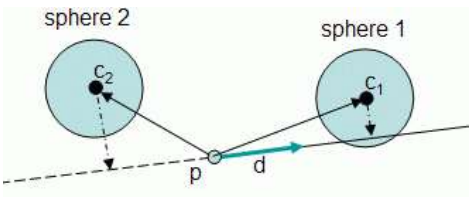


The first question is whether the ray intersects the sphere or not. In order to find out, the distance between the center of the sphere and the ray must be computed. If that distance is larger than the radius of the sphere then there is no intersection.

To compute the distance two cases are considered: either the center of the sphere projects on the ray, or it doesn't. A simple dot product will do the trick.

Let *v* be the vector that goes from *p* to *c* (the center of the sphere). Then if

- *d* . *v* > 0 // the distance to the ray can be computed (Sphere 1)
- *d* . *v* <= 0 // the sphere is behind the ray (Sphere 2)



Case 1: The center of the sphere projects on the ray

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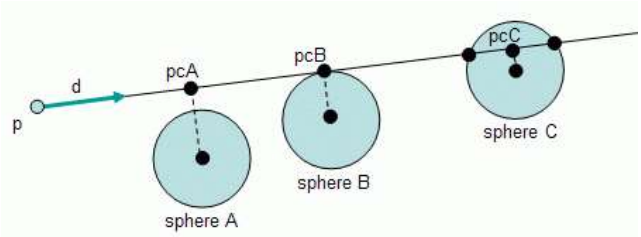
The following figure shows the three possible cases: no intersection (sphere A), single intersection (sphere B), and two intersections (sphere C).

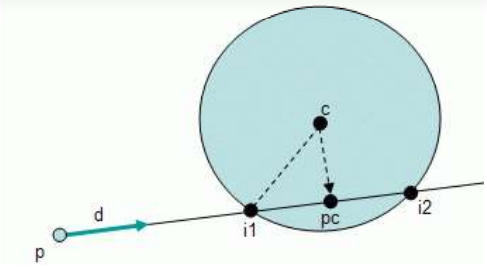


First compute the projection of the center of the sphere on the ray. Let *pc* be that projection.

If the distance from *pc* to the ray is greater than the ray then there is no intersection (sphere A in the above figure).

If the distance from *pc* to the ray is equal to the radius of the sphere, then the intersection is a single point: *pc* (sphere B).

The former case requires a little bit extra work.



The point *pc* has already been found. So the task at hand is to find *i1* and/or *i2*. The following procedure gives us *i1*, and finding *i2* is left as an (easy) exercise to the reader.

If the distance from *p* to *i1* is known, lets called *di1*, then finding *i1* is achieved with the following equation:

```
i1 = p + d * di1
```

Hence all there is left to do is to find *di1*.

The triangle defined by *i1*, *pc* and *c* is a right triangle (i.e. on of the angles is 90 degrees), hence the Pythagorean theorem applies. Let

```
a = |c - i1| = radius
b = |pc - c|
c = |pc - i1|
```

The only unknown is c, and it can be computed as:

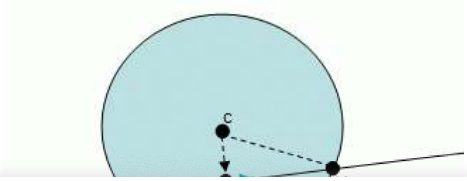
```
c^2 = a^2 - b^2
```

Then

```
di1 = | pc - p | - c
```

The previous computation of *di1* assumed that the origin of the ray was not inside the sphere. If *p* is inside the sphere then the computation of *di1* is slightly different:

```
di1 = | pc - p | + c
```



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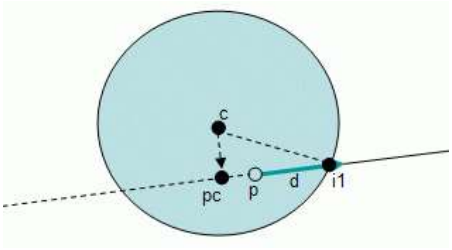
Case 2: The center of the sphere is behind the ray

If it is assumed that the origin of the ray is outside the sphere then there is no possible intersection. Otherwise, there is an intersection if the distance from *p* to *c* is less than or equal to the radius. If is equal then the intersection is the point *p* itself.

If the origin of the ray is inside the sphere a similar reasoning to the case presented previously may be applied. Note that the projection is done on the line, and not on the ray. The following figure shows the relevant points involved.



Computing the distance from *pc* to *i1* is exactly the same as before. Let *dist* be that distance. The value of *di1* can be computed as:

```
di1 = dist - | pc - p |
```

Algorithm

So here it is the full algorithm to compute the intersection between a ray and a sphere. It is assumed that the ray is defined by an origin *p*, and a direction *d* (unit vector), and that the sphere has a center *c*, and a radius *r*.

```
vpc = c - p  // this is the vector from p to c

if ((vpc . d) < 0) // when the sphere is behind the origin p
                    // note that this case may be dismissed if it is
                    // considered that p is outside the sphere
        if (|vpc| > r)

                   // there is no intersection

        else if (|vpc| == r)

                intersection = p

        else // occurs when p is inside the sphere

                pc = projection of c on the line
                // distance from pc to i1
                dist = sqrt(radius^2 - |pc - c|^2)
                di1 = dist - |pc - p|
                intersection = p + d * di1
```

```
pc = projection of c on the line
if (| c - pc | > r)

        // there is no intersection

else
        // distance from pc to i1
        dist = sqrt(radius^2 - |pc - c|^2)

        if (|vpc| > r) // origin is outside sphere

                di1 = |pc - p| - dist

        else // origin is inside sphere

                di1 = |pc - p| + dist

        intersection = p + d * di1
```

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5 Responses to "Ray-Sphere Intersection"

1. **Joao Oliveira** says:
   17/01/2015 at 3:36 PM

   Thanks for the tutorial, very simple and easy to understand.

   Small correction, when you say:
   "If the distance from pc to the ray is greater than the ray then there is no intersection (sphere A in the above figure)."

   Shouldn't it be:
   "If the distance from pc to the ray is greater than the radius of the sphere then there is no intersection (sphere A in the above figure)."

   ?

   Reply

2. **Rob A** says:
   10/07/2014 at 8:17 PM

   For the case where you have two intersections of the sphere
   to the line: If the angle at c-pc to the line denoted by
   p->d is always 90 degrees, then isn't the distance from i1
   to pc always identical to the distance from pc to i2?

   Reply

3. **Dylan** says:
   09/12/2013 at 2:21 AM

   Not sure what you mean by this: "If the distance from pc to the ray is greater than the ray then there is no

Reply

4. **Jakob Folkesson** says:
22/11/2013 at 10:04 AM

Everything is good, and the tutorial is perfect except for the projected point on the line

pc you never say how to calculate it and without it the algoritm cant be used

Reply

ARF says:
27/11/2013 at 5:44 PM

Hi Jakob,

Thanks for the feedback. pc is the projection of the center of the sphere on the ray segment. There is a link in the text to the page that describes the projection operation, that's why I didn't mention it in here.

Reply

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