Fundamentals of Signal Processing and Data Analysis Homework 8

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1 Proof of the Fourier Transform Identity for Cross-Correlation

$$\mathscr{F}[f(n) \otimes g(n)] = \mathscr{F}[f(n)]^* \cdot \mathscr{F}[g(n)]$$

Where $f(n) \otimes g(n)$ is the cross-correlation, $\mathscr{F}$ denotes the Fourier transform, and * indicates the complex conjugate.

We begin with the definition of the Fourier transform of the cross-correlation:

$$\mathscr{F}[f(n) \otimes g(n)] = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} f^*(m)\, g(m+n)\, dm \right] e^{-i\omega n}\, dn$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f^*(m)\, g(m+n)\, e^{-i\omega n}\, dm\, dn$$

$$= \int_{-\infty}^{\infty} f^*(m) \left[\int_{-\infty}^{\infty} g(m+n)\, e^{-i\omega n}\, dn \right] dm$$

Let $\varphi = m + n$, then $n = \varphi - m$, so $dn = d\varphi$:

$$= \int_{-\infty}^{\infty} f^*(m) \left[\int_{-\infty}^{\infty} g(\varphi)\, e^{-i\omega(\varphi - m)}\, d\varphi \right] dm$$

$$= \int_{-\infty}^{\infty} f^*(m)\, e^{i\omega m}\, dm \cdot \int_{-\infty}^{\infty} g(\varphi)\, e^{-i\omega\varphi}\, d\varphi$$

$$= \left(\int_{-\infty}^{\infty} f(m)\, e^{-i\omega m}\, dm \right)^* \cdot \int_{-\infty}^{\infty} g(\varphi)\, e^{-i\omega\varphi}\, d\varphi$$

$$= \mathscr{F}[f(n)]^* \cdot \mathscr{F}[g(n)]$$