From Data Fitting to Integral Transforms

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Abstract

This article systematically explores the theoretical framework and methodological applications of data fitting and integral transforms. The study centers on orthogonal projection in function spaces and explains the role of the Gram–Schmidt orthogonalization in constructing basis functions. Through comparative experiments with typical bases such as polynomials, Fourier bases, and exponential functions, the influence of the choice of basis functions on fitting performance is revealed. Furthermore, the work derives the essence of Fourier transform as an orthogonal projection, extends it to the integral expression of the Laplace transform, and establishes a new perspective for solving differential equations via various integral transforms. Finally, it briefly introduces the application of vector inner products in statistics.

Keywords: Data Fitting; Gram–Schmidt Orthogonalization; Fourier Transform; Laplace Transform; Integral Transforms; Projection in Function Spaces

Contents

1 Fitting

Basic Definition

Fitting refers to the process of finding a function $f(x)$ that approximately describes a given dataset $\{x_i, y_i\}_{i\in\mathbf{N}}$ such that each $f(x_i)$ is as close as possible to the corresponding y_i. A common measure for the fitting performance is the sum of squared residuals

$$\sum_{i=1}^{N} |f(x_i) - y_i|^2.$$

Similarly, to compare the difference between two functions $f(x)$ and $g(x)$, a similar criterion can be used, whose continuous form is written as

$$\int_a^b |f(x) - g(x)|^2 \, dx.$$

In the vector space of functions, we define the inner product as

$$\langle f, g \rangle = \int_a^b f(x)g(x) \, dx,$$

and then define the norm

$$\|f(x)\| = \sqrt{\int_a^b f^2(x) \, dx},$$

so that $\|f - g\|$ naturally represents the distance between f and g.

How to Perform Fitting

In high school, we learned to fit data points using the least squares method, but this method is only applicable to linear relationships, which are relatively rare in nature.

The least squares method uses an equation of the form

$$y = \hat{a} \cdot 1 + \hat{b} \cdot x,$$

taking $\{1, x\}$ as the basis to fit the data. Later, through Taylor expansion, we began using $\{1, x, x^2, x^3, \dots\}$ as basis functions to approximate functions. Furthermore, we introduced the Fourier series

$$\{1, \cos x, \cos 2x, \cos 3x, \dots, \sin x, \sin 2x, \sin 3x, \dots\}$$

as a fitting tool.

For example, consider $f(x) = 3x^2 + 2x - 1$. If the least squares method is used, it can only capture the general trend over a finite interval; however, if the Taylor series is used, the first three terms can fully represent the information contained in $f(x)$, yielding precise results in both inversion and prediction. Conversely, if the Fourier series is applied, then within a given closed interval

$$\sum_{i=-n}^{n} C_i e^{j\omega_i x}$$

converges absolutely to $f(x) = 3x^2 + 2x - 1$; but outside that interval, since the Fourier series is composed of sine and cosine functions exhibiting clear periodicity while $f(x)$ is non-periodic, the fitting fails. This illustrates the differences arising from the choice of fitting bases. Although we cannot always select the perfect basis for prediction, using an appropriate basis within a given range can effectively reflect the variation in the data or function.

2 Gram–Schmidt Orthogonalization

Method for Function Fitting

We use a set of basis functions $\{v_1(x), v_2(x), v_3(x), \ldots, v_n(x)\}$ to fit a target function. Any function in the function space $\mathbf{V}$ spanned by this set can be represented as

$$u(x) = \sum_{i=1}^{n} C_i v_i(x).$$

Our goal is to determine a $u(x)$ such that the sum of squared residuals

$$\sum_{i=1}^{N} |f(x_i) - u(x_i)|^2$$

is minimized. A weight function $r(x)$ can also be introduced to account for the different importance of data points, in which case the sum of squared residuals becomes

$$\sum_{i=1}^{N} |f(x_i) - u(x_i)|^2 r(x)$$

or its continuous form

$$\int_a^b |f(x) - u(x)|^2 r(x)\, dx.$$

For simplicity, we usually take $r(x) = 1$. Thus, the expression

$$\int_a^b |f(x) - u(x)|^2\, dx$$

reflects the distance between f and u in the overall space $\mathbf{W}$. To minimize

$$\int_a^b |f(x) - u(x)|^2\, dx,$$

u must be the orthogonal projection of f onto the space $\mathbf{V}$.

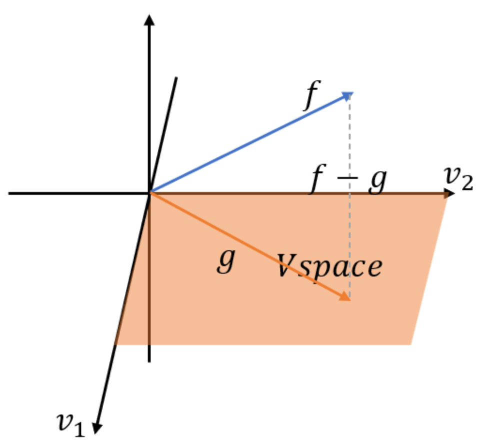

Figure 1: Schematic of Orthogonal Projection

Constructing an Orthonormal Basis

In a vector space equipped with an inner product, we can use the Gram–Schmidt orthogonalization process to convert the basis $\{v_1(x), v_2(x), v_3(x), \dots, v_n(x)\}$ into an orthonormal basis $\{u_1(x), u_2(x), u_3(x), \dots, u_n(x)\}$:

$$\textbf{Step 1:}\quad u_1(x) = \frac{v_1(x)}{\sqrt{\langle v_1, v_1\rangle}} = \frac{v_1(x)}{\sqrt{\int_a^b v_1(x)^2\,dx}},$$

$$\textbf{Step 2:}\quad u_2'(x) = v_2(x) - \langle v_2, u_1\rangle\, u_1(x), \quad u_2(x) = \frac{u_2'(x)}{\sqrt{\langle u_2', u_2'\rangle}},$$

$$\vdots$$

$$\textbf{Step n:}\quad u_n'(x) = v_n(x) - \sum_{i=1}^{n-1}\langle v_n, u_i\rangle\, u_i(x), \quad u_n(x) = \frac{u_n'(x)}{\sqrt{\langle u_n', u_n'\rangle}}.$$

It is easy to verify that $\langle u_i, u_j\rangle = \delta_{ij}$. For example:

$$\textbf{Example:}\quad \langle u, v\rangle = \int_{-1}^{1} u(x)v(x)\,dx,$$

$$\{1, x, x^2\} \Rightarrow \left\{\frac{1}{\sqrt{2}}, \frac{\sqrt{3}\,x}{\sqrt{2}}, \frac{3\sqrt{10}}{4}x^2 - \frac{\sqrt{10}}{4}\right\},$$

$$\{1, \cos\pi x, \cos 2\pi x\} \Rightarrow \left\{\frac{1}{\sqrt{2}}, \cos\pi x, \cos 2\pi x\right\}.$$

Obtaining the Projection

With the orthonormal basis at hand, we can project f onto $\mathbf{V}$, with the projection denoted by $P_V f$:

$$P_V f = \sum_{i=1}^{n}\langle f, u_i\rangle\, u_i.$$

In this way, the distance between f and the space $\mathbf{V}$ (i.e. the sum of squared residuals $\int_a^b |f(x) - P_V f(x)|^2\,dx$) is minimized.

For example, when we use the basis $\{\sin\pi x, \sin 2\pi x, \sin 3\pi x, \dots \sin n\pi x\}$ to fit the function

$$u(x) = \begin{cases} 0, & -1 < x < 0, \\ 1, & 0 \le x < 1, \end{cases}$$

it is clear that this is exactly the process of a Fourier series expansion; because the terms are orthogonal, each newly added term exhibits the shape of a sine function.

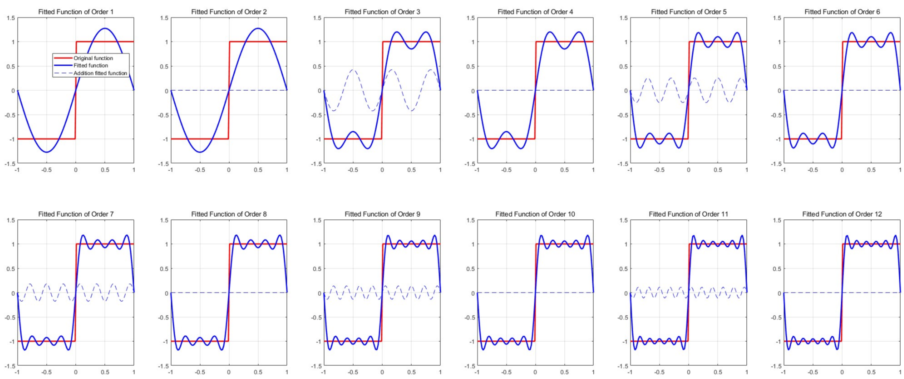

Figure 2: Schematic of Sine Series Fitting

The figure below shows the expansion effect when using the basis

$$\left\{\sin\!\Big(\Big(\frac{1-(-1)^n+2n}{4}\Big)\pi x+\frac{\pi\,(1+(-1)^n)}{4}\Big)\right\}_{0\le n\le N}$$

(i.e. a basis composed of alternating cosine and sine functions). It can be seen that for the cosine terms, the additional components are all zero because the cosine function is even while the square wave to be fitted is odd; on a symmetric interval they are naturally orthogonal, resulting in a zero projection. This shows that the cosine functions cannot reflect the characteristics of the square wave, and we need to supplement with a set of odd function bases to fit the target function.

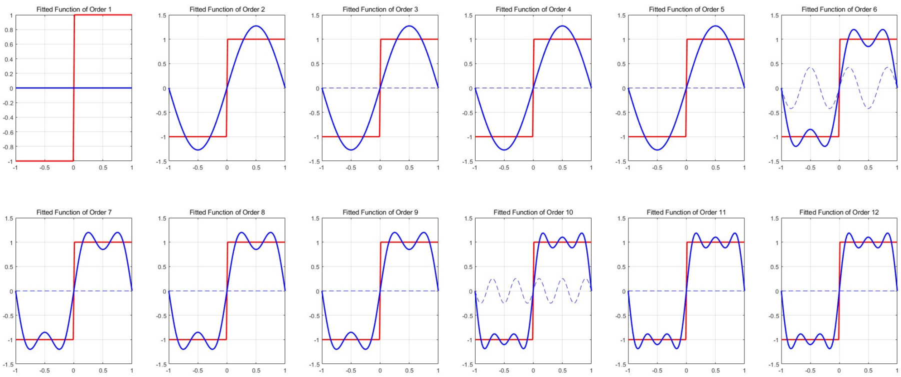

Figure 3: Fourier Expansion with Alternating Cosine and Sine Bases

Similarly, when using the basis $\{1, x, x^2, x^3, \dots x^n\}$ (after orthonormalization) for fitting, the first few terms exhibit obvious polynomial characteristics, while the later terms, being linear combinations of several polynomial functions, display ambiguous polynomial characteristics.

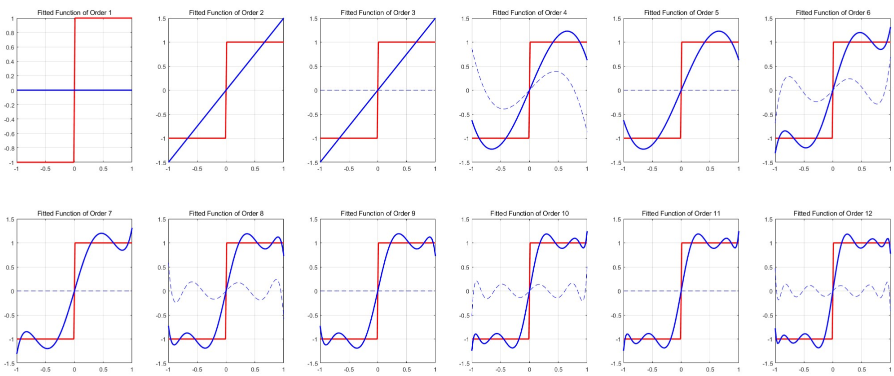

Figure 4: Schematic of Polynomial Basis Fitting

In addition, other functions can be used for fitting, for example the basis $\{1, e^x, e^{2x}, e^{3x}, \dots e^{nx}\}$ (after orthonormalization).

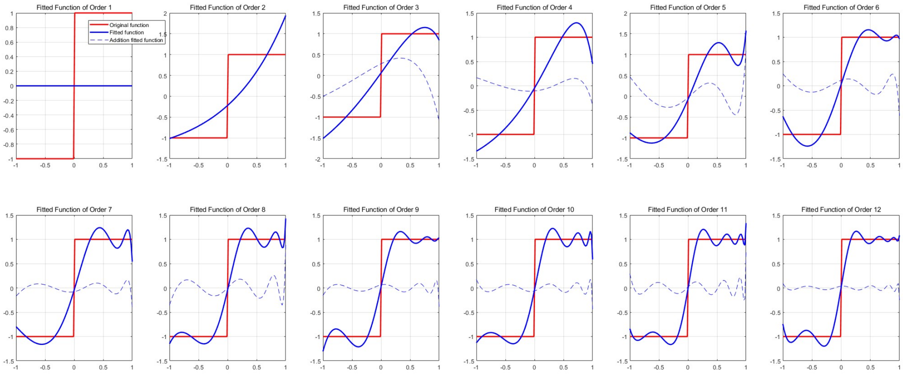

Figure 5: Schematic of Exponential Function Fitting

The figure below shows the fitting of $\sin(x)$ on the interval $[2\pi, 6\pi]$ using a logarithmic basis

$$\{\ln(x), \ln(x+1), \ln(x+2), \ln(x+3), \ldots, \ln(x+n)\}.$$

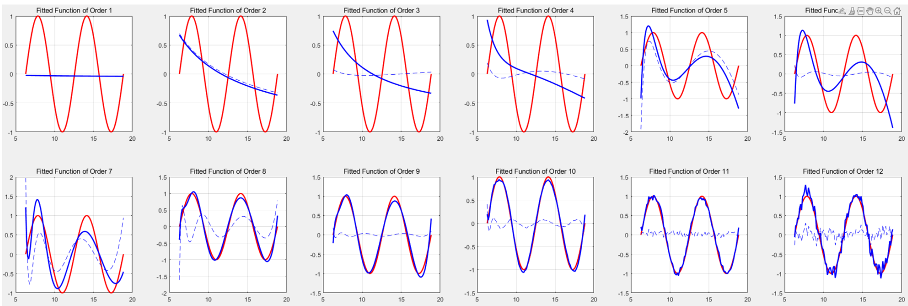

Figure 6: Schematic of Logarithmic Basis Fitting

Moreover, we can combine different bases. For example, using

$$\{1, x, x^2, x^3, x^4, x^5\} + \{e^x, e^{2x}, e^{3x}, e^{4x}, e^{5x}, e^{6x}\}$$

to fit $\sin(x)$ over the interval $[-\pi, \pi]$. The left figure shows the fitting process when the polynomial basis is used first and the exponential basis is introduced at the seventh term; the right figure shows the reverse order. It can be seen that if the polynomial basis is used first, the fitting effect is more efficient.

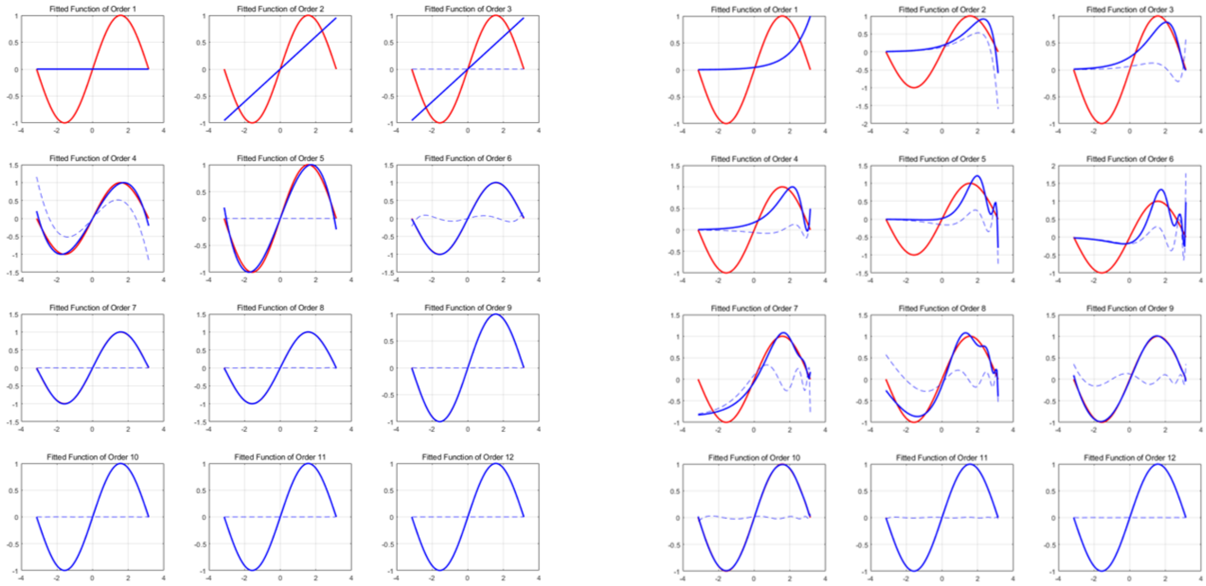

Figure 7: Comparison of $\sin(x)$ Fitting with Different Basis Combination Orders

In practical fitting, we can choose any basis $\{\varphi_n\}_{n\in\mathbf{N}}$ to span the function space $\mathbf{V}$, and then obtain the projection representation of f on this basis via the projection method. Commonly used bases include $\{x^n\}_{n\in\mathbf{N}}$, $\{\cos(n-1)x\}_{n\in\mathbf{N}}$, $\{\sin nx\}_{n\in\mathbf{N}}$, and $\{\cos(n-1)x, \sin nx\}_{n\in\mathbf{N}}$.

3 Integral Transforms

Fourier Form under Integral Transforms

The projection of the function f onto the space V is given by

$$P_V f = \sum_{n=1}^{N} \langle \varphi_n, f \rangle \, \varphi_n.$$

If we assume that $P_V f$ approximates f well (uniformly convergent on a compact interval and pointwise convergent on an open interval), then it can be written as

$$f = \sum_{n=1}^{N} \langle \varphi_n, f \rangle \, \varphi_n.$$

Using the aforementioned inner product definition, we fit the function f defined on $[-\frac{T}{2}, \frac{T}{2}]$ using the sine basis

$$\left\{ \sin \frac{n \cdot 2\pi}{T} x \right\}_{n\in\mathbf{N}},$$

and first orthonormalize this basis to obtain

$$\left\{ \sqrt{\frac{2}{T}} \sin \frac{n \cdot 2\pi}{T} x \right\}_{n\in\mathbf{N}}.$$

Then the projection of f in this space is

$$P_V f = \sum_{n=1}^{N} \langle \varphi_n, f \rangle \, \varphi_n$$
$$= \frac{2}{T} \sum_{n=1}^{N} \left(\int_{-\frac{T}{2}}^{\frac{T}{2}} f(\tau) \sin \frac{n \cdot 2\pi}{T} \tau \, d\tau \right) \sin \frac{n \cdot 2\pi}{T} x.$$

Using $T = \frac{2\pi}{\omega_0}$, this can be rewritten as

$$P_V f = \frac{\omega_0}{\pi} \sum_{n=1}^{N} \left(\int_{-\frac{T}{2}}^{\frac{T}{2}} f(\tau) \sin(n\omega_0 \tau) \, d\tau \right) \sin(n\omega_0 t).$$

This is the sine Fourier expansion of f. When $T \to \infty$ (i.e. $\omega_0 \to 0$), letting $\omega_0 = d\omega$ and $n\omega_0 = \omega$, we have

$$P_V f = \frac{1}{\pi} \int_0^{N \cdot \omega_0} \left(\int_{-\infty}^{\infty} f(\tau) \sin(\omega\tau) \, d\tau \right) \sin(\omega t) \, d\omega.$$

Since the outer integral originally runs from $\omega_0 = 0$ to $N\omega_0$, to achieve sufficient fitting accuracy, we approximate with an infinite number of terms and change the upper limit to ∞. Because the inner and outer functions are odd functions of ω and their product is even, the integral form can also be adjusted to

$$P_V f = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} f(\tau) \sin(\omega\tau) \, d\tau \right) \sin(\omega t) \, d\omega.$$

This is the origin of the sine expression of the Fourier transform. Recalling the initial version

$$P_V f = \sum_{n=1}^{N} \langle \sqrt{\frac{2}{T}} \sin \frac{n \cdot 2\pi}{T} x, f \rangle \sqrt{\frac{2}{T}} \sin \frac{n \cdot 2\pi}{T} x,$$

we can view it as a superposition of odd functions; therefore, the projection of f onto V is still an odd function.

Similarly, we can obtain the projection of f onto the space

$$W = \textbf{span} \left\{ \cos \frac{n \cdot 2\pi}{T} x \right\}_{0 \le n \le N},$$

which is

$$P_W f = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} f(\tau) \cos(\omega\tau) \, d\tau \right) \cos(\omega t) \, d\omega.$$

Since the sine basis reflects the odd part of f and the cosine basis reflects its even part, $P_V f$ and $P_W f$ are orthogonal to each other. By adding the two, we obtain a more complete representation of f:

$$P_V f + P_W f = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} f(\tau) \Big[\sin(\omega\tau) \sin(\omega t) + \cos(\omega\tau) \cos(\omega t) \Big] \, d\tau \right) d\omega$$
$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} f(\tau) \cos(\omega t - \omega\tau) \, d\tau \right) d\omega.$$

Given that $j^2 = -1$and the inner function is an odd function of ω, the following Cauchy principal value integral equals 0.

$$\frac{j}{2\pi}\int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} f(\tau)\sin(\omega t - \omega\tau)\,d\tau\right)d\omega = 0.$$

Therefore,

$$\begin{aligned}
P_V f + P_W f &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} f(\tau)\cos(\omega t - \omega\tau)\,d\tau\right)d\omega + \frac{j}{2\pi}\int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} f(\tau)\sin(\omega t - \omega\tau)\,d\tau\right)d\omega \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} f(\tau)\Big[\cos(\omega t - \omega\tau) + j\sin(\omega t - \omega\tau)\Big]d\tau\right)d\omega \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} f(\tau)e^{j\omega(t-\tau)}d\tau\right)d\omega \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} f(\tau)e^{-j\omega\tau}d\tau\right)e^{j\omega t}d\omega.
\end{aligned}$$

This yields the standard Fourier integral expression:

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} f(\tau)e^{-j\omega\tau}\,d\tau\right)e^{j\omega t}\,d\omega.$$

Discussion of Convergence

In the above examples, the bases we employed are all continuous functions, so $P_V f + P_W f$ is also continuous, reflecting the continuity of f. Under appropriate conditions, it can be proven that the projection converges to f either pointwise or uniformly.

It is important to note that the Fourier transform also has limitations. For instance, projecting a function onto the sine function space only retrieves its odd part and cannot fully capture all the information of the function. For example, when projecting $g(x) = x + x^2$ onto the sine space, the coefficients obtained by taking the inner products with the orthonormal bases only contain the x term, while the x^2 component is discarded, leading to loss of the corresponding information.

Comparison of Coefficients

In the projection process using a finite-dimensional basis, different functions may have the same projection, making it difficult to distinguish the original functions. To resolve this, an infinite-dimensional basis can be used to distinguish different functions by comparing their projection coefficients. If all the projection coefficients of a function in an infinite-dimensional space are determined, then the projection uniquely identifies the original function. The Laplace transform is based on this idea for solving differential equations.

Laplace Form under Integral Transforms

In a finite-dimensional space, the projection under a standard orthonormal basis $\{v_i\}_{0\le i\le n}$ results in a linear combination of these bases, and due to the linear independence and uniqueness of the bases, regardless of the order of orthogonalization, the coefficients for the original bases in the final projection vector remain the same.

When using the basis $\{e^{-st}\}_{s\in\mathbf{N}}$ to span a function space, we can think of the distance between adjacent s values as infinitesimal, so that the basis spans a sufficiently large finite-dimensional space. We can arbitrarily select one of the e^{-st} terms as the object of the final orthogonalization. Except for the final standard basis obtained in the last step, the other standard bases do not contain the linear term of e^{-st}; hence, e^{-st} appears only in the final projection computation.

Let the standard basis obtained from the final orthogonalization be

$$u_s = c\,e^{-st} + \sum a_{si} u_i,$$

where c, a_{si}, and u_i are inherent properties of the system and are independent of the input function f. The projection of f onto u_s is

$$\langle f, c\,e^{-st} + \sum a_{si} u_i \rangle \Big(c\,e^{-st} + \sum a_{si} u_i \Big),$$

and the projection of f in this function space is

$$P_V f = \langle f, c\,e^{-st} + \sum a_{si} u_i \rangle e^{-st} + \sum_{initial}^{final-1} \gamma_{s'} e^{-s't},$$

where $\gamma_{s'}$ and $\langle f, c\,e^{-st} + \sum a_{si} u_i \rangle$ represent the coefficients in front of each e^{-st}. In the coefficient of the first term e^{-st}, the part related to both f and s is $\langle f, c\,e^{-st} \rangle$, and since c is an inherent attribute of the system independent of f and s, we can use $\langle f,\, e^{-st} \rangle$ to reflect or represent the characteristic projection coefficient of e^{-st}. Here we define the inner product $\langle f,\, g \rangle = \int_0^\infty f(t)g(t)\,dt$. Thus, the characteristic projection coefficient obtained is

$$\langle f,\, e^{-st} \rangle = \int_0^\infty f(t) e^{-st}\,dt.$$

So, we can interpret the Laplace transform as a process that obtains the coefficient related to e^{-st}. Through this process, we obtain the relation

$$\int_0^\infty f'(t) e^{-st}\,dt = -f(0) + s \int_0^\infty f(t) e^{-st}\,dt,$$

which establishes the relationship between f and its derivatives $f^{(n)}$, and thus connects the two sides of a differential equation such as $\sum c_n f^{(n)}(t) = G(t)$. This gives the characteristic projection coefficients of f under e^{-st}, and finally, by consulting tables, the corresponding original function is obtained. Note that sometimes the same characteristic projection coefficient corresponds to different original functions; for example, the delta function projection coefficients of $\lim_{n\to\infty} \frac{n}{\sqrt{\pi}} e^{-n^2x^2}$ and $\lim_{n\to\infty} \frac{n}{\pi} \operatorname{sinc}^2(nx)$ are both 1. The common property of these delta functions is that $\int_{-\infty}^{\infty} \delta(x - x_0) f(x)\,dx = f(x_0)$. A projection coefficient of 1 reflects precisely this property of the functions.

Generalization

The above process does not utilize the special functional properties of $\{e^{-st}\}$, so similar methods apply to both the Fourier transform and the Laplace transform. We can also employ transforms such as

$$\int_0^\infty f(x) x^s\,dx,$$

(involving power functions), or

$$\int_0^\infty f(x) s^x\,dx,$$

(involving exponential functions), or

$$\int_a^b f(x) \ln(x + s)\,dx,$$

(involving logarithmic functions) to establish relations related to power, exponential, or logarithmic functions, thereby further obtaining the relationship between f and its nth derivative $f^{(n)}$.

Recall the projection process we employed:

$$P_V f = \sum_{i=1}^{n} \langle f, u_i \rangle \, u_i.$$

and the expression for the Fourier integral transform:

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} f(\tau) e^{-j\omega\tau} \, d\tau \right) e^{j\omega t} \, d\omega.$$

We can consider the inner integral as the process of taking an inner product to obtain the projection coefficients, and the outer integral as the process of summing the products of these coefficients with the orthonormal basis. In this way, the integral transform is closely linked with the projection process.

4 Applications in Statistics

Inner Product and Angle in a Vector Space

In a vector space $\mathbf{V}$, the inner product of two vectors f and g is defined as

$$\langle f, g \rangle = \|f\| \, \|g\| \cos\theta,$$

where θ is the angle between the two vectors. Hence,

$$\cos\theta = \frac{\langle f, g \rangle}{\|f\| \, \|g\|}.$$

When $\cos\theta = 1$, $\theta = 0$; the vectors f and g are in the same direction, meaning f can be written as a scalar multiple of g, $f = \lambda g,\ \lambda \in \mathbb{R}$, and the two vectors are completely linearly dependent. When $\theta = \frac{\pi}{2}$, $\langle f, g \rangle = 0$; the vectors f and g are orthogonal, indicating that they are mutually independent in the space.

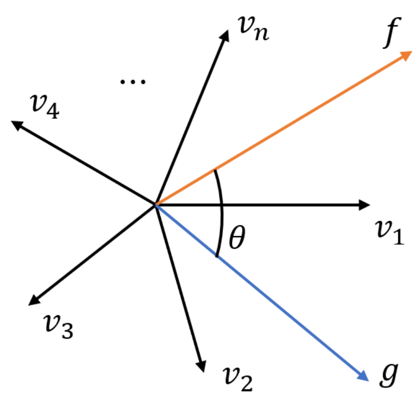

Figure 8: Illustration of the angle between two vectors

Definition of the Correlation Coefficient

Accordingly, we define the correlation coefficient between two vectors v_1 and v_2 as

$$r = \frac{\langle v_1, v_2 \rangle}{\|v_1\| \, \|v_2\|}.$$

Using the inner-product definition in a function space, let

$$r = \frac{\displaystyle\int_a^b f(x)\, g(x)\, \mathrm{d}x}{\sqrt{\displaystyle\int_a^b f^2(x)\, \mathrm{d}x}\sqrt{\displaystyle\int_a^b g^2(x)\, \mathrm{d}x}} = \frac{\displaystyle\sum_{i=0}^{n} f(x_i)\, g(x_i)}{\sqrt{\displaystyle\sum_{i=0}^{n} f^2(x_i)}\sqrt{\displaystyle\sum_{i=0}^{n} g^2(x_i)}}.$$

Pearson Correlation Coefficient

If we are interested in the correlation of centered variations of two datasets, we first subtract their respective means $\bar{x}, \bar{y}$, obtaining the classical Pearson correlation coefficient:

$$r = \frac{\displaystyle\sum_{i=0}^{n} \big(x_i - \bar{x}\big)\,\big(y_i - \bar{y}\big)}{\sqrt{\displaystyle\sum_{i=0}^{n} \big(x_i - \bar{x}\big)^2}\sqrt{\displaystyle\sum_{i=0}^{n} \big(y_i - \bar{y}\big)^2}}.$$

This coefficient measures the degree of linear correlation between two datasets after removing overall level differences. Its range is $[-1, 1]$, where ± 1 correspond to perfect positive and negative correlation, respectively, and 0 indicates no linear correlation.

5 Appendix

```
n = 12;              % Fitting order
a = 2*pi; b = 6*pi;  % Interval
orifun = @(x) sin(x);% Function to be fitted
createfun = @(ii,x) log(x+ii-1);
% Options for basis:
% Exponential: exp((ii-1)*x)
% Polynomial: x.^(ii-1)
% Sine: sin(ii*2*pi/(b-a)*(x-(a+b)/2))
% Cosine: cos((ii-1)*2*pi/(b-a)*(x-(a+b)/2))
% Logarithm: log(ii+x)

x = linspace(a,b,100);

fun = cell(1,n);       % Basis functions (not orthonormalized)
normalfun = cell(1,n); % Orthonormalized basis functions

% Generate the required bases
for ii = 1:n
    fun{ii} = @(x) createfun(ii,x);
    normalfun{ii} = fun{ii};
end

% Normalize the first vector
coeff = integral(@(x) normalfun{1}(x).^2, a, b);
normalfun{1} = @(x) normalfun{1}(x) / sqrt(coeff);
% Orthonormalize the n vectors
for ii = 2:n
    for jj = 1:ii-1
        coeff = integral(@(x) normalfun{ii}(x) .* normalfun{jj}(x), a, b
           );
        normalfun{ii} = @(x) normalfun{ii}(x) - coeff .* normalfun{jj}(x
           );
    end
    coeff = integral(@(x) normalfun{ii}(x).^2, a, b);
    normalfun{ii} = @(x) normalfun{ii}(x) / sqrt(coeff);
end

% Fitting process
coeff = zeros(1, n);
fitfun = @(x) 0;

% Create figure layout
figure;
rows = ceil(n/6);
cols = 6;
% Iterative fitting
for ii = 1:n
    coeff(ii) = integral(@(x) orifun(x) .* normalfun{ii}(x), a, b);
    fitfun = @(x) fitfun(x) + coeff(ii) .* normalfun{ii}(x);
    % Plot each fitting curve
    subplot(rows, cols, ii);
    plot(x, orifun(x), 'r', 'LineWidth', 2, 'DisplayName', 'Original␣
       function');
    hold on;
    plot(x, fitfun(x), 'b', 'LineWidth', 2, ...
         'DisplayName', ['Fitted␣function␣(Order␣' num2str(ii) ')']);
```

```
    contribution = @(x) coeff(ii) .* normalfun{ii}(x);
    plot(x, contribution(x), '--b', 'DisplayName', ...
        ['Additional␣fitted␣function␣(Order␣' num2str(ii) ')']);
    % Set title and legend for each subplot
    title(['Fitted␣Function␣of␣Order␣' num2str(ii)]);
    grid on;
end
```