**Fundamentals of Planetary Science Homework 6**

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# 1 Dynamo

## 1.1 The self-excited dynamo process for the generation of planetary magnetic fields

In the early stages of a planet's formation, certain fundamental conditions were present—most importantly, the existence of a conductive fluid in motion, such as Fe and Ni in the core.

Now imagine that a tiny magnetic field springs up. We don't consider its origin in detail, since magnetic fields are ubiquitous in the universe. For example, frictional motion can generate currents, and high temperatures can separate protons and electrons, both of which can contribute to initial electromagnetic fields.

This small magnetic field can be amplified by the motion of the conductive fluid. You can imagine the magnetic field lines being stretched and tangled by the flow, as if they were being pulled from an infinite source. As the lines become longer and more chaotic, the magnetic field becomes stronger.

According to Maxwell's equations, electric and magnetic fields are closely related. When conductive fluid moves through an existing magnetic field, it cuts through the magnetic field lines, generating electric currents (by Faraday's law). These electric currents then produce new magnetic fields (by Ampere's law), which can further reinforce the original magnetic field.

In this circual loop, the motion of the conductive fluid generates electric currents. These currents then maintain and strengthen the magnetic field. This process is the core of the self-excited dynamo. The resulting magnetic field moves with the fluid and gives rise to the global magnetic field observed.

## 1.2 Possible reasons for the absence of a magnetic field on Venus

Venus may lack a magnetic field because it doesn't meet the fundamental conditions required for a dynamo. While the planet has a conductive material, it lacks the necessary fluid motion.

The core of Venus is completely solidified, likely because the temperature is too low to keep it molten.

Due to Venus is more close to Solar and having a thick atmosphere, the core temperature may be higher than the Curie point. This means every magnetism will be lost.

Additionally, the core could be partially fluid, but it remains stable due to the planet's slow rotation.

# 2 Magnetopause distance of a planet with a diapole magnetic field

## 2.1 Derive the Formula for Magnetopause Distance

For a dipole magnetic field, the components are:

$$B_r = \frac{2\mu_0 m}{4\pi} \cdot \frac{\cos\theta}{r^3}, \quad B_\theta = \frac{\mu_0 m}{4\pi} \cdot \frac{\sin\theta}{r^3}$$

At the equator, let the magnetic field be $B = B_E$. Then:

$$B_r = 2B_E \cdot \frac{R_E^3}{r^3} \cos\theta, \quad B_\theta = B_E \cdot \frac{R_E^3}{r^3} \sin\theta$$

After the solar wind compresses the magnetic field, we can add a mirror dipole. Then, at the magnetopause:

$$B = 2B_\theta = 2B_E \cdot \frac{R_E^3}{r^3}$$

The solar wind pressure is equal to the magnetic pressure:

$$\rho_{sw} v_{sw}^2 = \frac{B^2}{2\mu_0}$$

Substitute the value of $B$:

$$\rho_{sw} v_{sw}^2 = \frac{4B_E^2 \cdot \frac{R_E^6}{r^6}}{2\mu_0}$$

Solve for $r$:

$$r = \left( \frac{2B_E^2}{\mu_0 \rho_{sw} v_{sw}^2} \right)^{1/6} R_E$$

### 2.2 Estimate the magneticpause of Jupiter and Saturn

| Planet | $B_E$ (T) | Radius ($R$) | density (1/cm$^3$) | $v_{sw}$ (km/s) | $r_{mp}$ (in $R$) |
|---|---|---|---|---|---|
| Jupiter | $4.3 \times 10^{-4}$ | $R_J$ | 0.4 atom | 400 | $\approx 37$ |
| Saturn | $2.1 \times 10^{-5}$ | $R_S$ | 0.1 atom | 400 | $\approx 17$ |

**Table 1:** Solar wind parameters and estimated magnetopause distances for Jupiter and Saturn

## 3 Calculate the cyclotron frequency of electron and proton in a magnetic field of 10 Gauss

The cyclotron frequency $f_B$ is given by:

$$f_B = \frac{qB}{2\pi m}$$

For a magnetic field $B = 10\,\text{Gauss} = 10^{-3}\,\text{T}$, the cyclotron frequency of electron and proton are:

| Particle | Charge $q$ (C) | Mass $m$ (kg) | Cyclotron Frequency $f_B$ (Hz) |
|---|---|---|---|
| Electron | $1.6 \times 10^{-19}$ | $9.11 \times 10^{-31}$ | $2.80 \times 10^7$ |
| Proton | $1.6 \times 10^{-19}$ | $1.67 \times 10^{-27}$ | $1.52 \times 10^4$ |

**Table 2:** Charge, mass, and cyclotron frequency of electron and proton in a 10 Gauss magnetic field

# 4 Plasma Frequency and Detectability of Jupiter's Cyclotron Emissions

The plasma frequency is given by:

$$\omega_{pe} = \left(\frac{ne^2}{m_e \varepsilon_0}\right)^{\frac{1}{2}} = 5.64 \times 10^7 \,\text{rad/s}, \quad f_{pe} = \frac{\omega_{pe}}{2\pi} = 8.97 \times 10^6 \,\text{Hz}$$

The electron cyclotron frequency near Jupiter is:

$$f_B = \frac{qB}{2\pi m_e} = 1.20 \times 10^7 \,\text{Hz}$$

### Conclusion

Since the cyclotron frequency $f_B \approx 12.0\,\text{MHz}$ is greater than the plasma cutoff frequency $f_{pe} \approx 9.0\,\text{MHz}$, Jupiter's cyclotron maser emissions can be detected by ground-based radio telescopes.

# 5 Exoplanet Detection Methods

## 5.1 1. Radial Velocity Method (Doppler Spectroscopy)

**Principle:**
The radial velocity method detects the Doppler shift in the spectrum of the host star caused by the gravitational influence of an orbiting planet. As the star and planet both orbit their common center of mass, the star exhibits periodic motion toward and away from the observer, causing periodic Doppler shifts in the star's spectral lines.

- **Blue shift:** Occurs when the star moves toward the observer.
- **Red shift:** Occurs when the star moves away from the observer.

**What Can Be Inferred:**

- **Orbital period** (derived from the periodicity of the Doppler signal).
- **Orbital eccentricity** (obtained from the shape of the velocity curve).
- **Semi-major axis** (calculated using Kepler's third law).

**Observational Biases:**

- The method favors detecting massive planets that are in close orbits to their host stars, as the signal is stronger.
- The technique is biased toward edge-on systems. If the orbit is not edge-on, the Doppler signal diminishes due to the sine function dependence ($\sin i$).
- The method is less effective for detecting planets around highly active or fast-rotating stars, as their spectral lines can be noisy and difficult to distinguish.
- The radial velocity method is not as sensitive to Earth-mass planets, particularly those in wide orbits.

## 5.2 2. Transit Method

**Principle:**
The transit method involves measuring the periodic dimming of a star's light as a planet passes

in front of it (from the observer's line of sight). This dip in brightness is caused by the planet obscuring a small portion of the star's light. The periodicity and magnitude of the dip can be used to derive the planet's orbital and physical properties.

**What Can Be Inferred:**

- **Planetary radius,** derived from the depth of the brightness dip.
- **Orbital period,** determined from the time between transits.
- **Orbital inclination,** which must be near edge-on for the planet to transit the star.

**Observational Biases:**

- The method requires the planet's orbital inclination to be close to edge-on, meaning that the orbit must align such that the planet passes directly between the observer and the star.
- The transit method favors large planets that are close to their host stars because they cause deeper and more frequent dips in brightness.
- False positives can occur in some cases, such as when an eclipsing binary star system produces similar periodic dips in brightness.
- Detection becomes increasingly difficult for planets with long orbital periods, as transits become infrequent.