Non-Relativistic Equations of the Internal Structure of a Neutron Star

Joseph Soliz*
California State University, Long Beach
(Dated: 18 August 2020)

In this undergraduate research project, nine questions were given to answer about the internal structure of neutron stars. Using research from Silbar, Griffiths, and Glendenning, solutions are created to understand the interior of this type of star. Ranging from quantum mechanics, Newtonian mechanics, thermodynamics and astrophysics are non-relativistic equations solved mathematically. Using these provides an understanding in the Fermi energy of the star to the combating pressures inside to where it would be in danger of collapse. While some results might have errors, the provide an general insight that other undergraduate students can use in knowing what a neutron star is like.

INTRODUCTION

Talk about research paper, Undergraduate research paper. In an published paper titled "Neutron stars for undergraduates", Richard R. Silbar and Sanjay Reddy of Los Alamos National Laboratory provide calculations the structure of white dwarf and neutron stars for an undergraduate thesis. Covering different areas of physics from thermodynamics to quantum statistics to nuclear physics to special and general relativity. The computations for solving the coupled structure differential equations are illustrated using a symbolic computational package of Mathematica. Through all this Silbar and Reddy provide a guide for undergraduate physics students in developing computational skills and understanding physics of equations of states including for degenerate stars. [1] Using this as a starting guide, "Neutron Stars for Undergraduates" gives an outlook into an provided undergraduate research assignment on understanding the internal structure of a neutron star.

Assigned from Professor Prashanth Jaikumar of California State Univeristy, Long Beach, this research solves some solutions of how the internal structure of a neutron star is set up as shown in Figure 1. Finding the highest filled neutron state in the star, (n_f), will provide the energy of the state known as the Fermi energy, $epsilon_f$, along with the internal energy of the star (U). From learning the degeneracy of the star, the degeneracy pressure can be computed and upon learning the star's gravitational self-energy is the star's gravitational pressure calculated. Balancing the two pressures to zero gives the equilibrium radius on where this equilibrium. Lastly, a "danger of collapse" condition is derived to understand when the neutron star becomes unstable and collapses in on itself.

1. Find the highest filled neutron state in the star (n_F).
2. Compute the energy of this state, which is the Fermi energy ε_F.
3. Compute the internal energy of the star (U), in terms of the Fermi energy.
4. Compute the degeneracy pressure using $P = -\frac{\partial U}{\partial V}$
5. Compute the star's gravitational self-energy.
6. From the gravitational self-energy, calculate the star's gravitational pressure, using the same method as above.
7. Balance the two pressures by requiring that they sum to zero at equilibrium.
8. From the equilibrium equation, find the equilibrium radius.
9. Derive a "danger of collapse" condition.

Figure 1: Questions of assignment provided by Professor Prashanth Jaikumar.

RESEARCH

For pure neutron star, a real neutron star consists not just of neutrons but contains a small fraction of protons and electrons. The degenerate particles in these stars are neutrons where a highly dense state of fermionic matter in which particles must occupy high states of kinetic energy to satisfy the Pauli exclusion principle. [1] In Pauli exclusion principle, the energy states in quantum mechanics play a role. Two or more identical fermions cannot occupy the same quantum state within a quantum system simultaneously. If all the available low-energy levels for the decay proton are already filled by the protons already present, then the Pauli exclusion principle takes over and prevents the decay from taking place. [2] So this would suggest that the highest filled neutron state will depend on what the spin states are of the fermions.

A fermion gas in which all quantum states below a given energy level are filled is called a fully degenerate fermion gas. The Fermi gas model ignores the strong nucleon-nucleon interactions, which contributes to energy density. The Fermi energy is a concept in quantum mechanics usually referring to the energy difference between the highest and lowest occupied single-particle states in a quantum system of non-interacting fermions at absolute zero temperature. [2] The Fermi energy plays into the potential internal energy of the neutron star.

From upon understanding the energy of state of the degeneracy star, the degeneracy pressure then comes into play. Degenerate matter is usually modelled as an ideal Fermi gas, an ensemble of non-interacting fermions, where particles limited to a finite volume may take only a discrete set of energies, called quantum states. Pauli exclusion principle prevents identical fermions from occupying the same quantum state. At lowest total energy, all the lowest energy quantum states are filled, i.e. full degeneracy. Degeneracy pressure remains non-zero even at absolute zero temperature, adding particles or reducing the volume forces the particles into higher-energy quantum states. Compression force is required which manifest resisting pressure. [1] The key feature is degeneracy pressure does not depend on the temperature but only on the density of the fermions. Degeneracy pressure keeps dense stars in equilibrium, independent of the thermal structure of the star.

In book called *Special and General Relativity: with Applications to White Dwarfs, Neutron Stars and Black Holes* by Norman K. Glendenning, the star's gravitational self-energy would be the gravitational binding energy. It's minimum energy that must be added to a system to cease being a gravitational bound state. In virial theorem, gravitational binding energy of a star is about two times its internal thermal energy in order for hydrostatic equilibrium to maintain. As a star becomes more relativistic, gravitational binding energy approaches zero and star becomes unstable. Another term to for this type of self-energy is gravitational binding energy where the minimum energy is be added to the system of a star in order for it to cease being in a gravitational bound state. [3]

Both the degeneracy pressure and the star's gravity illustrate what ever star goes through internally in a balance state of *hydrostatic equilibrium.* In any star, the gravitational force due to the mass of the star and the gas pressure due to energy generation in the core of the star balance with each other. This balance is self-regulating: if the rate of energy generation in the core slows down, gravity wins out over pressure and the star begins to contract.[3] This contraction increases the temperature and pressure of the stellar interior, which leads to higher energy generation rates and a return to equilibrium.

Where a star, particularly a neutron star, is under threat of collapse seems to involve the status of it's hydrostatic equilibrium. Basically, where gravity begins to overpower the pressure from the Fermi Energy. Each star has a limit of how much it can contain in density, know as "maximum mass", where mass becomes so overpowering on the pressure that gravity wins. [3] To balance this, it is required that the increment in pressure is large enough, yet the rate of change of pressure with respect to energy density is related to the speed of sound. In Newtonian, this is unbounded and cannot exceed the speed of light that puts a bound on the pressure increment associated with changes in density. In this situation, any increase in density will result in an additional gravitational attraction that cannot be compensated for by the corresponding increment in pressure. Including general relativistic corrections, since they act to amplify gravity, at the maximum mass their density becomes incredibly compact. [1]

In General Theory of Relativity, a maximum mass star with central density a few times above nuclear density are no stable configurations, no matter what the equation of state is. Consider $dp/d\epsilon$ greater than or less than 0, where pressure must rise as the density or at least remain constant as in the mixed phase of a first-order phase transition in a single-component substance.[1] Thus the maximum mass of the neutron star will depend on state of equilibrium of the star and where it is in danger of collapse.

RESULTS

To find the neutron state, consider a particle in a box that gives $\sqrt{(n_x)^2 + (n_y)^2 + (n_z)^2}$ which can be considered for the quantum number magnitude of n_f.[2] Finding n_f in terms of N, total number of neutrons, where we use the density of $\rho =$ N/V. In calculating section of an sphere of neutrons, we get an octant or 1/8 in which it fills up the k-space of the spherical shell. $(1/8)((4/3)\pi k_f^3) = Nq/2(Pi/v)$ By replacing k-space with n-space, the octant and the volume of the neutron spherical shell are considered. For fermions in Pauli exclusion principle, only two of them can occupy any given state where the factor of 2 represents both spin up and spin down are available in each quantum state. [2]

So then, $N = 2(1/8)((4/3)\pi n_f^3)$ where n_f also represent the radius of neutron star space. Simplified for n_f, (1)

$$n_f = (\frac{3N}{\pi})^{1/3}$$

Corresponding to kinetic energy of the momentum in relativistic terms where $\epsilon_f = k_f c$ relates to finding the Fermi Energy. In the energy level, the nth wave solution in quantum mechanics for neutron is the absolute momentum $p = (hn_f)/(2V^{1/3})$. For fermi energy, in relation to kinetic energy, $\epsilon(n) = p^2/2m = h^2n^2/8mV^{2/3}$ where we substitute n with the result of equation (1). [4] Here the energy state of the Fermi Energy is computed in non-relativistic terms. (2)

$$\epsilon_f = \frac{h^2}{8m}(\frac{3N}{\pi V})^{2/3}$$

After finding the Fermi energy, this can then be applied to understanding the internal energy (U) of a neutron star. Integrate energy of all of the filled energy levels, core is degenerate with levels starting at the ground state and continuous to n_f, and integrate by spherical coordinates. Spherical coordinates should be important since this taking in account of the volume of the sphere of the star.

The process starts out with U=

$$2\int_0^{n_f}\int_0^{\pi/2}\int_0^{\pi/2}\epsilon(n)n^2 dnd\theta d\phi$$

=

$$2\int_0^{n_f}\int_0^{\pi/2}\int_0^{\pi/2}\frac{h^2n^2}{8mV^{2/3}}n^2 dnd\theta d\phi$$

Taking care of the integration of $d\theta$ and $d\phi$, the outcome is $\pi/2$. =

$$2\int_0^{n_f}\frac{h^2}{8mV^{2/3}}\frac{\pi}{2}n^4 dn$$

=

$$\frac{\pi h^2}{8mV^{2/3}}\frac{(n_f)^5}{5}$$

A more simplified version of this is placing equation (1) and (2) for n_f and ϵ_f. U =

$$\frac{\pi h^2}{8mV^{2/3}}\frac{(n_f)^5}{5}$$

=

$$\frac{\pi h^2}{8mV^{2/3}}\frac{(n_f)^2}{5}\frac{3N}{\pi}$$

=

$$\frac{3}{5}N(\frac{h^2n_f^2}{8mV^{2/3}}$$

. Overall,This include kinetic and Fermi energy together. (3)

$$U=\frac{\pi h^2}{8mV^{2/3}}\frac{(n_f)^5}{5}=\frac{3}{5}N\epsilon_f$$

As part of hydrostatic equilibrium, compute the degeneracy pressure of the neutron star using $P=-(\partial U/\partial V)$.

$$P=-\frac{\partial U}{\partial V}=-\frac{\partial}{\partial V}[(\frac{\pi h^2}{8mV^{2/3}})\frac{n_f^5}{5}]$$

=

$$(-\frac{\partial}{\partial V}V^{-2/3})[(\frac{\pi h^2}{8m})\frac{n_f^5}{5}]$$

=

$$\frac{2}{3}V^{-5/3}[(\frac{\pi h^2}{8m})\frac{n_f^5}{5}]$$

=

$$\frac{2}{3}\frac{1}{V^{5/3}}[(\frac{\pi h^2}{8m})\frac{1}{5}(\frac{3N}{\pi})^{5/3}]$$

= (4)

$$\frac{\pi h^2}{20m}(\frac{3}{\pi})^{2/3}(\frac{N}{V})^{5/3}$$

or a simplified version

$$P_d=\frac{2}{3}(\frac{U}{V})$$

. In fact, the reduced solution for the degeneracy pressure matches with Fermi gas pressure of electron degeneracy. In electron degeneracy pressure, Pauli exclusion principle disallows two identical half-integer spin particles (electrons and all other fermions) from simultaneously occupying the same quantum state. Result is an emergent pressure against compression of matter into smaller volumes of space.

To compute this, consider integrating both hydrostatic equilibrium and gravitational potential energy for a uniform density neutron star. [3] Considering

$$dM(r)=(4\pi)r^3\rho dr$$

of the spherical shell radius where the gravitational energy occurs with

$$M(r)=(4\pi/3)r^3\rho$$

of the radius of the interior of the star. Since energy for a shell is the negative of the gravitational potential energy is

$$dU=-G\frac{M(r)dM(r)}{r}$$

, then use this to integrate all shells of the star.

$$U_g=-G\int_0^R\frac{M(r)dM(r)}{r}=-G\frac{16}{3}\pi^2\rho^2\int_0^R r^4 dr$$

=

$$-G\frac{16}{15}\pi^2\rho^2R^5$$

. Insert the uniform density of the neutron star with the star's radius, R, $\rho=\dfrac{M}{\frac{4}{3}\pi R^3}$. =

$$-G\frac{16}{15}\pi^2R^5(\frac{M}{\frac{4}{3}\pi R^3})^2$$

= (5)

$$-\frac{3}{5}(\frac{GM^2}{R})$$

This is the neutron star's gravitational self-energy where the negative represent the vector downward to the core due to gravity.

Now to calculate the star's gravitational pressure from the gravitational self-energy (5), using the same method above.

$$P_g = -\frac{\partial U_g}{\partial V} = -\frac{\partial}{\partial V}[-\frac{3}{5}GM^2(\frac{4\pi}{3V})^{1/3}]$$

=

$$\frac{3}{5}GM^2(\frac{4\pi}{3})^{1/3}\frac{\partial}{\partial V}(V^{-1/3})$$

=

$$\frac{3}{5}GM^2(\frac{4\pi}{3})^{1/3}[-\frac{1}{3}V^{-4/3}]$$

= (6)

$$P_g = -\frac{1}{5}(\frac{GM^2}{R})(\frac{4\pi}{3V})^{1/3}$$

Just like gravitational energy, gravitational pressure is negative which makes the area of the star more compact.

Balance the two pressures by requiring that they sum to zero at equilibrium. For the two pressures to balance with each other, they must sum with each other to equal zero. The equilibrium between the degeneracy pressure of the Fermi energy and the gravitational pressure would need to stay constantly be maintained. (7)

$$P_d + P_g = 0$$

Note that this is all based on hydrostatic equilibrium. Since the equation of P_g is negative, that would imply P_d has to be positive. In order for this work, we would need to look at where the equilibrium is maintained, viz the equilibrium radius. From the equilibrium equation, find the equilibrium radius. To solve this, this plug in (4) and (6) to have them equal to each other and solve for R. Where V is the volume of the entire star and M = mN.

$$P_d + P_g = 0 \Rightarrow P_d = -P_g$$

$$\frac{2}{5}\frac{h^2}{8m}(\frac{3}{\pi})^{2/3}(\frac{N}{V})^{5/3} = \frac{GM^2}{5}(\frac{4\pi}{3})^{1/3}V^{-4/3}$$

$$\frac{h^2}{8m}(\frac{N}{V})^{5/3} = \frac{GM^2}{2}(\frac{\pi}{3})(4)^{1/3}V^{-4/3}$$

$$\frac{h^2}{8m} = \frac{GM^2}{2}(\frac{\pi}{3})(4V)^{1/3}(N)^{-5/3}$$

$$\frac{h^2}{8m} = \frac{GM^2}{2}(8)^{1/3}(\frac{\pi}{3})^{4/3}R(N)^{-5/3}$$

$$\frac{h^2}{8m}(\frac{M}{m})^{5/3}\frac{2}{GM^2(8)^{1/3}} = R(\frac{\pi}{3})^{4/3}$$

$$\frac{2h^2(M)^{-1/3}}{8m^{8/3}G(8)^{1/3}} = R(\frac{\pi}{3})^{4/3}$$

(8)

$$R_{eq} = \frac{3}{8}(\frac{(\frac{3}{2})^{1/3}h^2}{G\pi^{4/3}m^{8/3}})M^{-1/3}$$

Graphing this shows as the mass increases, the radius the balances the two pressures exponentially decreases. Graph

The equilibrium plays an important role in this situation with the equilibrium radius that was derived in equation (8). So to derive a "danger of collapse" condition, consider the rest mass of energy-momentum relation in special relativity, $E = mc^2$. Estimate when pressures of degeneracy and gravitational pressure are unstable by assuming neutrons become relativistic when their average energy is similar to their rest mass. $E > mc^2$ and $\frac{U}{N} > mc^2$ will give the maximum mass of instability.

$$U = \frac{3}{5}N\epsilon_f = \frac{3}{5}N[\frac{h^2}{8m}(\frac{3N}{\pi V})^{2/3}]$$

$$\frac{U}{N} = \frac{3}{5}[\frac{h^2}{8m}(\frac{3N}{\pi V})^{2/3}]$$

In volume of star, place the equilibrium radius in it that will be applied later.

$$V = \frac{4}{3}\pi R_{eq}^3 = \frac{4}{3}\pi[(\frac{3}{8})\frac{(\frac{3}{2})^{1/3}h^2(M)^{-1/3}}{G\pi^{4/3}m^{8/3}}]^3$$

=

$$\frac{4\pi}{3}(\frac{3}{8})^3(\frac{3}{2})\frac{h^6(M)^{-1}}{G^3\pi^4m^8}$$

Now place this in the volume of the internal energy for $(U/N) > mc^2$.

$$\frac{3}{5}\frac{h^2}{8m}(\frac{3N}{\pi V})^{2/3} > mc^2$$

$$\frac{3}{5}\frac{h^2}{8m}[\frac{3M}{\pi m}(\frac{8}{3})^3\frac{G^3\pi^3m^8M}{h^6}]^{2/3} > mc^2$$

$$\frac{1}{5}\frac{8}{(3)^{1/3}}(\frac{G^2\pi^{4/33}m^{11/3}M^{4/3}}{h^2}) > mc^2$$

$$M^{4/3} > \frac{(3)^{1/3}(5)}{8}(\frac{h^2c^2}{G^2\pi^{4/3}m^{8/3}})$$

(9)

$$M_{unstable} > [\frac{3(5)^3}{2^9}]^{1/4}(\frac{hc}{G})^{3/2}(\frac{1}{\pi m^2})$$

The solution provides where the neutron star's mass

cause the hydrostatic equilibrium to collapse in on itself. To know the exact mass this type of star reaches it's limit, plugin the mass of a neutron. In equation (9), m is the mass of a neutron particle of 1.675×10^{-27} kg. The result of the unstable mass of the star:

$$M_{unstable} > 1.989 \times 10^{30}kg \Rightarrow M_{unstable} > 8.58M_{sun}$$

ANALYSIS

Analysis of the Fermi Energy and Internal Energy Equation (1) is basic since built on the number of particles, N, in the neutron state. Based on Pauli exclusion principle, only two fermions can occupy any given state that they fill up one octant of a sphere in the wave vector of space. This is determined by the fact that each pair of electrons requires a volume π^3/V of

$$\frac{1}{8}(\frac{4\pi}{3}k_f^3 = \frac{Nq}{2}(\frac{\pi^3}{V})$$

. Thus

$$k_f = (3\rho\pi^2)^{1/3}$$

where

$$\rho = \frac{Nq}{V}$$

is the free electron density.[2] So the using this steps provides a correct solution on the highest filled neutron state.

Using Equation (1), Equation (2) is provided that gives the formula for Fermi energy of the neutron star. The only problem is if it should consider to have the constant of speed of light, c, since the neutron star has such heavy gravity that relativity would be involved, unless you are looking at this non-relativistic terms. At higher densities, such as in a neutron star, the electrons become fully relativistic. For neutron stars, the equation of state should match to a low-density equation of state that describes matter and should be normalized. At the causal limit at high density and normalized in near nuclear saturation density as a causal limit equation of state this provides the most resistance to collapse and therefore yields an upper bound on the limiting mass.[1] Still, equation [2] provides an efficient solution for Fermi Energy since this formula can be found in statistical mechanics so would be manageable.[4]

Internal energy of equation (3) does seem to work out especially in terms of Fermi energy. Since the entire volume of the star is a sphere, it is logical to use spherical coordinates in order to calculate the entire volume of the star. Beside the internal energy of fermions, thermal energy would be the potential energy in a star as well. In virial theorem, the average over time of the total kinetic energy of a stable system of discrete particles, bound by potential forces, with that of the total potential energy of the system. [3] Half of the gravitational potential energy released if the star contracts is converted into thermal energy, while the other half is lost from the star. Thus, the internal energy that was calculate is part the balance to maintain the structure of the neutron star. It is also where pressures and gravity come in.

Equations (5) provides a very general equation on the binding energy every star has due to gravitational forces. Both pressures, equation (4) of the Fermi degeneracy pressure and equation (6) of $P = -\partial U/\partial V$ are derived from their respective potential energy. Using these equations illustrate the state of the neutron star's internal pressure through both the virial theorem and hydrostatic equilibrium. In fact, the summation of Equation (7) to equal zero shows the relation how the two pressures of Fermi energy and the star's gravity find balance with each other. This matches with the concept of hyrdostatic equilibrium where external forces such as gravity are balanced by a pressure-gradient force.

In *Neutron Stars for Undergraduates*, the coupled nonlinear equations for p(r) and M(r) can be integrated from r = 0 for a starting value of p_0 to the point where p(R) = 0, to determine the star mass M = M(R) and radius R for this value of p0. These equations invoke a balance between gravitational forces and the internal pressure.[1]. That is where equation (8) comes in that shows an area where these two are at equilibrium based from equations (4), (6), and (7). Of course, the pressure may not be sufficient to withstand the gravitational attraction. Thus the structure equations imply there is a maximum mass that a star can have.

Equation (9) seems to be the only solution that seems wrong. Although all the mathematics shown in Results does check out, when plug-in the mass of a neutron, the mass becomes unstable when it's greater than 1.989×10^{30}

kg or 8.58 M_{sun}. Yet, Neutron stars cannot have masses that exceed value of first estimate of 3.14 M_{sun}, approximately 3 times less than what equation (9) provided. Additions of any other constraints in the form of a realistic equation of state, the bound can only become smaller. Any compact object of mass greater than 3 M_{sun} cannot be a neutron star unless it is rotating at or near its Kepler frequency. In fact, rotating spin does play a role in maintain structure of a neutron star. Maximum mass does represent an upper limit to mass that a non-rotating neutron star can be. So then, consider that when a neutron star collapses into a black hole, it's a ultra-relativistic case. The neutron star in ultra-relativity will suffer a situation where the degeneracy pressure scaled with the some power as gravitational pressure. [1]

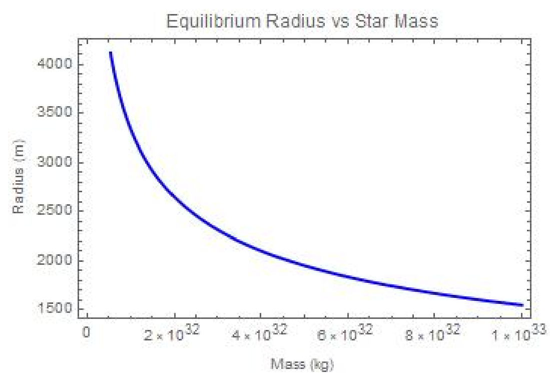

Figure 2: Rough draft of graph of the equilibrium radius. Shows as the mass of the star increases, the radius of the balance between the degeneracy pressure and gravity shrinks down exponentially.

Going back to the equilibrium radius, Figure 2 shows a basic plot that the area of where the degeneracy and gravitational pressure are in balance shrinks down. Using Mathematica, it is shown that even at a smaller radius of 4,000 m near 2×10^{32} the equilibrium radius is very small. From 2×10^{32} to 1×10^{33}, the radius shrinks down exponentially to near zero. This does illustrate the gravity is becoming stronger for the Fermi degeneracy to combat with that not only it becomes more and more compact but struggling to remain stable. If this graph was shown were the maximum radius exponent on the plot was the radius of our sun it would show an exponential slope the equilibrium radius decreases the more mass is placed in. So either there was a miscalculation in the derived equation or the star is on the extreme edge of collapsing.

SUMMARY

In *Neutron Stars for Undergraduates*, Richard R. Silbar and Sanjay Reddy provide detailed descriptions on calculating the internal structure of neutron stars through quantum statistics, nuclear physics, and general relativity. Yet, the results provided in this paper take a more direct approach on the solutions by using basics of quantum mechanics and Newtonian mechanics in a non-relativistic approach. This means if the neutron star was looked at in non-relativistic terms, it still has to obey the fundamental laws of physics from quantum physics to thermodynamics to gravitational physics.

When any ferimon is in a state of energy, it has to obey the Pauli Exclusion Principle where two or more identical ones cannot occupy the same quantum state. If a neutron is to be at it's highest state in a neutron state, then it has to be where an octant of the spherical shell of neutrons and a factor of 2 for spin up and down is represented. Using kinetic energy for Fermi energy provides an equation that's used in statistical mechanics to explain the number of fermions inside the Fermi sphere.[4] From this, the internal energy of the entire neutron star is computed to provide the potential energy of the Fermi degeneracy.

Dependent on the internal energy calculated, the degeneracy pressure is computed where the fermions compress inside the neutron star and produce energy. Based on the calculations of the gravitational binding energy, the pressure from the gravitational forces of the star is derived. Both of these two illustrate the forces at work inside a neutron star where we have gravity pushing inwards to the core while the Fermi energy exerts outward counteracting the gravitational pressure. This both illustrates hydrostatic equilibrium which is important in the structure of stars. In fact, it logically makes sense that the summation of the two pressures equal zero in order to maintain equilibrium. Knowing this provides the area where the two forces meet called the equilibrium radius which seems to shrink the more mass the star has as shown in Figure 2.

Yet, the most questionable result is solution for when the maximum mass of the neutron star is unstable. Although the mathematics involved seem to check out, the result of what happens when you plug-in the mass of a neutron seems off. According to *Special and General Relativity: with Applications to White Dwarfs, Neutron Stars and Black Holes*, the maximum mass for a neutron star is approximately 3 M_sun yet the results from equation (9) make it to be around 8 9 M_sun. This would imply that an error accorded in the results. A likely answer would be that more had to be done when understanding the equilibrium between the degeneracy pressure and the gravitational pressure. Or that from the very beginning when calculation the Fermi energy, relativistic corrections had to be taken into consideration. Since neutron stars do have extreme properties, they create very strong gravitational forces that the inside of them are intensely compact. [3]

Either way, the research done on this show how the fundamental forces of physics behave in a neutron star. Inside the star, hydrostatic equilibrium is pushed to the very limit where the Fermi energy counteracts against gravity to keep the star intact. If, however, the star's mass reaches it's maximum limit, then star collapses in on itself to either explode or turn into a black hole. [**?**] Although more could have been done such as using Mathematica more often as Silbar and Reddy told in their paper and relativistic corrections, this does show another approach that undergraduate students could use in understanding neutron stars. Using non-relativistic terms does obey the laws of basic physics but does seem to have flaws in understanding the movement of collapse. Hopefully, this research can guide the next group of undergraduate students in understanding the internal structure of neutron stars.

ACKNOWLEDGEMENT

- Dr. Prashanth Jaikumar. Professor of Nuclear Astrophysics, Quantum Chromodynamics (QCD), and Neutron Stars at California State University, Long Beach. Aided in providing guidance on research in neutron stars and formalization.

REFERENCES

* Joseph.Soliz@student.csulb.edu

[1] Silbar, Richard R.; Reddy, Sanjay *Neutron Star for Undergraduates.* American Journal of Physics, Volume 72, Issue 7, July 2004.

[2] Griffiths, David J. *Introduction to Quantum Mechanics.* Pearson Prentice Hall, 2nd Edition, Upper Saddle River, NJ 2005.

[3] Glendenning, Norman K. *Special and General Relativity: with Applications to White Dwarfs, Neutron Stars and Black Holes.* First ed., Springer, 2007.

[4] Sethna, James P. *Statistical Mechanics: Entropy, Order Parameters, and Complexity.* Oxford University Press, 2012. Pgs. 150-151.