

# Measuring the Hubble Constant, $H_0$ , and Age of the Universe through Philips Relation on the Peak Magnitudes of Type Ia Supernovae

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## ABSTRACT

Over a month long observation at the Mount Laguna Observatory (MLO), an undetermined supernova 2020aaajl and Type Ia supernova 2020zhh were measured through B, V, and R band. Using the data processed with the World Coordinate System (WCS), the aperture photometry determines the light curves of the targets of the observed magnitude over a period of time. Extracting the B and V band, the maximum peak over a 15 day period ( $\delta m_{15}$ ) is used in the Philips Relation. From this, and including other supernovae observations from 2020 and 2021, the maximum absolute magnitude and distances of the objects are determined and used in the distance-velocity relation in Hubble's law. The outcome where three Hubble's constant:  $H_0 = 59.82 \approx 6.65$  km/s/Mpc,  $H_0 = 69.50 \approx 8.90$  km/s/Mpc, and  $H_0 = 74.89 \approx 7.07$  km/s/Mpc. Using third Hubble constant, the age of the universe was calculate at 13.06 billion years old.

**Keywords:** Hubble Constant, Mount Laguna Observatory (MLO), aperture photometry, light curves, Type Ia supernova, Philips relation, SN2021abzd, SN2020adlt

## 1. INTRODUCTION

### 1.1. Hubble Constant

Since astronomer Edwin Hubble postulated that the universe is ever-expanding, an initial value of the expansion rate of the cosmos was proposed to determine the speed at which the universe is expanding. Known as the Hubble Constant ( $H_0$ ), it is a unit of in kilometres per second over mega-parsecs (km/s/Mpc) that describes how fast the universe is expanding at different distances from a particular point in space. It is a constant in the linear relation between the speed of recession of a distant object and its current distance from the observer called Hubble's Law. (Carroll & Ostlie 2017) Using this parameter gives astronomers a way to measure the scale of the universe as well as to determine it's age.

This plays a role in establishing the cosmic ladder in which a series of measures are made through a succession of methods to determine the distance of celestial objects. Using the absolute magnitude of an objects as a standard candle, a series of them establish a cosmic ladder to establish an approximation of the size of the universe. In turn, through Hubble's Law, we can mea-

sure what the constant vale of  $H_0$ . More importantly, this can also assist in determining the age of the universe. This is calculated based on  $age = 1/H_0$  (Rubin 2021) (Carroll & Ostlie 2017)

Yet, it is still debated on what the actual value of the constant is since there has been conflicting research that shows the measured parameter varies. One value of the Hubble constant was from observations through the Hubble Space Telescope where, Adam G Riess et al. approximated  $H_0 = 73.24 \pm 1.74$  km/s/ Mpc. Yet, the latest measurements on the Hubble constant have been observed through the Hubble Space Telescope by looking at Cepheid Variables in the Large Magellanic Cloud (LMC) by the same team. The Cepheid distance ladder with detached eclipsing binaries (DEBs) and the luminosity of SNe Ia established  $H_0 = 74.22 \pm 1.82$  km/s/ Mpc with systematic uncertainties. 2019ApJ...876...85R This shows more precise measurements are required to be taken on the cosmological expansion to obtain accuracy. One method that was used in this project was in regard using light curves through the Philips relation.

### 1.2. Type Ia Supernovae and Philips Relation

In 1993, astronomer M. Philips derived the absolute magnitudes in the B, V, and I bands for nine Type Ia supernovae using host galaxy distances estimated by the surface brightness fluctuations. These type of supernovae have proven to be very effective standard candles at distances to 1 Gpc. Carroll17 Type Ia supernovae appear to be similar to each other in the thermonuclear explosion in the Chandrasekhar limit making them easy to have consistent measurements. Rubin (2021) Using the Cepheid distances on the absolute magnitude over time, the light curves can be used as relation to find the Hubble constant.

M. Philips found a relation that the absolute magnitudes appear to be tightly correlated with the initial rate of decline of the B light curve. This implies that the connection between peak luminosity and light curve shape can provide the peak maximum of the absolute magnitude of the Type Ia supernova. The technique on the absolute magnitude of the supernovae can be used in this matter on cosmological expansion. The peak luminosity of the supernova fit is correlated with the decline in magnitudes from maximum peak of the light curve to 15 days after peak ( $\delta m_{15}$ ) known as the Philips Relation. Phillips (1993) Using this method assist in calculating the maximum absolute magnitude of the supernova that can provide the distance and velocity of the light that provides the Hubble constant.

### 1.3. Project

To determine the Hubble constant  $H_0$ , light curves from various Type Ia supernovae are used from the observations at the Mount Laguna Observatory (MLO). A series of observations was made from 11-09-2021 to 12-04-2021 on recent catalog supernova. In this paper two type Ia supernovae were used for the majority of the project, 2021abzd and 2021adlt. Each image data of the targets were processed as well as provided the World Coordinate System (WCS) to provide geometric transformations on each set of coordinates. Aperture photometry is used to extract the B and V band light curves. Similar to Philips, this calculates the absolute magnitude of each supernova over time. Using a template from E. Y. Hsiao et al. (2007), the maximum B-band of the phases are used to determine the decline rate parameter,  $\Delta m_{15}$ . Adding the Philips relation, absolute magnitudes for 2021abzd and 2021adlt are determined through the peak apparent magnitudes of the light curve fits. Using all light curve data to plot the distances (Mpc) against redshift velocity (km/s) to determined slope and its uncertainty of the Hubble Constant and the age of the universe.

## 2. DATA

### 2.1. Mount Laguna Observatory

Nine observations of the supernovae were done at the Mount Laguna Observatory (MLO) from 2021-11-09 to 2021-12-04. Operated by the Department of Astronomy at San Diego State University, CCD images were taken through the 1-meter telescope has a 13.5-micron pixels while the focal length of the telescope is about 7772 mm. The on-sky pixel size of the 1-meter MLO telescope is approximately 0.358 arcsec/pix. Quimby20 Each one taken in a different optical filter in the camera that will be applied to create the RGB image: R-band, V-band, and B-band. R-band provides the image in red color to the visible spectrum. V-band can be seen as the green color since it lines in the middle of the spectrum. B-band is the blue color of the image.

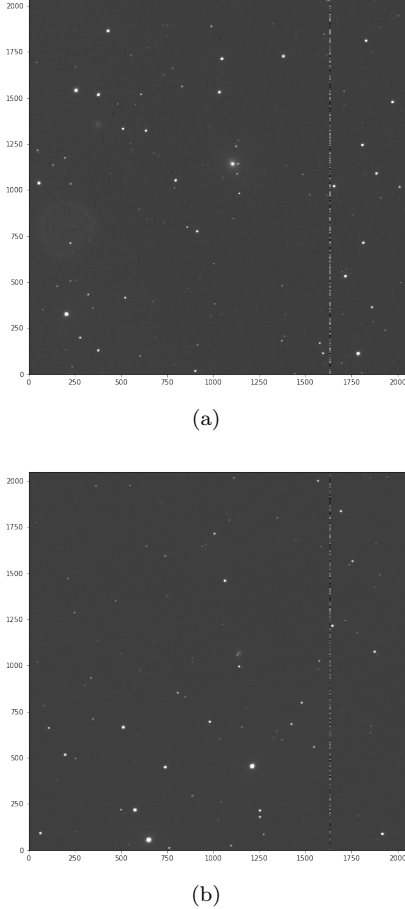
Weather conditions tend to vary where during observations in early November and early December, clear skies provided clear seeing on targets. Mid-November was challenging were cloudy skies would appear that would effect visibility of target that two observation nights had to be called off. The full-width half maximum in the images tend to sway from 4.3 to 6.5. Even the peak tend to be not comprehensible likely due to weather conditions or the supernovae dimmed over time. In either case, seven of the nine observations created acceptable images of 2021abzd and 2021adlt.

### 2.2. Processing Images and WCS

To process the images requires to go through three steps on removing any noise and pixel-to-pixel differences from CCD images: master bias, flat field frame, and calibrated image. To trim and correct a raw image for any floating bias, the bias level must be measure and subtracted from the CCD image. This involves creating a bias frame to make sure the number counts are accounted for in any offset and avoid any negative counts. This will involve making an overscan to remove small variations in the bias level from frame to frame. Flat fields are made to deal with variations in the sensitivity of pixels by creating a uniform field of the pixels to calibrate the illumination and remove much of the affects from the detectors. By stacking each exposure to create a master flat frame for each filter, they are combined with CCD FITS image to create a processed calibrated image. These calibrated images are done by taking the median value of each pixel to help remove any noise and cosmic rays in the CCD image.

After the processing the calibrated images, the FITS image data is then uploaded through the World Coor-

dinate Systems (WCS). (Explain more) In order to get a proper science frame on the image data, solutions are recommend to process the information for the aperture photometry. This will be crucial in order to process the photometric data on the targets. Figure 1 shows two examples of the targets during the first observation night 2021-11-09 through the B-filter.



**Figure 1.** Each image is an calibrated CCD image taken on 2021-11-09 from MLO. (a) SN 2021abzd. (b) SN 2021adlt.

### 2.3. Light Curves

To obtain the light curves of the targets requires using aperture photometry by measuring the zero-point of the target start to the background. The brightness for each object is measured through the magnitude of the supernova and then compared the magnitude of the other stars in the background. This based on the formula of the apparent magnitude in stars.

$$m_{obj} - m_{ref} = -2.5 \log_{10} \left( \frac{f_{obj}}{f_{ref}} \right) \quad (1)$$

In equation 1,  $m_{obj}$  is the magnitude of the main object star with a flux of  $f_{obj}$  and  $m_{ref}$  is the magnitude of another star in this case your reference with a flux of  $f_{ref}$ . The two fluxes over each of the stars represent the flux density that is being calculated. In this case, it flux density is the counts that are calculated through the aperture. (Sanquist 2021) (Carroll & Ostlie 2017)

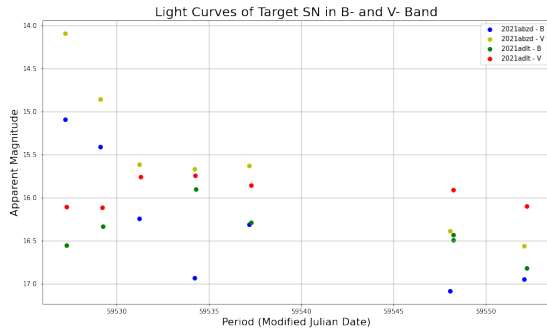
To measure the counts of the reference stars to the supernovae, an aperture has to be set in order to measure the light of the focal plane area Sanquist (2021). For reference stars, the APASS catalog "II/336/apass9" records all the stars in the cosmos with data on the their magnitude in V and B band filters. The counts of the circular aperture are measured on both the reference stars and target based on provide x, y coordinates. It is apparent that the aperture is fully contained on the image by defining the shape of the radius as well as an annulus for the background. From gathering data in the aperture and annulus, the counts and its uncertainties are measure from the difference of the aperture sum and background counts.

Where the counts of the aperture are calculated similarly to ratio of the fluxes to the target star and reference star where the the image zero-point,  $zp$ , acts as our reference to provide the absolute magnitude for the target,  $m = zp - 2.5 \log(counts)$ . The counts are then applied to measure the magnitude of each provided star based on the zeropoints in the CCD image. Basically it takes the flux magnitude equation and simplifies down to  $zp = m + \log_{10}(counts)$  where  $m$  is the apparent magnitude. The flux ratio is condensed to the counts that were measured. The signal-to-noise ratio of the counts and it's uncertainties are greater than 3 for accurate detection and stars are removed from edge of the image. However, in this case, the signal-to-noise ratio had to be reduced down to be greater than 0 in order to obtain all 7 observation nights. It is likely due to weather conditions, the counts were off and reduced when clouds passed by or that there were some errors during the processing of the CCD images.

From the photometric data of 2021abzd and 2021adlt, the light curves can be given on each supernova in the B-band and V-band. Yet, it is important to take note of any galactic extinction that could have interfered with the magnitude. This is where the absorption and scattering of electromagnetic radiation from dust and gas can cause a variation in the magnitude of objects between an emitting astronomical object and the observer. Carroll17 1993ApJ...413L.105P Information the extinction in coordinates of the supernova targets are provided

by the IRSA Dust Extinction Service Queries in the CTIO B and V absorption. Taking account of these, Figure 2 shows the light curves of the supernovae. What shows is that each supernovae seems to be have been observed at different times of it's peak brightness. The data on 2021abzd seems to show that the peak magnitude has already passed during the first day of observation. Instead it declines in both wavelengths as it steadying concave downward in a negative slope. On the other hand, 2021adlt shows an convex upward slope over time. This would indicate that the Type 1a supernova was observed during it's peak magnitude.

Table 1: CTIO absorption values		
Target Area	B	V
2021abzd	0.193	0.142
2021adlt	0.561	0.413



**Figure 2.** Light curve plot of the modified Julian date over the apparent magnitude of the observations of 2021abzd and 2021adlt with the galactic extinction accounted.

#### 2.4. SN1a Data

Besides the data from SN 2021abzd and SN 2021adlt, other observations of type 1a supernova were used in the later portion of this project. These as well as the two main targets are applied to the distance-velocity relation that Edwin Hubble used to calculate the rate of the universes expansion. While the distances were calculated as discussed in the Philips Relation subsection, the redshift on each supernova is used to obtain the redshift velocity of the object. Using  $z \approx v/c$ , the redshift can provide the speed at which the object is moving based on the speed of light,  $c$ , at  $3 \times 10^5$  km/s. (Carroll & Ostlie 2017) This based on Information on the redshift of each supernovae candidate is obtain from the Transient Name Server (TNS) provided by the International Astronomical Union (IAU). (cite) Table 2 lists all the

redshift, recessional velocity, and calculated distance of each supernova.

In order to get a more accurate calculation of the Hubble's constant, it was decided to also include observations of Type 1a Supernovae from November to December 2020. Gathered from a previous course in Astronomical Techniques using the CCD camera of the 1-meter telescope at MLO. The ones selected were based on those that provided information on the redshift of the target. Five supernova including the two targets were chosen and eight candidates were selected from the 2020 observations. Further discussion on how their distances are calculated are discussed in the Philips Relation and the outcome of their contribution to the Hubble constant discussed in the subsection of the same name.

### 3. ANALYSIS

#### 3.1. Best Fit Parameters

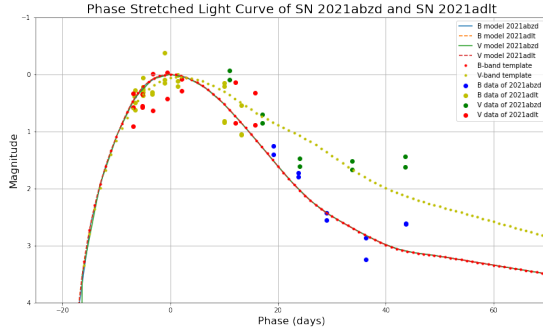
To determine where 2021abzd and 2021adlt data are in a Type Ia supernova template, the best fit model parameters of the target light curves are determined. The maximum apparent B-band magnitude at the peak of the light curve (peakmag), the date of maximum B-band brightness (jdmax), and the decay in which how fast the template rises and fades known as the stretch parameter are what needs to be found. One way to do this is take observer day of the observed magnitudes into template phase based on  $phase = (jdobs - jdmax)/stretch$ .

In order to solve this, we use a spectral template of Type Ia Supernovae done by Eric Hsiao et al. 2007. Each template provides a table of magnitudes relative to B-band maximum at certain phases, for in this case the B- and V-band magnitudes. 2007ApJ...663.1187H These applied in our model to calculate the parameters need to create a model of the light curves as a template. The template data on the phase and filter from the Hsiao et al. 2007 are interpolate to a function in the model to determine the peak magnitude. The parameters are then found using the  $\chi^2$  method from the model and the observation data of the target supernova. The best fit parameters are shown in Table 2 of the SNIa candidates.

#### 3.2. Stretched Phase Diagram

Using the best fit parameters, we can now apply the model of our supernovae along with to see your the light curves fit on the phased diagram. Figure 3 shows the light curves of the model and observation data of 2021abzd and 2021adlt aligned Hsiao's spectral template. Using the peak magnitude of the target in B and V band, the created models seem to match with the B and V template. 2007ApJ...663.1187H The observa-

tion data fitted with the model does prove a previous statement on the initial light curves plotted in Figure 2. In both bands, 2021abzd magnitude appears to decrease over time while observations of 2021adlt seem to have taken place during it's maximum magnitude phase. It should be noted that this is without the galactic extinction considered. It was tried to be include it the observation magnitudes but some of the data of the targets already had their extinction processed before submission of the shared data was given. When the extinction was subtracted with the observed data in the graph this caused the data to jump off the fit model. So only the galactic extinction for 2021abzd was applied in the the figure while 2021adlt was left only. In either case, the model template does seem to coincide well with the observation data.



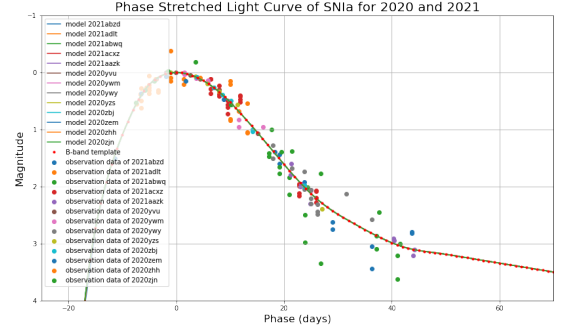
**Figure 3.** A stretched phase light curve of the observation data of 2021abzd and 2021adlt. These are compared with the template model of the B-band and V-band.

Figure 4 is now the phased light curve of all the applied SN data from 2020 and 2021. As before, the B-band template of Hsiao et al. is used to the interpolate function to provide the  $jd_{max}$ , stretch, and peakmag of each supernovae. The figure shows that all the observation data of each target fits along the B-band template as well as the calculated model of each light curve. While some light curves to appear to be off, all observations of these targets take place during the supernova's peak magnitude to it's gradual descent in brightness over time.

### 3.3. Philips Relation and Distances

Once the apparent B-band maximum magnitude and the date during the peak brightness are found, peak-mag, at B-band maximum, we can find the  $\delta m_{15}(B)$  magnitude through the Philips relation. Made by Mark Phillips (1993), this is determined through least-square fits of

$$M_{max} = a + b\Delta m_{15}(B), \quad (2)$$



**Figure 4.** A stretched phase light curve of the observation data of supernovae from 2020 and 2021. These are compared with the template model of the B-band.

where the parameters  $a$  and  $b$  represent each filter band. The difference of the peak magnitude at phase 0, i.e. the  $jd_{max}$ , to the next fifteen days in the phase light curve is  $\delta m_{15}(B)$ . (Phillips 1993)

In this case we calculated the absolute magnitude for the B-band and V-band of 2021abzd and 2021adlt. The parameters for B-band are  $a = -21.726(0.498)$  and  $b = 2.698(0.359)$  and V-band are  $a = -20.883(0.417)$  and  $b = 1.949(0.292)$ . (Phillips 1993) Applying these, we get an absolute peak magnitude ( $M_{max}$ ) of -14.036 mag for 2021abzd and -19.799 mag for 2021adlt in the B-band. In the V-band, 2021abzd has  $M_{max} = -15.328$  mag and 2021adlt has  $M_{max} = -19.491$  mag. Table 2 shows further the  $\delta m_{15}(B)$  for the other supernovae.

Thus from obtaining the  $M_{max}$ , we can now find the distance of the supernovae using the distance modulus.

$$d = 10^{\frac{m - M + 5}{5}} \quad (3)$$

In the modulus,  $m$  is the apparent magnitude in which is the median apparent of the target. The results of the equation is provided in parsecs and then divided by  $10^6$  to give in megaparsecs (Mpc). (Carroll & Ostlie 2017) In the B-band, the distance of 2021abzd was calculated to be 3.505 Mpc and the distance of 2021adlt is 49.806 Mpc. For the V-band, distance of 2021abzd is 7.339 Mpc and distance of 2021adlt is 136.624 Mpc. There seems to be a stark difference on the distance between B and V. Philips has suggested that the B-band would be the best wavelength to use since the slope of the correlation being steepest in B and flatten out more in the V band. This would imply that the distance would appear further too based in the Philips relation. Other issues could be that perhaps the processed data in the targets in the V-band are off. Further use of  $\delta m_{15}(B)$  is used on the other supernovae and uses only the B-band to calculate the distance.



**Table 2.** Type Ia Supernovae Data from 2020 and 2021 Observations

SN Target	$\delta m_{15}(B)$	Distance	z	velocity
	(mag)	(Mpc)		(km/s)
(1)	(2)	(3)	(4)	(5)
2021abzd	2.85024	3.505	0.018086	5425.8
2021adlt	0.71421	49.806	0.021	6300.0
2021abwq	0.96718	253.460	0.08	24000.0
2021acxz	0.69293	482.233	0.089	26700.0
2021aazk	0.90487	293.249	0.018086	19500.0
2020yvu	0.87805	97.552	0.01	3000.0
2020ywm	1.24421	248.974	0.047553	14265.9
2020ywy	0.42278	446.969	0.04	12000.0
2020yzs	1.2518	253.445	0.0672	20160.0
2020zbj	0.72168	479.389	0.066	19800.0
2020zem	1.35845	100.564	0.034	10200.0
2020zhhh	0.88897	165.644	0.04	12000.0
2020zjn	0.79265	257.480	0.04	12000.0

NOTE—All of these were taken from observations in B-band.

### 3.4. Hubble's Constant

Using all the data on the calculated distances and the velocities of the redshift of the supernovae, the distance-velocity relation can be used to determine the Hubble constant. In order to find  $H_0$ , the information needs to be simultaneously fit the data points by using the least-squares method to find the relative parallax and proper motion in RA and Dec. Basically, a system of equations to calculate the best fit in the least square model  $Y = p * X$ . (Osorz 2021) From this model, we can calculate the least squares fit to find the parameters of the Hubble's constant and it's uncertainties. Figure 5 shows three attempts at plotting Hubble's Law in the distance-velocity relation. In fact, the best fit line is an illustration of Hubble's Law,  $v = H_0 d$ . This is the linear relation between the speed of recession of a distant object and its current distance from the observer. (Carroll & Ostlie 2017) Using this, we can get the slope to determine what  $H_0$ .

First attempt is by using all the observation data of the five supernovae of 2021 and the eight supernovae from 2020. Using the least-squares method, we find the Hubble's constant  $H_0 = 59.82 \approx 6.65$  km/s/Mpc. This appears low to other calculations. Figure 5(a) does seem to show three data points of supernovae beyond 400 Mpc far off the best fit line of Hubble's law. These could be

weighing down the slope. To check, Figure 5(b) shows a second attempt at using only the 5 targets from the 2021 observations. Even though there does not appear to be enough data to create a proper fit, the Hubble constant manages to come out as  $H_0 = 69.50 \approx 8.90$  km/s/Mpc. The third attempt is then to keep the data from 2020 and remove the most distance objects that might be causing weight. Removing 2021acxz, 2020ywy, and 2020zbj, the Hubble constant is now at  $H_0 = 74.89 \approx 7.07$  km/s/Mpc with Figure 5(c) showing a better fit line of Hubble's law. This would indicate a greater results than the other previous attempts.

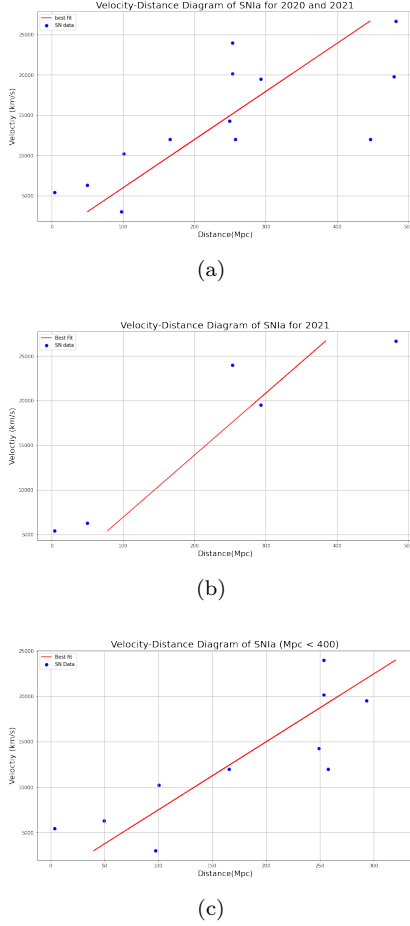
From this, the age of the universe can be determined. Since third  $H_0$  seems to be the best value, this is then plugged into the equation:

$$age = \frac{1}{H_0} \quad (4)$$

The results is the age of the universe is about 13.06 billion years old. This is short compared to the most recent calculation that but the cosmic age at  $13.787 \approx 0.020$  billion years from the Planck 2018 results.(Aghanim et al. 2020)

## 4. DISCUSSION AND CONCLUSION

Based on the Hubble's constant from Riess et al. 2019, the third attempt with the distance of the SNIa objects kept under 400 Mpc has the closest match to their  $H_0$  at



**Figure 5.** Each plot is the distance-velocity relation of Hubble’s Law of the SN Data. (a) Uses all SN data, (b) only contains 2021 data, and (c) has SN data from distances less than 400 Mpc.

$74.03 \approx 1.42 \text{ km s}^{-1} \text{ Mpc}^{-1}$ . (Riess et al. 2019) Even the second attempt with only 2021 observations was close to  $H_0$  value of  $66.93 \approx 0.62 \text{ km s}^{-1} \text{ Mpc}^{-1}$  by Planck Collaboration et al. (2016) based on  $\Lambda$ CDM.(Aghanim et al. 2016) This would indicate that based on the data used, it does seem the layout of the observation data made a difference on what the final results were. Basically the relation of the distance and redshift velocity affects the best fit of the the Hubble’s Law. The removal of 2021acxz, 2020wy, and 2020zjb likely had less to do with their distance and more how their velocities were low compared to how far they were. The age of the universe was

also close to the Planck results of 2018 but still off by 700 million years. If their velocities were higher, then they would have not weighted the slope and be closer to that of Riess et al. 2019 Hubble constant. Shows that Hubble’s Law is depend on the distance-velocity relation where if the object is further than the redshift should be greater. In either case, the final results provides a strong indication on the constant rate of expansion in the cosmos. From this project, using the standard can-

dle method with Type Ia supernovae are very effective in measuring Hubble’s Law. Having a narrow range of peak luminosity, particularly in the B-band, prove useful to measure their distance over the phase to see how far they are and how fast they appear to be moving away from the observer. Philips relation provides an accurate method in obtaining this data based on the spectral template of the light curves from the target. The main issue is simply that processing and calculating this data can be tedious and does require careful measurements on the model and data through  $\chi^2$  and least square fits. Perhaps if more supernovae candidates were observed and in a longer observation time period, a more concise  $H_0$  would have be calculated. Perhaps even adding nearby Cepheid variables and other objects as potential standard candles would provide a stronger measurement of  $H_0$ . Thus establishing a stable cosmic ladder to measure the universe’s expansion rate and more importantly, the age of the universe.

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