DCSA-Assignment6

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Q:證明 $\mathrm{MSE(OPAP-K)} = \mathrm{EMSE(OPAP-K)} =$

$$2^{-\mathrm{k}} * \left\{\left[2 * \left(\sum_{i=0}^{2^{k-1}-1} i^2\right)\right] + [-(2^{k-1})]^2\right\} = \frac{2^{2k-1}+1}{6}$$

proof:

$$2^{-k} * \left\{\left[2 * \left(\sum_{i=0}^{2^{k-1}-1} i^2\right)\right] + [-(2^{k-1})]^2\right\}$$

$$=2^{-\mathrm{k}} * \left\{\left[2 * \left(\frac{(2^{k-1}-1)*(2^{k-1}-1+1)*(2(2^{k-1}-1)+1)}{6}\right)\right] + [-(2^{k-1})]^2\right\}$$

$$=2^{-\mathrm{k}} * \left\{\left[2 * \left(\frac{(2^{k-1}-1)*(2^{k-1})*(2^{k}-1)}{6}\right)\right] + [-(2^{k-1})]^2\right\}$$

$$=2^{-\mathrm{k}} * \left\{\left[2 * \left(\frac{(2^{k-1}-1)*(2^{k-1})*(2^{k}-1)}{6}\right)\right] + 2^{2k-2}\right\}$$

$$=\left[2^{1-k} * \left(\frac{(2^{k-1}-1)*(2^{k-1})*(2^{k}-1)}{6}\right)\right] + 2^{k-2}$$

$$=\left[2^{1-k} * \left(\frac{(2^{2k-2}-2^{k-1})*(2^{k}-1)}{6}\right)\right] + 2^{k-2}$$

$$=\left(\frac{(2^{k-1}-1)*(2^{k}-1)}{6}\right) + 2^{k-2}$$

$$=\left(\frac{(2^{2k-1}-2^{k}-2^{k-1}+1)}{6}\right) + 2^{k-2}$$

$$=\left(\frac{(2^{2k-1}-2^{k}-2^{k-1}+1)}{6}\right) + \frac{2^{k-1}}{2}$$

$$=\frac{(2^{2k-1}-2^{k}-2^{k-1}+1+3*2^{k-1})}{6}$$

$$=\frac{(2^{2k-1}-2^{k}+2*2^{k-1}+1)}{6}$$

$$=\frac{(2^{2k-1}+1)}{6}$$ 得證