DCSA-Assignment5

資工四 4104056004 李家駿

Q:證明 $\mathrm{MSE(LSB-k)}=\frac{1}{2^k*2^k}*\sum_{i=0}^{2^k-1}\sum_{j=0}^{2^k-1}(i-j)^2=\frac{2^{2k}-1}{6}$

proof:

$$\frac{1}{2^k*2^k}\sum_{i=0}^{2^k-1}\sum_{j=0}^{2^k-1}(i-j)^2=\sum_{i=0}^{2^k-1}\sum_{j=0}^{2^k-1}(i^2-2ij+j^2)$$

$=\frac{1}{2^k*2^k}\sum_{i=0}^{2^k-1}\left(\sum_{j=0}^{2^k-1}i^2-2\sum_{j=0}^{2^k-1}ij+\sum_{j=0}^{2^k-1}j^2\right)$-----(1)

已知$\sum_{k=1}^{n}k^2=\frac{n(n+1)(2n+1)}{6}$

故(1)

$=\frac{1}{2^k*2^k}\sum_{i=0}^{2^k-1}\left(2^k*i^2-i*(2^k-1)2^k+\frac{1}{3}*(2^k-1)*(2^{k-1})*(2^{k+1}-1)\right)$

$=\frac{1}{2^k*2^k}(2^k*\frac{1}{3}*(2^k-1)*(2^{k-1})*(2^{k+1}-1)-\frac{(2^k-1)*2^k}{2}*(2^k-1)2^k+(2^k)*\frac{1}{3}*(2^k-1)*(2^{k-1})*(2^{k+1}-1))$

$=\frac{1}{2^k*2^k}(2^{\mathrm{k}+1}*\frac{1}{3}*(2^k-1)*(2^{k-1})*(2^{k+1}-1)-\frac{((2^k-1)*2^k)\hat{}2}{2})$

$=\frac{1}{3}*(2^k-1)*(2^{k+1}-1)-\frac{(2^k-1)^2}{2}$

$=\frac{2(2^k-1)*(2^{k+1}-1)-3(2^k-1)^2}{6}$

$=\frac{2^{2k+2}-2^{k+2}-2^{k+1}+2-3*2^{2k}+3*2^{k+1}-3}{6}$

$=\frac{2^{2k+2}-3*2^{2k}-1}{6}$

$=\frac{4*2^{2k}-3*2^{2k}-1}{6}$

$=\frac{2^{2k}-1}{6}$ 得證