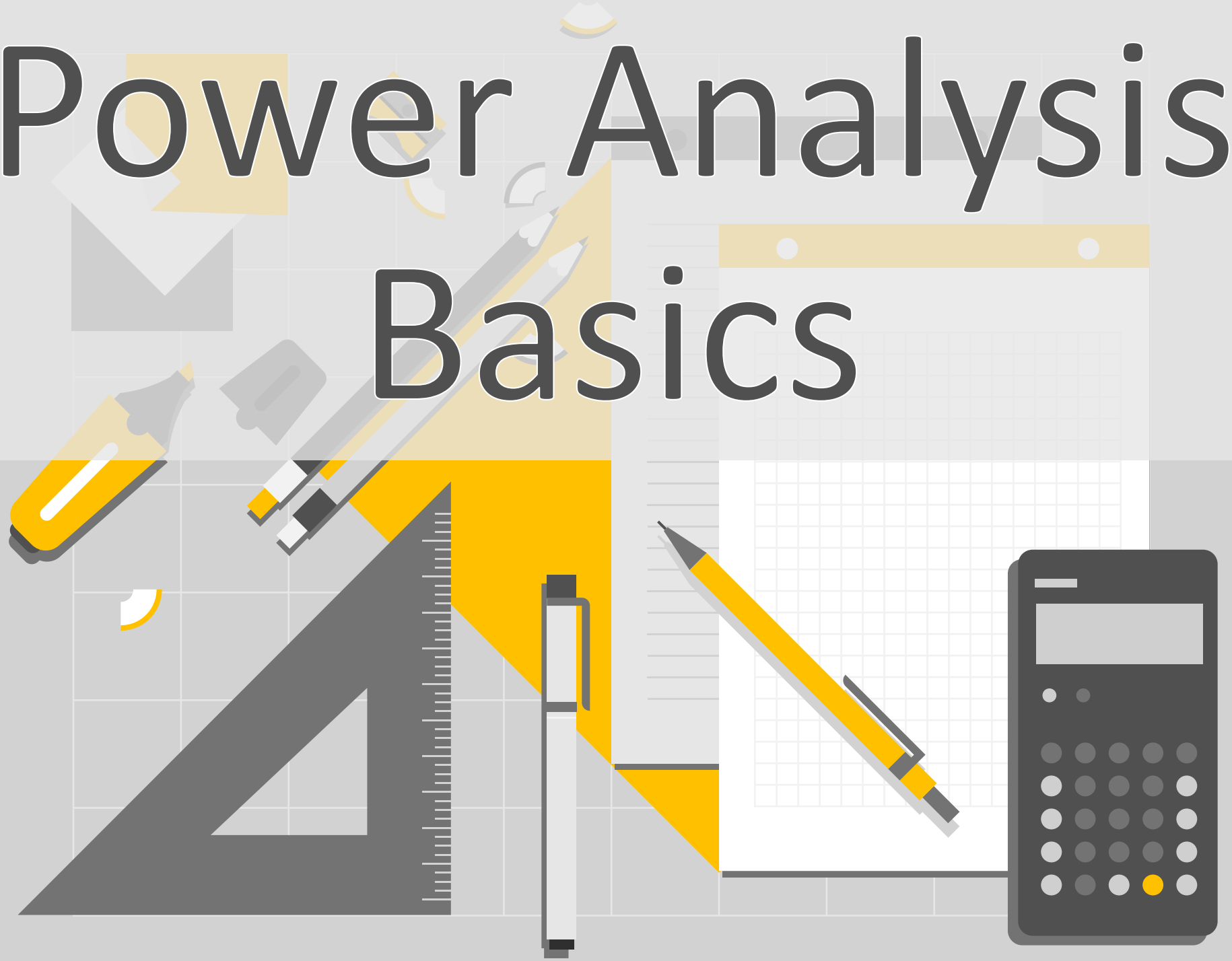
Power Analysis Basics

Dr. Mark Williamson



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1. What is Power Analysis?

- Power is the probability of detecting an effect, given that the effect is truly there [1]
  - Example, the effect of a treatment, measured by comparing the mean between a treatment group and a control group
- The most common use of power analysis is to determine sample sizes for experiments
  - Too few, you could miss a true effect
  - Too many, you could waste resources or unethically expose more patients to risk
- Requirements for some studies

2. Components needed.

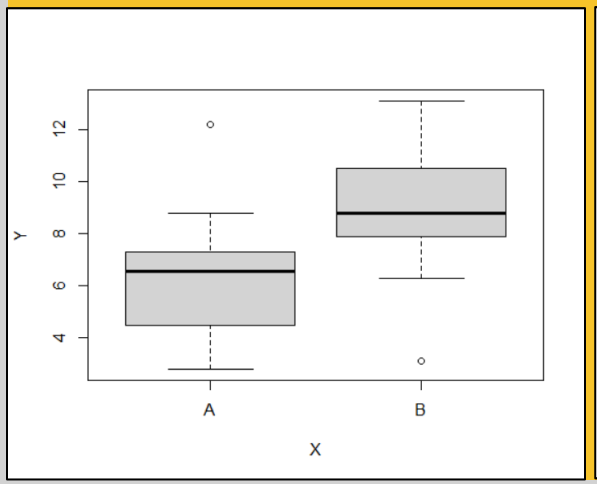
- In a basic power analysis, there are four components [2]:
  - Alpha (α) = risk of a Type I error (false positive)
  - Power = 1 – risk of a Type II error (false negative)
  - Sample size (n) = number of samples needed to detect the effect
  - Effect size = magnitude of the effect under the alternative hypothesis
- Alpha and Power have defaults:
  - Alpha = 0.05
  - Power = 0.80
- Effect size is the major unknown:
  - Can be estimated
  - Can be guessed

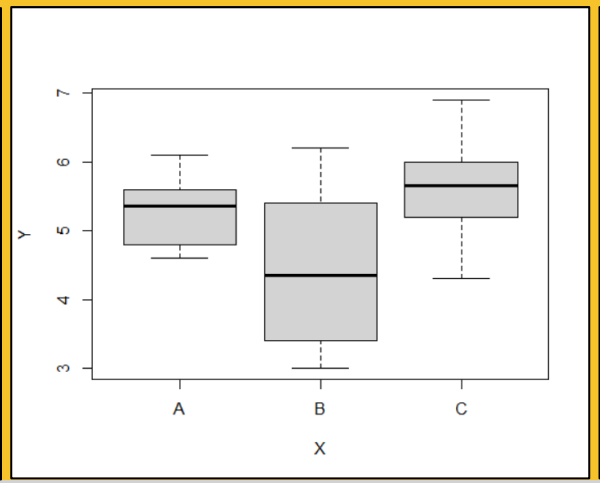
3. The Challenge of Effect Size.

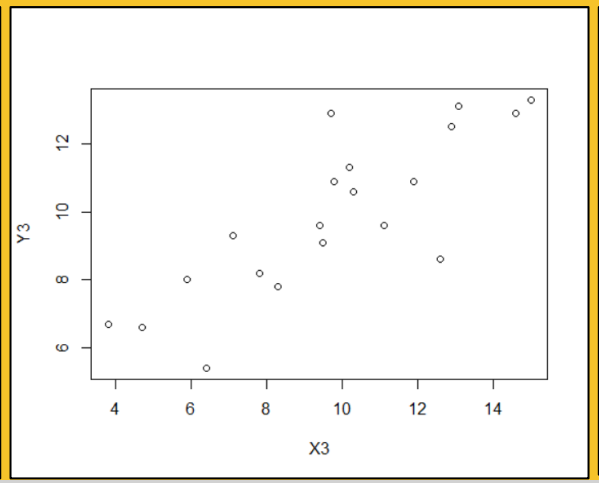
- Effect size matters the most for sample size
  - The higher the effect size, the fewer samples needed
- Calculating Effect size:
  - Prior studies (pilot, trial, etc.)
  - Similar studies (same or related field)
  - Other pertinent background information
- Guessing Effect size:
  - Rules of thumb for small, medium, large effects
  - Enter plausible values to effect size equation

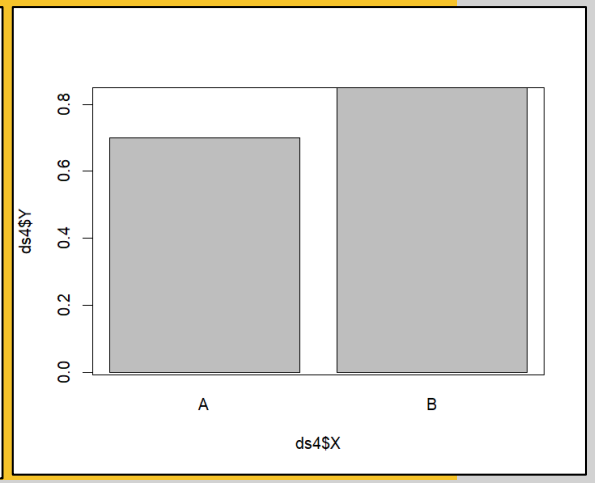
4. Basic Comparisons.

- Comparing 2 groups (T-test)
- Comparing 3+ groups (ANOVA)
- Correlation
- Proportions









5. Guessing Effect Size.

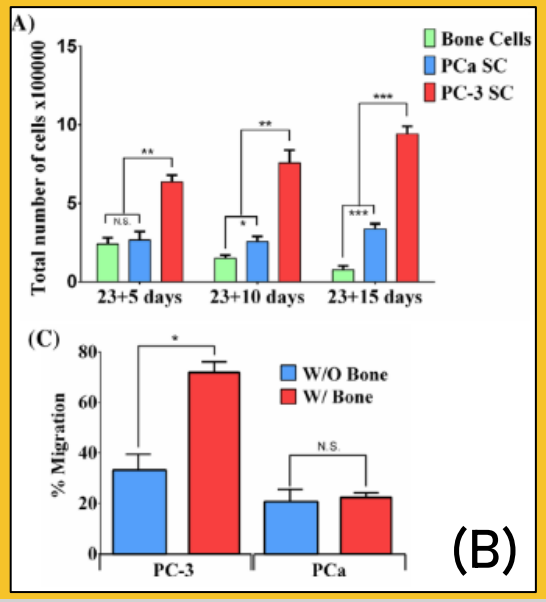
- Rule of thumb values [3,4]
- Enter plausible equation values
  - T-test: Two-fold difference, SD 1/2 of mean

$$d = \frac{10 - 5}{5} = 1$$

	Effect Size		
Test	**Small**	**Medium**	**Large**
T-test (d)	0.20	0.50	0.80
ANOVA (f)	0.10	0.25	0.40
Correlation (R)	0.10	0.30	0.50

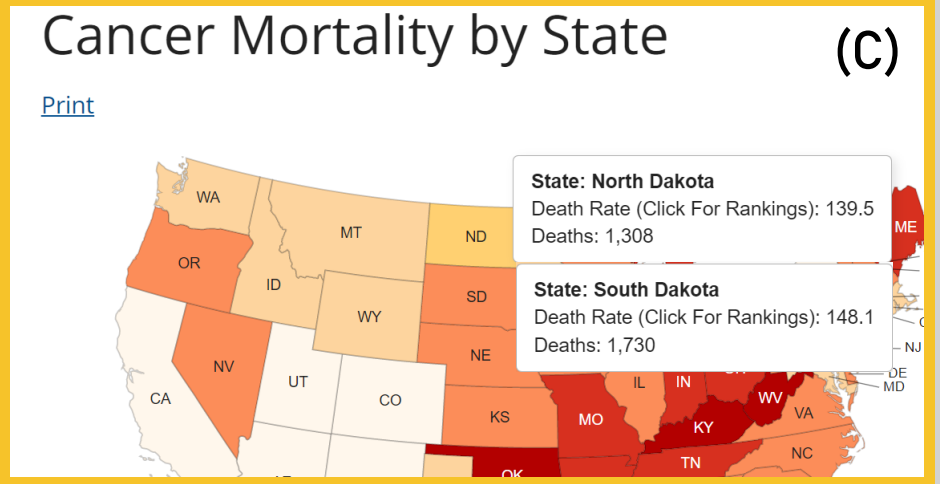
6. Calculating Effect Size.

- Raw data (A)
- Literature search (B)
- Summary statistics (C)



(A)

Treat	Control
6.8	4.7
3.2	3.3
3.4	5
5.1	2.5
7.9	2.4
7.4	1.9
8.8	0.6
4.4	2.6
5.1	7.2
8.2	5
mean 6	3.5
SD 2	1.9



7. Tools to Use.

- By hand with equations (!!!)
- G*Power[5]
- Online calculator
  - Powerandsamplesize.com[6]
  - GIGAcalculator[7]
  - Statistics Kingdom[8]
  - Sample-size.net[9]
- Software
  - R: package ‘pwr’
  - SPSS -> Analyze -> Power Analysis

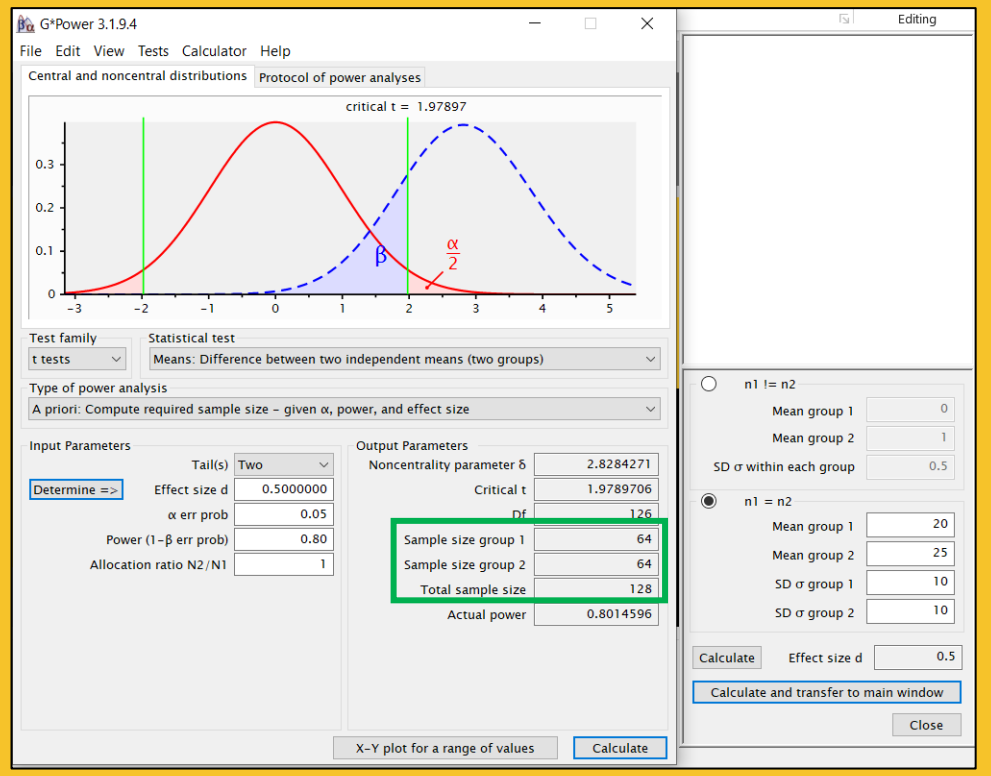



8. T-test Example.

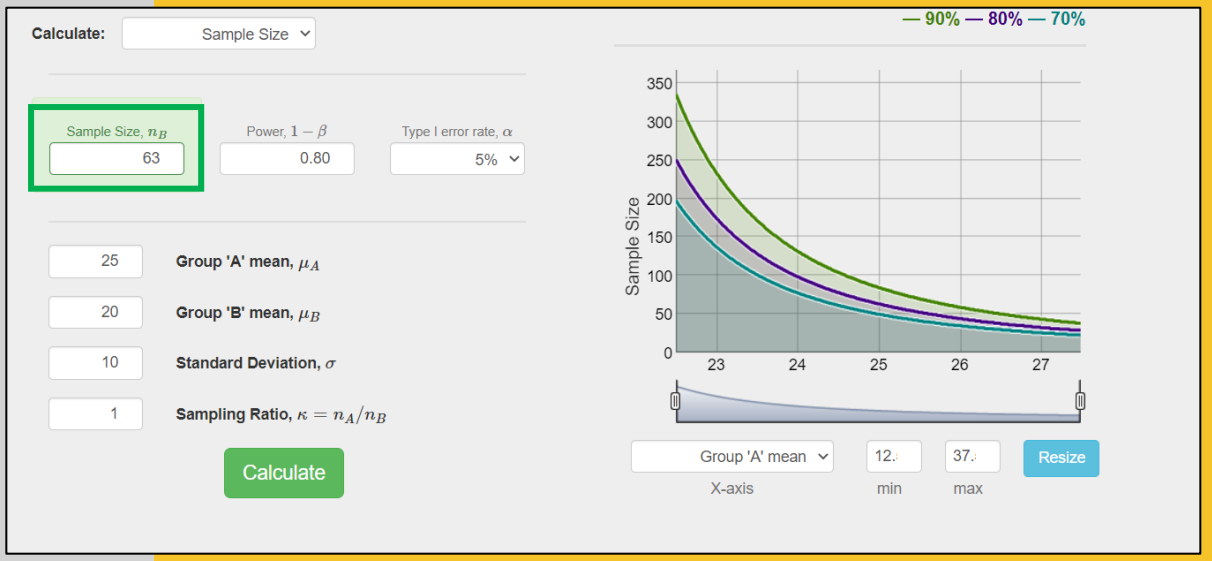
- Data:

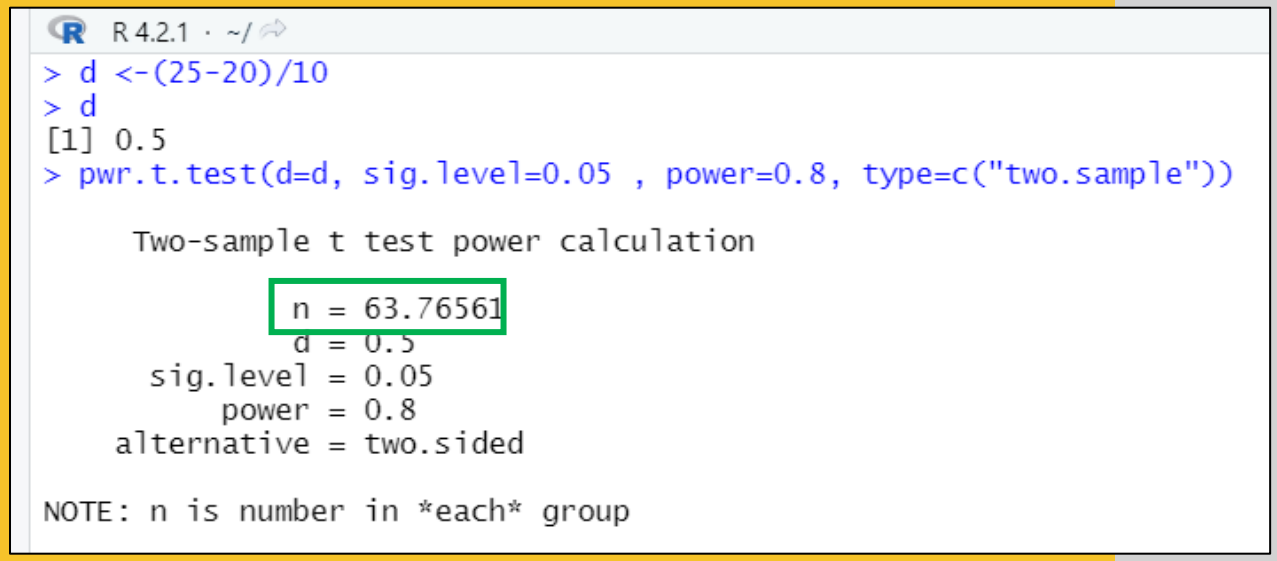
	Treat	Control
mean	20	25
std	10	10

- Method:
  - G*Power
  - Online calculator
  - R
  - SPSS



8. T-test Example.





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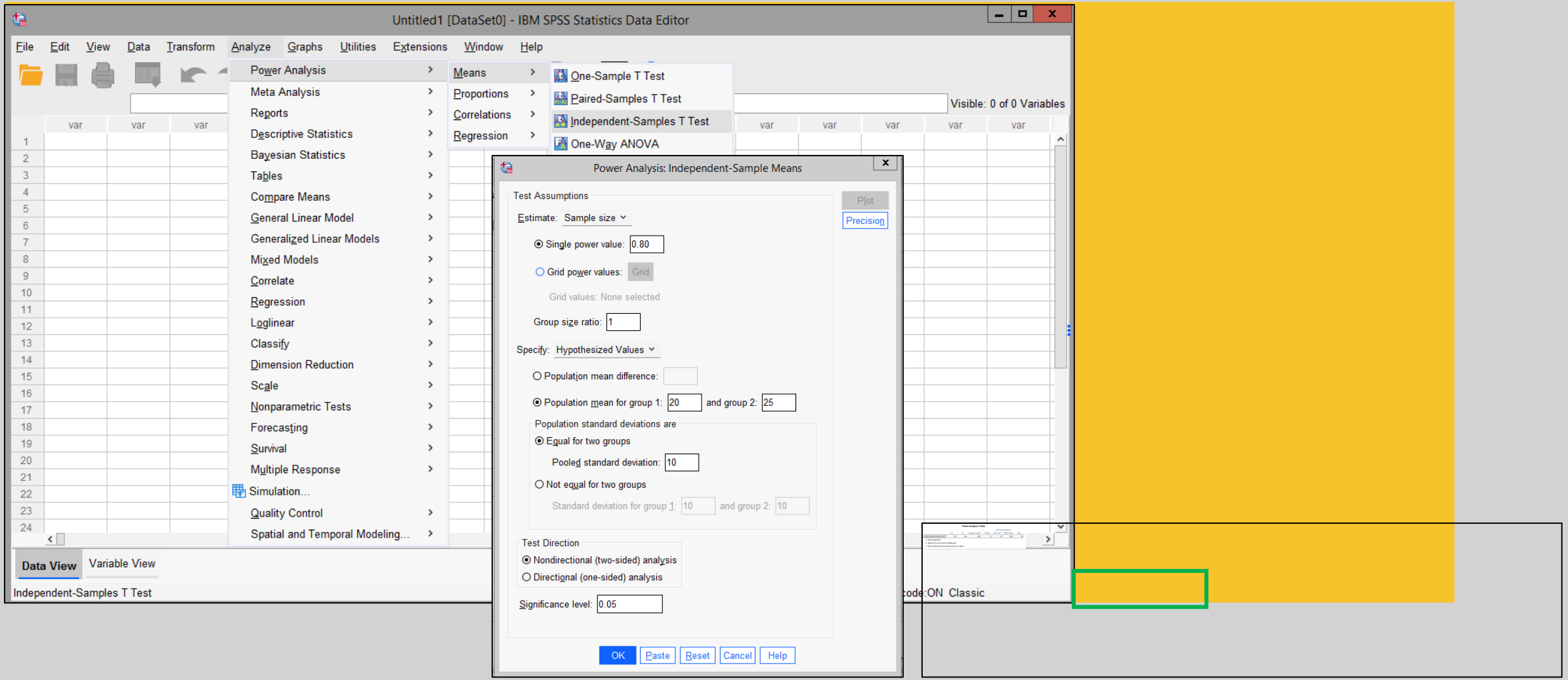


9. ANOVA Example.

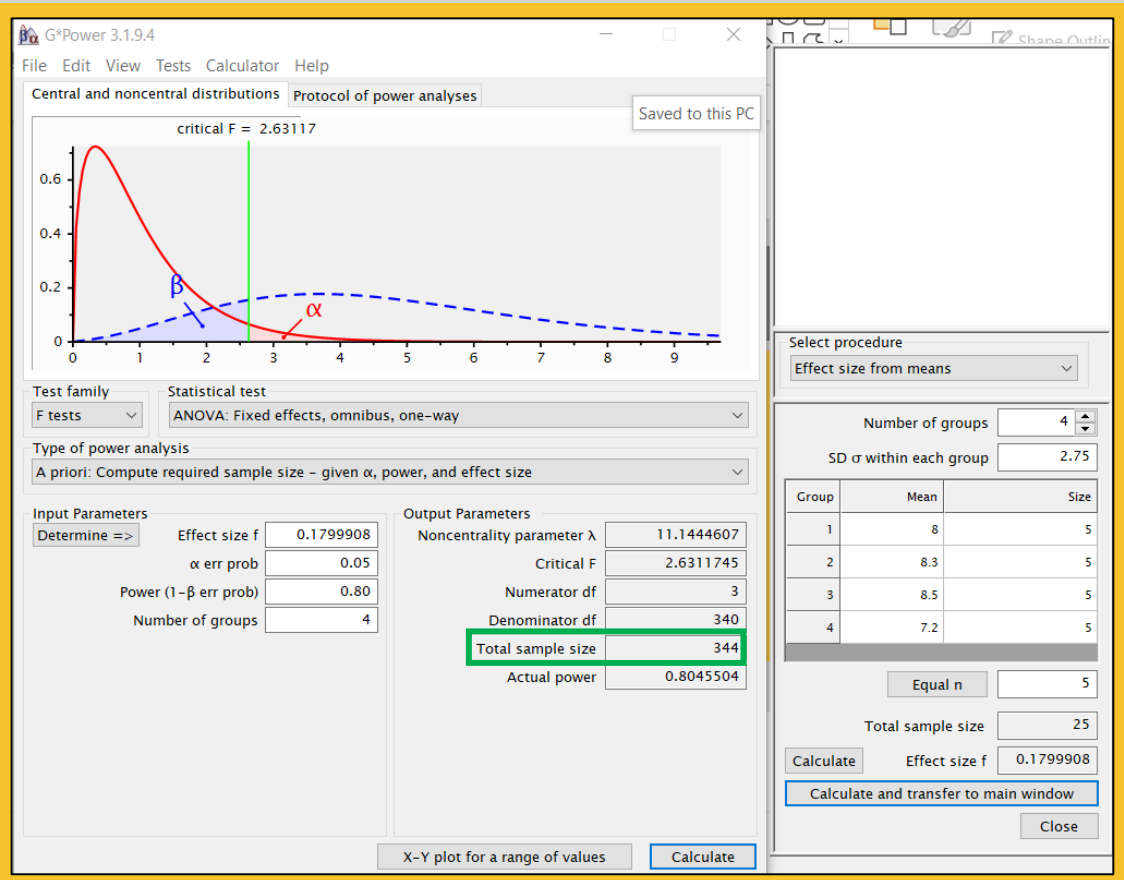
- Data:

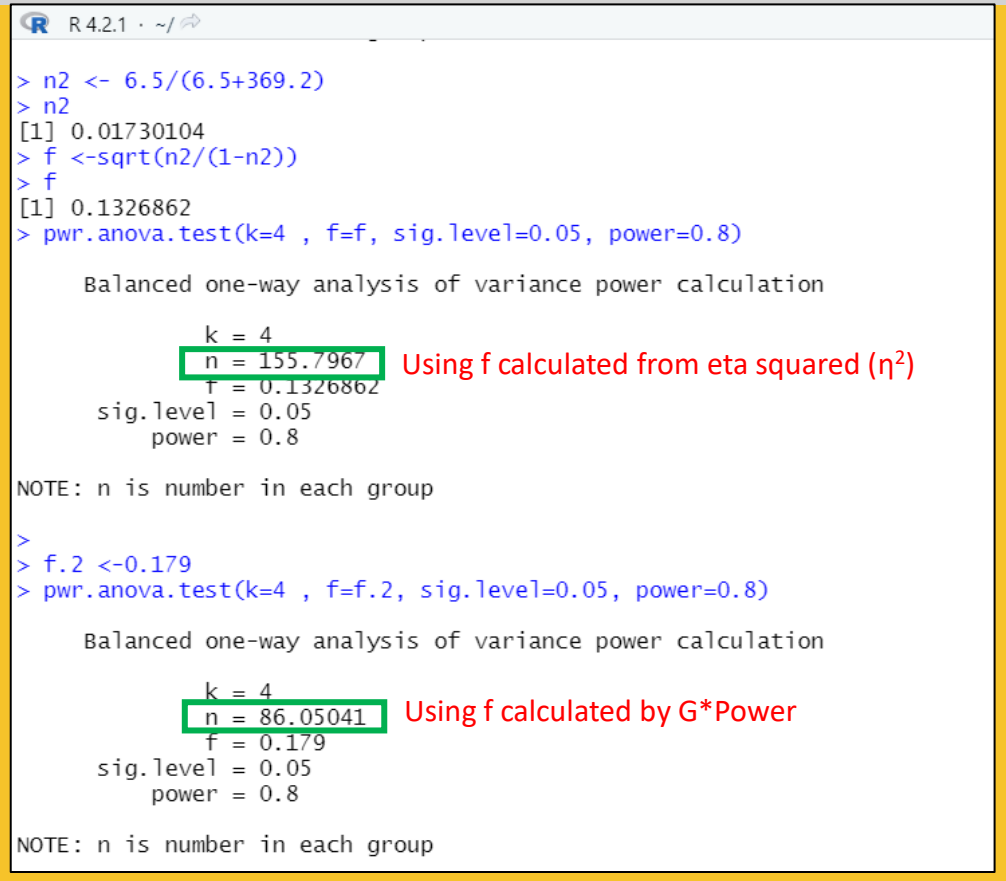
	A	B	C	D
mean	8.0	8.3	8.5	7.2
std	2.1	3.2	2.2	3.5

[10, 11]	Df	SS	Mean	F	p
X var	3	6.5	2.163	0.211	0.888
Residuals	36	369.2	2.2		

- Method:
  - G*Power
  - ~~Online calculator~~
  - R
  - SPSS

9. ANOVA Example.





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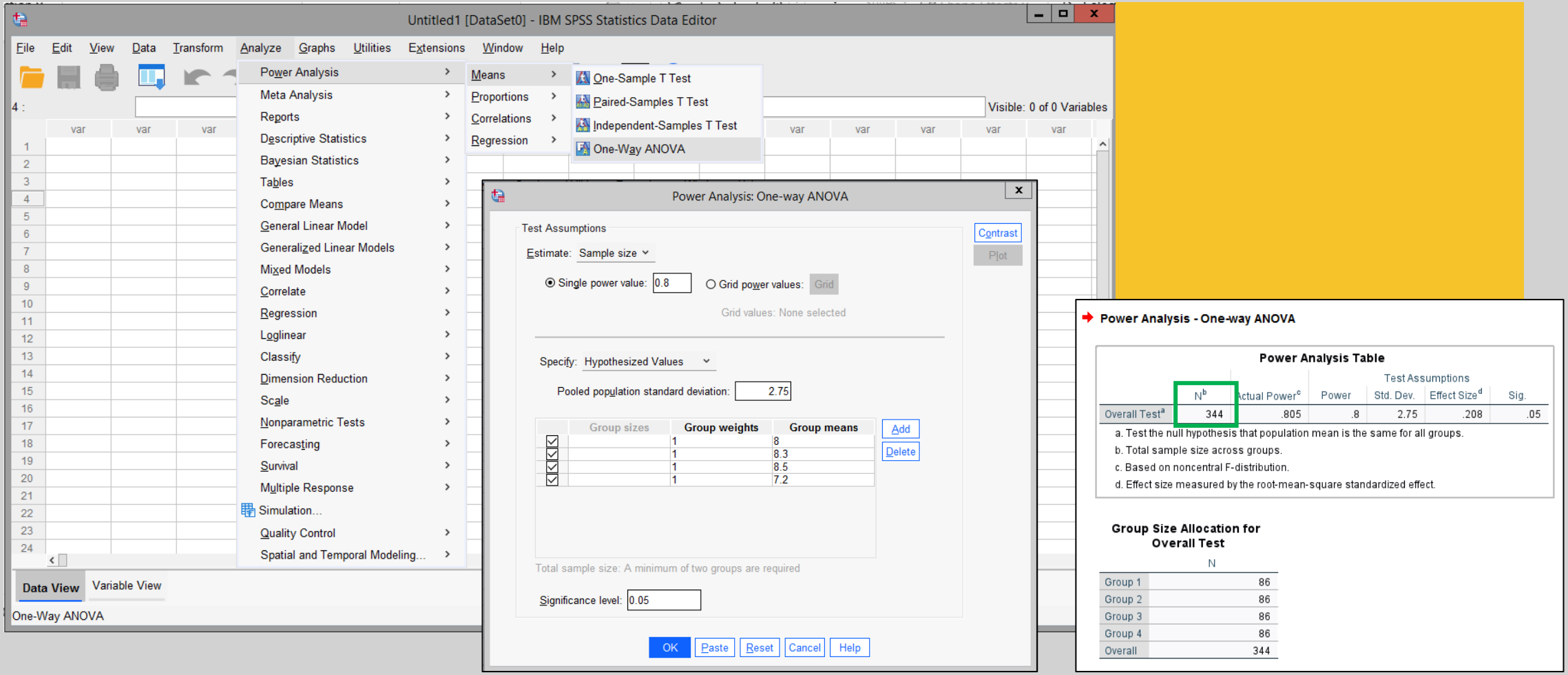
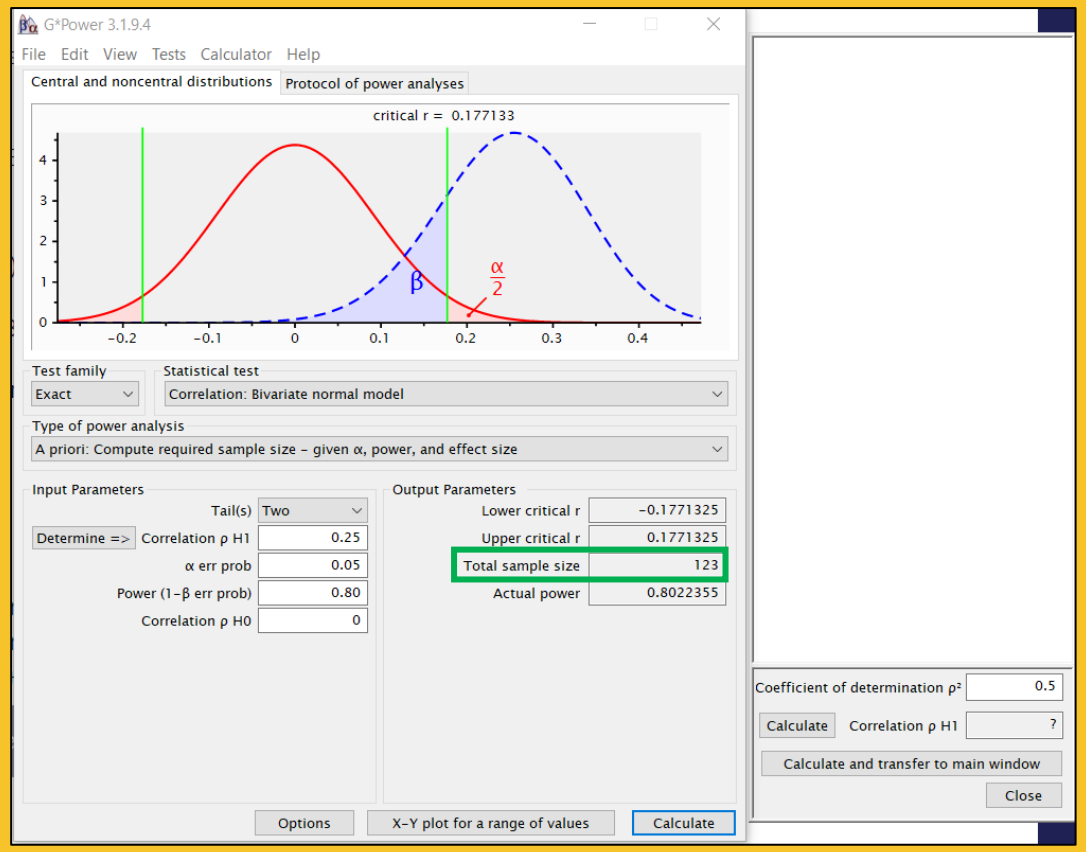


10. Correlation Example.

- Data:

	H1	H0
Correlation	0.25	0.00

- Method:
  - G*Power
  - Online calculator
  - R
  - SPSS



10. Correlation Example.

Correlation sample size

Total sample size required to determine whether a correlation coefficient differs from zero.

Instructions: Enter parameters in the **green** cells. Answers will appear in the **blue** box below.

α (two-tailed) =	0.05	Threshold probability for rejecting the null hypothesis. Type I error rate.
β =	0.20	Probability of failing to reject the null hypothesis under the alternative hypothesis. Type II error rate.
r =	0.25	The expected correlation coefficient.

Calculate

The standard normal deviate for $\alpha = Z_\alpha = 1.9600$

The standard normal deviate for $\beta = Z_\beta = 0.8416$

$C = 0.5 * \ln[(1+r)/(1-r)] = 0.2554$

Total sample size = $N = [(Z_\alpha+Z_\beta)/C]^2 + 3 =$ **123**

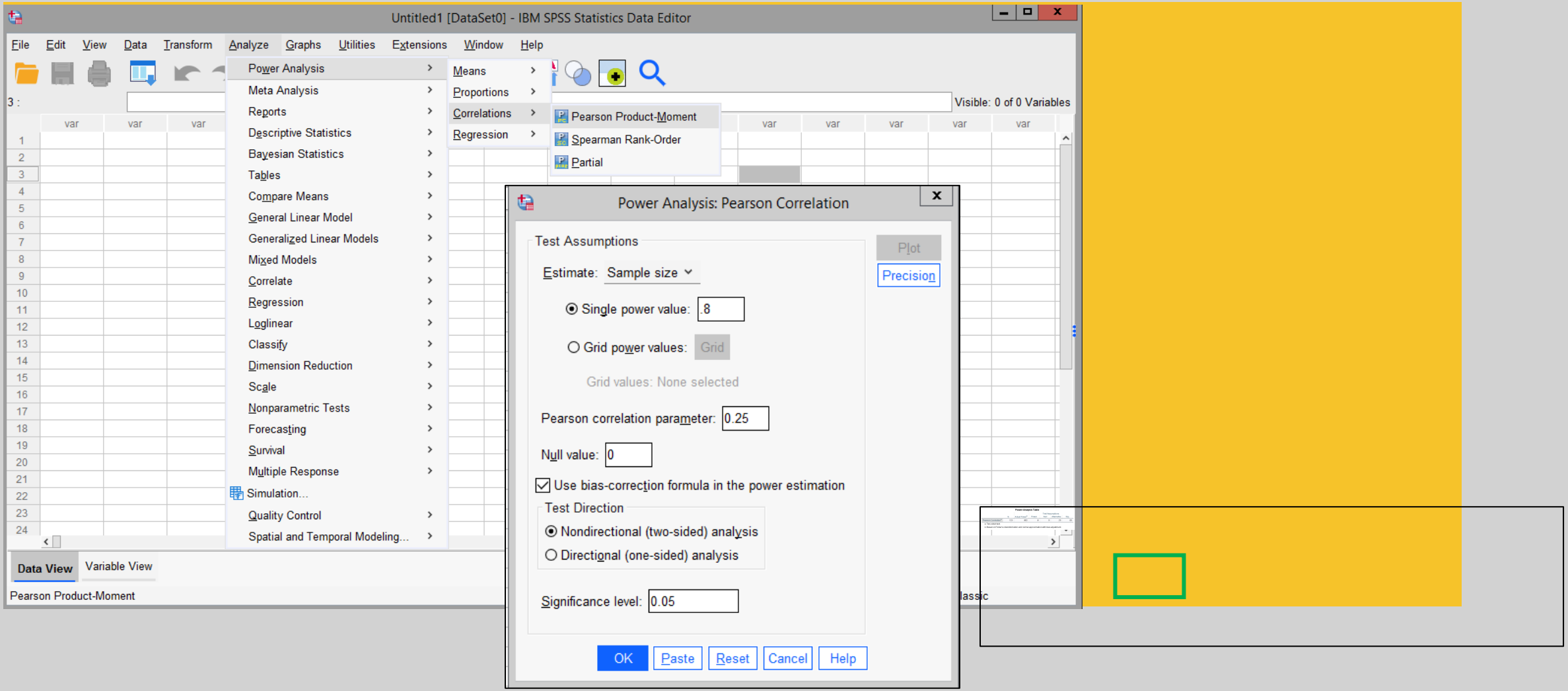
R 4.2.1 · ~/

```
> pwr.r.test(r=0.25, sig.level=0.05, power=0.80)

     approximate correlation power calculation (arctangh transformation)

              n = 122.4466
              r = 0.25
      sig.level = 0.05
          power = 0.8
    alternative = two.sided
```

10. Correlation Example.

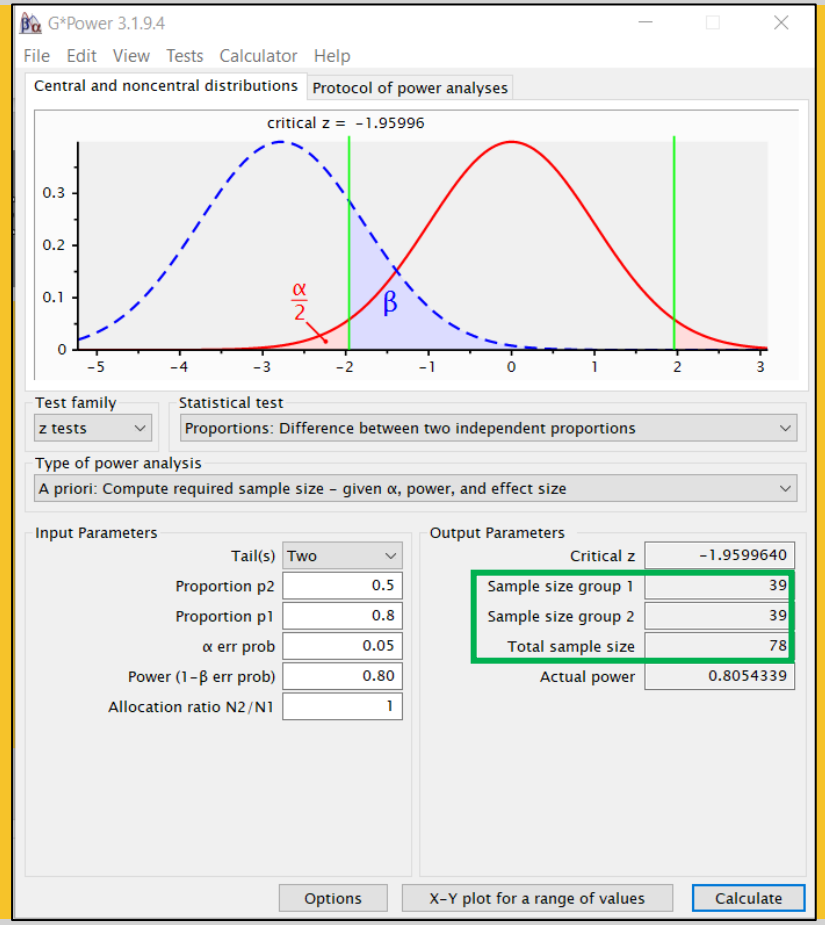


11. Proportion Example.

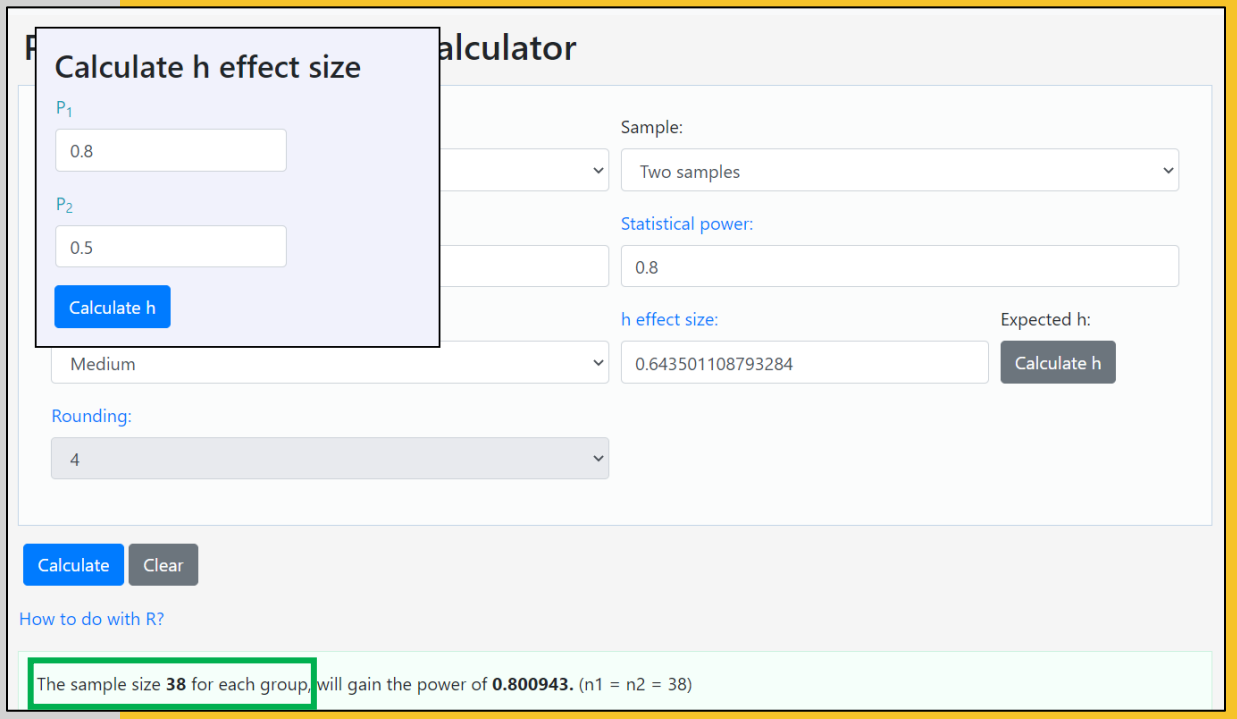
- Data:

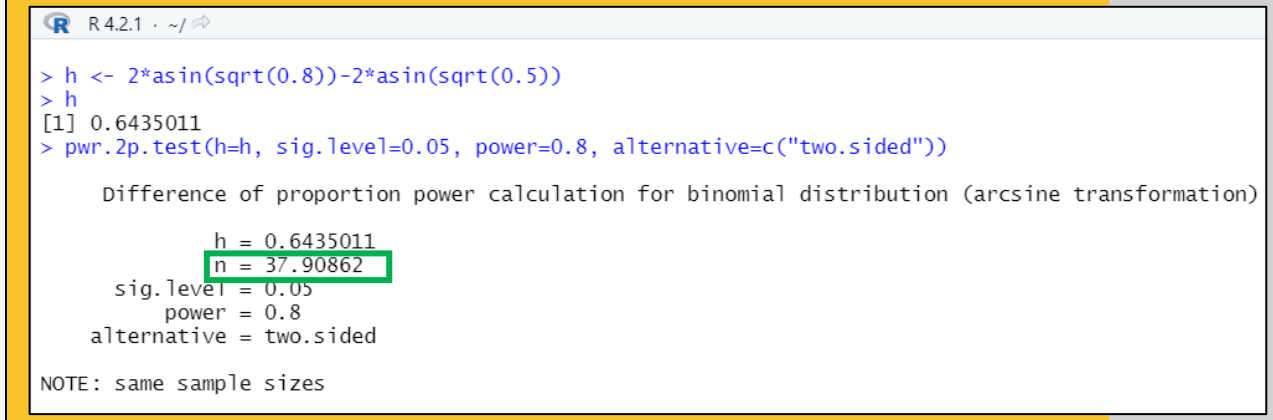
	Treat	Control
proportion	0.50	0.80

- Method:
  - G*Power
  - Online calculator
  - R
  - SPSS



11. Proportion Example.





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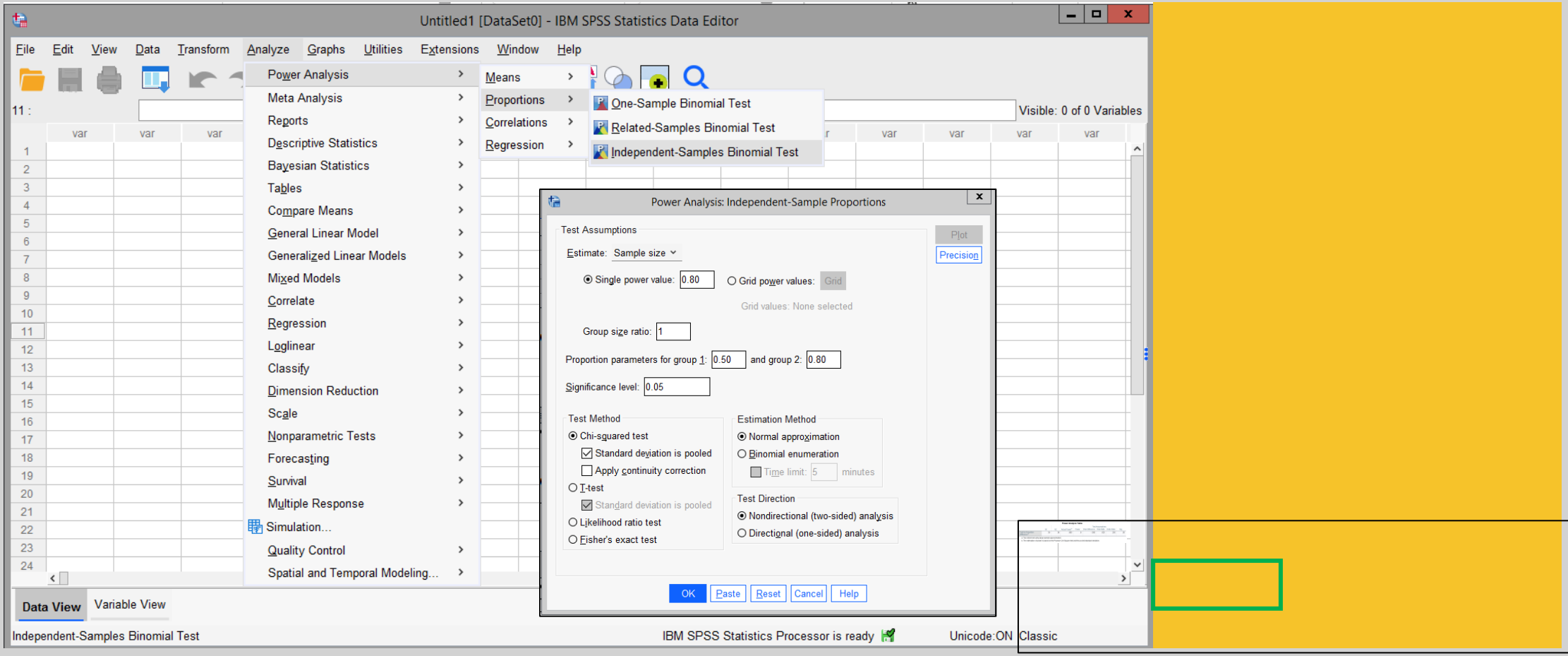
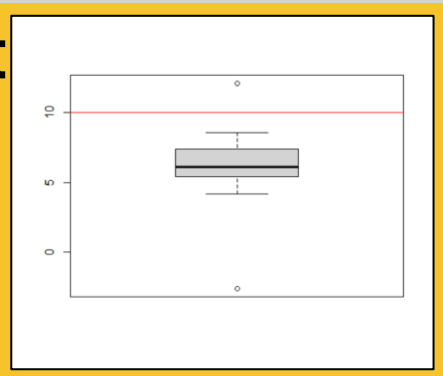
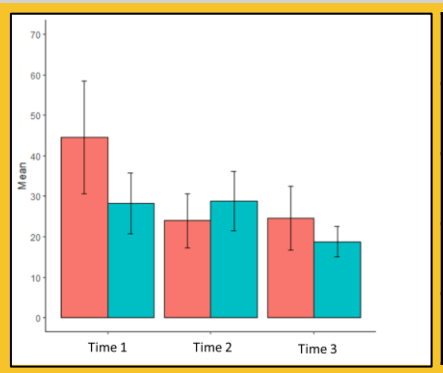


12. Additional Designs.

- 1-sample or Paired T-test
- Repeated Measures
- Regression
- Chi-squared
- …





Expected	Observed
9	56
3	18
3	7
1	7

$$Y = \beta_0 + \beta_1 * X_1 + \beta_2 * X_2 + \epsilon$$

"A Power Analysis for every analysis"

13. Practical Limitations.

- Understanding what goes in.
- Simulation sanity check. (A)
- Realistic patient/sample number.
- Balance of art and science.

(A)

Mean A	30
Mean B	22
Std	12

d = (30-22) / 12
d = 0.667

```
> pwr.t.test(d=0.667, sig.level=0.05, power=0.8, type=c("two.sample"))

     Two-sample t test power calculation

              n = 36.27042
              d = 0.667
      sig.level = 0.05
          power = 0.8
    alternative = two.sided

NOTE: n is number in *each* group
```

```
> Treat<-c(rep("A",37), rep("B",37))
> Response<-round(c(rnorm(37, mean=30, sd=12), rnorm(37, mean=22, sd=12)),1)
> ss1 <-data.frame(Treat=Treat, Response=Response)
> t.test(Response~Treat, data=ss1)

        Welch Two Sample t-test

data:  Response by Treat
t = 2.9245, df = 69.914, p-value = 0.004647
alternative hypothesis: true difference in means between group A and group B is not equal to 0
95 percent confidence interval:
  2.942929 15.565179
sample estimates:
mean in group A mean in group B
       31.42432        22.17027
```

14. Where to Next?

- More resources
  - Power Analysis in G*Power (videos and slides)
  - Power Analysis in R (videos and slides)
  - Power Analysis in R: GLMMs (videos and slides)
  - Advanced Power Analysis: Into the Weeds (video and slides)
- How to practice
  - Design mock experiments
  - Calculate/guess effect size
  - Get comfortable using calculator/software
- Handy handouts [12]
  - Effect sizes for common designs

DACCOTA Statistical Resources

The Biostatistics, Epidemiology, and Research Design Core (BERDC) offers a variety of statistical resources from both our core and other biostatistical cores.

Effect Sizes for common designs

Statistical Test	Effect size	Equation	Rule of thumb for effect sizes
1 sample t-test	Cohen's d	d = (mean – constant)/ SD	small=0.20, medium=0.50, large=0.80
2 sample t-test	Cohen's d	$d = (mean_1 - mean_2) / SD_{pooled}$	small=0.20, medium=0.50, large=0.80
Paired t-test	Cohen's d	$d = (mean_1 - mean_2) / SD_{pooled}$	small=0.20, medium=0.50, large=0.80
1-Way ANOVA	Eta squared Cohen's f	$\eta^2 = SS_{treatment} / SS_{total}$ $f = sqrt(\eta^2 / (1 - \eta^2))$	small=0.01, medium=0.05, large=0.14 small=0.10, medium=0.25, large=0.40
2-Way ANOVA	Eta squared Cohen's f	$\eta^2 = SS_{treatment} / SS_{total}$ $f = sqrt(\eta^2 / (1 - \eta^2))$	small=0.01, medium=0.06, large=0.14 small=0.10, medium=0.25, large=0.40
Repeated Measures ANOVA	Partial Eta squared Cohen's f	Partial $\eta^2 = SS_{effect} / (SS_{effect} + SS_{error})$ ...	small=0.01, medium=0.06, large=0.14 small=0.10, medium=0.25, large=0.40
1 proportion test	Cohen's h	$h = 2*asin(sqrt(prop_1)) - 2*asin(sqrt(prop_{const}))$	small=0.20, medium=0.50, large=0.80
2 proportions test	Cohen's h	$h = 2*asin(sqrt(prop_1)) - 2*asin(sqrt(prop_2))$	small=0.20, medium=0.50, large=0.80
Chi-squared test	Cohen's w	$w = sqrt(\sum (prop_{obs} - prop_{exp})^2 / prop_{exp})$	small=0.10, medium=0.30, large=0.50
Pearson Correlation	Correlation (R)	...	small=0.10, medium=0.30, large=0.50
Linear Regression (Entire Model)	F squared	$f^2 = R^2_{model} / (1 - R^2_{model})$	small=0.02, medium=0.15, large=0.35
Linear Regression (Ind. Predictor)	F squared	$f^2 = R^2_{increase} / (1 - R^2_{increase})$	small=0.10, medium=0.30, large=0.50

15. References and Acknowledgement.

DaCCoTA
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[1] https://stats.oarc.ucla.edu/other/mult-pkg/seminars/intro-power/Others
[2] https://www.coursehero.com/file/p7g5rdk/Power-Analysis-Components-There-are-four-components-to-a-power-analysis-three/
[3] https://www.statology.org/effect-size/
[4] https://www.spss-tutorials.com/effect-size/#anova
[5] https://www.psychologie.hhu.de/arbeitsgruppen/allgemeine-psychologie-und-arbeitspsychologie/gpower
[6] http://powerandsamplesize.com/Calculators/
[7] https://www.gigacalculator.com/calculators/power-sample-size-calculator.php
[8] https://www.statskingdom.com/statistical-power-calculators.html
[9] https://sample-size.net
[10] https://www.statology.org/eta-squared/
[11] https://www.statology.org/partial-eta-squared/
[12]https://docs.google.com/spreadsheets/d/1dqbPqj3VfiHC3oZE4azLypiFOQaeoj9HQ8Z5yjOvybs/edit#gid=0

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