

Power Analysis Basics

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1. What is Power Analysis?

- Power is the probability of detecting an effect, given that the effect is truly there [1]
 - Example, the effect of a treatment, measured by comparing the mean between a treatment group and a control group
- The most common use of power analysis is to determine sample sizes for experiments
 - Too few, you could miss a true effect
 - Too many, you could waste resources or unethically expose more patients to risk
- Requirements for some studies

2. Components needed.

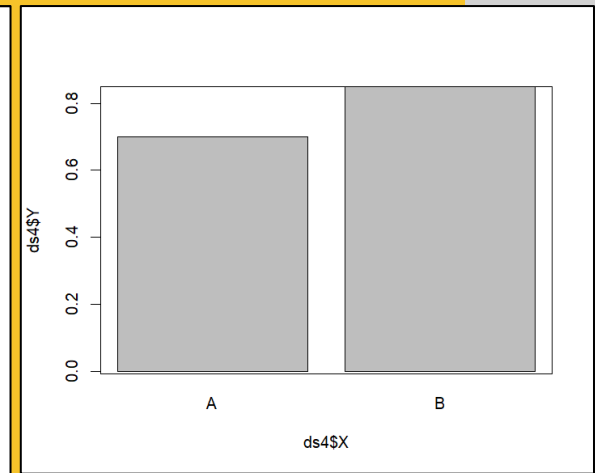
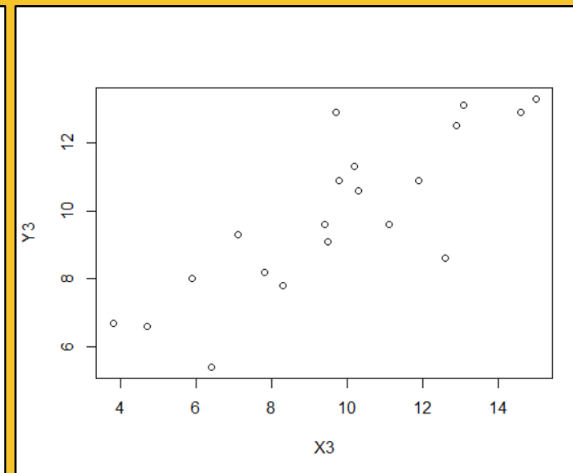
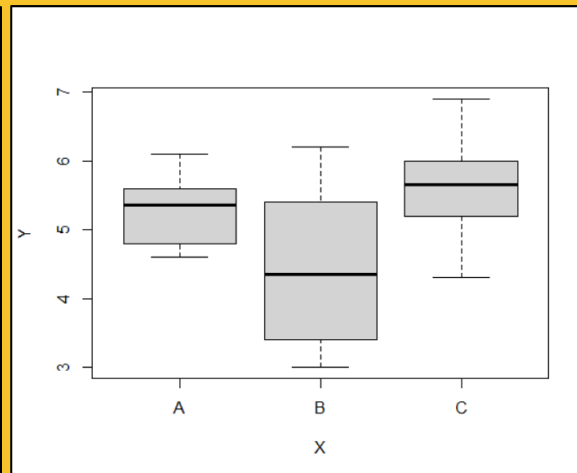
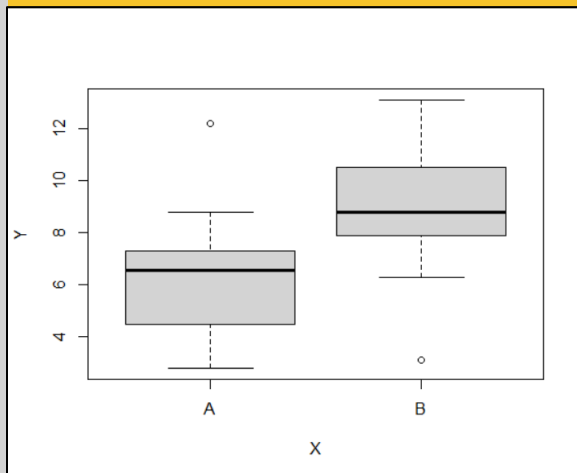
- In a basic power analysis, there are four components [2]:
 - Alpha (α) = risk of a Type I error (false positive)
 - Power = 1 – risk of a Type II error (false negative)
 - Sample size (n) = number of samples needed to detect the effect
 - Effect size = magnitude of the effect under the alternative hypothesis
- Alpha and Power have defaults:
 - Alpha = 0.05
 - Power = 0.80
- Effect size is the major unknown:
 - Can be estimated
 - Can be guessed

3. The Challenge of Effect Size.

- **Effect size matters the most for sample size**
 - The higher the effect size, the fewer samples needed
- **Calculating Effect size:**
 - Prior studies (pilot, trial, etc.)
 - Similar studies (same or related field)
 - Other pertinent background information
- **Guessing Effect size:**
 - Rules of thumb for small, medium, large effects
 - Enter plausible values to effect size equation

4. Basic Comparisons.

- Comparing 2 groups (T-test)
- Comparing 3+ groups (ANOVA)
- Correlation
- Proportions



5. Guessing Effect Size.

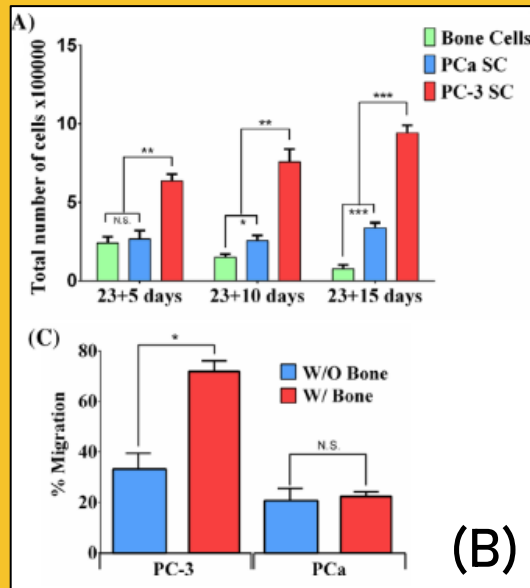
- Rule of thumb values [3,4]
- Enter plausible equation values
 - T-test: Two-fold difference, SD 1/2 of mean

$$d = \frac{10 - 5}{5} = 1$$

	Effect Size		
Test	Small	Medium	Large
T-test (d)	0.20	0.50	0.80
ANOVA (f)	0.10	0.25	0.40
Correlation (R)	0.10	0.30	0.50

6. Calculating Effect Size.

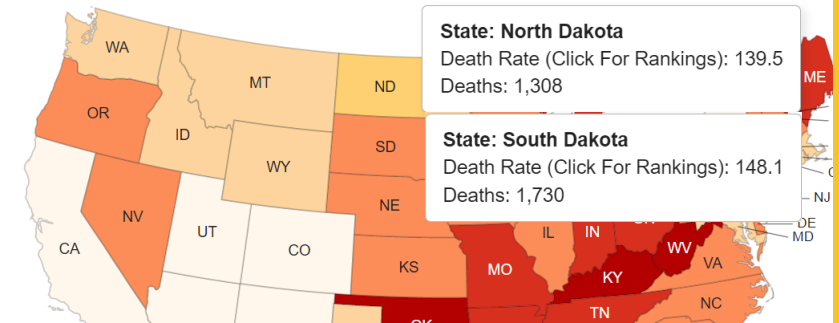
- Raw data (A)
- Literature search (B)
- Summary statistics (C)



	Treat	Control
	6.8	4.7
	3.2	3.3
	3.4	5
	5.1	2.5
	7.9	2.4
	7.4	1.9
	8.8	0.6
	4.4	2.6
	5.1	7.2
	8.2	5
mean	6	3.5
SD	2	1.9

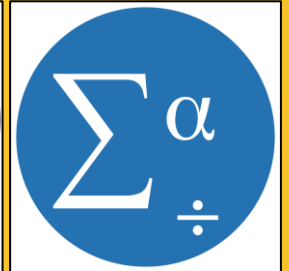
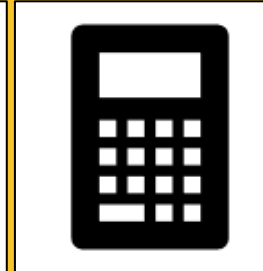
Cancer Mortality by State (C)

[Print](#)



7. Tools to Use.

- By hand with equations (!!!)
- G*Power[5]
- Online calculator
 - Powerandsamplesize.com[6]
 - GIGAcaculator[7]
 - Statistics Kingdom[8]
 - Sample-size.net[9]
- Software
 - R: package 'pwr'
 - SPSS -> Analyze -> Power Analysis



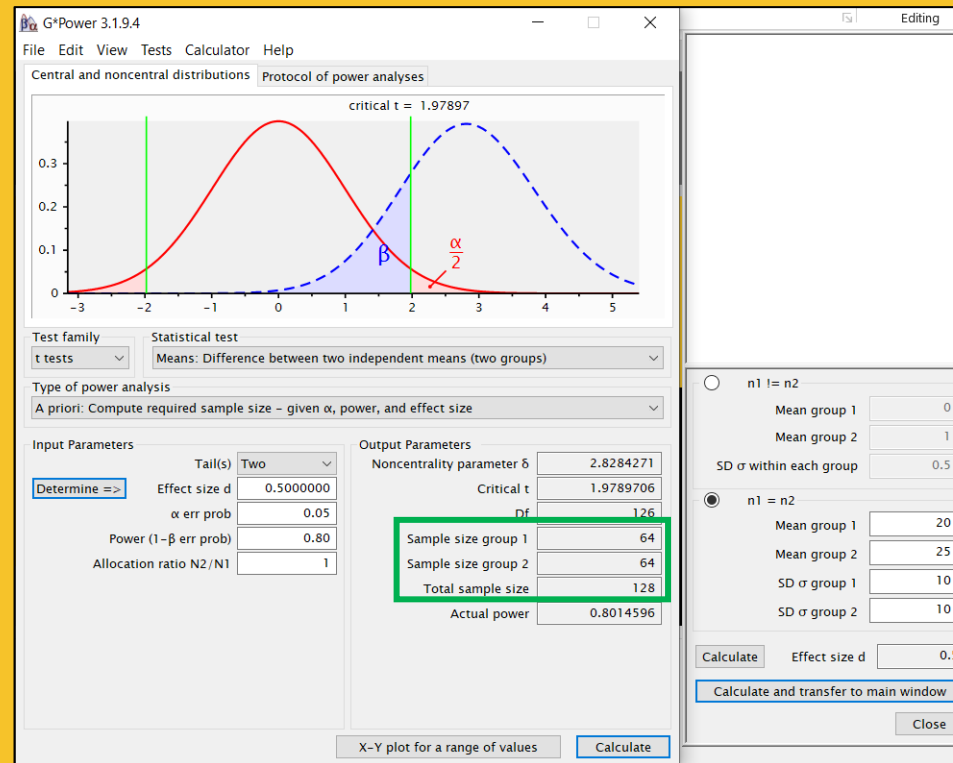
8. T-test Example.

- Data:

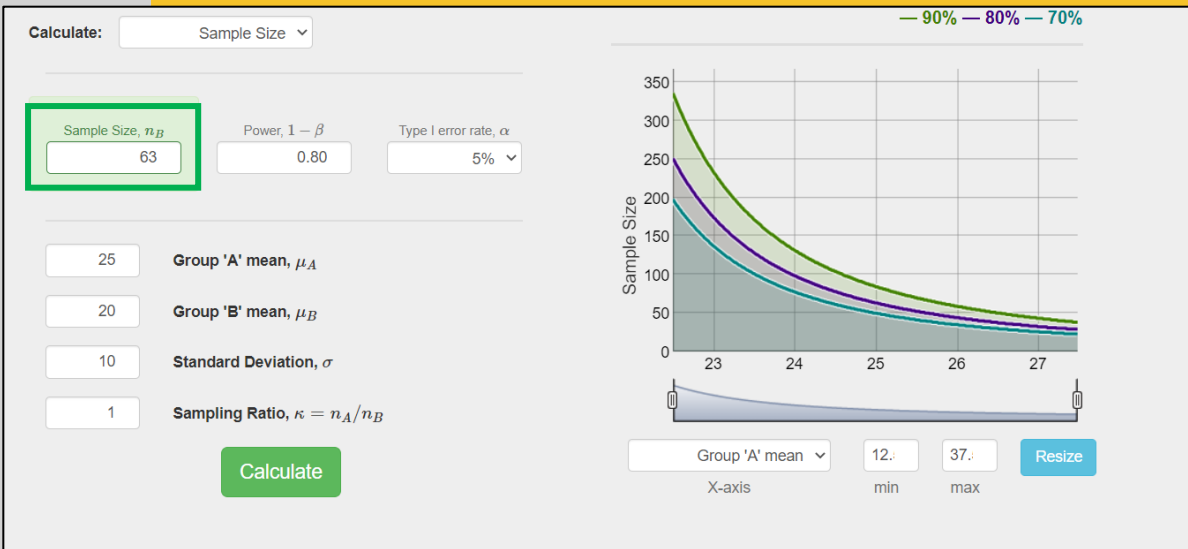
	Treat	Control
mean	20	25
std	10	10

- Method:

- G*Power
- Online calculator
- R
- SPSS



8. T-test Example.



```
R 4.2.1 · ~/
> d <- (25-20)/10
> d
[1] 0.5
> pwr.t.test(d=d, sig.level=0.05, power=0.8, type=c("two.sample"))
```

Two-sample t test power calculation

$n = 63.76561$
 $d = 0.5$
 $\text{sig.level} = 0.05$
 $\text{power} = 0.8$
 $\text{alternative} = \text{two.sided}$

NOTE: n is number in *each* group

8. T-test Example.

The screenshot shows the IBM SPSS Statistics Data Editor interface. The 'Analyze' menu is open, and the 'Independent-Samples T Test' option is selected. The 'Power Analysis: Independent-Sample Means' dialog box is displayed, showing the following settings:

- Test Assumptions:**
 - Estimate: Sample size (dropdown)
 - Single power value: 0.80 (selected)
 - Grid power values: Grid (disabled)
 - Grid values: None selected
 - Group size ratio: 1
- Specify: Hypothesized Values (dropdown)**
 - Population mean difference: (empty)
 - Population mean for group 1: 20 and group 2: 25 (selected)
- Population standard deviations are:**
 - Equal for two groups (selected)
 - Pooled standard deviation: 10
 - Not equal for two groups
 - Standard deviation for group 1: 10 and group 2: 10
- Test Direction:**
 - Nondirectional (two-sided) analysis (selected)
 - Directional (one-sided) analysis
- Significance level:** 0.05

The dialog box has buttons for OK, Paste, Reset, Cancel, and Help. The background shows the SPSS Data Editor window with the 'Data View' tab selected.

code: ON Classic

9. ANOVA Example.

- **Data:**

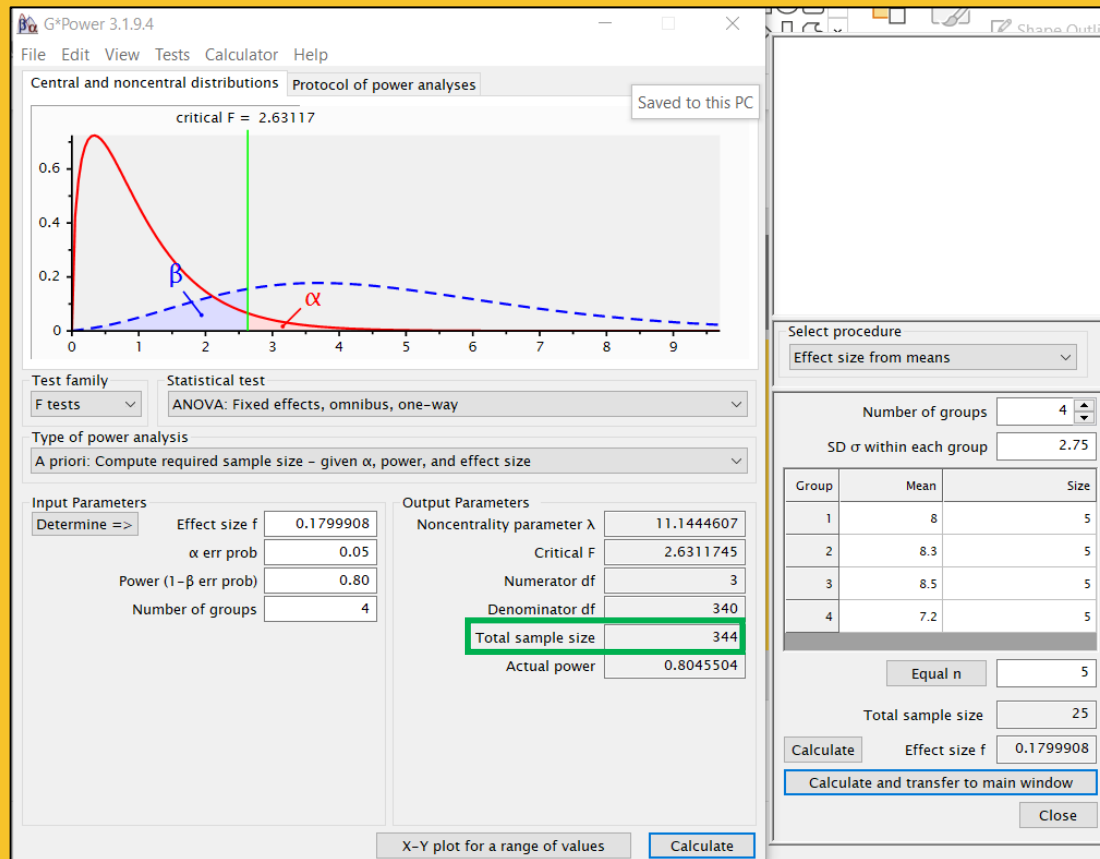
	A	B	C	D
mean	8.0	8.3	8.5	7.2
std	2.1	3.2	2.2	3.5

[10, 11]	Df	SS	Mean	F	p
X var	3	6.5	2.163	0.211	0.888
Residuals	36	369.2	2.2		

- **Method:**

- G*Power
- ~~Online calculator~~
- R
- SPSS

9. ANOVA Example.



R 4.2.1 · ~/

```
> n2 <- 6.5/(6.5+369.2)
> n2
[1] 0.01730104
> f <-sqrt(n2/(1-n2))
> f
[1] 0.1326862
> pwr.anova.test(k=4 , f=f, sig.level=0.05, power=0.8)
```

Balanced one-way analysis of variance power calculation

$k = 4$
 $n = 155.7967$ Using f calculated from eta squared (η^2)
 $f = 0.1326862$
 $\text{sig.level} = 0.05$
 $\text{power} = 0.8$

NOTE: n is number in each group

```
>
> f.2 <-0.179
> pwr.anova.test(k=4 , f=f.2, sig.level=0.05, power=0.8)
```

Balanced one-way analysis of variance power calculation

$k = 4$
 $n = 86.05041$ Using f calculated by G*Power
 $f = 0.179$
 $\text{sig.level} = 0.05$
 $\text{power} = 0.8$

NOTE: n is number in each group

9. ANOVA Example.

The screenshot shows the IBM SPSS Statistics Data Editor interface. The 'Analyze' menu is open, and the 'Power Analysis' option is selected. The 'Power Analysis: One-way ANOVA' dialog box is displayed, showing the following settings:

- Test Assumptions: Estimate: Sample size (Selected)
- Single power value: 0.8
- Grid power values: Grid
- Specify: Hypothesized Values
- Pooled population standard deviation: 2.75
- Group sizes: 1, 1, 1, 1
- Group weights: 8, 8.3, 8.5, 7.2
- Group means: 8, 8.3, 8.5, 7.2
- Significance level: 0.05

The dialog box also includes buttons for Contrast, Plot, Add, and Delete. The total sample size is calculated as 344.

Power Analysis - One-way ANOVA

Power Analysis Table						
	N ^b	Actual Power ^c	Power	Std. Dev.	Effect Size ^d	Sig.
Overall Test ^a	344	.805	.8	2.75	.208	.05

a. Test the null hypothesis that population mean is the same for all groups.
b. Total sample size across groups.
c. Based on noncentral F-distribution.
d. Effect size measured by the root-mean-square standardized effect.

Group Size Allocation for Overall Test

Group	N
Group 1	86
Group 2	86
Group 3	86
Group 4	86
Overall	344

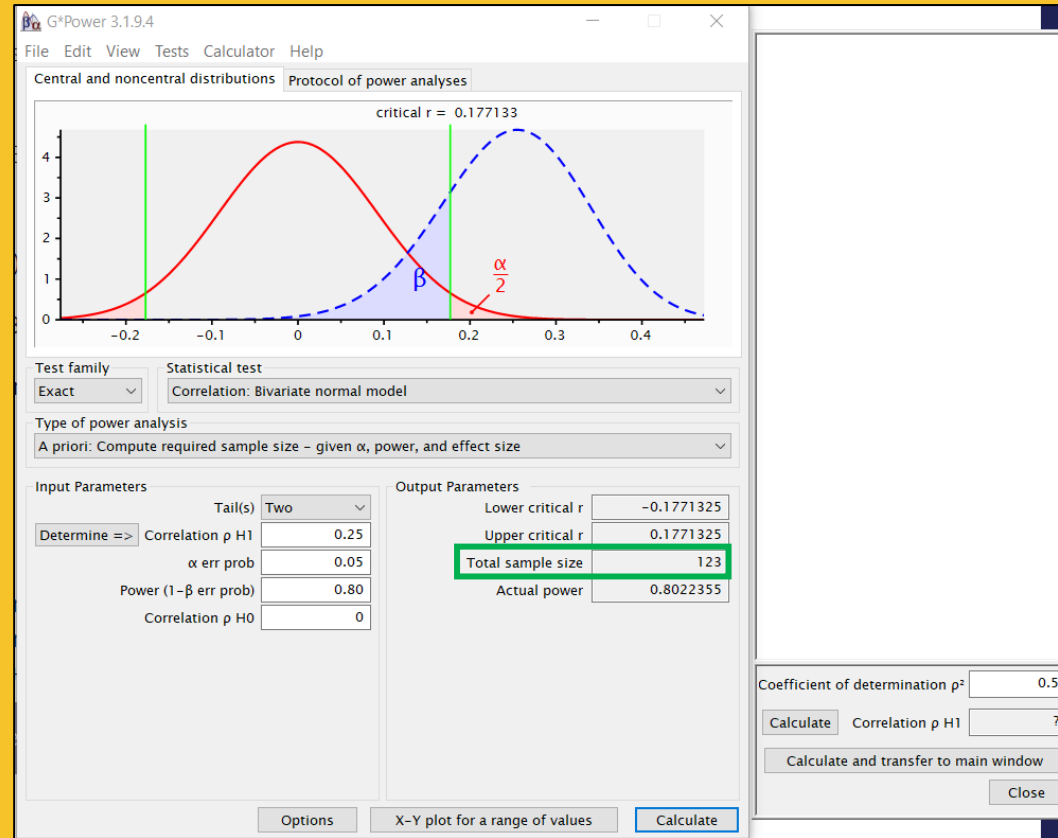
10. Correlation Example.

- Data:

	H1	H0
Correlation	0.25	0.00

- Method:

- G*Power
- Online calculator
- R
- SPSS



10. Correlation Example.

Correlation sample size

Total sample size required to determine whether a correlation coefficient differs from zero.

Instructions: Enter parameters in the **green** cells. Answers will appear in the **blue** box below.

α (two-tailed) = 0.05 Threshold probability for rejecting the null hypothesis. Type I error rate.
 β = 0.20 Probability of failing to reject the null hypothesis under the alternative hypothesis. Type II error rate.
 r = 0.25 The expected correlation coefficient.

Calculate

The standard normal deviate for $\alpha = Z_\alpha = 1.9600$

The standard normal deviate for $\beta = Z_\beta = 0.8416$

$C = 0.5 * \ln[(1+r)/(1-r)] = 0.2554$

Total sample size = $N = [(Z_\alpha + Z_\beta)/C]^2 + 3 = 123$

R 4.2.1 · ~/ ↻

```
> pwr.r.test(r=0.25, sig.level=0.05, power=0.80)
```

approximate correlation power calculation (arctangh transformation)

$n = 122.4466$

$r = 0.25$

sig.level = 0.05

power = 0.8

alternative = two.sided

10. Correlation Example.

The screenshot shows the IBM SPSS Statistics Data Editor interface. The menu path **Analyze > Correlations > Pearson Product-Moment** is highlighted. The **Power Analysis: Pearson Correlation** dialog box is open, showing the following settings:

- Test Assumptions:**
 - Estimate: Sample size
 - Single power value: .8
 - Grid power values: Grid
 - Grid values: None selected
 - Pearson correlation parameter: 0.25
 - Null value: 0
 - ☒ Use bias-correction formula in the power estimation
- Test Direction:**
 - ☒ Nondirectional (two-sided) analysis
 - ☐ Directional (one-sided) analysis
- Significance level:** 0.05

The dialog box has buttons for **OK**, **Paste**, **Reset**, **Cancel**, and **Help**. The **Plot** button is also visible. The background shows a data editor window with a grid of variables and a yellow sidebar.

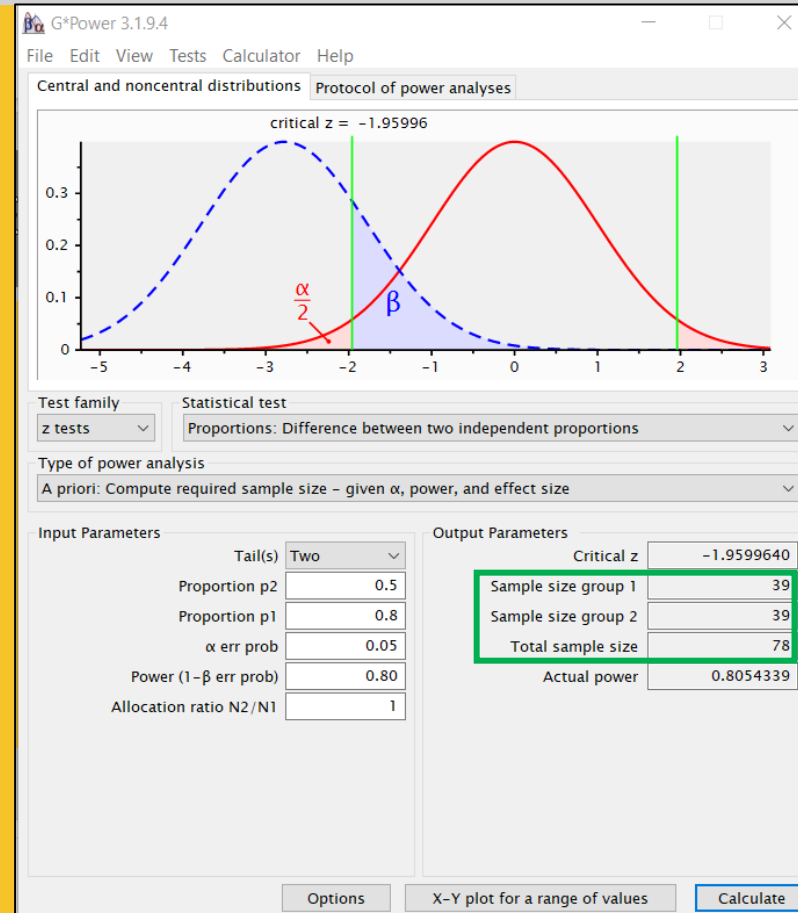
11. Proportion Example.

- Data:

	Treat	Control
proportion	0.50	0.80

- Method:

- G*Power
- Online calculator
- R
- SPSS



11. Proportion Example.

Calculate h effect size calculator

P_1
0.8

P_2
0.5

Calculate h

Sample:
Two samples

Statistical power:
0.8

h effect size:
0.643501108793284

Expected h:
Calculate h

Rounding:
4

Calculate Clear

How to do with R?

The sample size **38** for each group, will gain the power of **0.800943**. ($n_1 = n_2 = 38$)

```
R 4.2.1 · ~/
```

```
> h <- 2*asin(sqrt(0.8))-2*asin(sqrt(0.5))
> h
[1] 0.6435011
> pwr.2p.test(h=h, sig.level=0.05, power=0.8, alternative=c("two.sided"))
```

Difference of proportion power calculation for binomial distribution (arcsine transformation)

```
h = 0.6435011
n = 37.90862
sig.level = 0.05
power = 0.8
alternative = two.sided
```

NOTE: same sample sizes

11. Proportion Example.

The screenshot displays the IBM SPSS Statistics Data Editor interface. The menu bar at the top includes File, Edit, View, Data, Transform, Analyze, Graphs, Utilities, Extensions, Window, and Help. The 'Analyze' menu is open, showing a hierarchy: Power Analysis > Means > Proportions > One-Sample Binomial Test. The 'One-Sample Binomial Test' option is highlighted. Below the menu, the 'Power Analysis: Independent-Sample Proportions' dialog box is open. The 'Test Assumptions' section shows 'Estimate: Sample size' with a dropdown arrow. The 'Single power value' is set to 0.80, and the 'Grid power values' are set to 'Grid'. The 'Group size ratio' is 1. The 'Proportion parameters for group 1' is 0.50 and for group 2 is 0.80. The 'Significance level' is 0.05. The 'Test Method' section has 'Chi-squared test' selected, with 'Standard deviation is pooled' checked and 'Apply continuity correction' unchecked. The 'I-test' section has 'Standard deviation is pooled' checked. The 'Likelihood ratio test' and 'Fisher's exact test' are also listed. The 'Estimation Method' section has 'Normal approximation' selected, with 'Binomial enumeration' and 'Time limit: 5 minutes' also visible. The 'Test Direction' section has 'Nondirectional (two-sided) analysis' selected, with 'Directional (one-sided) analysis' also visible. The dialog box has 'OK', 'Paste', 'Reset', 'Cancel', and 'Help' buttons at the bottom. The status bar at the bottom indicates 'IBM SPSS Statistics Processor is ready' and 'Unicode:ON Classic'.

Untitled1 [DataSet0] - IBM SPSS Statistics Data Editor

File Edit View Data Transform Analyze Graphs Utilities Extensions Window Help

Power Analysis > Means > Proportions > One-Sample Binomial Test

Visible: 0 of 0 Variables

11 : var var var

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24

Describe Statistics
Bayesian Statistics
Tables
Compare Means
General Linear Model
Generalized Linear Models
Mixed Models
Correlate
Regression
Loglinear
Classify
Dimension Reduction
Scale
Nonparametric Tests
Forecasting
Survival
Multiple Response
Simulation...
Quality Control
Spatial and Temporal Modeling...

Power Analysis: Independent-Sample Proportions

Test Assumptions

Estimate: Sample size

Single power value: 0.80 Grid power values: Grid

Grid values: None selected

Group size ratio: 1

Proportion parameters for group 1: 0.50 and group 2: 0.80

Significance level: 0.05

Test Method

Chi-squared test

Standard deviation is pooled

Apply continuity correction

I-test

Standard deviation is pooled

Likelihood ratio test

Fisher's exact test

Estimation Method

Normal approximation

Binomial enumeration

Time limit: 5 minutes

Test Direction

Nondirectional (two-sided) analysis

Directional (one-sided) analysis

OK Paste Reset Cancel Help

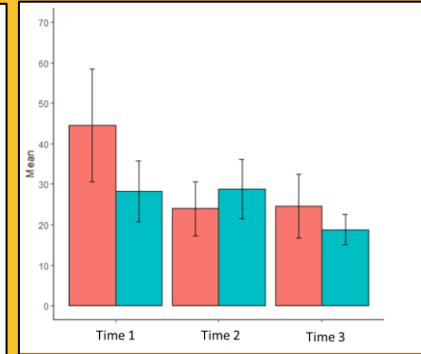
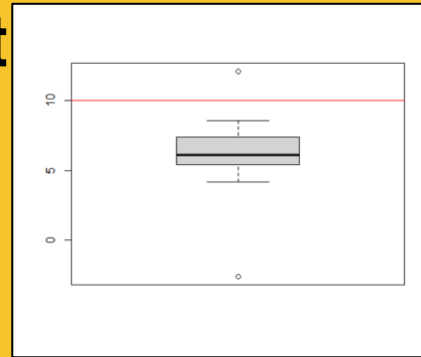
Data View Variable View

Independent-Samples Binomial Test

IBM SPSS Statistics Processor is ready Unicode:ON Classic

12. Additional Designs.

- 1-sample or Paired T-test
- Repeated Measures
- Regression
- Chi-squared
- ...



Expected	Observed
9	56
3	18
3	7
1	7

$$Y = \beta_0 + \beta_1 * X_1 + \beta_2 * X_2 + \epsilon$$

“A Power Analysis for every analysis”

13. Practical Limitations.

- Understanding what goes in.
- Simulation sanity check. (A)
- Realistic patient/sample number.
- Balance of art and science.



(A)

Mean A	30
Mean B	22
Std	12
$d = (30-22) / 12$ $d = 0.667$	

```
> pwr.t.test(d=0.667, sig.level=0.05, power=0.8, type=c("two.sample"))
```

Two-sample t test power calculation

```
      n = 36.27042
      d = 0.667
sig.level = 0.05
power = 0.8
alternative = two.sided
```

NOTE: n is number in *each* group

```
> Treat<-c(rep("A",37), rep("B",37))
> Response<-round(c(rnorm(37, mean=30, sd=12), rnorm(37, mean=22, sd=12)),1)
> ss1 <-data.frame(Treat=Treat, Response=Response)
> t.test(Response~Treat, data=ss1)
```

Welch Two Sample t-test

data: Response by Treat
t = 2.9245, df = 69.914, p-value = 0.004647
alternative hypothesis: true difference in means between group A and group B is not equal to 0
95 percent confidence interval:
2.942929 15.565179
sample estimates:
mean in group A mean in group B
31.42432 22.17027

14. Where to Next?

- More resources

- [Power Analysis in G*Power](#) (videos and slides)
- [Power Analysis in R](#) (videos and slides)
- [Power Analysis in R: GLMMs](#) (videos and slides)
- [Advanced Power Analysis: Into the Weeds](#) (video and slides)

- How to practice

- Design mock experiments
- Calculate/guess effect size
- Get comfortable using calculator/software

- Handy handouts [12]

- Effect sizes for common designs

DACCOTA Statistical Resources

The Biostatistics, Epidemiology, and Research Design Core (BERDC) offers a variety of statistical resources from both our core and other biostatistical cores.

Effect Sizes for common designs			
Statistical Test	Effect size	Equation	Rule of thumb for effect sizes
1 sample t-test	Cohen's d	$d = (\text{mean} - \text{constant}) / SD$	small=0.20, medium=0.50, large=0.80
2 sample t-test	Cohen's d	$d = (\text{mean}_1 - \text{mean}_2) / SD_{\text{pooled}}$	small=0.20, medium=0.50, large=0.80
Paired t-test	Cohen's d	$d = (\text{mean}_1 - \text{mean}_2) / SD_{\text{pooled}}$	small=0.20, medium=0.50, large=0.80
1-Way ANOVA	Eta squared Cohen's f	$\eta^2 = SS_{\text{treatment}} / SS_{\text{total}}$ $f = \sqrt{\eta^2 / (1 - \eta^2)}$	small=0.01, medium=0.05, large=0.14 small=0.10, medium=0.25, large=0.40
2-Way ANOVA	Eta squared Cohen's f	$\eta^2 = SS_{\text{treatment}} / SS_{\text{total}}$ $f = \sqrt{\eta^2 / (1 - \eta^2)}$	small=0.01, medium=0.06, large=0.14 small=0.10, medium=0.25, large=0.40
Repeated Measures ANOVA	Partial Eta squared Cohen's f	Partial $\eta^2 = SS_{\text{effect}} / (SS_{\text{effect}} + SS_{\text{error}})$...	small=0.01, medium=0.06, large=0.14 small=0.10, medium=0.25, large=0.40
1 proportion test	Cohen's h	$h = 2 * \text{asin}(\sqrt{\text{prop}_1}) - 2 * \text{asin}(\sqrt{\text{prop}_{\text{control}}})$	small=0.20, medium=0.50, large=0.80
2 proportions test	Cohen's h	$h = 2 * \text{asin}(\sqrt{\text{prop}_1}) - 2 * \text{asin}(\sqrt{\text{prop}_2})$	small=0.20, medium=0.50, large=0.80
Chi-squared test	Cohen's w	$w = \sqrt{(\sum (\text{prop}_{\text{obs}} - \text{prop}_{\text{exp}})^2) / \text{prop}_{\text{exp}}}$	small=0.10, medium=0.30, large=0.50
Pearson Correlation	Correlation (R)	...	small=0.10, medium=0.30, large=0.50
Linear Regression (Entire Model)	F squared	$F^2 = R^2_{\text{model}} / (1 - R^2_{\text{model}})$	small=0.02, medium=0.15, large=0.35
Linear Regression (Ind. Predictor)	F squared	$F^2 = R^2_{\text{increase}} / (1 - R^2_{\text{increase}})$	small=0.10, medium=0.30, large=0.50

15. References and Acknowledgement.

- [1] <https://stats.oarc.ucla.edu/other/mult-pkg/seminars/intro-power/Others>
- [2] <https://www.coursehero.com/file/p7g5rdk/Power-Analysis-Components-There-are-four-components-to-a-power-analysis-three/>
- [3] <https://www.statology.org/effect-size/>
- [4] <https://www.spss-tutorials.com/effect-size/#anova>
- [5] <https://www.psychologie.hhu.de/arbeitsgruppen/allgemeine-psychologie-und-arbeitspsychologie/gpower>
- [6] <http://powerandsamplesize.com/Calculators/>
- [7] <https://www.gigacalculator.com/calculators/power-sample-size-calculator.php>
- [8] <https://www.statskingdom.com/statistical-power-calculators.html>
- [9] <https://sample-size.net>
- [10] <https://www.statology.org/eta-squared/>
- [11] <https://www.statology.org/partial-eta-squared/>
- [12] <https://docs.google.com/spreadsheets/d/1dqbPqj3VfiHC3oZE4azLypiFOQaeoj9HQ8Z5yj0vybs/edit#gid=0>

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For the labs that use the Biostatistics, Epidemiology, and Research Design Core in any way, including this Module, please acknowledge us for publications: ***"Research reported in this publication was supported by DaCCoTA (the National Institute of General Medical Sciences of the National Institutes of Health under Award Number U54GM128729)"***