

Logic in Computer Science(Homework 1)

10175101148 Junjie Chen

September 19, 2019

1 Prove the following Theroems with nature deduction.

1.1 $\neg(p \wedge q) \dashv\vdash \neg q \vee \neg p$

Proof:

Left to right:

1	$\neg(p \wedge q)$	premise
2	$p \vee \neg p$	LEM
3	p	assumption
4	q	assumption
5	$p \wedge q$	$\wedge i$ 3, 4
6	\perp	$\neg e$ 1, 5
7	$\neg q$	$\neg i$ 4 – 6
8	$\neg q \vee \neg p$	$\vee i_1$ 7
9	$\neg p$	assumption
10	$\neg q \vee \neg p$	$\vee i_2$ 9

11 $\neg q \vee \neg p$ $\vee e$ 2, 3 – 8, 9 – 10

Right to left:

1	$\neg q \vee \neg p$	premise
2	$\neg q$	assumption
3	$p \wedge q$	assumption
4	q	$\wedge e_2$ 3
5	\perp	$\neg e$ 2, 4
6	$\neg(p \wedge q)$	$\neg i$ 3 – 5
7	$\neg p$	assumption
8	$p \wedge q$	assumption
9	p	$\wedge e_1$ 8
10	\perp	$\neg e$ 7, 9
11	$\neg(p \wedge q)$	$\neg i$ 8 – 10
12	$\neg(p \wedge q)$	$\vee e$ 1, 2 – 6, 7 – 11

So $\neg(p \wedge q) \dashv\vdash \neg q \vee \neg p$.

1.2 $p \rightarrow q \dashv\vdash \neg q \rightarrow \neg p$

Proof:

Left to right:

1	$p \rightarrow q$	premise
2	$\neg q$	assumption
3	$\neg p$	MT 1, 2
4	$\neg q \rightarrow \neg p$	\rightarrow i 2 – 3

Right to left:

1	$\neg q \rightarrow \neg p$	premise
2	p	assumption
3	$\neg\neg p$	$\neg\neg$ i 2
4	$\neg\neg q$	MT 1, 3
5	q	$\neg\neg$ e 4
6	$p \rightarrow q$	\rightarrow i 2 – 5

So $p \rightarrow q \dashv\vdash \neg q \rightarrow \neg p$.

1.3 $p \wedge q \rightarrow p \dashv\vdash r \vee \neg r$

Proof:

Left to right:

$$1 \quad r \vee \neg r \quad \text{LEM}$$

Right to left:

1	$p \wedge q$	assumption
2	p	\wedge e ₁ 1
3	$p \wedge q \rightarrow p$	\rightarrow i 1 – 2

So $p \wedge q \rightarrow p \dashv\vdash r \vee \neg r$.