Logic in Computer Science(Homework 4)

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1 Prove the following theroems with deduction rules.

1.1 $\forall x(P(x) \to \neg Q(x)) \vdash \neg(\exists x(P(x) \wedge Q(x)))$

Proof:

	1	$\forall x(P(x) \to \neg Q(x))$	premise
	2	$\exists x(P(x) \wedge Q(x))$	assumption
x_0	3	$P(x_0) \wedge Q(x_0)$	assumption
	4	$P(x_0)$	$\wedge e_1$ 3
	5	$Q(x_0)$	$\wedge e_2$ 3
	6	$P(x_0) \to \neg Q(x_0)$	$\forall x$ e 1
	7	$\neg Q(x_0)$	$\to$ e $4, 6$
	8	$\perp$	$\neg$e $5, 7$
	9	$\perp$	$\exists x$ e $2, 3-8$
	10	$\neg(\exists(P(x) \wedge Q(x)))$	$\neg$i $2-9$

1.2 $\forall x(P(x) \leftrightarrow x = b) \vdash P(b) \wedge \forall x \forall y(P(x) \wedge P(y) \rightarrow x = y)$

Proof:

	1	$\forall x(P(x) \leftrightarrow x = b)$	premise
	2	$b = b$	$=$ i
	3	$b = b \rightarrow P(b)$	$\forall x$ e 1
	4	$P(b)$	$\rightarrow$ e $2, 3$
x_0	5	$P(x_0) \rightarrow x_0 = b$	$\forall x$ e 1
y_0	6	$P(y_0) \rightarrow y_0 = b$	$\forall x$ e 1
	7	$P(x_0) \wedge P(y_0)$	assumption
	8	$P(x_0)$	$\wedge$e$_1$ 7
	9	$P(y_0)$	$\wedge$e$_2$ 7
	10	$x_0 = b$	$\rightarrow$ e $5, 8$
	11	$y_0 = b$	$\rightarrow$ e $6, 9$
	12	$x_0 = y_0$	$=$ e $10, 11$
	13	$P(x_0) \wedge P(y_0) \rightarrow x_0 = y_0$	$\rightarrow$ i $7 - 12$
	14	$\forall y(P(x_0) \wedge P(y) \rightarrow x_0 = y)$	$\forall y$ i $6 - 13$
	15	$\forall x \forall y(P(x) \wedge P(y) \rightarrow x = y)$	$\forall x$ i $5 - 14$
	16	$P(b) \wedge \forall x \forall y(P(x) \wedge P(y) \rightarrow x = y)$	$\wedge$i $4, 15$