Hall Effect Notebook - February 2024

by James Amidei, Zach Stedman, and Max Markgraf

In [10]:

```
import csv
import os
import numpy as np
import matplotlib.pyplot as plt
import scipy.stats as stats

def extract_data(directory):
    data_dict = {}
    for filename in os.listdir(directory):
        if filename.endswith('.csv'):
            filepath = os.path.join(directory, filename)
            with open(filepath, 'r') as csvfile:
                csv_reader = csv.reader(csvfile)

                for _ in range(8):
                    next(csv_reader)

                voltage = []
                current = []

                for row in csv_reader:
                    if len(row) >= 15:
                        voltage.append(row[1])
                        current.append(row[14])
                    else:
                        print(f'Skipping row {row}')
                data_dict[filename] = (voltage, current)
    return data_dict

dir_path = 'HallEffectFeb24'
data_dict = extract_data(dir_path)

#data_dict.keys()
#voltageBQ1, currentBQ1 = data_dict['HallBQ1.csv']

for filename, data in data_dict.items():
    voltage, current = data
    voltage_variable_name = f'{os.path.splitext(filename)[0]}_voltage'
    current_variable_name = f'{os.path.splitext(filename)[0]}_current'
    globals()[voltage_variable_name] = voltage
    globals()[current_variable_name] = current

#dir()
```

Finding the Resistance and Resistivity

$$V = RI \qquad \text{Ohm's law}$$

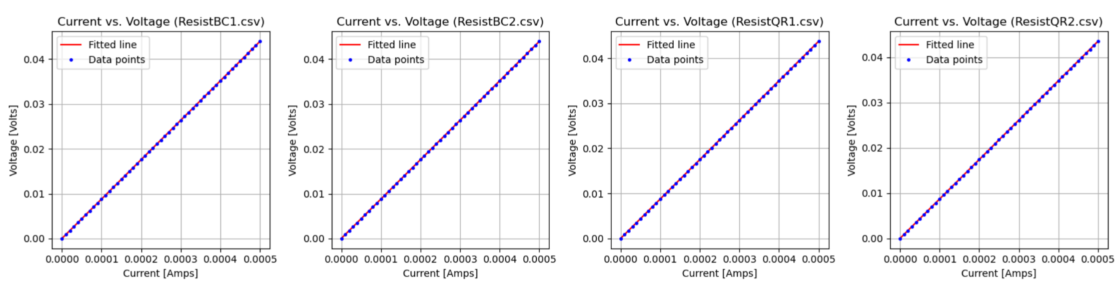
Since we can see a very linear relationship between the current and voltage in our when no magentic field is present, we can use Ohm's law to find the resistance. If we plot the values of the current on the x-axis, and the values for the voltage on the y-axis, our slope will be the resistance.

Fig. 3.1: (left) A sketch of a top view of the semiconductor chips from TeachSpin, showing the two 'stripes' and the four 'tabs' on the chip. The letters reference the connections to the 8 pins of the connector. (right) A pins'-eye view 'looking into' the connector, showing the lettered labelling of the 8 pins. Note that the circuit-board is at-bottom in this view.

In [11]:

```
data_sets = [('ResistBC1_current', 'ResistBC1_voltage', 'ResistBC1.csv'),
             ('ResistBC2_current', 'ResistBC2_voltage', 'ResistBC2.csv'),
             ('ResistQR1_current', 'ResistQR1_voltage', 'ResistQR1.csv'),
             ('ResistQR2_current', 'ResistQR2_voltage', 'ResistQR2.csv')]

fig, axs = plt.subplots(1, 4, figsize=(16, 4))

for i, (current_var, voltage_var, title) in enumerate(data_sets):
    I = np.asarray(globals()[current_var], dtype=np.float64)
    V = np.asarray(globals()[voltage_var], dtype=np.float64)

    slope, intercept, _, _, std_err = stats.linregress(I, V)

    axs[i].plot(I, intercept + slope * I, 'r', label='Fitted line')
    axs[i].errorbar(I, V, fmt='.', color='blue', label='Data points', mark
    axs[i].set_xlabel('Current [Amps]')
    axs[i].set_ylabel('Voltage [Volts]')
    axs[i].set_title(f'Current vs. Voltage ({title})')
    axs[i].grid(True)
    axs[i].legend()

    print(f"Linear fit for {title}: V = ({slope:.3f} +/- {std_err:.3f}) I
    # it may be worth stating that while there actually is an intercept wh
    # and it is most likely something that is not physical. Most likely it
    # in the computer or whatever.

plt.tight_layout()
plt.show()
```

```
Linear fit for ResistBC1.csv: V = (87.851 +/- 0.009) I + 0.000
Linear fit for ResistBC2.csv: V = (87.839 +/- 0.008) I + 0.000
Linear fit for ResistQR1.csv: V = (87.396 +/- 0.012) I + 0.000
Linear fit for ResistQR2.csv: V = (86.896 +/- 0.012) I + 0.000
```



In [12]:

```
R_avg = (87.85050897 + 87.83943163 + 87.39626631 + 86.89558920)/4
R_err_avg = (0.00868031 + 0.00827114 + 0.01175926 + 0.01198674)/4
print(f'Average resistance of the circuit: {R_avg} +/- {R_err_avg}')
```

```
Average resistance of the circuit: 87.4954490275 +/- 0.01017436249999999
9
```

Resistivity

We can find the resistivity with the following formula.

$$\rho = R\frac{A}{l} = R\frac{wt}{l}(\Omega * m)$$

That is, the resistivity ρ is equal to the resistance times the area (width times thickness) over length.

We were given the thickness on the semicondictors packagaing, but we had to measure the length and width ourselves. So there may be a need to revisit these calculations.

Error propagation formula

$$\frac{\sigma_\rho}{\rho} = \sqrt{\left(\frac{\sigma_R}{R}\right)^2 + \left(\frac{\sigma_A}{A}\right)^2 + \left(\frac{\sigma_l}{l}\right)^2}$$

In [13]:

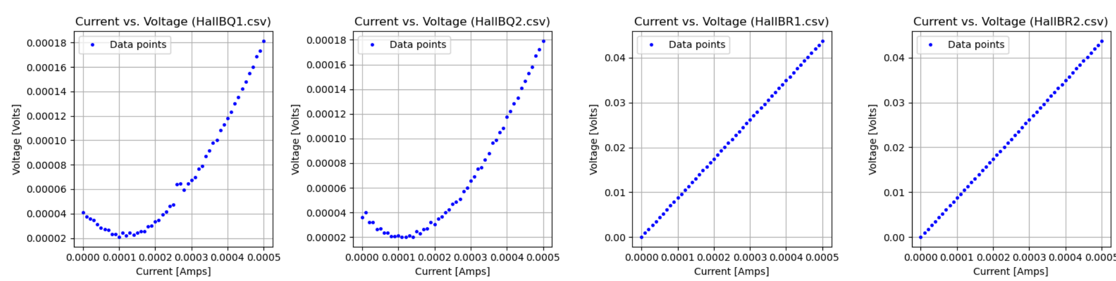
```
t = 500e-6 # 500 micro meters. +/- 15e-6
w = 0.003 # 0.3 cm +/- 0.0005 m
l = 0.018 # 1.8 cm +/- 0.0005 m


sigma_l = 0.0005

sigma_R = R_err_avg

A = w*t

sigma_A = A*np.sqrt(((0.0005)/w)**2 + (0.000015/t)**2 )

rho = (R_avg*(w*t))/l

sigma_rho = rho*np.sqrt((sigma_R/R_avg)**2 + (sigma_l/l)**2 + (sigma_A/A)*

print(f'Calculated resistivity is equal to {rho:0.5f} +/- {sigma_rho:0.4f}
```

```
Calculated resistivity is equal to 0.00729 +/- 0.0013
```

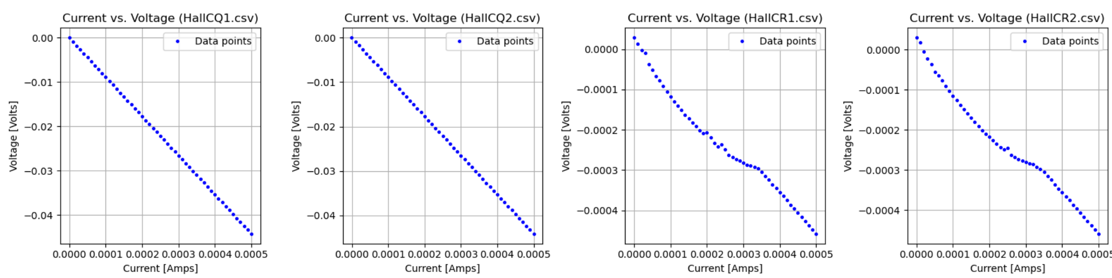
With magnetic field present

In [14]:

```
# When magnetic field is present
data_sets = [('HallBQ1_current', 'HallBQ1_voltage', 'HallBQ1.csv'),
             ('HallBQ2_current', 'HallBQ2_voltage', 'HallBQ2.csv'),
             ('HallBR1_current', 'HallBR1_voltage', 'HallBR1.csv'),
             ('HallBR2_current', 'HallBR2_voltage', 'HallBR2.csv')]

fig, axs = plt.subplots(1, 4, figsize=(16, 4))

for i, (current_var, voltage_var, title) in enumerate(data_sets):
    I = np.asarray(globals()[current_var], dtype=np.float64)
    V = np.asarray(globals()[voltage_var], dtype=np.float64)

    #slope, intercept, _, _, std_err = stats.linregress(I, V)

    #axs[i].plot(I, intercept + slope * I, 'r', label='Fitted line')
    axs[i].errorbar(I, V, fmt='.', color='blue', label='Data points', mark
    axs[i].set_xlabel('Current [Amps]')
    axs[i].set_ylabel('Voltage [Volts]')
    axs[i].set_title(f'Current vs. Voltage ({title})')
    axs[i].grid(True)
    axs[i].legend()

    #print(f"Linear fit for {title}: V = ({slope:.8f} +/- {std_err:.8f}) I

plt.tight_layout()
plt.show()
```

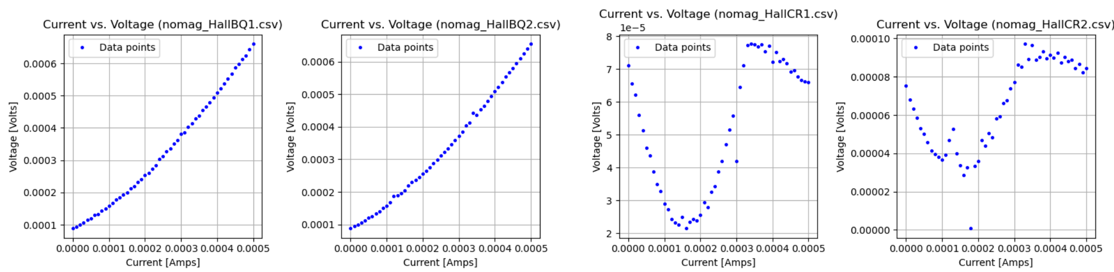


In [6]:
```
data_sets= [('HallCQ1_current', 'HallCQ1_voltage', 'HallCQ1.csv'),
            ('HallCQ2_current', 'HallCQ2_voltage', 'HallCQ2.csv'),
            ('HallCR1_current', 'HallCR1_voltage', 'HallCR1.csv'),
            ('HallCR2_current', 'HallCR2_voltage', 'HallCR2.csv')]

fig, axs = plt.subplots(1, 4, figsize=(16, 4))

for i, (current_var, voltage_var, title) in enumerate(data_sets):
    I = np.asarray(globals()[current_var], dtype=np.float64)
    V = np.asarray(globals()[voltage_var], dtype=np.float64)

    #slope, intercept, _, _, std_err = stats.linregress(I, V)

    #axs[i].plot(I, intercept + slope * I, 'r', label='Fitted line')
    axs[i].errorbar(I, V, fmt='.', color='blue', label='Data points', mark
    axs[i].set_xlabel('Current [Amps]')
    axs[i].set_ylabel('Voltage [Volts]')
    axs[i].set_title(f'Current vs. Voltage ({title})')
    axs[i].grid(True)
    axs[i].legend()

    #print(f"Linear fit for {title}: V = ({slope:.8f} +/- {std_err:.8f}) I

plt.tight_layout()
plt.show()
```



In [15]:

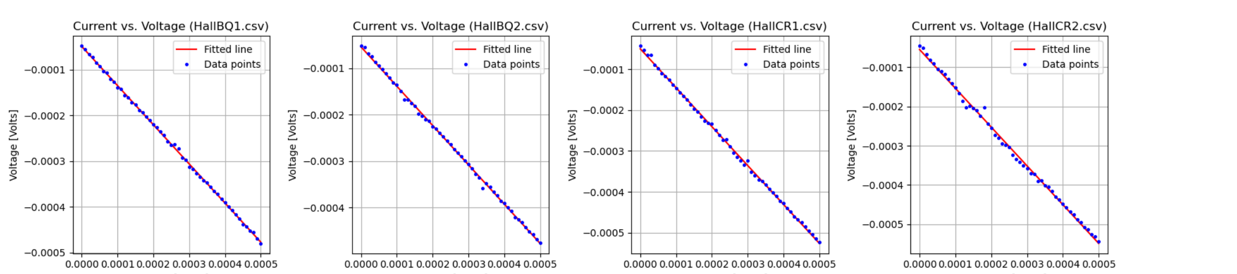
```
# When "horizontal components" without magnetic field present
data_sets = [('nomag_HallBQ1_current', 'nomag_HallBQ1_voltage', 'nomag_Hal
             ('nomag_HallBQ2_current', 'nomag_HallBQ2_voltage', 'nomag_Hal
             ('nomag_HallCR1_current', 'nomag_HallCR1_voltage', 'nomag_Hal
             ('nomag_HallCR2_current', 'nomag_HallCR2_voltage', 'nomag_Hal

fig, axs = plt.subplots(1, 4, figsize=(16, 4))

for i, (current_var, voltage_var, title) in enumerate(data_sets):
    I = np.asarray(globals()[current_var], dtype=np.float64)
    V = np.asarray(globals()[voltage_var], dtype=np.float64)

    #slope, intercept, _, _, std_err = stats.linregress(I, V)

    #axs[i].plot(I, intercept + slope * I, 'r', label='Fitted line')
    axs[i].errorbar(I, V, fmt='.', color='blue', label='Data points', mark
    axs[i].set_xlabel('Current [Amps]')
    axs[i].set_ylabel('Voltage [Volts]')
    axs[i].set_title(f'Current vs. Voltage ({title})')
    axs[i].grid(True)
    axs[i].legend()

    #print(f"Linear fit for {title}: V = ({slope:.8f} +/- {std_err:.8f}) I

plt.tight_layout()
plt.show()
```



In [8]:

```
# the idea here came from talking to Dr. Tagg. The components that were se
# which is why we were getting those weird shapes. The idea here was that
# components sans field, then substracting that from the measurements we t
# the new vertical Hall voltage than we initially were when this excentric
# As we can see from the results below, we get much more linear fits whihc

V_BQ1 = np.asarray(HallBQ1_voltage, dtype=np.float64) - np.asarray(nomag_H
V_BQ2 = np.asarray(HallBQ2_voltage, dtype=np.float64) - np.asarray(nomag_H
V_CR1 = np.asarray(HallCR1_voltage, dtype=np.float64) - np.asarray(nomag_H
V_CR2 = np.asarray(HallCR2_voltage, dtype=np.float64) - np.asarray(nomag_H


# When magnetic field is present
data_sets = [('HallBQ1_current', 'V_BQ1', 'HallBQ1.csv'),
             ('HallBQ2_current', 'V_BQ2', 'HallBQ2.csv'),
             ('HallBR1_current', 'V_CR1', 'HallCR1.csv'),
             ('HallBR2_current', 'V_CR2', 'HallCR2.csv')]

fig, axs = plt.subplots(1, 4, figsize=(16, 4))

for i, (current_var, voltage_var, title) in enumerate(data_sets):
    I = np.asarray(globals()[current_var], dtype=np.float64)
    V = np.asarray(globals()[voltage_var], dtype=np.float64)

    slope, intercept, _, _, std_err = stats.linregress(I, V)

    axs[i].plot(I, intercept + slope * I, 'r', label='Fitted line')
    axs[i].errorbar(I, V, fmt='.', color='blue', label='Data points', mark
    axs[i].set_xlabel('Current [Amps]')
    axs[i].set_ylabel('Voltage [Volts]')
    axs[i].set_title(f'Current vs. Voltage ({title})')
    axs[i].grid(True)
    axs[i].legend()

    print(f"Linear fit for {title}: V = ({slope:.3f} +/- {std_err:.3f}) I

plt.tight_layout()
plt.show()
```

```
Linear fit for HallBQ1.csv: V = (-0.857 +/- 0.003) I + -0.000
Linear fit for HallBQ2.csv: V = (-0.848 +/- 0.004) I + -0.000
Linear fit for HallCR1.csv: V = (-0.954 +/- 0.004) I + -0.000
Linear fit for HallCR2.csv: V = (-0.988 +/- 0.008) I + -0.000
```



Note:

~~The configurations that go from B - Q and C - R only measure the vertical component of the voltage. This is the Hall voltage which is he result of the force generated by the magnetic field.~~

B-Q and C-R were *not* perfectly vertical, which is why we needed to the sans field measurements to substract the strange features we saw in our initial data.

The two other configurations, B - R and C - Q, go diagonally across, so they have both a vertical and horizontal component. Compare the magnitudes of the voltage and resistance with those from the plots from B - C and Q - R. This shows that the Hall voltage with relatively small compared to the horizontal voltage across the chip

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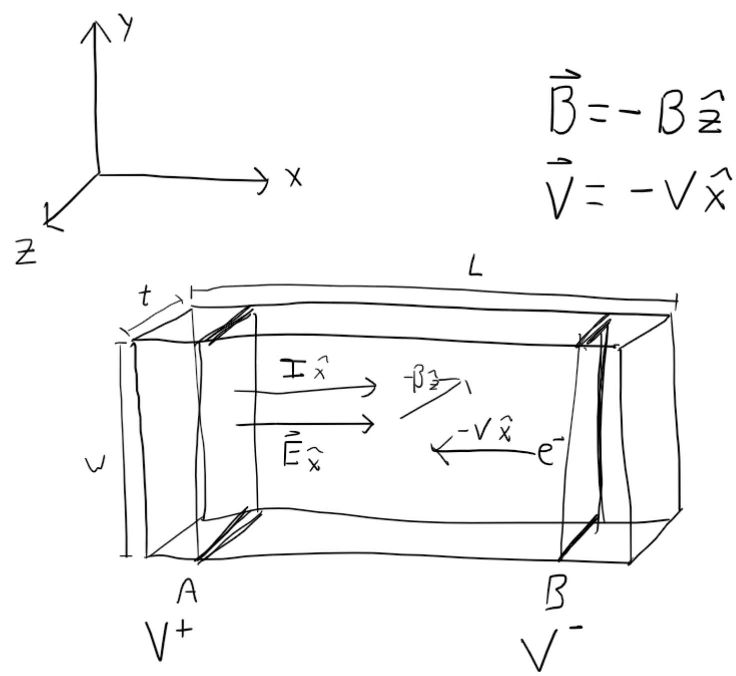
Hall Effect



Rough sketch of semiconductor configuration. The electric field ran from tabs A to D, meaning that the electric field moved from from A to B in the positive $\hat{x}$ direction. In the drawing above, we assumed the charge carriers were electrons, which would move against the current in the negative $\hat{x}$ direction. We placed the chip in the magentic such that the magnetic field went in the negative $\hat{z}$ direction.

$$q = -e$$
$$\vec{v} = -v\hat{x}$$
$$\vec{B} = -B\hat{z}$$

$$\overrightarrow{F_B} = q\vec{v} \times \vec{B} \rightarrow \overrightarrow{F_B} = -e(-v\hat{x}) \times (-B\hat{z})$$
$$\overrightarrow{F_B} = +evB\hat{y}$$

From this, calculation we'd exect to see a deflection in the positive $\hat{y}$ direction due to the magnetic field.

If the charge carriers were holes, we would simply flip the sign of the velocity to be in the direction of the electrical field.

$$q = e$$
$$\vec{v} = v\hat{x}$$
$$\vec{B} = -B\hat{z}$$

$$\overrightarrow{F_B} = q\vec{v} \times \vec{B} \rightarrow \overrightarrow{F_B} = -e(v\hat{x}) \times (-B\hat{z})$$
$$\overrightarrow{F_B} = -evB\hat{y}$$

Taking the equation for the current density, we can relate the velocity to both the quantity and magnitude of the individual charge carriers, as well as the geometry of the chip.

$$\vec{j} = \text{sign}(q)en\vec{v} = \text{sign}\frac{I}{A}\hat{x}$$

$$\vec{v} = \text{sign}(q)\frac{I}{qnA}\hat{x} = \text{sign}(q)\frac{I}{qnwt}\hat{x}$$

$\text{sign}(q)$ is just the sign of the charge carrier and represents either a $+$ or $-$.

Plugging this in the equation for the force due to the magnetic field, we get

$$\overrightarrow{F_B} = \text{sign}(q)q\vec{v} \times \vec{B} \rightarrow \overrightarrow{F_B} = \cancel{\text{sign}(q)}q\frac{I}{\cancel{\text{sign}(q)}q\ nwt}(\hat{x}) \times (-B\hat{z})$$

Using right hand rule,

$$\overrightarrow{F_B} = -\frac{IB}{nwt}\hat{y}$$

This magnetic force deflects charges upward until the above force due to the magnetic field is opposed by an electrostatic force pointing in the downwards $\hat{y}$ direction.

$$\overrightarrow{F_B} = q\overrightarrow{E_y}\hat{y} = q\frac{V_H}{w}\hat{y}$$

The term V_H is the Hall voltage.

Equating the electrostatic force with the force due to the magnetic field we get

$$q\frac{V_H}{w}\hat{y} = -\frac{IB}{nwt}\hat{y}$$

$$V_H = -\text{sign}(q)\frac{B}{qnt}I$$

The above has units of $\frac{T}{C \cdot m} \cdot A$, which reduces to

$$\frac{T}{C \cdot m} \cdot A \equiv \frac{\frac{kg}{s^2 \cdot A}}{A \cdot s \cdot m} \cdot A \equiv \frac{kg}{s^3 \cdot A^2 \cdot m} \cdot A \equiv \Omega \cdot A \equiv V$$

From above $\frac{B}{qnt}$ has units of Ω. This is called the Hall resistance: $R_H \equiv \frac{B}{net}$.

The Hall resistance is found empirically by making a linear fit of the Hall voltage V_H versus the current I. This is similar to how we found the initial resistance without the presence of the magnetic field.

We can then use this to solve for the carrier density.

$$n = \frac{B}{qtR_H} \quad \text{(carrier number)}$$

Mobility

In [16]:

```
q = 1.602e-19 # C magnitude of charge carriers
B = 0.095  # T magnetic field strength +/- 0.001: This was updated with th
t = 500e-6 # 500 micro meters
R_H_BQ = (0.857 + 0.848)/2
R_H_CR = (0.954 + 0.988)/2

sigma_B = 0.001
sigma_t = 0.000015
sigma_R_H_BQ = (0.003 + 0.004)/2
sigma_R_H_CR = (0.004 + 0.008)/2

n_BQ = B/(q*t*R_H_BQ)
n_CR = B/(q*t*R_H_CR)

sigma_n_BQ = n_BQ*np.sqrt((sigma_B/B)**2 + (sigma_t/t)**2 + (sigma_R_H_BQ/
sigma_n_CR = n_BQ*np.sqrt((sigma_B/B)**2 + (sigma_t/t)**2 + (sigma_R_H_CR/

#mobility
mu_BQ = (n_BQ*q*rho)**(-1)
mu_CR = (n_CR*q*rho)**(-1)

sigma_mu_BQ = mu_BQ*np.sqrt((sigma_n_BQ/n_BQ)**2 + (sigma_rho/rho)**2)
sigma_mu_CR = mu_CR*np.sqrt((sigma_n_CR/n_CR)**2 + (sigma_rho/rho)**2)

print(f'The Hall resistance measured from B-Q: {R_H_BQ} +/- {sigma_R_H_BQ}
print()
print(f'The Hall resistance measured from C-R: {R_H_CR} +/- {sigma_R_H_CR}
print()
print(f'The charge density from the Hall resistance measured from B-Q: {ro
print()
print(f'The charge density from the Hall resistance measured from C-R: {n_
print()
print(f'The mobility from BQ is {mu_BQ:0.3f} +/- {sigma_mu_BQ:0.3f}')
print()
print(f'The mobility from BQ is {mu_CR:0.3f} +/- {sigma_mu_CR:0.3f}')
```

```
The Hall resistance measured from B-Q: 0.8525 +/- 0.0035

The Hall resistance measured from C-R: 0.971 +/- 0.006

The charge density from the Hall resistance measured from B-Q: 139122284
8272503668736 +/- 44598599998452375552

The charge density from the Hall resistance measured from C-R: 122143921
5398877134848 +/- 45058995454692573184

The mobility from BQ is 0.615 +/- 0.107

The mobility from BQ is 0.701 +/- 0.123
```

Given the fact that the magnetic force is in the positive $\hat{y}$ direction due to the current in the positive $\hat{x}$ direction and magnetic field in the negative $\hat{z}$ direction, we can take this together with the negative hall voltage to say that our majority charge carriers must be traveling in the same direction as the current, meaning that they are positively charged holes.

In [ ]:

In [ ]:

In [ ]:

In [ ]: