Hall Effect Notebook - February 2024 ¶

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In [2]:

```
import csv
import os
import numpy as np
import matplotlib.pyplot as plt
import scipy.stats as stats

def extract_data(directory):
    data_dict = {}
    for filename in os.listdir(directory):
        if filename.endswith('.csv'):
            filepath = os.path.join(directory, filename)
            with open(filepath, 'r') as csvfile:
                csv_reader = csv.reader(csvfile)

                for _ in range(8):
                    next(csv_reader)

                voltage = []
                current = []

                for row in csv_reader:
                    if len(row) >= 15:
                        voltage.append(row[1])
                        current.append(row[14])
                    else:
                        print(f'Skipping row {row}')
                data_dict[filename] = (voltage, current)
    return data_dict

dir_path = 'HallEffectFeb24'
data_dict = extract_data(dir_path)

#data_dict.keys()
#voltageBQ1, currentBQ1 = data_dict['HallBQ1.csv']

for filename, data in data_dict.items():
    voltage, current = data
    voltage_variable_name = f'{os.path.splitext(filename)[0]}_voltage'
    current_variable_name = f'{os.path.splitext(filename)[0]}_current'
    globals()[voltage_variable_name] = voltage
    globals()[current_variable_name] = current

#dir()
```

Finding the Resistance and Resistivity

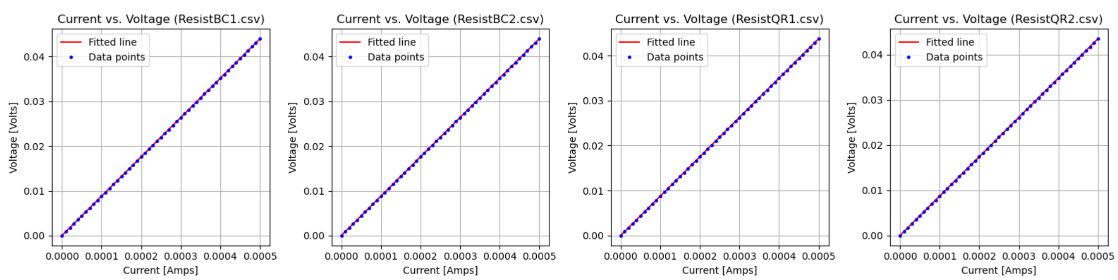
$$V = RI \qquad \text{Ohm's law}$$

Since we can see a very linear relationship between the current and voltage in our when no magentic field is present, we can use Ohm's law to find the resistance. If we plot the values of the current on the x-axis, and the values for the voltage on the y-axis, our slope will be the resistance.

Fig. 3.1: (left) A sketch of a top view of the semiconductor chips from TeachSpin, showing the two 'stripes' and the four 'tabs' on the chip. The letters reference the connections to the 8 pins of the connector. (right) A pins'-eye view 'looking into' the connector, showing the lettered labelling of the 8 pins. Note that the circuit-board is at-bottom in this view.

In [11]:
```
data_sets = [('ResistBC1_current', 'ResistBC1_voltage', 'ResistBC1.csv'),
             ('ResistBC2_current', 'ResistBC2_voltage', 'ResistBC2.csv'),
             ('ResistQR1_current', 'ResistQR1_voltage', 'ResistQR1.csv'),
             ('ResistQR2_current', 'ResistQR2_voltage', 'ResistQR2.csv')]

fig, axs = plt.subplots(1, 4, figsize=(16, 4))

for i, (current_var, voltage_var, title) in enumerate(data_sets):
    I = np.asarray(globals()[current_var], dtype=np.float64)
    V = np.asarray(globals()[voltage_var], dtype=np.float64)

    slope, intercept, _, _, std_err = stats.linregress(I, V)

    axs[i].plot(I, intercept + slope * I, 'r', label='Fitted line')
    axs[i].errorbar(I, V, fmt='.', color='blue', label='Data points', mark
    axs[i].set_xlabel('Current [Amps]')
    axs[i].set_ylabel('Voltage [Volts]')
    axs[i].set_title(f'Current vs. Voltage ({title})')
    axs[i].grid(True)
    axs[i].legend()

    print(f"Linear fit for {title}: V = ({slope:.8f} +/- {std_err:.8f}) I

plt.tight_layout()
plt.show()
```

```
Linear fit for ResistBC1.csv: V = (87.85050897 +/- 0.00868031) I + 0.000
01437
Linear fit for ResistBC2.csv: V = (87.83943163 +/- 0.00827114) I + 0.000
01538
Linear fit for ResistQR1.csv: V = (87.39626631 +/- 0.01175926) I + 0.000
03408
Linear fit for ResistQR2.csv: V = (86.89558920 +/- 0.01198674) I + 0.000
03139
```



In [4]:
```
R_avg = (87.85050897 + 87.83943163 + 87.39626631 + 86.89558920)/4
R_err_avg = (0.00868031 + 0.00827114 + 0.01175926 + 0.01198674)/4
print(f'Average resistance of the circuit: {R_avg} +/- {R_err_avg}')
```

```
Average resistance of the circuit: 87.4954490275 +/- 0.01017436249999999
9
```

Resistivity

We can find the resistivity with the following formula.

$$\rho = R\frac{A}{l} = R\frac{wt}{l}(\Omega * m)$$

That is, the resistivity ρ is equal to the resistance times the area (width times thickness) over length.

We were given the thickness on the semicondictors packagaing, but we had to measure the length and width ourselves. So there may be a need to revisit these calculations.

In [5]:

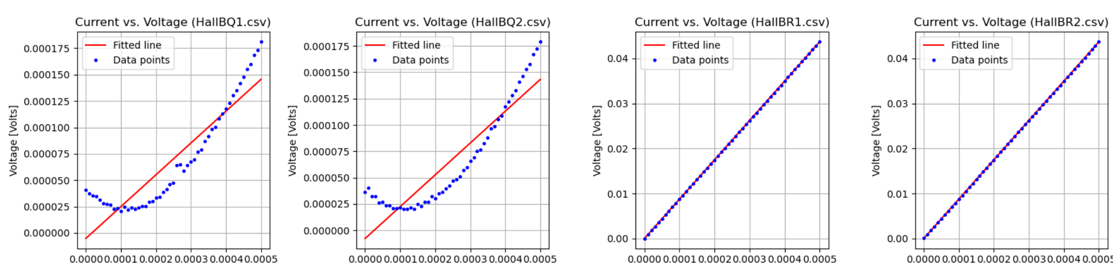
```
R_avg_l = R_avg - R_err_avg # lower bound of resistance
R_avg_u = R_avg + R_err_avg # upper bound of resistance

t = 500e-6 # 500 micro meters
w = 0.003 # 0.3 cm
l = 0.018 # 1.8 cm

rho_l = R_avg_l * (w*t)/l
rho = R_avg * (w*t)/l
rho_u = R_avg_u * (w*t)/l

print('Lower resistivity: ', rho_l)
print()
print('Middle resistivity: ', rho)
print()
print('Upper resistivty: ', rho_u)
print()
print('Calculated resistivity is equal to ', rho, '+/-', rho_u-rho_l)
```

```
Lower resistivity:  0.0072904395554166685

Middle resistivity:  0.007291287418958335

Upper resistivty:  0.0072921352825

Calculated resistivity is equal to  0.007291287418958335 +/- 1.695727083
3313845e-06
```

With magnetic field present

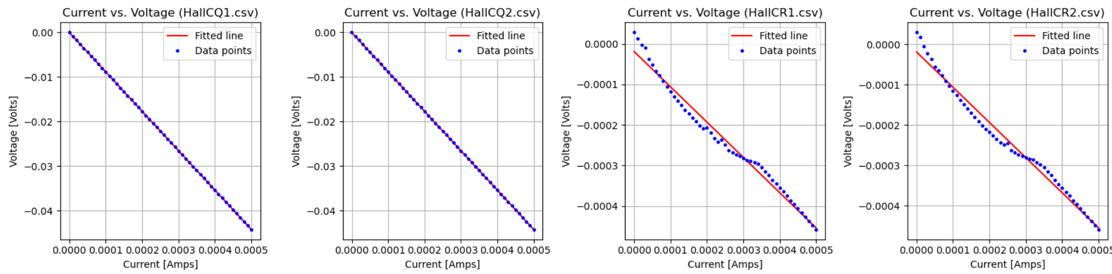
In [9]:
```
# When magnetic field is present
data_sets = [('HallBQ1_current', 'HallBQ1_voltage', 'HallBQ1.csv'),
             ('HallBQ2_current', 'HallBQ2_voltage', 'HallBQ2.csv'),
             ('HallBR1_current', 'HallBR1_voltage', 'HallBR1.csv'),
             ('HallBR2_current', 'HallBR2_voltage', 'HallBR2.csv')]

fig, axs = plt.subplots(1, 4, figsize=(16, 4))

for i, (current_var, voltage_var, title) in enumerate(data_sets):
    I = np.asarray(globals()[current_var], dtype=np.float64)
    V = np.asarray(globals()[voltage_var], dtype=np.float64)

    slope, intercept, _, _, std_err = stats.linregress(I, V)

    axs[i].plot(I, intercept + slope * I, 'r', label='Fitted line')
    axs[i].errorbar(I, V, fmt='.', color='blue', label='Data points', mark
    axs[i].set_xlabel('Current [Amps]')
    axs[i].set_ylabel('Voltage [Volts]')
    axs[i].set_title(f'Current vs. Voltage ({title})')
    axs[i].grid(True)
    axs[i].legend()

    print(f"Linear fit for {title}: V = ({slope:.8f} +/- {std_err:.8f}) I

plt.tight_layout()
plt.show()
```

```
Linear fit for HallBQ1.csv: V = (0.30150992 +/- 0.01898714) I + -0.00000
524
Linear fit for HallBQ2.csv: V = (0.30199204 +/- 0.01927253) I + -0.00000
780
Linear fit for HallBR1.csv: V = (87.38262204 +/- 0.01258746) I + 0.00001
283
Linear fit for HallBR2.csv: V = (87.36505117 +/- 0.01337833) I + -0.0000
0041
```



In [10]:
```
data_sets= [('HallCQ1_current', 'HallCQ1_voltage', 'HallCQ1.csv'),
            ('HallCQ2_current', 'HallCQ2_voltage', 'HallCQ2.csv'),
            ('HallCR1_current', 'HallCR1_voltage', 'HallCR1.csv'),
            ('HallCR2_current', 'HallCR2_voltage', 'HallCR2.csv')]

fig, axs = plt.subplots(1, 4, figsize=(16, 4))

for i, (current_var, voltage_var, title) in enumerate(data_sets):
    I = np.asarray(globals()[current_var], dtype=np.float64)
    V = np.asarray(globals()[voltage_var], dtype=np.float64)

    slope, intercept, _, _, std_err = stats.linregress(I, V)

    axs[i].plot(I, intercept + slope * I, 'r', label='Fitted line')
    axs[i].errorbar(I, V, fmt='.', color='blue', label='Data points', mark
    axs[i].set_xlabel('Current [Amps]')
    axs[i].set_ylabel('Voltage [Volts]')
    axs[i].set_title(f'Current vs. Voltage ({title})')
    axs[i].grid(True)
    axs[i].legend()

    print(f"Linear fit for {title}: V = ({slope:.8f} +/- {std_err:.8f}) I

plt.tight_layout()
plt.show()
```

```
Linear fit for HallCQ1.csv: V = (-88.34269547 +/- 0.01472905) I + -0.000
01791
Linear fit for HallCQ2.csv: V = (-88.31050937 +/- 0.01627855) I + -0.000
01862
Linear fit for HallCR1.csv: V = (-0.87094484 +/- 0.01719977) I + -0.0000
1890
Linear fit for HallCR2.csv: V = (-0.87235488 +/- 0.01736898) I + -0.0000
1922
```

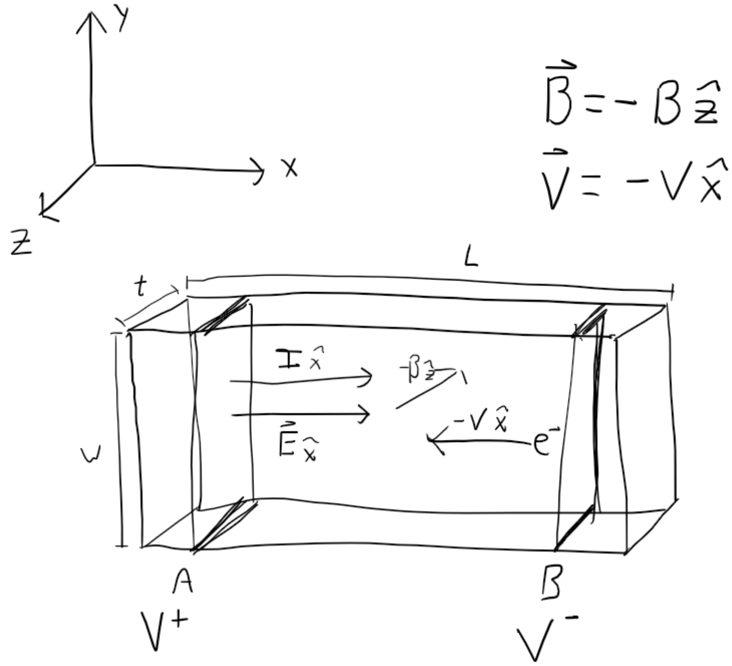


Note:

The configurations that go from B - Q and C - R only measure the vertical component of the voltage. This is the Hall voltage which is he result of the force generated by the magnetic field.

The two other configurations, B - R and C - Q, go diagonally across, so they have both a vertical and horizontal component. Compare the magnitudes of the voltage and resistance with those from the plots from B - C and Q - R. This shows that the Hall voltage with relatively small compared to the horizontal voltage across the chip.

Hall Effect



Rough sketch of semiconductor configuration. The electric field ran from tabs A to D, meaning that the electric field moved from from A to B in the positive $\hat{x}$ direction. In the drawing above, we assumed the charge carriers were electrons, which would move against the current in the negative $\hat{x}$ direction. We placed the chip in the magentic such that the magnetic field went in the negative $\hat{z}$ direction.

$$q = -e$$
$$\vec{v} = -v\hat{x}$$
$$\vec{B} = -B\hat{z}$$

$$\overrightarrow{F_B} = q\vec{v} \times \vec{B} \rightarrow \overrightarrow{F_B} = -e(-v\hat{x}) \times (-B\hat{z})$$
$$\overrightarrow{F_B} = +evB\hat{y}$$

From this, calculation we'd exect to see a deflection in the positive $\hat{y}$ direction due to the magnetic field.

If the charge carriers were holes, we would simply flip the sign of the velocity to be in the direction of the electrical field.

$$q = e$$
$$\vec{v} = v\hat{x}$$
$$\vec{B} = -B\hat{z}$$

$$\overrightarrow{F_B} = q\vec{v} \times \vec{B} \rightarrow \overrightarrow{F_B} = -e(v\hat{x}) \times (-B\hat{z})$$
$$\overrightarrow{F_B} = -evB\hat{y}$$

Taking the equation for the current density, we can relate the velocity to both the quantity and magnitude of the individual charge carriers, as well as the geometry of the chip.

$$\vec{j} = \text{sign}(q)en\vec{v} = \text{sign}\frac{I}{A}\hat{x}$$

$$\vec{v} = \text{sign}(q)\frac{I}{qnA}\hat{x} = \text{sign}(q)\frac{I}{qnwt}\hat{x}$$

$\text{sign}(q)$ is just the sign of the charge carrier and represents either a $+$ or $-$.

Plugging this in the equation for the force due to the magnetic field, we get

$$\overrightarrow{F_B} = \text{sign}(q)q\vec{v} \times \vec{B} \rightarrow \overrightarrow{F_B} = \cancel{\text{sign}(q)}q\frac{I}{\cancel{\text{sign}(q)}q\ nwt}(\hat{x}) \times (-B\hat{z})$$

Using right hand rule,

$$\overrightarrow{F_B} = -\frac{IB}{nwt}\hat{y}$$

This magnetic force deflects charges upward until the above force due to the magnetic field is opposed by an electrostatic force pointing in the downwards $\hat{y}$ direction.

$$\overrightarrow{F_B} = q\overrightarrow{E_y}\hat{y} = q\frac{V_H}{w}\hat{y}$$

The term V_H is the Hall voltage.

Equating the electrostatic force with the force due to the magnetic field we get

$$q\frac{V_H}{w}\hat{y} = -\frac{IB}{nwt}\hat{y}$$

$$V_H = -\text{sign}(q)\frac{B}{qnt}I$$

The above has units of $\frac{T}{C\cdot m} \cdot A$, which reduces to

$$\frac{T}{C \cdot m} \cdot A \equiv \frac{\frac{kg}{s^2\cdot A}}{A \cdot s \cdot m} \cdot A \equiv \frac{kg}{s^3 \cdot A^2 \cdot m} \cdot A \equiv \Omega \cdot A \equiv V$$

From above $\frac{B}{qnt}$ has units of Ω. This is called the Hall resistance: $R_H \equiv \frac{B}{net}$.

The Hall resistance is found empirically by making a linear fit of the Hall voltage V_H versus the current I. This is similar to how we found the initial resistance without the presence of the magnetic field.

We can then use this to solve for the carrier density.

$$n = \frac{B}{\cdot R}$$

In [23]:

```
q = 1.602e-19 # C magnitude of charge carriers
B = 0.0925 # T magnetic field strength +/- 0.0005
t = 500e-6 # 500 micro meters

resistance_BQ_avg = (0.30150992 + 0.30199204)/2
resistance_err_BQ_avg = (0.01898714 + 0.01927253)/2
resistance_CR_avg = (0.87094484 + 0.87235488)/2
resistance_err_CR_avg = (0.01719977 + 0.01736898)/2

n_BQ_l = B/q*t*(resistance_BQ_avg - resistance_err_BQ_avg)
n_BQ = B/q*t*resistance_BQ_avg
n_BQ_u = B/q*t*(resistance_BQ_avg + resistance_err_BQ_avg)

n_CR_l = B/q*t*(resistance_CR_avg - resistance_err_CR_avg)
n_CR = B/q*t*resistance_CR_avg
n_CR_u = B/q*t*(resistance_CR_avg + resistance_err_CR_avg)

print(f'The Hall resistance measured from B-Q: {resistance_BQ_avg} +/- {re
print()
print(f'The Hall resistance measured from C-R: {resistance_CR_avg} +/- {re
print()
print(f'The charge density from the Hall resistance measured from B-Q: {n_
print()
print(f'The charge density from the Hall resistance measured from C-R: {n_
```

```
The Hall resistance measured from B-Q: 0.30175098 +/- 0.0191298349999999
98

The Hall resistance measured from C-R: 0.87164986 +/- 0.0172843749999999
98

The charge density from the Hall resistance measured from B-Q: 871159976
59176.03 +/- 11045628823345.797

The charge density from the Hall resistance measured from C-R: 251646729
244694.12 +/- 9980054229088.656
```

In [ ]:

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In [ ]:

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