Taylor-Couette Experiment

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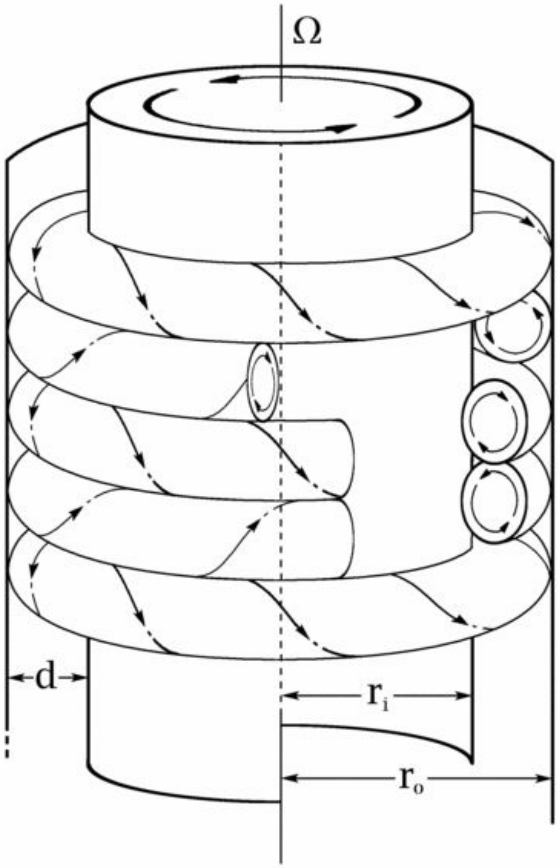

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Main Idea

- The Navier-Stokes equation describes the flow of Newtonian fluids.
- For an incompressible fluid, in a solid container, both the continuity equation and the no-slip boundary condition apply.
- For sufficiently small angular velocities this force is negligible.
- There exists a critical angular velocity where the laminar state experiences instability as it transitions to a second laminar state.

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \vec{u} + \vec{g}$$

(Navier-Stokes Equation)

$$\rho = \text{constant}$$

$$\nabla \cdot \vec{u} = 0.$$

$$u_\phi(r_1) = \Omega_1 r_1 \quad u_\phi(r_2) = \Omega_2 r_2$$

(No-slip boundary condition)

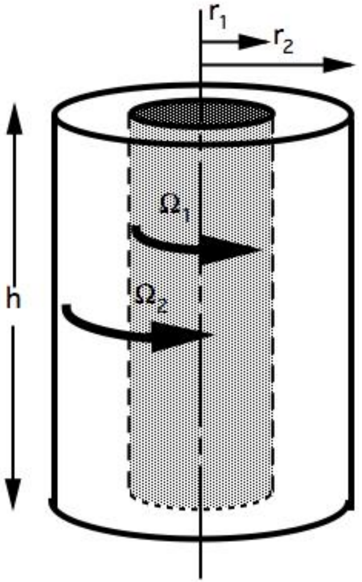
$$u_\phi(r) = Ar + B\frac{1}{r}$$

$$A = \frac{r_2^2\Omega_2 - r_1^2\Omega_1}{r_2^2 - r_1^2} \quad B = \frac{r_1^2 r_2^2(\Omega_1 - \Omega_2)^2}{r_2^2 - r_1^2}$$

Instrumentation and Data Collection

- A water-glycerol mixture was placed in between two 52 cm tall cylinders
- Each cylinder was connected to a motor that was controlled by a python script.
- The speed of each motor was varied by choosing a specific Reynold's number.
- **Re_c = 68.46** (critical Reynolds number)

$$Re \equiv \frac{\Omega r d}{\nu} \quad \text{(Reynolds Number)}$$

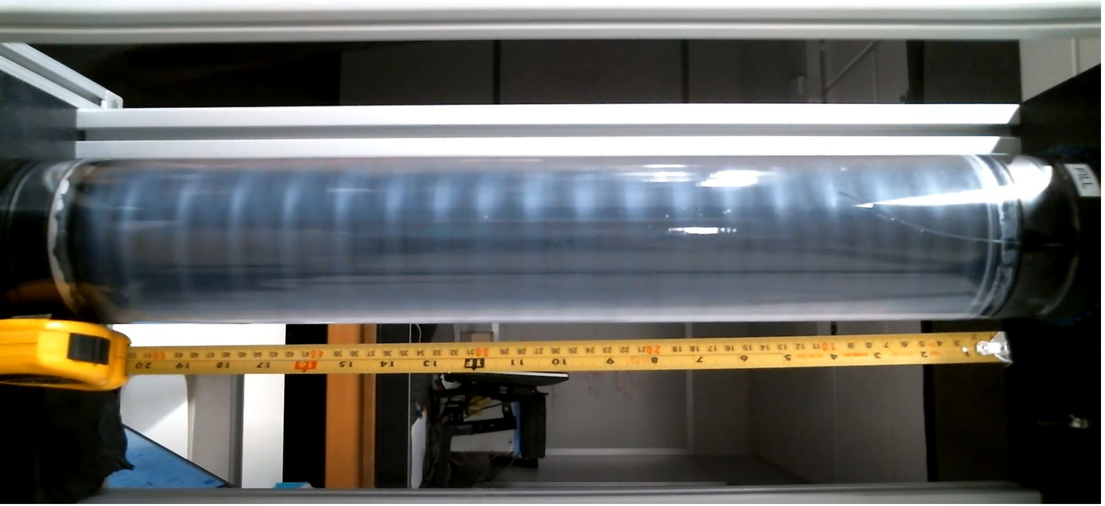


inner radius $r_1 = 2.911$ cm.

outer radius $r_2 = 4.445$ cm.

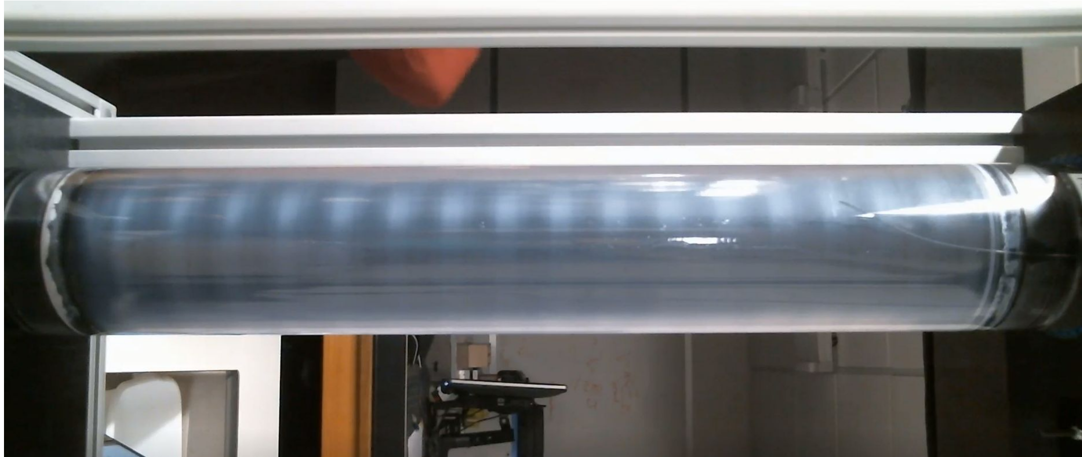
height $h = 52$ cm.

gap $d \equiv r_2 - r_1 = 1.534$ cm.



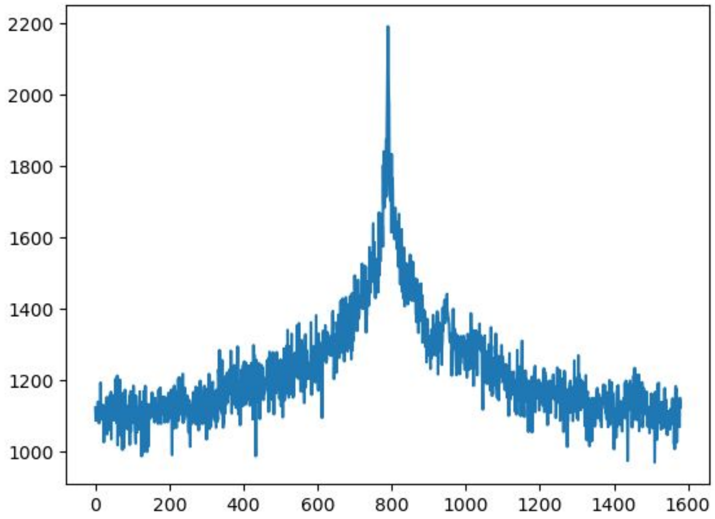
Left: Example of strong Taylor vortices . Re = 75.01

Right: Taylor vortices at critical Reynolds number. Re = 68.46



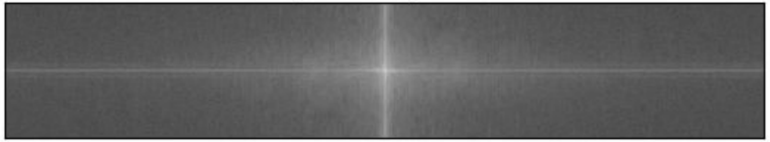
Data Analysis

- An image of the Taylor vortices at the critical Reynolds number was put through a Fourier transform in python.
- This was used, along with pixel width of the image, to find the wavelength.
- The calculated wavelength was approximately **3.15 cm**.



Input Image

Magnitude Spectrum



Special Topic - Heat Transfer

- The Nusselt number is the ratio of convective to conductive heat transfer at a boundary in a fluid.
- Lower Nusselt numbers indicate largely conductive heat transfer, while higher values indicate largely convective.
- For forced convection, the Nusselt number is proportional the the Reynolds number to the power of some constant.

$$Nu = C \cdot Re^m \cdot Pr^n$$

$Re =$ Reynolds number

$Pr =$ Prandtl number $= \dfrac{\text{kinematic viscosity}}{\text{thermal diffusivity}}$

$C, m, n =$ constants

References

Lueptow, Richard. “Taylor-Couette flow”, 2009. http://www.scholarpedia.org/article/Taylor-Couette_flow

“Nusselt Number”. https://en.wikipedia.org/wiki/Nusselt_number