B6

Monday, April 8, 2024 1:19 PM

1.) I can identify the integral and differential forms of Maxwell's equations that describe the magnetic field in vacuum and explain the terms in those equations

2.) I recognize that the Biot-Savart law, Ampere's law, and Maxwell's fourth law all describe the magnetic field and can explain how they are related

3.) I can show that the curl of all the magnetic fields I calculate is not zero

1.)

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} \qquad \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\oint \vec{B} \cdot d\vec{A} = 0 \qquad \vec{\nabla} \cdot \vec{B} = 0$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \left(I_{enc} + \varepsilon_0 \frac{\partial \Phi_E}{\partial t} \right) \qquad \vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

~~In a vacuum~~ (no current or charge / no current density or charge density)

$$\oint \vec{E} \cdot d\vec{A} = 0 \qquad \vec{\nabla} \cdot \vec{E} = 0$$

$$\oint \vec{B} \cdot d\vec{A} = 0 \qquad \vec{\nabla} \cdot \vec{B} = 0$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{\partial \Phi_B}{\partial t} \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{\partial \Phi_E}{\partial t} \qquad \vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

~~We can see in a vacuum away, away from the source, the presence of an electric field still is associated with the presence of a magnetic field and vice versa. This is expressed in the contour integral around the boundary of a gaussian surface equaling the negative of the change in magnetic flux over time/the curl of the electric field equally the change in the magnetic field over time and in the contour integral of the magnetic field around the boundary of a gaussian surface equaling the change of the electric flux over time times the constants for vacuum permittivity and permeability/the curl of the magnetic field equaling the change in the electric field over time times the constants for vacuum permittivity and permeability.~~

The fourth equation is also known as Ampere's law and can be used to solve problems in the same way the Biot-Savart law can be used.

Ex. (for long current carrying wire)

This example is just a first step. Ultimately we need to look at these equations in a vacuum, i.e. no current or current density.

Biot-Savart

$$B = \frac{\mu_0}{4\pi} \int \frac{I\, d\vec{l} \times \hat{r}}{r^2} = \frac{\mu_0 I}{2\pi d}$$

Limits set because the integral describes a gaussian surface (in this case a cylinder of radius d) around the wire.

Ampere's law

$$\oint \vec{B} \cdot d\vec{s} = \oint_0^{2\pi d} \frac{\mu_0 I}{2\pi d}\, ds = \frac{\mu_0 I}{2\pi d} \int_0^{2\pi d} ds = \frac{\mu_0 I}{2\pi d}(2\pi d) = \mu_0 I_{enc}$$

The current here is the current that is enclosed by the gaussian surface, so it can written with the "enc" subscript.

$$I_{enc} = \int \vec{J} \cdot d\vec{A}$$

The enclosed current can be described at the integral of the volume current density integrated with respect to the surface area of an surface bounded by the loop.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \int \vec{J} \cdot d\vec{A}$$

Stokes theorem

$$\left(\oint \vec{B} \cdot d\vec{s} = \int (\vec{\nabla} \times \vec{B}) \cdot d\vec{A} \right)$$

$$\int (\vec{\nabla} \times \vec{B}) \cdot dA = \mu_0 \int \vec{J} \cdot d\vec{A}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

From here we can see that we have a non-zero curl of the magnetic field. (B6-3)