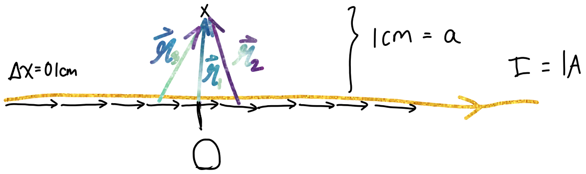
B4-1,2,3

Monday, April 8, 2024 12:46 AM

B4-1	B-Field Calc 2	I can numerically find the magnetic field of any current distribution at an arbitrary location and draw a diagram that shows all quantities used in my calculuation
B4-2*	B-Field Calc 2	Given an arbitrary current distiribution, I can set up an integral expression that describes the magnetic field for a selected location. I can identify appropriate integration limits and which quantities are constant and which are changing. I can identify locations where that integral will be easy to evaluate.
B4-3	B-Field Calc 2	I can compare a calculation of a magnetic field with a rough estimate of a magnetic field to check my calculation



From the right hand rule, we know that the magnetic field is going to be out of the page at point x. In fact, it will be out of the page everywhere above the wire and into the page everywhere below.

$$A \times B = \langle A_y B_z - A_z B_y,\ A_z B_x - A_x B_z,\ A_x B_y - A_y B_x \rangle$$

$$\vec{r}_1 = \langle 0, 1\text{ cm}, 0 \rangle$$

$$\vec{r}_2 = \langle -0.15\text{ cm}, 1\text{ cm}, 0 \rangle$$

$$\vec{r}_3 = \langle 0.15\text{ cm}, 1\text{ cm}, 0 \rangle$$

$$r_1 = 1\text{ cm}$$

$$r_2 = 1.01\text{ cm}$$

$$r_3 = 1.01\text{ cm}$$

$$d\vec{l} = \langle \Delta x, 0, 0 \rangle$$

$$d\vec{l} \times \vec{r}_1 = \langle 0, 0, \Delta x(1\text{ cm}) \rangle = \langle 0, 0, 0.1\text{ cm}^2 \rangle$$

$$d\vec{l} \times \vec{r}_2 = \langle 0, 0, \Delta x(1\text{ cm}) \rangle = \langle 0, 0, 0.1\text{ cm}^2 \rangle$$

$$d\vec{l} \times \vec{r}_3 = \langle 0, 0, \Delta x(1\text{ cm}) \rangle = \langle 0, 0, 0.1\text{ cm}^2 \rangle$$

$$B_1 = (10^{-7}) \frac{(1\text{ A})\langle 0, 0, 0.0001\text{ m}^2 \rangle}{(0.01\text{ m})^3} = \langle 0, 0, 1 \times 10^{-5}\text{ T} \rangle$$

$$B_2 = (10^{-7}) \frac{(1\text{ A})\langle 0, 0, 0.0001\text{ m}^2 \rangle}{(0.0101\text{ m})^3} = \langle 0, 0, 9.7 \times 10^{-6}\text{ T} \rangle$$

$$B_2 = (10^{-7}) \frac{(1\text{ A})\langle 0, 0, 0.0001\text{ m}^2 \rangle}{(0.0101\text{ m})^3} = \langle 0, 0, 9.7 \times 10^{-6}\text{ T} \rangle$$

$$B_{tot} \approx B_1 + B_2 + B_3 = \langle 0, 0, 2.94 \times 10^{-5}\text{ T} \rangle$$

Based on the three segments used, this is approximately the total field. As we can see above, the greater the horizontal component of the distance vector, the smaller the contribution to the magnetic field at the point. Meaning that the line segment with the greatest contribution to the field isdirectly vertical to it.

In order to solve the contribution of every part of the wire, we can assume the wire is infinite and integrate along the path of the wire. We saw when cross multiplying above that the only contribution to the cross product was the vertical y-component (which was 1 cm; we can call this a as above in order to simplify our calculations) and the change along x (delta-x = 0.1 cm) in the z component. Additionally, we have a constant current. This allows us to take the integral form of the Biot-Savart equation below and write it as

$$B = \int \frac{\mu_0}{4\pi} \frac{I\, \vec{dl} \times \vec{r}}{r^{3/2}} \Rightarrow \frac{\mu_0 I a}{4\pi} \int_{-\infty}^{\infty} \frac{dx}{\sqrt{x^2+a^2}}\, \hat{z}$$

(looked this integral up)

$$= \frac{\mu_0 I a}{4\pi} \left(\frac{x}{a^2\sqrt{x^2+a^2}} \right) \Bigg|_{x=-\infty}^{x=\infty} \hat{z}$$

$$= \frac{\mu_0 I}{4\pi} \left(\lim_{x\to\infty} \frac{x}{a\sqrt{x^2+a^2}} - \lim_{x\to-\infty} \frac{x}{a\sqrt{x^2+a^2}} \right) \hat{z}$$

$$\lim_{x\to\infty} \frac{1}{a} \frac{1}{\sqrt{1+\frac{a^2}{x^2}}} = \frac{1}{a} \qquad / \qquad \lim_{x\to-\infty} \frac{1}{a} \frac{1}{\sqrt{1+a^2/x^2}} = -\frac{1}{a}$$

$$= \frac{\mu_0 I}{4\pi} \left[\frac{1}{a} - \left(-\frac{1}{a} \right) \right] \hat{z} = \frac{\mu_0 I}{4\pi} \left(\frac{2}{a} \right) = \frac{\mu_0 I}{2\pi a} \hat{z}$$

$$B = \frac{\mu_0 I}{2\pi a} \hat{z} \quad \text{(for an infinite wire)}$$

Plugging in the values and comparing to the approximate answer above

$$B = \frac{(4\pi \times 10^{-7})(1A)}{2\pi\,(0.01\,m)} \hat{z} = 2 \times 10^{-5}\, T\, \hat{z}$$

$$B = 2\,94 \times 10^{-5}\, T\, \hat{z}$$

$$B_{approx} = 2.94 \times 10^{-5}\ T\ \hat{z}$$

$$\frac{\left| 2.94 \times 10^{-5} - 2 \times 10^{-5} \right|}{2 \times 10^{-5}} \simeq 0.47$$

So our approximation was roughly 47% off from the answer for an infinite wire. However, we did get to the correct order of magnitude. This seems to be a decent approximation for using only three points and relatively large wire segments (i.e. about 10% of the vertical distance).