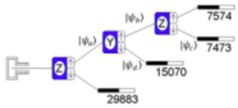
# James Amidei - Homework 2

**3.** **[4 points]** Consider the chained Stern-Gerlach setup shown at right. The thermal source creates a beam of spin-1/2 particles. Three students are discussing the states following some of the measurements:

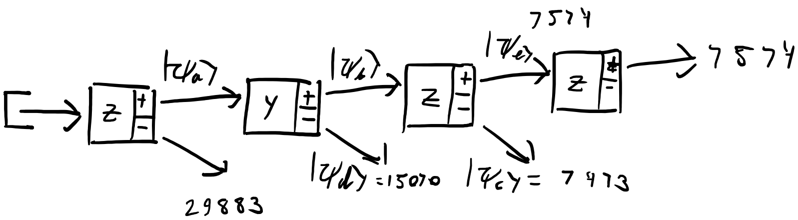



**Student A:** "The y-analyzer (Analyzer 2) changes its incoming particles from being all plus-z to an output $|\psi_b\rangle$, which is a mixture of half of the particles being in the plus-z state and half being in the minus-z state. That's why the farthest right z-analyzer (Analyzer 3) measures half up and half down."

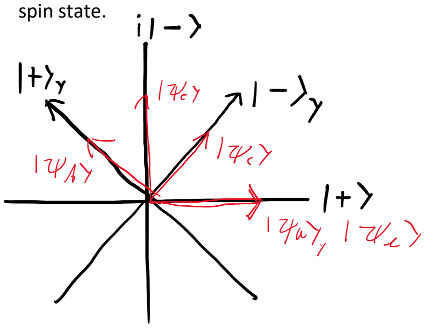
**Student B:** "No, that's not right. All the particles entering Analyzer 3 are in the same state: $|\psi_b\rangle$ is a specific superposition of up-z and down-z states so the particles are in both the up-z and down-z states at the same time."

**Student C:** "There's no way to tell the difference between those arguments, both say that the last analyzer measures 50-50, so this discussion is more about philosophy than physics."

Describe a simple experiment (or experiments) whose results show that student B is correct. Explain clearly how the experimental results not only show that student B is correct, but that students A and C are *incorrect*. Sketch SG-experiment diagrams (like the one above) **and** Hilbert Space diagrams as part of your explanation.




By adding another z analyzer at the end, after the second z analyzer, we are able to show that student A must be incorrect, because if the analyzer changed the input as it measured it like they claim, then we'd expect the output of the final channel to be a 50/50 split of spin up and down in the z direction. The fact that it will return an output with 100% correlation to its input shows us that we're measuring a specific spin state.




For questions 4-6, please use **Dirac notation** (bras and kets, not matrices).

**4.** **[6 points total]** Consider the following vectors:

$$|\psi_1\rangle = \frac{\sqrt{3}}{2}|+\rangle - \frac{i}{2}|-\rangle \qquad and \qquad |\psi_2\rangle = \frac{1}{3}|+\rangle + \frac{2}{3}\underbrace{e^{-i\frac{3\pi}{4}}}_{=1}|-\rangle$$

a) **[1 point]** Could these two vectors be valid descriptions of physical states? Explain.

b) **[3 points]** For *each* state, find a normalized ket $|\phi_n\rangle$ that is orthogonal to it. Use the convention discussed in class, that the coefficient of the $|+\rangle$ ket is real and positive.

c) **[2 points]** Calculate the inner products $\langle\psi_1|\psi_2\rangle$ and $\langle\psi_2|\psi_1\rangle$. How are these numbers related? Is this consistent with the idea of an inner product being a "dot" product? Explain.

a.) $|\psi_1\rangle$: yes, because it is normalized

$$\langle\psi_1|\psi_1\rangle = \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

$|\psi_2\rangle$: No, because it is not normalized

$$\langle\psi_2|\psi_2\rangle = \frac{1}{9} + \frac{4}{9} = \frac{5}{9} \neq 1$$

b.)

b.)

$$|\varphi_n\rangle = a|+\rangle + b|-\rangle$$

$$\langle\varphi_n|\psi_1\rangle = 0 = \langle\varphi_n|\psi_2\rangle$$

$$\langle\varphi_n|\varphi_n\rangle = 1 = a^*a + b^*b = |a^2| + |b^2|$$

$$\langle\varphi_n|\psi_1\rangle = \begin{pmatrix} a^* & b^* \end{pmatrix}\begin{pmatrix} \frac{\sqrt{3}}{2} \\ -\frac{i}{2} \end{pmatrix} = \frac{\sqrt{3}}{2}a^* - \frac{i}{2}b^* = 0$$

$$\frac{\sqrt{3}}{2}a^* = \frac{i}{2}b^*$$

$$a = -\frac{i}{\sqrt{3}}b \leftarrow a^* = \frac{i}{\sqrt{3}}b^*$$

$$|\varphi_n\rangle = -\frac{i}{\sqrt{3}}b|+\rangle + b|-\rangle$$

$$\langle\varphi_n| = \frac{i}{\sqrt{3}}b^*\langle+| + b^*\langle-|$$

$$\langle\varphi_n|\varphi_n\rangle = -\frac{i^2}{3}b^*b + b^*b$$

$$= \left(\frac{1}{3}+1\right)b^*b = \underline{\frac{4}{3}b^*b = 1}$$

$$b^*b = \frac{3}{4}$$

$$a^*a + \frac{3}{4} = 1 \rightarrow a^*a = \frac{1}{4}$$

$$|b|^2 = \frac{3}{4} \qquad |a|^2 = \frac{1}{4}$$

$$|b| = \frac{\sqrt{3}}{2} \qquad |a| = \frac{1}{2}$$

$$\langle\varphi_n|\psi_1\rangle = \begin{pmatrix} a^* & b^* \end{pmatrix}\begin{pmatrix} \frac{\sqrt{3}}{2} \\ -\frac{i}{2} \end{pmatrix} = \frac{\sqrt{3}}{2}a^* - \frac{i}{2}b^* = 0$$

$$\langle \psi_n | \psi_1 \rangle = (a \quad b)\begin{pmatrix} \\ -\frac{i}{2} \end{pmatrix} = \ldots$$

$$\langle \psi_n | \psi_1 \rangle = \begin{pmatrix} \frac{1}{2} i & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} \frac{\sqrt{3}}{2} \\ -i/2 \end{pmatrix}$$

$$a = -\frac{i}{2} \qquad b = \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{2} i \cdot \frac{1}{2} - \frac{\sqrt{3}}{2} \cdot \frac{i}{2}$$

$$\frac{\sqrt{3}}{4} i - \frac{\sqrt{3}\, i}{4} = 0$$

$$\psi_n = -\frac{i}{2} |+\rangle + \frac{\sqrt{3}}{2} |-\rangle$$

$$\psi_2 = \frac{1}{3} |+\rangle + \frac{2}{3} e^{-i\frac{3\pi}{4}} |-\rangle$$

$$\psi_2^* \psi = 5/9 \qquad \frac{2}{3} \cdot \frac{9}{5} = \frac{2}{5} \cdot 3 = \frac{6}{5}$$

$$|\psi_2\rangle = \frac{1/3}{\sqrt{5}/3} |+\rangle + \frac{2/3}{\sqrt{5}/3} e^{-i\frac{3\pi}{4}} |-\rangle$$

$$= \frac{1}{\sqrt{5}} |+\rangle + \frac{2}{\sqrt{5}} e^{-i\frac{3\pi}{4}} |-\rangle$$

$$\langle \psi_n | \psi_2 \rangle = \frac{1}{\sqrt{5}} a^* + \frac{2}{\sqrt{5}} b^* e^{-i\frac{3\pi}{4}} = 0$$

$$a^* = 2 b^* e^{-i\frac{3\pi}{4}}$$

$$\langle \psi_n | = 2 b^* e^{-i\frac{3\pi}{4}} \langle + | + b^* \langle - |$$

$$| \psi_n \rangle = 2 b e^{i\frac{3\pi}{4}} |+\rangle + b |-\rangle$$

$$\langle \psi_n | \psi_n \rangle = 2^2 b^* b + b^* b = 1 = a^* a + b^* b$$

$$\langle \varphi_n | \varphi_n \rangle = 2^2 b^* b + b^* b = 1 = a^* a + b^* b$$

$$b^* b (4+1) = 1$$

$$b^* b = \frac{1}{5}$$

$$|b| = \frac{1}{\sqrt{5}}$$

$$a^* a = |a|^2 = \frac{4}{5} \Rightarrow |a| = \frac{2}{\sqrt{5}}$$

$$\langle \varphi_n | \psi_2 \rangle = \frac{1}{\sqrt{5}} a^* + \frac{2}{\sqrt{5}} b^* e^{-i\frac{3\pi}{4}} = 0$$

$$\langle \psi_2 | \varphi_n \rangle = 0$$

$$\langle \psi_2 | \varphi_n \rangle = \left( \frac{1}{\sqrt{5}} \langle +| + \frac{2}{\sqrt{5}} e^{i\frac{3\pi}{4}} \langle -| \right) \left( \frac{2}{\sqrt{5}} x |+\rangle + \frac{1}{\sqrt{5}} y |-\rangle \right)$$

$$= \frac{2}{5} x + \frac{2}{5} e^{i\frac{3\pi}{4}} y = 0$$

$$x = -e^{i\frac{3\pi}{4}} \qquad y = 1$$

$$= -\frac{2}{5} e^{i\frac{3\pi}{4}} + \frac{2}{5} e^{i\frac{3\pi}{4}} = 0$$

$$\boxed{|\varphi_n\rangle = -\frac{2}{\sqrt{5}} e^{i\frac{3\pi}{4}} |+\rangle + \frac{1}{\sqrt{5}} |-\rangle}$$

c.)

$$\langle \psi_1 | \psi_2 \rangle = \left( \frac{\sqrt{3}}{2} \langle +| + \frac{i}{2} \langle -| \right) \left( \frac{1}{3} |+\rangle + \frac{2}{3} e^{-i\frac{3\pi}{4}} |-\rangle \right)$$

$$= \frac{\sqrt{3}}{6} + \frac{2ie^{-i\frac{3\pi}{4}}}{6}$$

$$= \frac{\sqrt{3}}{6} + \frac{2ie}{6}$$

$$\langle \psi_2 | \psi_1 \rangle = \left( \frac{1}{3} \langle + | + \frac{2}{3} e^{i3\pi/4} \langle - | \right) \left( \frac{\sqrt{3}}{2} | + \rangle - \frac{i}{2} | - \rangle \right)$$

$$= \frac{\sqrt{3}}{6} - \frac{2i}{6} e^{i\frac{3\pi}{4}}$$

$$\langle \psi_1 | \psi_2 \rangle \neq \langle \psi_2 | \psi_1 \rangle$$

But,

$$\langle \psi_1 | \psi_2 \rangle = \left( \langle \psi_2 | \psi_1 \rangle \right)^*$$

Consistent with "dot product" for real numbers. When flipping order with complex numbers, return conjugate.

**5.** **[5 points total]** Consider the following vectors:

$$a: |+\rangle - 2e^{i\frac{\pi}{4}} |-\rangle \qquad and \qquad b: |+\rangle - ie^{-i\pi} |-\rangle$$

a) **[1 point]** Normalize each expression to create states $|\psi_a\rangle$ and $|\psi_b\rangle$. Follow the standard convention for the coefficients of kets.

b) **[1 point]** For **each** state, find the probability of measuring $S_z = -\hbar/2$.

c) **[1.5 points]** For **only** $|\psi_a\rangle$, find the probability of measuring $S_x = -\hbar/2$.

d) **[1.5 points]** For **only** $|\psi_b\rangle$, find the probability of measuring $S_y = -\hbar/2$.

a.)

$$\langle \psi_a | \psi_a \rangle = 1 + \left| 2e^{i\frac{\pi}{4}} \right|^2 = 1 + 4 = 5$$

$$|\psi_a\rangle = \frac{1}{\sqrt{5}} |+\rangle - \frac{2}{\sqrt{5}} e^{i\frac{\pi}{4}} |-\rangle$$

$$\langle \psi_b | \psi_b \rangle = 1 + \left| ie^{-i\pi} \right|^2 = 1 + 1 = 2$$

$$|\psi_b\rangle = \frac{1}{\sqrt{2}} |+\rangle - \frac{i}{\sqrt{2}} e^{-i\pi} |-\rangle$$

b.)

$$\langle - | \psi_a \rangle = -\frac{2}{\sqrt{5}} e^{i\frac{\pi}{4}} \qquad |\langle - | \psi_a \rangle|^2$$

$$P_a = \frac{4}{5} \Rightarrow 80\%$$

$$\langle - | \psi_b \rangle = -\frac{i}{\sqrt{2}} e^{-i\pi} \qquad |\langle - | \psi_b \rangle|^2$$

$$P_b = \frac{1}{2} \Rightarrow 50\%$$

c.)

$$_x\langle - | \psi_a \rangle = \left(\frac{1}{\sqrt{2}}\langle + | - \frac{1}{\sqrt{2}}\langle - |\right)\left(\frac{1}{\sqrt{5}}|+\rangle - \frac{2}{\sqrt{5}} e^{i\pi/4}|-\rangle\right)$$

$$= \frac{1}{\sqrt{10}} + \frac{2}{\sqrt{10}} e^{i\pi/4}$$

$$\underline{P_a} = |_x\langle - | \psi_a \rangle|^2 = \left|\frac{1}{\sqrt{10}} + \frac{2}{\sqrt{10}} e^{i\pi/4}\right|^2 = \frac{1}{10} + \frac{4}{10} = \frac{5}{10}$$

$$\underline{= \frac{1}{2} \Rightarrow 50\%}$$

d.)

$$_y\langle - | \psi_b \rangle = \left(\frac{1}{\sqrt{2}}\langle + | + i\frac{1}{\sqrt{2}}\langle - |\right)\left(\frac{1}{\sqrt{2}}|+\rangle - \frac{i}{\sqrt{2}} e^{-i\pi}|-\rangle\right)$$

$$= \frac{1}{2} + \frac{1}{2} e^{-i\pi}$$

$$P_b = |\langle - | \psi_b \rangle|^2 = \left|\frac{1}{2} + \frac{1}{2} e^{-i\pi}\right|^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \Rightarrow \underline{50\%}$$

$$P_b = |{}_y\langle - | \psi_b \rangle|^2 = \left|\frac{1}{2} + \frac{1}{2} e^{-i\pi}\right|^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \Rightarrow 50\%$$

6. **[3 points]** Consider a general state $|\psi\rangle$. Show that the probability of obtaining a particular spin measurement (in *any* direction) is unaffected by changing the state to $|\phi\rangle = e^{i\beta}|\psi\rangle$. Discuss the physical significance of this result, and how it is related to our standard convention for the coefficient of the $|+\rangle$ ket.

$$P = |{}_y\langle - | \psi \rangle|^2 = \left| \begin{bmatrix} \frac{1}{\sqrt{2}} & +\frac{i}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \right|^2$$

$$= \left| \frac{1}{\sqrt{2}} a + \frac{i}{\sqrt{2}} b \right|^2$$

$$= \frac{1}{2} a a^* + \frac{1}{2} b b^*$$

$$A = a e^{i\beta} \qquad B = b e^{i\beta}$$

$$P = |{}_y\langle - | \varphi \rangle|^2 = \left| \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} \right|^2$$

$$= \frac{1}{2} A A^* + \frac{1}{2} B B^* = \frac{1}{2} a e^{i\beta} a^* e^{-i\beta} + \frac{1}{2} b e^{i\beta} b^{-i\beta}$$

$$= \frac{1}{2} a a^* + \frac{1}{2} b b^*$$

The complex conjugate makes the additional exponential term disappear.

The exponential term we introduced can be thought of as introducing a phase across the entire wavefunction. Since this applies to the whole thing, it does not affect the relative phase of the different components to each other, meaning that the probability is unchanged.

7. **[10 points total]** Matrix representations:

a) **[3 points]** Given:

$$\hat{S}_y|\pm\rangle_y = \pm\frac{\hbar}{2}|\pm\rangle_y \qquad (Experimental\ SG\ results)$$
$$|\pm\rangle_y \doteq \frac{1}{\sqrt{2}}\binom{1}{\pm i} \qquad (derived\ in\ question\ 6)$$

derive the **matrix** representation of $\hat{S}_y$ (in the $z$-basis). As always, show your work clearly.

While we normally work in the $z$-basis, this is a convention - there is nothing truly special about the $z$ direction. So instead, **let's work in the $y$-basis.**

b) **[1 point]** How do you represent the states $|\pm\rangle_y$ and the operator $\hat{S}_y$ in the $y$-basis (using matrix notation)? Explain.

c) **[1.5 points]** How do you represent the state $|+\rangle$ (meaning $|+\rangle_z$) in the $y$-basis? Explain.

d) **[2.5 points]** How do you represent the state $|+\rangle_x$ in the $y$-basis? Explain. *Hint:* Consider your answer to question 6 regarding overall phase.

e) **[2 points]** Calculate the bracket ${}_x\langle+|+\rangle$ using matrices in the y-basis. Show that this is the same result you would get using the z-basis.

a.)

$$\hat{S}_y = \begin{pmatrix} a & b \\ c & d \end{pmatrix}_z \qquad |+\rangle_y = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$$

$$|-\rangle_y = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$$

$$\hat{S}_y|+\rangle_y = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\left(\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}\right) = \frac{1}{\sqrt{2}}\begin{pmatrix} a+ib \\ c+id \end{pmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{pmatrix} a+ib \\ c+id \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} \hbar/2 \\ i\hbar/2 \end{pmatrix}$$

$$a+ib = \frac{\hbar}{2}$$

$$c+id = i\frac{\hbar}{2}$$

$$\hat{S}_y|-\rangle_y = \frac{1}{\sqrt{2}}\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ -i \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} a-ib \\ c-id \end{pmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{pmatrix} a-ib \\ c-id \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} -\hbar/2 \\ \hbar/2\, i \end{pmatrix}$$

$$a-ib = -\frac{\hbar}{2} \qquad a+ib = \frac{\hbar}{2}$$

$$c-id = i\frac{\hbar}{2} \qquad c+id = i\frac{\hbar}{2}$$

$$\begin{array}{rl} & a+ib = \frac{\hbar}{2} \\ + & a-ib = -\frac{\hbar}{2} \\ \hline & 2a+0 = 0 \\ & a = 0 \end{array} \qquad \begin{array}{rl} & c+id = i\frac{\hbar}{2} \\ - & c-id = i\frac{\hbar}{2} \\ \hline & 0+2id = 0 \\ & d = 0 \end{array}$$

$$2a + 0 = 0 \qquad 0 + 2id = 0$$
$$a = 0 \qquad d = 0$$

$$ib = \frac{\hbar}{2} \qquad c = i\frac{\hbar}{2}$$
$$b = -i\frac{\hbar}{2}$$

$$\hat{S}_y = \begin{pmatrix} 0 & -i\frac{\hbar}{2} \\ i\frac{\hbar}{2} & 0 \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

b.) Representing y in the y basis ought to look identical to z in the z basis.

$$\hat{S}_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \hat{S}_y = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$
in z $\qquad$ in y

c.)

$${}_y\langle +|+\rangle_y = \left(\frac{1}{\sqrt{2}}\langle +| - \frac{i}{\sqrt{2}}\langle -|\right)\left(|+\rangle_y\right)$$

$${}_y\langle -|+\rangle_y = \left(\frac{1}{\sqrt{2}}\langle +| + \frac{i}{\sqrt{2}}\langle -|\right)\left(|+\rangle_y\right)$$

$$|+\rangle \xrightarrow{\text{in } y} \begin{pmatrix} {}_y\langle +|+\rangle_y \\ {}_y\langle -|+\rangle_y \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

d $\quad |+\rangle_x$ in y

$$|+\rangle_x = \frac{1}{\sqrt{2}}\left(|+\rangle_y + |-\rangle_y\right)$$

$${}_y\langle +|+\rangle_x = \left(\frac{1}{\sqrt{2}}\langle +| - \frac{i}{\sqrt{2}}\langle -|\right)\left(\frac{1}{\sqrt{2}}|+\rangle_y + \frac{1}{\sqrt{2}}|-\rangle_y\right)$$

$${}_y\langle +|+\rangle_x = \left(\frac{1}{\sqrt{2}}\langle +| - \frac{i}{\sqrt{2}}\langle -|\right)\left(\frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle\right)$$

$$= \frac{1}{2} - \frac{i}{2} = \frac{1-i}{2}$$

$${}_y\langle -|+\rangle_x = \left(\frac{1}{\sqrt{2}}\langle +| + \frac{i}{\sqrt{2}}\langle -|\right)\left(\frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle\right)$$

$$= \frac{1}{2} + \frac{i}{2} = \frac{1+i}{2}$$

*in y*

$$|+\rangle_x \to \begin{pmatrix} {}_x\langle +|+\rangle_x \\ {}_y\langle -|+\rangle_x \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1-i \\ 1+i \end{pmatrix}$$

c.)

*in y basis*

$${}_x\langle +|+\rangle = \frac{1}{2}\begin{pmatrix} 1-i & 1+i \end{pmatrix}\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

$$= \frac{1}{2}\left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)$$

$$= \frac{1}{2}\left(\frac{2}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}$$

*in z basis*

$${}_x\langle +|+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}(1+0) = \frac{1}{\sqrt{2}}$$

**8.** **[6 points total]** Suppose we act the $\hat{S}_z$ operator on the general state $|\psi\rangle = a\,|+\rangle + b\,|-\rangle$.

a) **[1 point]** What kind of *object* (bra, ket, operator, number, etc.) is $\hat{S}_z\,|\psi\rangle$?

b) **[1.5 points]** Determine an expression for $\hat{S}_z\,|\psi\rangle$ using *matrix notation*. Is your result an *eigenequation*?

c) **[2 points]** Consider the following discussion between three students:

**Student A:** "I think this formula tells you the experimental results of measuring the z-component of spin on some general input state."

**Student B:** "I disagree. I think it tells you what the resulting quantum **state** is after you measure the z-component of spin. That's different from an experimental result, which would be a number (with dimensions), not a state."

**Student C:** "I think you're both over-interpreting this expression. It's a mathematical equation, it doesn't mean anything physically."

With which student, if any, do you agree? Discuss *both* what you agree with and what you don't.

d) **[1.5 points]** Based on your answers above, can you explain why I **strongly** disagree with the claim that $\hat{O}|\psi\rangle$ means "a measurement of $\hat{O}$ on the state $|\psi\rangle$"?

a.) An operator acting on a ket returns a ket.

b.)

$$\hat{S}_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad |\psi\rangle = a|+\rangle + b|-\rangle = \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\hat{S}_z|\psi\rangle = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} a \\ -b \end{pmatrix}$$

This not an eigenequation because the relative sign between the two components changes.

c.) Student B is correct because applying an operator to a wave function simply returns another wave function (which in turn describes a state) rather than an eigenvalue from the wave function, as student function (which in turn describes a state) rather than an eigenvalue from the wave function, as student A suggests. Student C seems confused as to the difference between taking an measurement and describing a state with the wave function, which contains multiple possible measurement values.

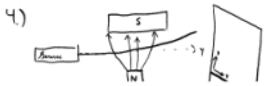
d.) For the same reason touched on above, an operator acting on a state/wave function produces another state. Each state contains multiple possible values which can be measured, whereas a measurement causes the wave function/state to collapse to one of its possible eigenvalues.

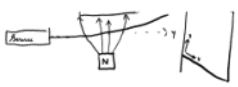
After each homework, you should review both your answers and the solutions to make sure that you understand all the questions and how to answer them correctly. Going forward, on each homework you will receive credit for reviewing your previous homework. Please review your homework submission and the solution set and reach out to me if you have any questions or concerns.

1. **[1.5 points]** Going forward, part of each homework will be submitting a correction from the previous week. Please select a problem from the prior homework for which you had the wrong answer and:
   i. Identify the question number you are correcting
   ii. State/copy your original wrong answer
   iii. Explain where your original reasoning was incorrect, the correct reasoning for the problem, and how it leads to the right answer.

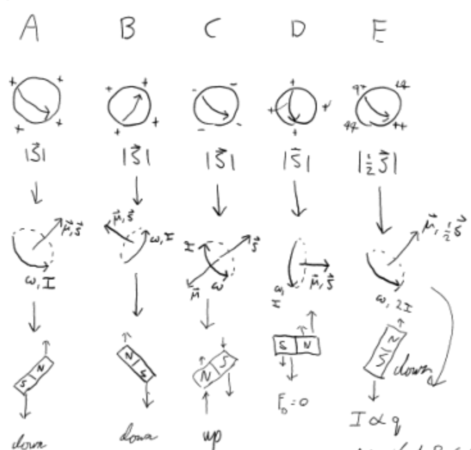
   If you got all the answers correct, that's great! Please review your work anyway and discuss which problem you found the most interesting, enjoyable, and/or helpful and why.

4.)




a.)




$I \propto q$

so, if A, B, C, D have the same charge and current $I$, then E has current $2I$

$F \cong \mu_z \frac{\partial B}{\partial z}$

in direction of $\vec{\mu}$

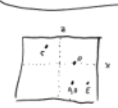
$\vec{\mu} = I\vec{A} = \frac{q}{2\pi r/v} \pi r^2 \hat{\mu} = \frac{q v r \hat{\mu}}{2} = g\frac{q}{2m}\vec{S}$

$\vec{F} = g\frac{q}{2m}\vec{S}\,\nabla\vec{B} \Rightarrow |\vec{F}| \propto q|\vec{S}||\nabla\vec{B}|$

$F_z \cong g\frac{q}{2m} S_z \frac{\partial B}{\partial z}$ $\quad |F_E| \propto 2q|\frac{1}{2}\vec{S}||\nabla\vec{B}| = q|\vec{S}||\nabla\vec{B}|$

$|\vec{F}_A| = |\vec{F}_B| = |\vec{F}_C| = |\vec{F}_E| > |\vec{F}_D| = 0$ on with sign $F_C > F_D = 0 > F_A = F_B = F_E$

b.)




$\vec{F} = q\vec{v} \times \vec{B}$ ← Lorentz Force

Here I think I made a dumb mistake when listing the values of the forces relative to each other. I listed them exactly opposite of each other. The correct order is

$$F_A = F_D = F_E > F_D = 0 > F_C$$

I made this mistake because I thought a downward deflection equaled a negative value for the force. I forgot that the force is directly proportional to the gradient of the magnetic field, which pointed in the -z direction.

**2.** **[1.5 points]** Select another problem from the prior homework for which you had the wrong answer and correct it as above. If you got all the answers correct (or have nothing left to correct) that's great! Please review your work anyway and discuss which problem you found the *least* interesting, enjoyable, and/or helpful and why.

7.) $|\psi\rangle = \frac{2}{\sqrt{5}}|+\rangle_x - \frac{1}{\sqrt{5}}|-\rangle_x$

a.)

$P_{x+} = \frac{1}{2} = 0.5$

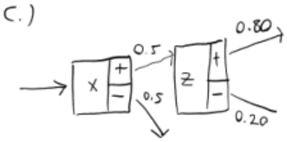
$P_{x-} = \frac{1}{2} = 0.5$

b.)

$P_{z+} = |{}_x\langle +|\psi\rangle|^2 = \frac{4}{5} = 0.80$

$P_{z-} = |{}_x\langle -|\psi\rangle|^2 = \frac{1}{5} = 0.20$

c.)



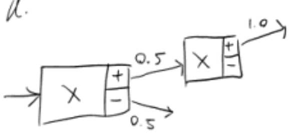

50% chance of $S_x = +\frac{\hbar}{2}$

80% chance of $S_z = +\frac{\hbar}{2}$

&

20% chance of $S_z = -\frac{\hbar}{2}$

d.




50% chance of $S_x = +\frac{\hbar}{2}$

then

100% chance of $S_x = +\frac{\hbar}{2}$ from second analyzer

In part A, I was reckless and forgot that the state was given to us in the x basis. So the probability would be the absolute value squared of the amplitudes on both of the components.

$$P_+ = \frac{4}{5} \quad P_- = \frac{1}{5}$$

In part B, we would use the definition of spin-up and spin-down for x in the z basis like so

$$|\psi\rangle = \frac{2}{\sqrt{5}}\left(\frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle\right) - \frac{1}{\sqrt{5}}\left(\frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle\right)$$

$$= \frac{2}{\sqrt{10}}|+\rangle + \frac{2}{\sqrt{10}}|-\rangle - \frac{1}{\sqrt{10}}|+\rangle + \frac{1}{\sqrt{10}}|-\rangle$$

$$= \frac{1}{\sqrt{10}}|+\rangle + \frac{3}{\sqrt{10}}|-\rangle$$

$$P_{+z} = \frac{1}{10} \qquad P_{-z} = \frac{9}{10}$$

For part C, since our particle is first in the x basis, we'd expect a 50/50 chance of either spin-up or spin-down in the z direction.

For part D, this answer is basically correct.

I would say the reason I got this problem wrong, as well as the one before it had to do with not paying close enough attention to what was given in the problem. I should have understood that the state was given to us in the x basis, which would have made the calculations easier and more correct.