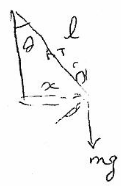
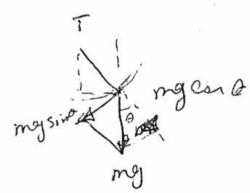
chapter (7)

2.13





$ma = F$

$\cancel{m}\, l\ddot{\theta} = -\cancel{m}g\sin\theta$

$T = mg\cos\theta$

where $\ddot{\theta} = \frac{d^2\theta}{dt^2}$

$l\frac{d^2\theta}{dt^2} + g\sin\theta = 0$

For small θ, $\sin\theta \simeq \theta$

$l\frac{d^2\theta}{dt^2} + g\theta = 0$

$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$ where $\omega = \sqrt{\frac{g}{l}}$

Simple harmonic oscillator

$\theta = A\sin\omega t + B\cos\omega t$

if at $t=0$ $\theta = 0$ Then $0 = 0 + B \Rightarrow B = 0$

$\theta = A\sin\omega t$ $\boxed{A = \text{Max of } \theta}$ and $\boxed{\theta(0) = A}$

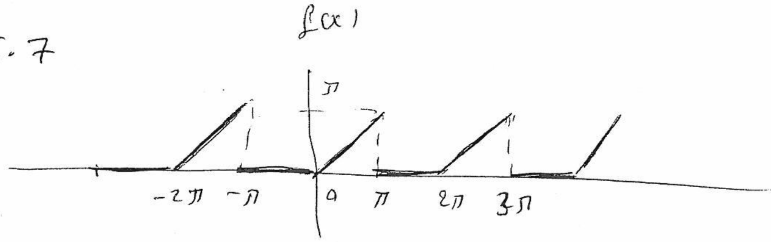
$\begin{cases} x = l\sin\theta \\ \theta \simeq \sin\theta \end{cases} \Rightarrow x = l\theta$

$x = l(A\sin\omega t)$

$\boxed{x = Al\sin\omega t}$

Chapter (7)

5.7



$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$$

$$a_n = \frac{1}{\pi}\int_0^{\pi} x \cos nx \, dx =$$

$u = x \quad du = \cos nx \, dx$

$dv = dx \quad u = \frac{\sin nx}{n}$

$$= \frac{1}{\pi}\left[x \frac{\sin nx}{n}\Big|_0^{\pi} - \frac{1}{n}\int \sin nx \, dx \right]$$

$$= \frac{1}{\pi}\left[x \frac{\sin nx}{n} + \frac{1}{n^2}\cos nx \right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{nx \sin nx + \cos nx}{n^2} \right]_0^{\pi} = \frac{\cos n\pi - 1}{n^2\pi} = \begin{cases} 0 & \text{even } n \\ & n \neq 0 \\ \frac{-2}{n^2\pi} & \text{odd } n. \end{cases}$$

$$a_0 = \frac{1}{\pi}\int_0^{\pi} x \, dx = \frac{\pi}{2}$$

$$b_n = \frac{1}{\pi}\int_0^{\pi} x \sin nx \, dx = \frac{1}{\pi}\left[\frac{\sin nx - nx \cos nx}{n^2} \right]_0^{\pi}$$

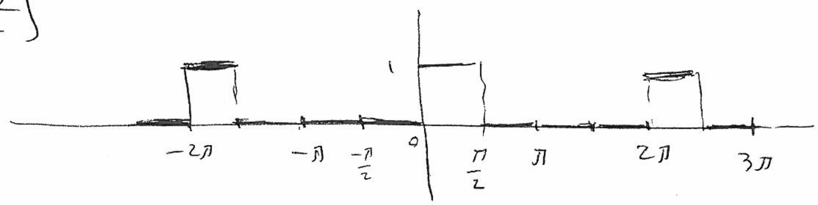
$$= -\frac{\cos n\pi}{n} = \begin{cases} \frac{-1}{n} & \text{even } n \\ \frac{1}{n} & \text{odd } n \end{cases}$$

$$f(x) = \frac{\pi}{4} - \frac{2}{\pi}\left[\cos x + \frac{\cos 3x}{3^2} + \cdots \right]$$

$$+ \left[\sin x - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \cdots \right]$$

Chapter 7

6.2



at $x = \pm\pi$, $x = -\pi/2$ $f(x)$ is Continuous so Fourier Series Converges to value $f(x) = 0$.

at $x = \pm 2\pi$, $x = 0$, $x = \pi/2$ $f(x)$ is not Continuous so Fourier Series Converges to midpoint ~~$(1/2)$~~ of the Jump, $\frac{1}{2}$.

Chapter (7)

$x \to dx$

$e^{ax}dx \to \frac{e^{ax}}{a}$

7.7

$$\int x e^{ax} = \frac{x e^{ax}}{a} - \frac{1}{a}\int e^{ax} = \frac{x e^{ax}}{a} - \frac{e^{ax}}{a^2} = \frac{a x e^{ax} - e^{ax}}{a^2} = \frac{e^{ax}}{a^2}(ax-1)$$

$$C_0 = \frac{1}{2\pi}\int_0^{\pi} x\,dx = \frac{1}{2\pi}\left(\frac{\pi^2}{2}\right) = \frac{\pi}{4}$$

$$C_n = \frac{1}{2\pi}\int_0^{\pi} x e^{-inx}\,dx = \frac{1}{2\pi}\,\frac{e^{-inx}}{(-in)^2}(-nx-1)\Big|_0^{\pi}$$

$$= \frac{1}{-2\pi n^2}\left[e^{-in\pi}(-in\pi-1)+1\right] = \begin{cases} \frac{i}{2n} & \text{even } n \neq 0 \\ -\left(\frac{1}{\pi n^2} + \frac{i}{2n}\right) & \text{odd } n \end{cases}$$

$$f(x) = \frac{\pi}{4} - \left(\frac{1}{\pi} + \frac{i}{2}\right)e^{ix} - \left(\frac{1}{\pi} - \frac{i}{2}\right)e^{-ix}$$
$$+ \frac{i}{4}\left[e^{2ix} - e^{-2ix}\right] - \left(\frac{1}{9\pi} + \frac{i}{6}\right)e^{3ix} - \left(\frac{1}{9\pi} - \frac{i}{6}\right)e^{-3ix} + \cdots$$

Note that $C_{-n} = \overline{C_n}$.

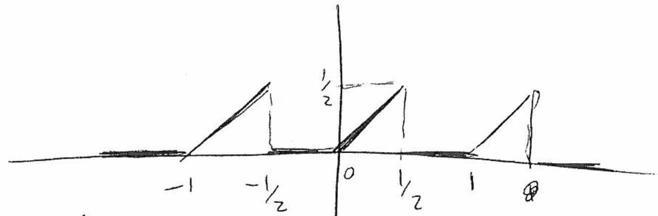
use: $\sin nx = \frac{e^{inx} - e^{-inx}}{2i}$, $\cos nx = \frac{e^{inx} + e^{-inx}}{2}$

Then

$$f(x) = \frac{\pi}{4} - \frac{2}{\pi}\sum_{n=\text{odd}} \frac{1}{n^2}\cos nx - \sum_{1}^{\infty} \frac{(-1)^n}{n}\sin nx$$

chapter 7

$\boxed{8.19}$



$$f(x)=\begin{cases} 0 & -\frac{1}{2}<x<0 \\ x & 0<x<\frac{1}{2} \end{cases}$$

$l=\frac{1}{2}$

$$a_0 = 2\int_0^{1/2} x\,dx = \frac{1}{4}$$

$$a_n = 2\int_0^{1/2} x \cos\frac{n\pi x}{(1/2)} = 2\int_0^{1/2} x\cos 2n\pi x\,dx$$

$$= 2\left(\frac{1}{2n\pi}\right)^2 \Big[(\cos 2n\pi x + 2n\pi x \sin 2n\pi x)\Big]_0^{1/2}$$

$$= \frac{1}{2n^2\pi^2}(\cos n\pi - 1) = \begin{cases} 0 & \text{even } n \neq 0 \\ -\frac{1}{n^2\pi^2} & \text{odd } n \end{cases}$$

$$b_n = 2\int_0^{1/2} x \sin 2n\pi x\,dx$$

$$= 2\left(\frac{1}{2n\pi}\right)^2 \Big[\sin 2n\pi x - 2n\pi x\cos 2n\pi x\Big]_0^{1/2}$$

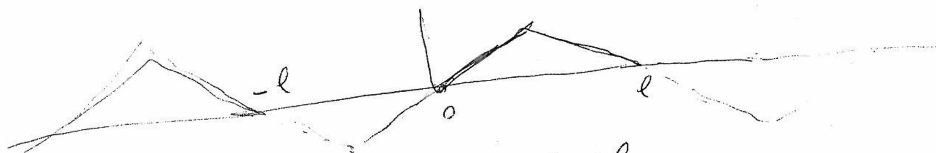
$$= \frac{1}{2n^2\pi^2}(-n\pi\cos n\pi) = -\frac{1}{2n\pi}(-1)^n$$

$$f(x) = \frac{1}{8} - \frac{1}{\pi^2}\sum_{\substack{1 \\ n\,\text{odd}}}^{\infty} \frac{1}{n^2}\cos 2n\pi x - \frac{1}{2\pi}\sum_{1}^{\infty}\frac{(-1)^n}{n}\sin 2n\pi x$$

chapter ~~8~~ 7

9.23

Fourier Sine Series: (~~odd~~ extend as odd function on $-l, l$)



$$f(x,0) = \begin{cases} \frac{2hx}{l} & 0 < x < \frac{l}{2} \\ \frac{2h}{l}(l-x) & \frac{l}{2} < x < l \end{cases}$$

$$b_n = \frac{2}{l}\left[\int_0^{l/2} \frac{2h}{l} x \sin\frac{n\pi x}{l}\,dx + \int_{l/2}^{l} \frac{2h}{l}(l-x)\sin\frac{n\pi x}{l}\,dx \right]$$

$$= \frac{4h}{l^2}\frac{l^2}{n^2\pi^2}\left(\sin\frac{n\pi x}{l} - \frac{n\pi x}{l}\cos\frac{n\pi x}{l}\right)\Big]_0^{l/2} - \frac{4h}{l}\frac{l}{n\pi}\cos\frac{n\pi x}{l}\Big|_{l/2}^{l}$$

$$- \frac{4h}{l^2}\frac{l^2}{n^2\pi^2}\left(\sin\frac{n\pi x}{l} - \frac{n\pi x}{l}\cos\frac{n\pi x}{l}\right)\Big]_{l/2}^{l}$$

$$= \frac{4h}{n^2\pi^2}\left[\sin\frac{n\pi}{2} - \frac{n\pi}{2}\cos\frac{n\pi}{2}\right] - \frac{4h}{n\pi}\left(\cos n\pi - \cos\frac{n\pi}{2}\right)$$

$$- \frac{4h}{n^2\pi^2}\left[\sin n\pi - n\pi\cos n\pi - \sin\frac{n\pi}{2} + \frac{n\pi}{2}\cos\frac{n\pi}{2}\right]$$

$$= \frac{8h}{n^2\pi^2}\sin\frac{n\pi}{2} = \frac{8h}{n^2\pi^2}\begin{cases} 1 & n = 1+4k \\ 0 & n \text{ even} \\ -1 & n = 3+4k \end{cases}$$

$$f(x) = \frac{8h}{\pi^2}\left(\sin\frac{\pi x}{l} - \frac{1}{9}\sin\frac{3\pi x}{l} + \frac{1}{25}\sin\frac{5\pi x}{l} - \cdots\right)$$

Chapter 7

11.7

$$\text{Average of } |f(x)|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi}(1+x)^2\,dx = \frac{1}{2\pi}\int_{-\pi}^{\pi}(1+2x+x^2)\,dx$$

$$= \frac{1}{2\pi}\left(2\pi+0+\frac{2\pi^3}{3}\right) = 1+\frac{\pi^2}{3}$$

Fourier Series Prob 5.8 :

$$f(x) = 1+\sum \frac{2}{n}(-1)^{n+1}\sin nx$$

Parseval's Theorem

$$1+\frac{\pi^3}{3} = 1+\frac{1}{2}\sum_{1}^{\infty}\frac{4}{n^2} \Rightarrow \boxed{\frac{1}{n^2} = \frac{\pi^2}{6}}$$

The Same as Example 2 Page 333

Problem (8)
Find the Fourier cosine transform of

$$F(x) = \begin{cases} 1 & 0 < x < \frac{\pi}{2} \\ 0 & otherwise \end{cases}$$

Chapter ~~4.5~~ 4.16

Problem 8 Find the Fourier cosine transform of $f(x) = \begin{cases} 1 & 0 < x < \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases}$

$$g_c(\alpha) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \cos \alpha x \, dx = \sqrt{\frac{2}{\pi}} \int_0^{\pi/2} \cos \alpha x \, dx$$

Prob (11) $g(\alpha) = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} e^{-i\alpha x} \cos x \, dx$

~~$f(x) = 0$~~ The limit of integral is $-\pi/2$ to $\pi/2$ because $f(x) = 0$ for $|x| > \pi/2$

$$g(\alpha) = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} e^{-i\alpha x} \frac{e^{ix} + e^{-ix}}{2} \, dx$$

$$= \frac{1}{4\pi} \left[\frac{e^{-i(\alpha+1)x}}{-i(\alpha+1)} + \frac{e^{-i(\alpha-1)x}}{-i(\alpha-1)} \right]_{-\pi/2}^{\pi/2}$$

$$e^{i\pi/2} = i \Rightarrow e^{i(\alpha+1)\pi/2} = i e^{i\alpha\pi/2}$$

$$g(\alpha) = \frac{i e^{i\alpha\pi/2} - (-i) e^{-i\alpha\pi/2}}{4\pi i(\alpha+1)} + \frac{-i e^{-\alpha\pi/2} - i e^{-i\alpha\pi/2}}{4\pi i(\alpha-1)}.$$

~~Chapter 15~~

~~4.16~~ Contin.

$$g(\alpha) = \frac{\cos(\alpha\pi/2)}{2\pi}\left(\frac{1}{\alpha+1} - \frac{1}{\alpha-1}\right) = \frac{\cos(\alpha\pi/2)}{\pi(1-\alpha^2)}$$

$$\cancel{f(x) = \frac{1}{\pi}\int_{-\infty}^{\infty} \cos}$$

$$f(x) = \frac{1}{\pi}\int_{-\infty}^{\infty} g(\alpha)\, e^{i\alpha x}\, d\alpha$$

$$= \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\cos(\alpha\pi/2)}{1-\alpha^2}\, e^{i\alpha x}\, d\alpha$$

$$f(x) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\cos(\alpha\pi/2)}{1-\alpha^2}\,(\cos\alpha x + i\sin\alpha x)\, d\alpha$$

$g(\alpha)$ is even $\quad \int_{-\infty}^{\infty} \underbrace{g(\alpha)\sin\alpha x}_{\text{odd}}\, d\alpha = 0$

$\int_{-\infty}^{\infty} g(\alpha)\cos\alpha x\, d\alpha$ is even $= 2\int_0^{\infty} g(\alpha)\cos\alpha x\, d\alpha$

$$f(x) = \frac{2}{\pi}\int_0^{\infty} \frac{\cos(\alpha\pi/2)}{1-\alpha^2}\cos\alpha x\, dx$$

The same as f_c Eq. (I)