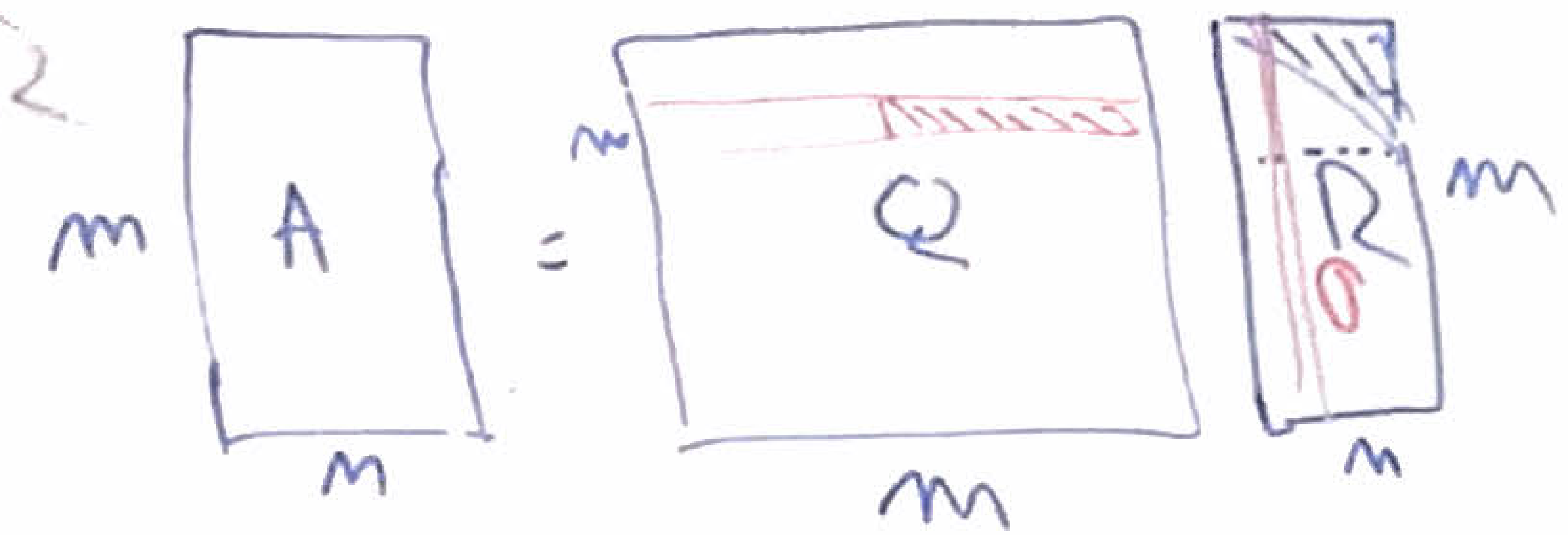
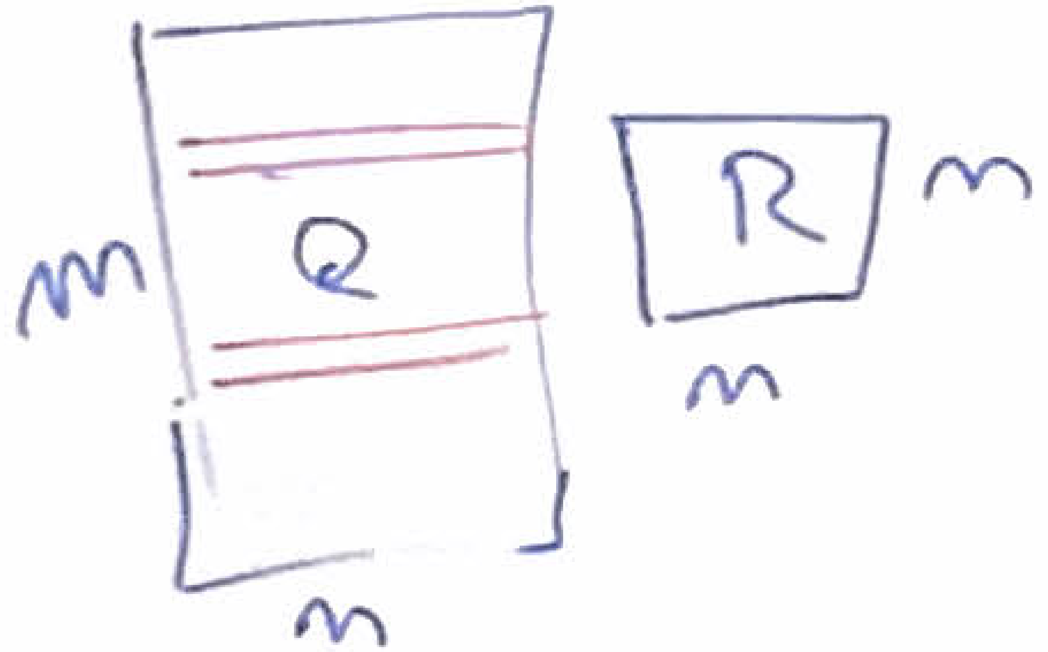
QR

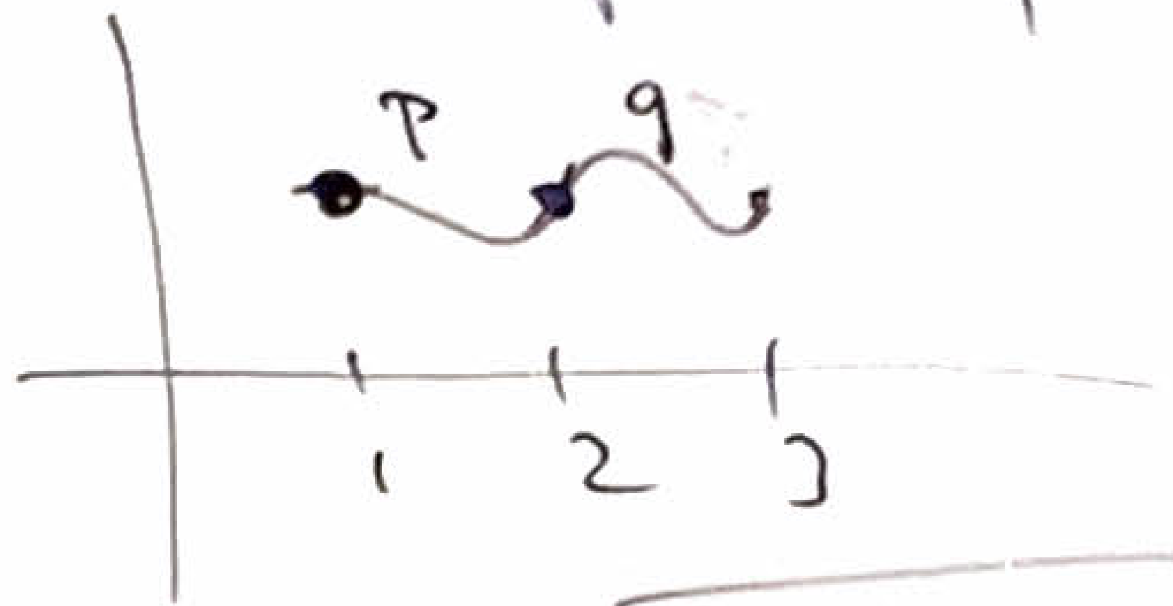


"economy QR"



Piecewise Hermite interpolation

x	f	f'
1	1	0
2	1	1
3	1	1



$p \in P_3$: $p(1)=1$ $p'(1)=0$ $\boxed{p(2)=1 \quad p'(2)=1}$

$p(x) = a_3(x-1)^3 + a_2(x-1)^2 + a_1(x-1) + a_0$ at $x=1$

$p_1(1) = \boxed{a_0 = 1}$

$p_1'(1) = 3a_3(x-1)^2 + 2a_2(x-1) + a_1 = \boxed{a_1 = 0}$ at $x=1$

$p_1(2) = a_3 1^3 + a_2 1^2 + \underbrace{a_1 1}_{=0} + \underbrace{a_0}_{=1} = 1$

$p'(2) = 3a_3 \underbrace{(x-1)^2}_{=1} + 2a_2 \underbrace{(x-1)}_{1} + \underbrace{a_1}_{=0} = 1$

$a_2 + a_3 = 0 \quad) \cdot -2$

$2a_2 + 3a_3 = 1$

$0 + a_3 = 1$

$a_2 = -1$

$a_3 = 1$

Natural cubic spline

x	f
1	1
2	1
3	1

f', f'' continuous

$f''(1) = f''(3) = 0$

$f(x) = 1$

because: satisfies conditions
natural cubic spline is unique

$p(x) = 1(x-1)^3 - (x-1)^2 + 0(x-1) + 1$

Clamped cubic spline

x	f	f'	f''
0	0	0	any
1	1	continuous	continuous
2	0	0	any

$$f(x) = a_3(x-1)^3 + a_2(x-1)^2 + a_1(x-1) + a_0 \quad x \in [0,1]$$

$$f'(x) = 3a_3(x-1)^2 + 2a_2(x-1) + a_1$$

$$f''(x) = 6a_3(x-1) + 2a_2$$

$$f(x) = b_3(x-1)^3 + b_2(x-1)^2 + b_1(x-1) + b_0 \quad x \in [1,2]$$

$$f'(x) = 3b_3(x-1)^2 + 2b_2(x-1) + b_1$$

$$f(0)=0 \Rightarrow -a_3 + a_2 - a_1 + a_0 = 0$$

$$f(2)=0 \Rightarrow b_3 + b_2 + b_1 + b_0 = 0$$

$$f(1)=1 \Rightarrow a_0 = 1 \quad b_0 = 1$$

$$f' \text{ cont} \Rightarrow a_1 = b_1$$

$$f'' \text{ cont} \Rightarrow a_2 = b_2$$

$$f'(0)=0 \Rightarrow 3a_3 - 2a_2 + a_1 = 0$$

$$f'(2)=0 \Rightarrow 3b_3 + 2b_2 + b_1 = 0$$

7a Least squares for $y = ax + b$ given

x	1	2	3
y	1	0	1

$$|(a1 + b) - 1|^2 + |(a2 + b) - 0|^2 + |(a3 + b) - 1|^2 \to \min$$

$$\Bigg\| \underbrace{\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a \\ b \end{bmatrix}}_{u} - \underbrace{\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}}_{f} \Bigg\|^2 \to \min_{a,b}$$

$$\|Au - f\|^2 \to \min$$

normal equation

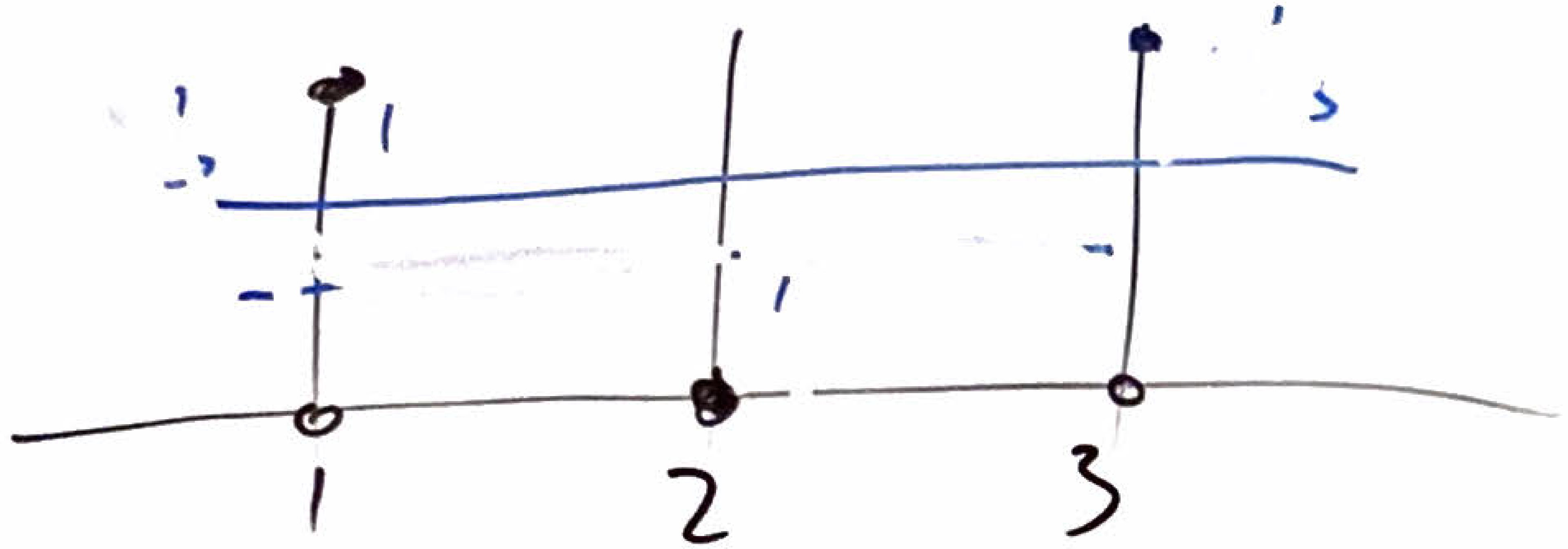
$$A^T(Au - f) = 0$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$-\tfrac{1}{2} \left(\begin{bmatrix} 14 & 6 \\ 6 & 3 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \right.$$

$$14a + 6b = 4$$
$$6a + 3b = 2 \quad \cdot\left(-\tfrac{1}{2}\right)$$

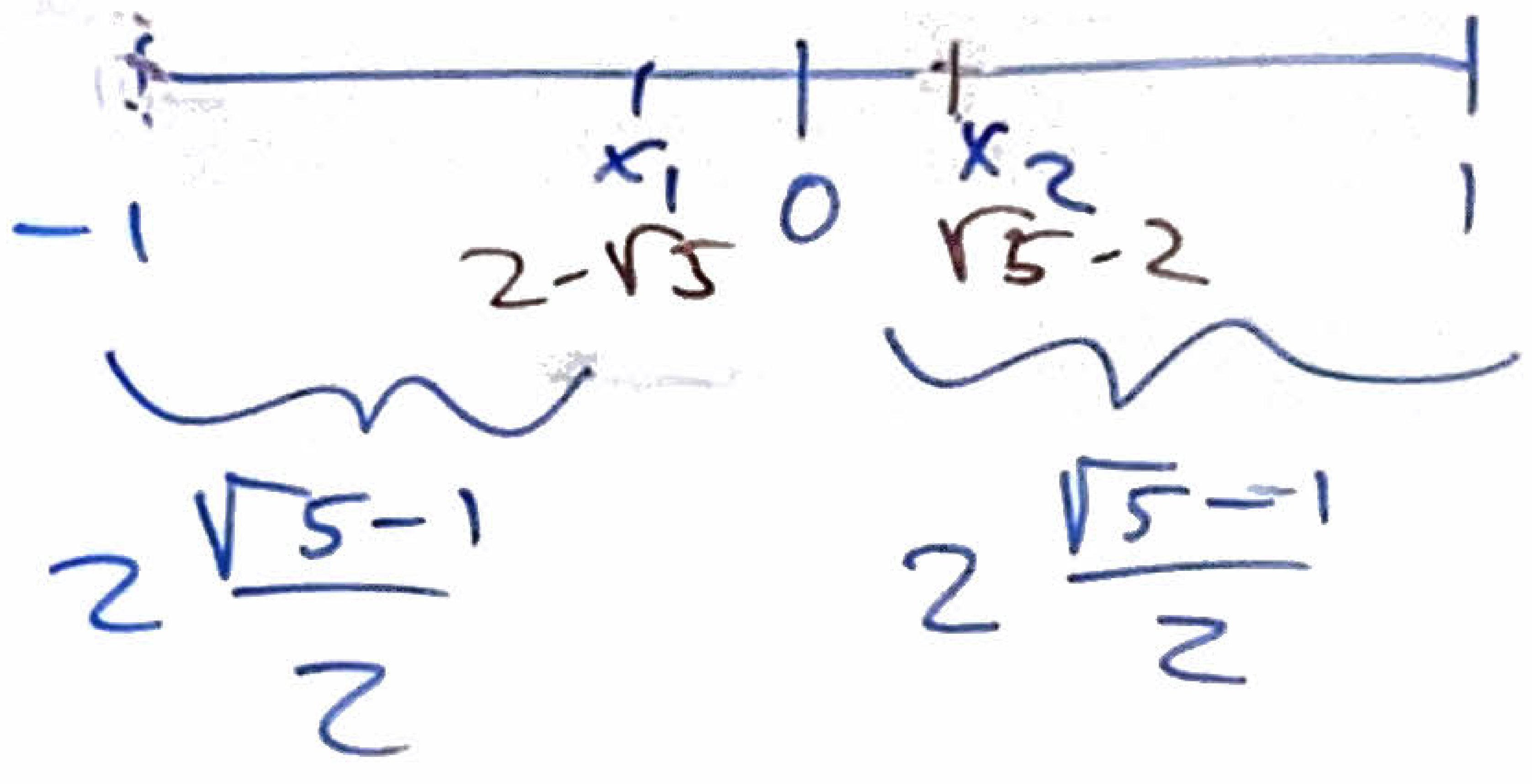
$$14a + 6b = 4 \Rightarrow b = \frac{2}{3}$$
$$-1a + 0b = 0 \Rightarrow a = 0$$



8b

$$[a, b] = [-1, 1]$$

$$x_1 = a + \frac{\sqrt{5}-1}{2}(b-a) \qquad x_2 = a + \frac{\sqrt{5}+1}{2}(b-a)$$



$$x_1 = -1 + \frac{3-\sqrt{5}}{2}2 = 2-\sqrt{5}$$

$$x_2 = 1 + \frac{3-\sqrt{5}}{2}(-2)$$

$$= 1 - (3-\sqrt{5}) = -2 + \sqrt{5}$$

$$x_2 = a + \frac{\sqrt{5}-1}{2}(b-a)$$

$$= -1 + \frac{\sqrt{5}-1}{2}2 = -2 + \sqrt{5}$$