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Taylor-Couette Experiment Paragraphs

Main Idea and Introduction

The Navier-Stokes equation does a decent job of describing the dynamics of Newtonian fluids. The equation itself follows directly the continuity equation (i.e. conservation of mass) and Newton's second law. On this slide, on the left-hand side, there is the convection derivative of the velocity (i.e. the derivative with respect to a coordinate system which tells us that the particle moves to a new location with a different velocity). On the right-hand side, we can see the sum of the pressure force, the viscous force, and the gravitational force, all divided by the density of the fluid. Take note of the no-slip boundary condition, which tells us that along the surface of one of the two cylinders the fluid is stationary relative to the cylindrical surface. This is due to the fact that the fluid "sticks" to the surface of a given cylinder. This creates a velocity gradient across the fluid. Additionally, we can see that a solution exists, where each constant depends upon the radii and angular velocities of each cylinder.

In the case that the inner cylinder is in motion while the outer is motionless, as was the case in our experiment, we should also consider the centrifugal force due to the azimuthal (radial) velocity. This is the result of the inertia related to the centripetal acceleration. In most cases, the centrifugal force is balanced by the radial pressure force. If the fluid is perturbed inward or outward from its original radius, it reaches a region where the local restoring force due to the pressure is slightly less or more than the inward or outward inertia of the particle, which is dependent on the initial position of the particle due to the conservation of angular momentum. What this means is that a perturbed fluid particle will continue in the direction of its perturbation. At low angular velocities, any perturbations are damped by viscosity. However, since pressure force decrease with increasing radius, as well as decreasing angular velocity, when the velocity gradient is made more extreme by high inner rotational speed, this the pressure force Is no longer able to suppress these perturbations. In this case, fluid particles that are perturbed outward continue till they hit the outer cylinder wall and begin to move inward and then back again. The same happens for particles that are perturbed inward. This is why we get the toroidal shape of the Taylor vortices.

Also of note is that this critical angular velocity is referred to in terms of the dimensionless Reynolds number, which is proportional to the Angular velocity, radius, difference between the radii of the two cylinders divided kinematic viscosity. So, we should understand this critical angular velocity in terms of a critical Reynold's number, which describes the transition between laminar to turbulent flow.

Instrumentation and Data Collection

For the actual experiment, a water-glycerol mixture (with a ratio of about 0.81) was placed in between two concentric, differently rotating cylinders of the listed dimensions. Each cylinder was connected to a motor which themselves were connected to a computer where their speed could be controlled via a python script.

In order to find the critical Reynolds number for this experiment, we started at the predicted value given to us in the pre-lab write up, i.e. 75.3439. While this Reynolds number certainly produced strong Tayor vortices, we varied also checked several other Reynolds numbers below it in order to see where these vortices first began to form.

By starting from a Reynolds number of 65 and working our way back up to a Reynolds number of 75 by steps of 2.5, we eventually found our best candidate for the critical Reynolds number between 67.5 and 70. In that range we were able to narrow it down to above 68 and found the critical Reynolds number at 68.46 while performing a separate run after a cool down period. However, it's worth noting that when we went back to verify this critical Reynolds number the Taylor vortices appeared extremely faint but became significant around 69. So we can probably only say that our critical Reynolds number appears between 67.5 and 70.

Data Analysis

Since the main point of this experiment was to identify the critical Reynolds number, there wasn't much more to do. However, we decided to go about calculating the critical wavenumber and corresponding wavelength of the Taylor vortices to verify that it is in fact approximately two times the difference between the two cylindrical radii.

We did this by taking a photo of the Couette flow at the critical Reynolds number and counting the number of pixels in the image, cropping it, and performing a Fourier transform on the image in order to get the frequencies. After this we took the number of pixels and the real, physical height of the cylinders to find the length of the photo. From there we were able to divide 2 pi by this length to find the difference in wavenumber between two frequencies and then eventually the actual wavenumber of the main frequency. The wave number we got from this was approximately 2 cm, which gave us a wavelength of roughly 3.1 cm, which is almost exactly 2 times 1.534 cm as we saw on the Instrumentation slide. Additionally, this seems to agree with the physical measurements we took while collecting the data that you can see on one of the photographs on the previous slide.

Special Topic

One topic that makes use of Taylor-Couette flow, and the Reynolds number specifically, is heat transfer. In heat transfer, the Nusselt number is a dimensionless quantity that is the ratio of convective to conductive heat transfer. This means that for small (i.e. <1) Nusselt numbers, the hat transfer is predominantly conductive. While for larger Nusselt numbers, the heat transfer is predominantly convective and thus more efficient.

Large Nusselt numbers are typically associated with turbulent flow, in much the same way as the Reynolds number was in our attempt at the Taylor-Couette experiment. Additionally, for systems where there is forced convection and where the Nusselt number is high (i.e., ~100<), it's proportional to the Reynolds number of the fluid system. In that way we can see how the Reynold's number is used to describe the transition of a fluid state from laminar to turbulent. This actual transition is described more accurately by the Taylor number, which is proportional to the square of the Reynold's number.