HW (1)

Chapter (4), Sec (4) Prob (3)

Page: 155 $\frac{1}{i}+\frac{1}{o}=\frac{1}{f} \Rightarrow \frac{-di}{i^2}-\frac{do}{o^2}=0$ $f=$ Const.

$do = 10.1 - 10 = 0.1$

$di = \frac{-i^2}{o^2}\,do \Rightarrow di = \frac{-15^2}{10^2}(0.1) = -0.225$

$i = i + di = 15 + (-0.225) = 14.775$

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$u = f(x-ct) + g(x+ct)$ $\begin{cases} y = x - ct \\ z = x + ct \end{cases}$

$\frac{\partial y}{\partial x} = 1, \frac{\partial y}{\partial t} = -c, \frac{\partial z}{\partial x} = 1, \frac{\partial z}{\partial t} = c$

$$\begin{cases} \frac{\partial u}{\partial x} = \frac{\partial f}{\partial y}\frac{\partial y}{\partial x} + \frac{\partial g}{\partial z}\frac{\partial z}{\partial x} = \frac{df}{dy} + \frac{dg}{dz} = f' + g' \\ \frac{\partial u}{\partial t} = \frac{\partial f}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial g}{\partial z}\frac{\partial z}{\partial t} = -c\frac{df}{dy} + c\frac{dg}{dz} = -cf' + cg' \end{cases}$$

$$\begin{cases} \frac{\partial^2 u}{\partial x^2} = \frac{d^2 f}{dy^2}\underbrace{\frac{\partial y}{\partial x}}_{1} + \frac{d^2 g}{dz^2}\underbrace{\frac{\partial z}{\partial x}}_{1} = f'' + g'' & (1) \\ \frac{\partial^2 u}{\partial t^2} = -c\frac{d^2 f}{dy^2}\underbrace{\frac{\partial y}{\partial t}}_{-c} + c\frac{d^2 g}{dz^2}\underbrace{\frac{\partial z}{\partial t}}_{c} = c^2 f'' + c^2 g'' & (2) \end{cases}$$

$(1), (2) \Rightarrow (f'' + g'') = \frac{1}{c^2} c^2 (f'' + g'')$

$$\Rightarrow \boxed{\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}}$$ Wave equation

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$y = mx + b$	observed points:
$x_1 = -1 \quad y_1 = -m + b$	$y_{01} = -2$
$x_2 = 0 \quad y_2 = b$	$y_{02} = 0$
$x_3 = 1 \quad y_3 = m + b$	$y_{03} = 3$

$\cancel{f(m,b) = [-2-(-m+b)]^2}$

least squares: Minimize:

$$(y_1 - y_{01})^2 + (y_2 - y_{02})^2 + (y_3 - y_{03})^2$$

$$f(m,b) = (-2+m-b)^2 + b^2 + (3-m-b)^2$$

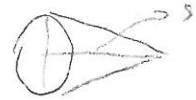
$$\begin{cases} \dfrac{\partial f}{\partial m} = 2(-2+m-b) - 2(3-m-b) = 0 \\[2ex] \dfrac{\partial f}{\partial b} = -2(-2+m-b) + 2b - 2(3-m-b) = 0 \end{cases} = \begin{cases} 2m - 5 = 0 \\[2ex] 3b - 1 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} m = 5/2 \\ b = 1/3 \end{cases} \qquad \left.\begin{cases} \dfrac{\partial^2 f}{\partial m^2} = 2 > 0 \\[2ex] \dfrac{\partial^2 f}{\partial b^2} = 3 > 0 \\[2ex] \dfrac{\partial^2 f}{\partial m \partial b} = 0 \end{cases}\right\} \Rightarrow f_{mm} f_{bb} = 6 > f_{mb}^2 = 0$$

(Test for minimum value)

the eq. of line:

$$\boxed{\begin{aligned} y &= mx + b \\ &= \frac{5}{2}x + \frac{1}{3} \end{aligned}}$$



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$$V = \pi r^2 l + \frac{1}{3}\pi r^2\sqrt{s^2-r^2}$$

$$A = \pi r^2 + 2\pi r l + \frac{1}{2}(2\pi r s) = \pi(r^2+2rl+rs)$$

$$F = V + \lambda A$$
$$= \pi\left[r^2 l + \frac{1}{3}r^2\sqrt{s^2-r^2} + \lambda(r^2+2rl+rs)\right]$$

(1) $$\frac{\partial F}{\partial r} = \pi\left[2rl + \frac{2}{3}r\sqrt{s^2-r^2} - \frac{1}{3}r^2\frac{r}{\sqrt{s^2-r^2}} + \lambda(2r+2l+s)\right] = 0$$

$$\frac{\partial F}{\partial l} = \pi\left[r^2+2r\lambda\right] = 0 \Rightarrow \lambda = \frac{-r}{2}$$ (2)

$$\frac{\partial F}{\partial s} = \pi\left[\frac{1}{3}r^2\frac{s}{\sqrt{s^2-r^2}} + \lambda r\right] = 0 \Rightarrow \lambda = \frac{-rs}{3\sqrt{s^2-r^2}}$$ (3)

(4) $$A = \pi(r^2+2rl+rs)$$ A is given

4 equations 4 unknowns (r, l, s, λ)

(2),(3): $$\frac{-r}{2} = \frac{-rs}{3\sqrt{s^2-r^2}} \xrightarrow{r\neq 0} \boxed{r = \frac{\sqrt{5}}{3}s}$$ (5)

~~Sub (1)~~ Substitute (5) into (1) $\quad l = \left(\frac{1+\sqrt{5}}{3}\right)s$

$$A = \pi\left[\frac{5}{9}s^2 + 2\frac{\sqrt{5}}{3}\frac{(1+\sqrt{5})}{3}s^2 + \frac{\sqrt{5}}{3}s^2\right]$$

$$A = \pi\left[\frac{5}{9} + \frac{2\sqrt{5}+10}{9} + \frac{\sqrt{5}}{3}\right]s^2 = \left(\frac{15+5\sqrt{5}}{9}\right)s^2$$

$$s = \frac{3\sqrt{A}}{\sqrt{\pi(15+5\sqrt{5})}}, \quad r = \frac{\sqrt{5A}}{\sqrt{\pi(15+5\sqrt{5})}}$$

$$l = \frac{(1+\sqrt{5})\sqrt{A}}{\sqrt{\pi(15+5\sqrt{5})}}$$

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$$x = e^{z} \Rightarrow dx = e^{z}\,dz \Rightarrow \frac{dz}{dx} = e^{-z}$$

$$\frac{dy}{dx} = \frac{dy}{dz}\,\frac{dz}{dx} = e^{-z}\frac{dy}{dz}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dz}\left(\frac{dy}{dx}\right)\underbrace{\frac{dz}{dx}}_{e^{-z}} = \frac{d}{dz}\left(e^{-z}\frac{dy}{dz}\right)e^{-z}$$

$$= e^{-z}\left[e^{-z}\frac{d^2y}{dz^2} - e^{-z}\frac{dy}{dz}\right] = e^{-2z}\left[\frac{d^2y}{dz^2} - \frac{dy}{dz}\right]$$

$$x^2\left[\frac{d^2y}{dx^2}\right] + 2x\left[\frac{dy}{dx}\right] - 5y = 0$$

$$e^{2z}\left[e^{-2z}\left(\frac{d^2y}{dz^2} - \frac{dy}{dz}\right)\right] + 2e^{z}\left[e^{-z}\frac{dy}{dz}\right] - 5y = 0$$

$$\frac{d^2y}{dz^2} - \frac{dy}{dz} + 2\frac{dy}{dz} - 5y = 0$$

$$\boxed{\frac{d^2y}{dz^2} + \frac{dy}{dz} - 5y = 0}$$

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$L(q, \dot{q})$ $\quad dL = \dot{p}\, dq + p\, d\dot{q}$

$$dL - d(p\dot{q}) = \dot{p}\, dq + \cancel{p\, d\dot{q}} - \cancel{p\, d\dot{q}} - \dot{q}\, dp$$

$$\underbrace{d(L - p\dot{q})}_{-H} = \dot{p}\, dq - \dot{q}\, dp \quad \Rightarrow \quad d(H) = \dot{q}\, dp - \dot{p}\, dq$$

$$-H = L - p\dot{q} \quad \Rightarrow \quad \boxed{H = p\dot{q} - L}$$

$H(p, q)$

$$\frac{\partial H}{\partial p} = \dot{q}, \quad \cancel{\frac{\partial H}{\partial q} = \dot{p}} \qquad \frac{\partial H}{\partial q} = -\dot{p}$$

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(12.8) $$\frac{dT}{dt} = 100\left(\frac{-2}{\sqrt{\pi}}\right) e^{-\left(\frac{8}{\sqrt{t}}\right)^2} \left(\frac{-4}{t^{3/2}}\right)$$

$$dT = \cancel{15.73 - 17} = 17 - 15.73 = 1.27$$

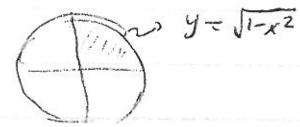
$$\frac{1.27}{dt} = 100\left(\frac{-2}{\sqrt{\pi}}\right) e^{\frac{-64}{64}} \left(\frac{-4}{64^{3/2}}\right)$$

$$= \frac{800}{\sqrt{\pi}} \frac{e^{-1}}{8^3} = \frac{100\, e^{-1}}{\sqrt{\pi}\, 8^2} = \frac{100\, e^{-1}}{\sqrt{\pi}\, 64}$$

$$\Rightarrow \boxed{dt = 3.9}$$

Chapter 5 Sec (4) Prob (14) Page (226)

$r^2 = x^2 + y^2$



$$I = \int_{\theta=0}^{\pi/2} d\theta \int_{r=0}^{1} e^{-r^2}\, r\,dr = \frac{\pi}{2}\left(-\frac{1}{2} e^{-r^2}\right)\Big|_0^1 = \frac{\pi}{4}\left(1 - e^{-1}\right)$$

$$= \frac{\pi}{4}\left(\frac{e-1}{e}\right)$$

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$$z = x^2 + y^2$$

$$\sec\gamma = \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1} = \sqrt{4x^2 + 4y^2 + 1}$$

$$A = \iint \sec\gamma\, dx\,dy = \iint \sqrt{4(x^2+y^2)+1}\, dx\,dy$$

$x^2 + y^2 = r^2$

cylinder eq: $x^2 + y^2 = 9 \Rightarrow r^2 = 9 \quad r = 3$

$$A = \int_{r=0}^{3} \int_{\theta=0}^{2\pi} \sqrt{4r^2+1}\; r\,dr\,d\theta = 2\pi \frac{1}{12}\left(4r^2+1\right)^{3/2}\Big|_0^3$$

$$= \frac{\pi}{6}\left(37^{3/2} - 1\right)$$