Standard 13 (?)

standard 13.pdf

Standard 13 (Thermal)

Paramagnetism

"up": $-\mu\vec{B}$ "down": $+\mu\vec{B}$

$$Z = \sum_s e^{-\beta E(s)} = e^{+\beta\mu B} + e^{-\beta\mu B} = 2\cosh(\beta\mu B)$$

$$P_\uparrow = \frac{e^{+\beta\mu B}}{Z} = \frac{e^{+\beta\mu B}}{2\cosh(\beta\mu B)}$$

$$P_\downarrow = \frac{e^{-\beta\mu B}}{Z} = \frac{e^{-\beta\mu B}}{2\cosh(\beta\mu B)}$$

$$\bar{E} = \sum_s E(s)P(s) = (-\mu B)P_\uparrow + (+\mu B)P_\downarrow = -\mu B(P_\uparrow - P_\downarrow)$$

average dipole energy

$$\rightarrow = -\mu B\frac{e^{\beta\mu B} - e^{-\beta\mu B}}{2\cosh(\beta\mu B)} = -\mu B\frac{2\sinh(\beta\mu B)}{2\cosh(\beta\mu B)} = -\mu B\tanh(\beta\mu B)$$

For N number of dipoles, the total energy is

$$U = -N\mu B\tanh(\beta\mu B) = -N\mu B\tanh\left(\frac{\mu B}{kT}\right)$$

The magnetization is

$$M = N\mu\tanh(\beta\mu B) = N\mu\tanh\left(\frac{\mu B}{kT}\right)$$

Helmholtz Free Energy

$$F \equiv U - TS$$

$$F = -kT\ln(Z)$$

$$F = -kT\ln(2\cosh(\beta\mu B))$$

$$= -kT\ln\left(e^{+\beta\mu B} + e^{-\beta\mu B}\right)$$

This is only for 1 paramagnet

For N paramagnets, the partition function would be

$$Z_{tot} = \left(Z_{\uparrow} Z_{\downarrow}\right)^N$$

$$Z = \left(2\cosh\left(\beta\mu B\right)\right)^N$$

$$= 2^N \cosh^N(\beta\mu B)$$

This gives us the Helmholtz free energy

$$F = -kT\ln\left(2^N \cosh^N\left(\frac{\mu B}{kT}\right)\right)$$