$2x+3y=8$
$3x-4y=-5$

$$\left[\begin{array}{cc|c} 2 & 3 & 8 \\ 3 & -4 & -5 \end{array}\right] \quad -\frac{3}{2}$$

triangular

$$\left[\begin{array}{cc|c} 2 & 3 & 8 \\ 0 & -\frac{17}{2} & -17 \end{array}\right]$$

gausian elimination

$3-\frac{3}{2}\cdot 2=0 \quad -4-\frac{3}{2}\cdot 3=-4-\frac{9}{2}=-\frac{17}{2} \quad -5-\frac{3}{2}8=-5-\frac{24}{2}=-17$

back substitution

$2x+3y=8$

$-\frac{17}{2}y=-17 \Rightarrow y=2$

$2x+3\cdot 2=8$

$2x=8-6=2$

$x=1$

On a computer? same

for triangular system, back substitution:

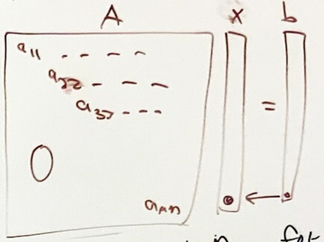
$a_{11}x_1+a_{12}x_2 \cdots + a_{1n}x_n = b_1$

$a_{22}x_2+\cdots+a_{2n}x_n=b_2$

$\vdots$

$a_{n-1,n-1}x_{n-1}+a_{n-1,n}x_n=b_{n-1}$

$a_{nn}x_n=b_n$



$a_{nn}x_n=b_n \Rightarrow x_n=b_n/a_{nn}$

$a_{n-1,n-1}x_{n-1}=b_{n-1}-a_{n-1,n}x_n$

compute

know

for $k=n$, the sum is empty

$k=n$

$$\boxed{b_k=\frac{1}{a_{kk}}\left(b_k-\sum_{i=k+1}^{n}a_{ki}x_i\right)}$$

for $i=k+1,\ldots,n$

$b_k=b_k-a_{ki}*x_i$

$b_k=b_k/a_{kk}$

replace b by solution

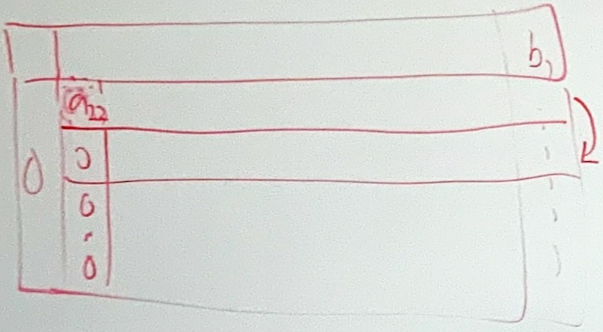
$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$-\frac{a_{21}}{a_{11}}$$

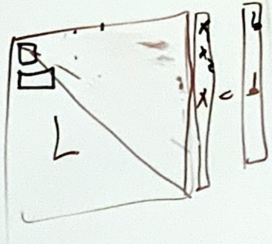
$$0 = a_{21}x_1 + a_{22}x_2 + \dots a_{2n}x_n = b_2$$

$$-\frac{a_{31}}{a_{11}}$$

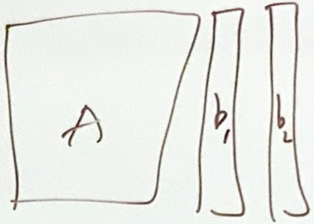
$$0 = a_{31}x_1 + a_{32}x_2 + \dots a_{3n}x_n = b_3$$



$$-\frac{a_{32}}{a_{22}}$$



multiple right hand sides



$$Ax_\ell = b_\ell, \ \ell = 1, \dots$$

instead of repeating elimination

use decomposition: $A = LU$ ← upper triangular

↑ lower triangular

$Ax = b$

$\Leftrightarrow LUx = b$

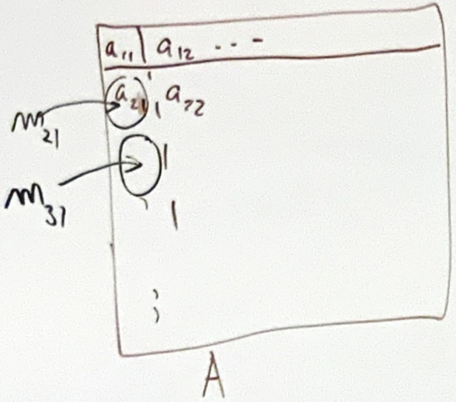
$Ly = b$ ← forward substitution

$Ux = y$ ← back substitution

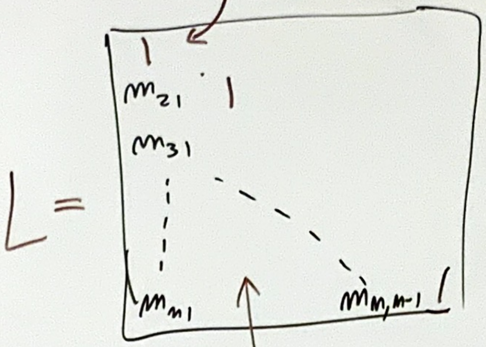
LU decomposition from gauss elimination

solve for additional right hand sides

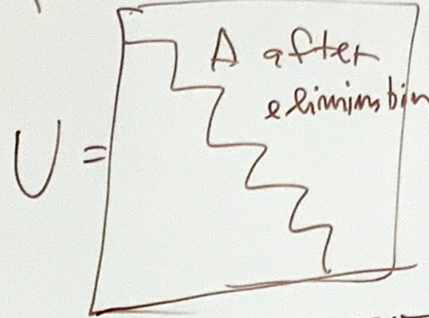
$m_{21} = \frac{a_{21}}{a_{11}}, \quad a_{2*} = a_{2*} - m_{21} a_{1*}$



add 1 on diagonal



multipliers



$LU = A$

note elimination