Physics 3120

Homework (8)

Chapter (13)

1.2, 2.1, 2.15, 3.5, 4.6, 5.14, 6.3, 7.15, 8.4, 8.9, 9.2, 9.3

Chap 13

1.2:

$$u = \sin(x - vt)$$

$$\frac{\partial u}{\partial x} = \cos(x - vt) \qquad \frac{\partial u}{\partial t} = -v\cos(x - vt)$$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x - vt) \qquad \frac{\partial^2 u}{\partial t^2} = -v^2\sin(x - vt)$$

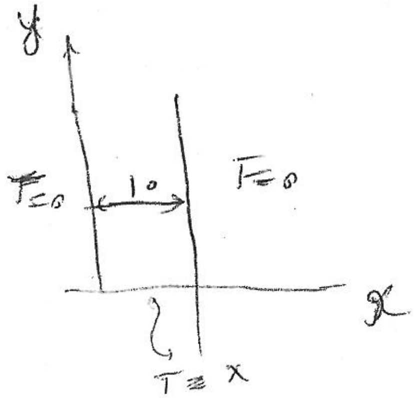
Thus $\dfrac{\partial^2 u}{\partial x^2} = \dfrac{1}{v^2}\dfrac{\partial^2 u}{\partial t^2}$

$u = u(x,t)$ if $u = f(x+vt)$

$$\frac{\partial^2 u}{\partial x^2} = f''(x+vt), \quad \frac{\partial^2 u}{\partial t^2} = v^2 f''(x+vt) \Rightarrow \frac{\partial^2 u}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2}$$

and a similar way for $f(x-vt)$

2.1



$$X : \begin{cases} \sin \lambda x \xrightarrow[x=10]{T=0} \lambda_n = \frac{n\pi}{10} \\ \cos \lambda x \to T=0 \text{ @ } x=0 \quad \text{No cosn} \end{cases}$$

$T \to 0$, $y \to \infty$ no $\to e^{+\lambda_n y}$

$$T_{(x,y)} = \sum b_n e^{-\frac{n\pi y}{10}} \sin\left(\frac{n\pi}{10}x\right)$$

$T = x$, $y = 0$

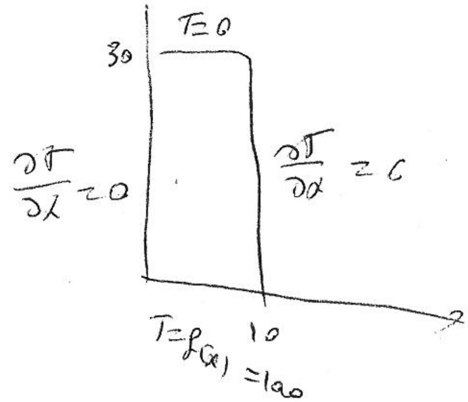
$$x = \sum b_n \sin\left(\frac{n\pi}{10}x\right)$$

$$b_n = \frac{2}{10}\int_0^{10} x \sin\frac{n\pi x}{10}\,dx = -\frac{20}{n\pi}(-1)^n$$

$$\Rightarrow T(x,y) = \frac{20}{\pi}\sum \frac{(-1)^{n+1}}{n} e^{-\frac{n\pi y}{10}} \sin\frac{n\pi x}{10}$$

Chap 13

$\boxed{2.18}$



$\frac{\partial T}{\partial x} = 0 \rightarrow$ ~~we~~ we have
@ x=0 $\cos \lambda_n x$

F$\Rightarrow$ $\frac{\partial T}{\partial x}$ $\Rightarrow$ $\lambda_n = \frac{n\pi}{10}$
@ x=10

$$T = \sum A_n \sinh \frac{n\pi}{10}(30-y) \cos \frac{n\pi x}{10}$$

→ we might ~~gess~~ guess this as solution but it is not since there is no const term here

Note

f(x) at y=0 should be given as a Casine Faurier expantion what is the constant term of that ?

$$\frac{1}{X}\frac{d^2x}{dx^2} = -\frac{1}{Y}\frac{d^2y}{dy^2} = -k^2 \qquad k \geq 0$$

$$X'' + k^2 X = 0 \quad \text{and} \quad Y'' - k^2 Y = 0$$

$k=0$ $\quad X' = \text{Cons} \Rightarrow X^0 = a_x x + b_x$, $Y'' = 0 \Rightarrow Y = a_y y + b_y$

$\frac{\partial T}{\partial x} \rightarrow X' = a_x = 0$

$a_x = 0$

$k=0$ $\quad T_0 = XY = a_x(a_y y + b_y) = a_0 y + b_0 \Rightarrow$ T=0, y=30 $\quad 0 = a_0 30 + b_0$

$\Downarrow$

$b_0 = -30a_0$

$T_0 = a_0 y - 30a_0 = a_0(30-y)$

$$T = a_0(30-y) + \sum_1^\infty A_n \sinh \frac{n\pi}{10}(30-y) \cos \frac{n\pi x}{10}$$

y=0 $\quad f(x) = \underbrace{30a_0}_{\frac{a_0}{2}} + \sum_1^\infty \underbrace{A_n \sin 3n\pi}_{a_n} \cos \frac{n\pi x}{10} = \frac{a_0}{2} + \sum a_n \cos \frac{n\pi x}{10}$

2.15

$f(x) = 100 \longrightarrow a_n = 0$

$a_0 = 200 \implies a = \frac{a_0}{60} = \frac{20\cancel{0}}{6\cancel{0}} = \frac{10}{3}$

$T = a(30-y)$

$= \frac{10}{3}(30-y)$

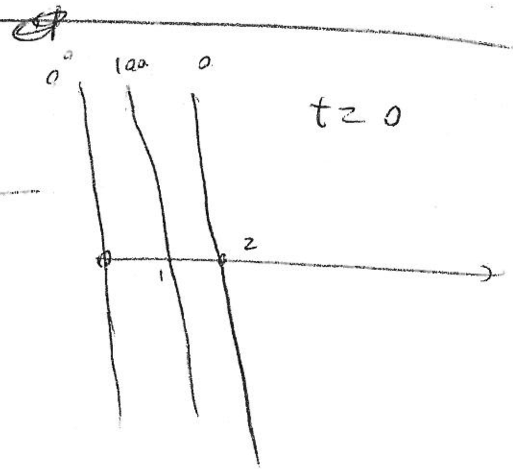
$f(x) = x$

$a_0 = \frac{2}{10}\int_0^{x} x\,dx = 10 \qquad a = \frac{a_0}{60} = \frac{1}{6}$

$a_n = \frac{2}{10}\int_0^{10} x\cos\frac{n\pi x}{10}\,dx = \begin{cases} 0 & \text{even } n \\ -\frac{40}{n^2\pi^2} & \text{odd } n \end{cases}$

$T = \frac{1}{6}(30-y) - \frac{40}{\pi^2}\sum_{\text{odd } n} \frac{1}{n^2 \sinh 3n\pi} \sinh\frac{n\pi}{10}(30-y)\cos\frac{n\pi x}{10}$

$\boxed{\cancel{3.15}}$ (3.5)



$t = 0$

~~interbys~~

Steady → State :

$u_0 = ax + b$

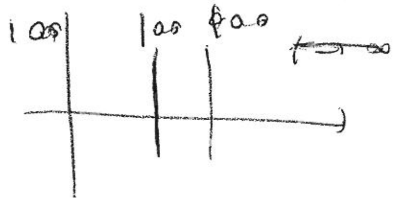
for each slab

1st slab: $u_0 = 0,\ x = 0$; $u_0 = 100,\ x = 1 \implies$

2nd slab: $u_0 = 100,\ x = 1$; $u_0 = 0,\ x = 2$

$\implies u_0 = \begin{cases} 100x & 0 < x < 1 \\ 100(2-x) & 1 < x < 2 \end{cases}$

Line



$\implies u_f = 100$

3.5

$$\boxed{\begin{array}{l}\nabla^2 F + k^2 F = 0 \\ \dfrac{dT}{dt} = -k^2\alpha^2 T\end{array}}$$

$k=0$

$\nabla^2 F = 0 \qquad \dfrac{d^2F}{dx^2} = 0 \qquad F = ax+b$

$\dfrac{dT}{dt} = 0 \qquad T = \text{Const}$

$\Big\} \Rightarrow u = ax+b$

$k>0$

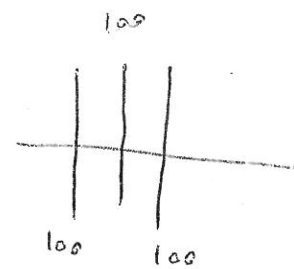
$$u = \begin{cases} e^{-k^2\alpha^2 t}\ \text{Sen}\, kx & k>0 \\ e^{-k\alpha^2 t}\ \text{Cos}\, kx & k>0 \\ ax+b & k=0 \end{cases}$$

$$u = ax+b + \sum_k e^{-k^2\alpha^2 t}\left(b_k \,\text{Sen}\, kx + a_k \,\text{Cos}\, kx\right)$$

$\left.\begin{array}{l} u \to u_f \\ t \to \infty \end{array}\right\} \Rightarrow \quad ax+b = u_f \quad \left.\begin{array}{l} a=0 \\ b=100 \end{array}\right.$

$\qquad\qquad\qquad = 100$

$$u = 100 + \sum_k e^{-k^2\alpha^2 t}\left(b_k \,\text{Sen}\, kx + a_k \,\text{Cos}\, kx\right)$$



$u=100$ at $x=0$ and $u=100$ $x=2$ for all t

$x=0 \quad \cancel{100} = \cancel{100} + \sum e^{-k^2\alpha^2 t}\left(b_k \,\text{Sen}\, k\cdot 0 + a_k \,\text{Cos}\, k\cdot 0\right) \Rightarrow a_k = 0$

$x=2$
$T=100 \quad \cancel{100} = \cancel{100} + \sum e^{-k^2\alpha^2 t}\left(b_k \,\text{Sen}\,(k\, 2)\right)$

$\text{Sen}\, 2k = 0 \qquad k_n = \dfrac{n\pi}{2}$

$$u = 100 + \sum_{n=1}^{\infty} b_n\, e^{-\left(\frac{n\pi\alpha}{2}\right)^2 t}\ \text{Sen}\ \frac{n\pi x}{2}$$

$\boxed{3.5}$

$t=0 \qquad u=u_0$

$$u_0 = 100 + \sum_{n=1} b_n \sin\frac{n\pi x}{2}$$

$$\underbrace{u_0 - 100}_{u_0 - u_f} = \sum_{n=1} b_n \sin\left(\frac{n\pi}{2}x\right)$$

$$u_0 - u_f = \begin{cases} 100x - 100 = 100(x-1) & 0<x<1 \\ 100(2-x) - 100 = -100(x-1) & 1<x<2 \end{cases}$$

$$\frac{b_n}{100} = \frac{2}{2}\left[\int_0^1 (x-1)\sin\frac{n\pi x}{2}\,dx - \int_1^2 (x-1)\sin\frac{n\pi x}{2}\,dx\right.$$

$$\Rightarrow b_n = 100\begin{cases} 0 & \text{even } n \\ \frac{8}{n^2\pi^2} - \frac{4}{n\pi} & n = 1+4k \\ -\frac{8}{n^2\pi^2} - \frac{4}{n\pi} & n = 3+4k \end{cases}$$

$$= 400\begin{cases} 0 & \text{even} \\ \frac{2}{n^2\pi^2} - \frac{1}{n\pi} & n = 1+4k \\ -\frac{2}{n^2\pi^2} - \frac{1}{n\pi} & n = 3+4k \end{cases}$$

Chapter 13

4.6

9

~~eq 4.9, 4.11~~

eq 4.9: $y = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l} \sin\frac{n\pi v t}{l}$

4.11 $\left.\frac{\partial y}{\partial t}\right|_{t=0} = \sum_{n=1}^{\infty} \underbrace{B_n \frac{n\pi v}{l}}_{b_n} \sin\frac{n\pi x}{l} = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l} = V(x)$

$$V(x) = \begin{cases} h & \frac{l}{2} - w < x < \frac{l}{2} + w \\ 0 & \text{otherwise} \end{cases}$$

$$b_n = \frac{2}{l}\int_{(l/2)-w}^{l/2+w} h \sin\frac{n\pi x}{l}\, dx = \frac{2h}{l}\left(-\frac{l}{n\pi}\cos\frac{n\pi x}{l}\right)\Big|_{(l/2)-w}^{(l/2)+w}$$

$$= -\frac{2h}{n\pi}\left[\cos\left(\frac{n\pi}{2} + \frac{n\pi w}{l}\right) - \cos\left(\frac{n\pi}{2} - \frac{n\pi w}{l}\right)\right]$$

$\cos x - \cos y = -2\sin\frac{(x+y)}{2}\sin\frac{(x-y)}{2}$

$$= \frac{4h}{n\pi}\sin\frac{n\pi}{2}\sin\frac{n\pi w}{l}$$

$$B_n = \frac{l}{n\pi v} b_n = \frac{4hl}{n^2\pi^2 v}\sin\frac{n\pi}{2}\sin\frac{n\pi w}{l}$$

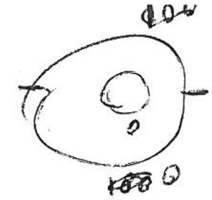
$$y = \frac{4hl}{\pi^2 v}\sum_{1}^{\infty}\frac{1}{n^2}\sin\frac{n\pi}{2}\sin\frac{n\pi w}{l}\sin\frac{n\pi x}{l}\sin\frac{n\pi v t}{l}$$

$$\sin\frac{n\pi}{2} = \begin{cases} 0 & \text{even} \\ 1, -1 & \text{odd} \end{cases}$$

Chapter 13

(5.14)

$r_1 = 1$
$r_2 = 2$



The same as 5-13

$\nabla^2 u = 0$

$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\left(\frac{\partial^2 u}{\partial \theta^2}\right) = 0$

$u = R(r)\Theta(\theta)$

$\frac{r}{R}\frac{d}{dr}\left(r\frac{dR}{dr}\right) = -\frac{1}{\Theta}\frac{d^2\Theta}{d\theta^2} = n^2$

$n=0$
$\Theta'' = 0$
$\Theta' = Con$
$\Theta = Con\,\theta$
Periodic

$n=0$
$\frac{d}{dr}\left(r\frac{dR}{dr}\right) = 0$
$r\frac{dR}{dr} = Const$
$\frac{dR}{dr} = \frac{Const}{r}$
$R = Con \ln r$

$\boxed{\left\{\begin{matrix} r^n \\ r^{-n}\end{matrix}\right\}\left\{\begin{matrix} \sin n\theta \\ \cos n\theta\end{matrix}\right\}}$ for $n>0$ and $n=0$ $\left\{\begin{matrix} Const \\ \ln r \end{matrix}\right.$

in r direction

~~$u(r,\theta) = b\ln r + \sum_{n=1}^{\infty}(r^n - r^{-n})$~~

$T=0$
$r=1$

$A1^n + B1^{-n} = 0 \quad \Rightarrow \quad Ar^n - Ar^{-n} = A(r^n - r^{-n})$

$A = -B$

$a + b\ln r\Big|_{r=1} = 0 \Rightarrow a = 0$

$T=0$
$r=1$

$u(r,\theta) = b\ln r + \sum_{n=1}^{\infty}(r^n - r^{-n})(a_n\cos n\theta + b_n \sin n\theta)$

$r = 2$;

$b\ln 2 + \sum_{n=1}^{\infty}(2^n - 2^{-n})(a_n \cos n\theta + b_n \sin n\theta) = \begin{cases} 100 & 0<\theta<\pi \\ 0 & \pi<\theta<2\pi \end{cases}$

$100 \; 0<\theta<\pi$
$0 \; \pi<\theta<2\pi$

it should be odd

No why this one?

$b_n \ln 2 = 50 \Rightarrow b = \frac{50}{\ln 2}$

it should be odd

we assume the following
To simplify
$= 50 + \begin{cases} 50 \\ -50 \end{cases}$

Just for $b\ln 2$

odd

$a_n = 0$

~~$b\ln 2$~~ $+ \sum_n b(2^n - 2^{-n})\sin n\theta$

50

~~b~~

$= \begin{cases} 50 \\ -50 \end{cases}$

$b_n(2^n - 2^{-n}) = 50\,\frac{2}{\pi}\int_0^{\pi}\sin n\theta\, d\theta = -\frac{100}{\pi}\frac{\cos n\theta}{n}\Big|_0^{\pi} = \begin{cases} \frac{200}{n\pi} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$

$u(r,\theta) = 50\ln r \cdot \frac{200}{...} = \frac{r^n - r^{-n}}{...}$

Chap 13

6.3

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{1}{v^2}\frac{\partial^2 z}{\partial t^2} \qquad z = X Y . T$$

$$\frac{X''}{X} + \frac{Y''}{Y} = \frac{1}{v^2}\frac{\ddot{T}}{T}$$

$$\frac{X''}{X} = -k_x^2 \qquad \frac{Y''}{Y} = -k_y^2 \quad , \quad \frac{\ddot{T}}{T} = -(k_x^2 + k_y^2) v^2$$

$$X = \begin{cases} \sin k_x x \\ \cos k_x x \end{cases} \qquad Y = \begin{cases} \sin k_y y \\ \cos k_y y \end{cases} \quad , \quad T = \begin{cases} \sin(vt\sqrt{k_x^2 + k_y^2}) \\ \cos(vt\sqrt{k_x^2 + k_y^2}) \end{cases}$$

$$X\Big|_{x=0} = 0 \qquad X = \sin k_x x \qquad X\Big|_{x=a} = 0 \quad k_x = \frac{n\pi x}{a}$$

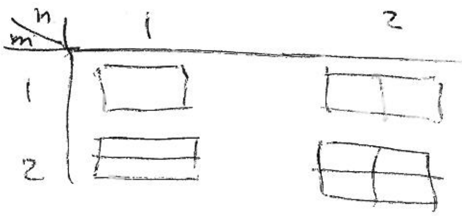
The same for Y

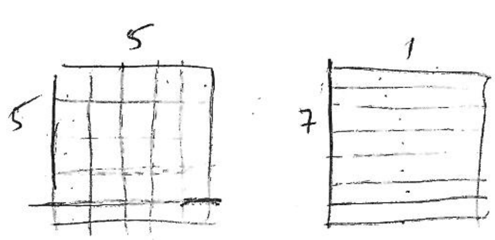
$$Y = \sin k_y y \qquad k_y = \frac{m\pi y}{a}$$

$$T = \begin{cases} \sin\left(vt\pi\sqrt{(\frac{n}{a})^2 + (\frac{m}{b})^2}\right) \\ \cos\left(vt\pi\sqrt{(\frac{n}{a})^2 + (\frac{m}{b})^2}\right) \end{cases}$$

frequencies

$$\nu_{nm} = \frac{v}{2}\sqrt{(\frac{n}{a})^2 + (\frac{m}{b})^2}$$





Square 

$$\nu_{nm} = \frac{v}{2a}\sqrt{n^2 + m^2} \qquad \nu_{55} = \nu_{71} = \frac{v}{2a}\sqrt{50}$$

$$\nu_{1,8} = \nu_{4,7} = \frac{v}{2a}\sqrt{65}$$

1,8, 4,7

$1^2 + 8^2 = 65 \qquad 4^2 + 7^2 = 16 + 49 = 65$

$$\nu_{9,2} = \nu_{7,6} = \frac{v}{2a}\sqrt{85}$$

9,2 , 7,6

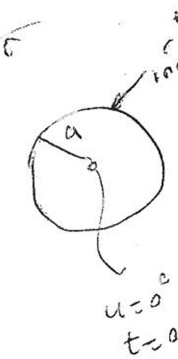
$9^2 + 2^2 = 85 \qquad 7^2 + 6^2 = 49 + 36 = 85$

7.15

$\nabla^2 u = \frac{1}{\alpha^2}\frac{\partial u}{\partial t}$ $u = F(r,\theta,\varphi)T$ $T = \left\{ e^{-k^2\alpha^2 t},\ \cancel{e^{+k^2\alpha^2 t}} \right.$

$u = F(r,\theta,\varphi)\, e^{-k^2\alpha^2 t}$

(2) $\nabla^2 F + k^2 F = 0$ $F = R(r)\,\Theta(\theta)\,\Phi(\varphi)$ (1)



Sub (1) into (2) and $\times \left[\frac{1}{R\Theta\Phi}\right]$

$$\frac{1}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) + \frac{1}{\Theta}\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right) - \frac{m^2}{\sin^2\theta} + k^2r^2 = 0$$

$$\frac{1}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) + k^2r^2 = l(l+1)$$

$$\frac{1}{\Theta}\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right) - \frac{m^2}{\sin^2\theta} + l(l+1) = 0 \Rightarrow P_m^l(\cos\theta)$$

$$2rR' + r^2R'' + (k^2r^2R - l(l+1))R = 0$$

eq 16.1 ch 12

$$R'' + \frac{2}{r}R' + \left(k^2 - \frac{l(l+1)}{r^2}\right)R = 0 \leftarrow$$ General Diff. Eq. having Bessel Fun. as Sol.

$1-2a = 2 \Rightarrow a = \frac{1}{2}$ $\quad k^2r^2 = b^2c^2r^{2(c-1)} = k^2r^2$

$2(c-1)=2$, $c-1=1$, $\boxed{c=1}$

$bc = k$, $\boxed{b=k}$

$-l(l+1) = a^2 - p^2c^2 = \frac{1}{4} - p^2$

$p^2 = \frac{1}{4} + l(l+1)$

$p^2 = \frac{1}{4} + l^2 + l = \left(l+\frac{1}{2}\right)^2$

$p = l + \frac{1}{2}$

only J since it should be finite for $r \to 0$

$R = r^{-1/2} J_p(kr)$ where $\boxed{p = l + \frac{1}{2}}$

$$u = r^{-1/2} J_p(kr)\, P_l^m(\cos\theta) \begin{Bmatrix}\sin m\varphi \\ \cos m\varphi\end{Bmatrix} e^{-k^2\alpha^2 t}$$

$u = 100$, $r = a$

No θ, φ dependent $\Downarrow$ $m=0$, $l=0$, $p=\frac{1}{2}$

$u = r^{-1/2} J_{1/2}(kr)\, e^{-k^2\alpha^2 t} \Rightarrow \boxed{u = j_0(kr)\, e^{-k^2\alpha^2 t}}$

$j_n = \sqrt{\frac{\pi}{2}}\, r^{-1/2} J_{\frac{(2n+1)}{2}}(r)$

Subtrac 100 — $\begin{cases} u_{new} = -100 & t=0 \\ u_{new} = 0 & t>0 \end{cases}$

$u_{new} = u$

7.15 Contin

12

u $\boxed{u = r^{-1} \sin kr \; e^{-k^2 \alpha^2 t}}$

for $t>0$

$\left.\begin{array}{l} u=0 \\ r=a \end{array}\right\} \Rightarrow \quad j_0(ka) = \frac{1}{ka}\sin ka = 0 \Rightarrow ka = n\pi \Rightarrow \boxed{k = \frac{n\pi}{a}}$

$$u = \sum c_n j_0\left(\frac{n\pi r}{a}\right) e^{-\left(\frac{n\pi\alpha}{a}\right)^2 t} \qquad \text{or} \qquad u = \frac{1}{r}\sum_n b_n \sin\frac{n\pi r}{a} e^{-\left(\frac{n\pi\alpha}{a}\right)^2 t}$$

$t=0 \rightarrow \quad -100 = \underbrace{\sum_n c_n j_0\left(\frac{n\pi r}{a}\right)}_{\text{Bessel's Seri}}$

$t=0 \quad -100r = \underbrace{\sum_n b_n \sin\frac{n\pi r}{a}}_{\text{Fouri Seri}}$

$$b_n = \frac{2}{a}\int_0^a -100r \sin\frac{n\pi r}{a} dr$$

$$\boxed{b_n = (-1)^n \frac{200a}{n\pi}}$$

$\boxed{U = u + 100}$ ← add 100° back

Bessell Series:

$$-100 = \sum_n c_n j_n\left(\frac{n\pi r}{a}\right) = \sum_n c_n \sqrt{\frac{\pi a}{2n\pi r}} J_{1/2}\left(\frac{n\pi r}{a}\right)$$

let $x = \frac{r}{a}$

$$-100\sqrt{x} = \sum_n \frac{c_n}{\sqrt{2n}} J_{1/2}(n\pi x)$$

multiply by $x J_{1/2}(m\pi x)$ and $\int_0^1$

$$-100\int_0^1 x^{3/2} J_{1/2}(m\pi x)\,dx = \frac{c_m}{\sqrt{2m}}\int_0^1 x J_{1/2}^2(m\pi x)\,dx \qquad (2)$$

RHS(2) ⟶

eq 19.10 page 523:

$$\int_0^1 x J_{1/2}^2(m\pi x)\,dx = \frac{1}{2} J_{3/2}^2(m\pi) \qquad \text{let } v = m\pi x \Rightarrow x = \frac{v}{m\pi}$$

LHS(2) ⟶

$$\int_0^1 x^{3/2} J_{1/2}(m\pi x)\,dx = \int_0^{m\pi}\left(\frac{v}{m\pi}\right)^{3/2} J_{1/2}(v)\frac{dv}{m\pi} = \frac{1}{(m\pi)^{5/2}} v^{3/2} J_{3/2}(v)\Big|_0^{m\pi}$$

$$= \frac{1}{m\pi} J_{3/2}(m\pi)$$

7.15 Contin

Sub into eq (2)

$$\frac{-100}{m\pi} J_{3/2}(m\pi) = \frac{c_m}{\sqrt{2m}} \frac{1}{2} J^2_{3/2}(m\pi)$$

$$c_m = \frac{-200\sqrt{2m}}{m\pi J_{3/2}(m\pi)}$$

17.4
Page 518

$$J_{3/2}(x) = \sqrt{\frac{2x}{\pi}}\, x\left(-\frac{1}{x}\frac{d}{dx}\right)\frac{\sin x}{x} = -\sqrt{\frac{2x}{\pi}}\,\frac{x\cos x - \sin x}{x^2}$$

$$J_{3/2}(m\pi) = -\sqrt{\frac{2m\pi}{\pi}}\,\frac{m\pi\cos m\pi - \overset{0}{\widehat{\sin m\pi}}}{(m\pi)^2} = -\sqrt{2m}\,\frac{(-1)^m}{m\pi}$$

$$c_m = 200(-1)^m$$

Plus 100

$$u = 200\sum_n (-1)^n \underbrace{J_0\left(\frac{n\pi r}{a}\right)}_{b} e^{-\left(\frac{n\pi\alpha}{a}\right)^2 t} + 100$$

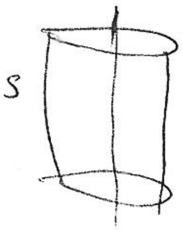
$$J_0(x) = \frac{\sin x}{x} \longrightarrow \text{eq } 17.4 \text{ Page } 518$$

$$u = \frac{200a}{\pi r}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\left(\frac{n\pi r}{a}\right) e^{-\left(\frac{n\pi\alpha}{a}\right)^2 t} + 100$$

the Same as
Fourier Series

8.4

Line Charge



Gauss's law

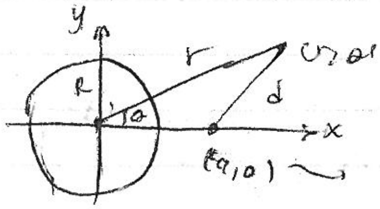
$$\int_S \bar{E}\cdot\vec{n}\,ds = 4\pi\underbrace{(\text{total charge inside } S)}_{k}$$

$\bar{E}\cdot\bar{n} = 0$ (on top and bottom)

for curved surface: $\int \bar{E}\cdot n\, da = 2\pi r \cdot 1 \cdot E$

$2\pi r\, E = 4\pi k \Rightarrow \boxed{E = \frac{2k}{r}}$

$E = -\nabla V \xrightarrow[V=-\int E\,dr]{} V = -2k\ln r = -k\ln r^2$ ①



$(a,0) \rightsquigarrow$ a line charge ⊥ to xy plane

The same as ① The potential due to line charge is

$V_k = -k\ln d^2$

$= -k\ln[r^2 - 2ra\cos\theta + a^2]$ ②

Now after grounding the cylinder, we have to add Laplace's eq.

Laplace's eq: $\left\{\begin{matrix} r^n \\ r^{-n}\end{matrix}\right\}\left\{\begin{matrix}\sin n\theta \\ \cos n\theta\end{matrix}\right\}$ $n \neq 0$ and $\left\{\begin{matrix}\text{Const.} \\ \ln r\end{matrix}\right\}$ $n=0$

→ Prob 5.4)

$r_{\to\infty} \to V$ (finite) $\Rightarrow$ No r^n

Const is the $n=0$ term of cosine series

$$V = V_k + A\ln r + \sum_{n=0}^{\infty} a_n r^{-n}\cos n\theta$$ ④

$$= -k\ln(r^2 - 2ra\cos\theta + a^2) + A\ln r + \sum_{n=0}^{\infty} a_n r^{-n}\cos n\theta$$ ③

$r\to\infty$ V = finite. $\Rightarrow -k\ln r^2 + A\ln r = 0 \Rightarrow -2k\ln r + A\ln r = 0$

~~$-2k\ln r + A\ln r = 0$~~

$\boxed{A = 2k}$

8.4 Canton

at $r=R$ $V=0$

(4) $$0 = -k\ln(R^2 - 2Ra\cos\theta + a^2) + 2k\ln R + \sum_{n=0}^{\infty} a_n R^{-n}\cos n\theta$$

$$\sum a_n R^{-n}\cos n\theta = k\ln\underbrace{\left[a^2\left(\frac{R^2}{a^2} - \frac{2R}{a}\cos\theta + 1\right)\right]}_{k\ln\left(\frac{R^2}{a^2} - \frac{2R}{a}\cos\theta + 1\right) + \underbrace{k\ln a^2}_{2k\ln a}} - 2k\ln R$$

$$\underbrace{2k\ln a - 2k\ln R}_{2k\ln\frac{a}{R}}$$

(5) $$\sum a_n R^{-n}\cos n\theta = k\ln\left[1 - \frac{2R}{a}\cos\theta + \left(\frac{R}{a}\right)^2\right] + 2k\ln\left(\frac{a}{R}\right)$$

Using complex formula: Chap 2 page 51, 71

$$z = re^{i\theta} \Rightarrow \ln z = \ln re^{i\theta} \Rightarrow \ln\underbrace{|z|}_{\text{absolute value}} = \ln(r) = \mathrm{Re}\ln z$$

Now let $z = (1-\rho e^{i\theta})$

$$\ln|1-\rho e^{i\theta}| = \mathrm{Re}\ln(1-\rho e^{i\theta})$$

$$2\ln|1-\rho e^{i\theta}| = \ln|1-\rho e^{i\theta}|^2 = \ln\left|(1-\rho e^{i\theta})\underbrace{(1-\rho e^{-i\theta})}_{\text{Complex conjugate}}\right|$$

$$= \ln(1 - 2\rho\cos\theta + \rho^2)$$

$\rho < 1$:

$$\ln(1-\rho e^{i\theta}) = -\sum_{n=1}^{\infty}\frac{1}{n}(\rho e^{i\theta})^n$$

eq 13.4
page 24

$$= -\sum_{n=1}^{\infty}\frac{1}{n}\rho^n e^{in\theta}$$

$$\mathrm{Re}\ln(1-\rho e^{i\theta}) = -\sum_{n=1}^{\infty}\frac{1}{n}\rho^n e^{in\theta}$$

(6) ~~in 63~~

$$\ln(1-2\rho\cos\theta+\rho^2) = 2\ln|1-\rho e^{i\theta}| = 2\mathrm{Re}\ln(1-\rho e^{i\theta}) = -2\sum_{n=1}^{\infty}\frac{1}{n}\rho^n\cos n\theta$$

for R.H.S eq ⑤ ~~R.H.S~~

8.41 Contd. use ⑥ with $\rho = \frac{R}{a}$ to write ⑤ as

$\rho = \frac{R}{a}$

$$\sum_{n=0}^{\infty} a_n R^{-n} \cos n\theta = K(-2)\sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{R}{a}\right)^n \cos n\theta + 2K \ln\frac{a}{R}$$

$$\boxed{a_0 = 2K \ln\frac{a}{R}}$$

$$a_n R^{-n} = -2K\,\frac{1}{n}\left(\frac{R}{a}\right)^n$$

$$\boxed{a_n = -2K \cdot \frac{1}{n}\left(\frac{R^2}{a}\right)^n}$$

~~eq ③~~

use again $\rho = \frac{R^2}{ar}$ for series at eq ③ ← R.H.S

(use ⑥ for $\rho = \frac{R^2}{ar}$)

~~V_0~~ =

$$\sum_{n=0} a_n r^{-n} \cos n\theta = 2K \ln\frac{a}{R} - \cancel{2K\sum_{n=1}\frac{1}{n}\left(\frac{R^2}{ar}\right)^n \cos n\theta}$$

$$= 2K\ln\frac{a}{R} + \sum_{n=1}\frac{-2K}{n}\left(\frac{R^2}{ar}\right)^n \cos n\theta = 2K\ln\frac{a}{R} - 2K\sum_{n=1}\frac{1}{n}\left(\frac{R^2}{ar}\right)^n \cos n\theta$$

ρ in eq ⑥ $\qquad \rho$

$$= 2K\ln\frac{a}{R} + K\ln\left[1 - \frac{2R^2}{ar}\cos\theta + \left(\frac{R^2}{ar}\right)^2\right]$$

$$V = -K\ln(r^2 - 2ra\cos\theta + a^2) + \underbrace{2K\ln r}_{K\ln r^2}$$

$$= 2K\ln\frac{a}{R} + K\ln\left[1 - \frac{2R^2}{ar}\cos\theta + \left(\frac{R^2}{ar}\right)^2\right]$$

$$= \underbrace{-K\ln(r^2 - 2ra\cos\theta + a^2)}_{\text{image method}} + K\ln\left[r^2 - \frac{2R^2}{a}r\cos\theta + \left(\frac{R^2}{a}\right)^2\right]$$

K at $(a, 0)$

$-K$ at $\frac{R^2}{a}$

$+2K\ln\frac{a}{R}$

$K\ln\left(\frac{a^2}{R^2}\right)$

$$= -K\ln(r^2 - 2ra\cos\theta + a^2) + K\left[\frac{a^2r^2}{R^2} ...\right.$$

8.9

Page 281 (text) [Green's Second identity]

$$\int (\Phi \nabla^2 \Psi - \Psi \nabla^2 \Phi)\, d\tau = \oint (\Phi \nabla \Psi - \Psi \nabla \Phi) \cdot \vec{n}\, d\sigma$$

$$\Psi = G(\vec{r}, \vec{r}') = -\frac{1}{4\pi |\vec{r} - \vec{r}'|} + F(r, r')$$

$$\Phi = u(\vec{r})$$

text eq 8.22, 8.23, 8.28

$$\nabla^2 \Phi = \nabla^2 u = f(\vec{r})$$

$$\nabla^2 F = 0$$

$$\nabla^2 \Psi = \nabla^2 G = \delta(\vec{r} - \vec{r}')$$

$\Psi = G = 0$ on the surface σ

Sub into (1) and integrate with respect to primed variables

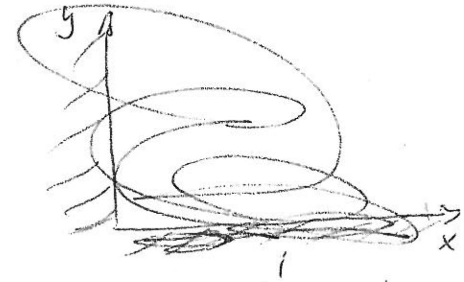
$$\int [u(\vec{r}')\, \delta(\vec{r} - \vec{r}') - G(\vec{r}, \vec{r}')\, f(\vec{r}')]\, d\tau' = \oint [u(\vec{r}') \nabla G - \underbrace{0}_{\text{on surface}}] \cdot n'\, d\sigma'$$

$$u(\vec{r}) = \int G(\vec{r}, \vec{r}')\, f(\vec{r}')\, d\tau' + \oint u(\vec{r}')\, \frac{\partial G}{\partial n'}\, d\sigma'$$

apter 15

9.2

$$u(x,0) = \begin{cases} 100(2-x) & 0<x<2 \\ 0 & x>2 \end{cases}$$

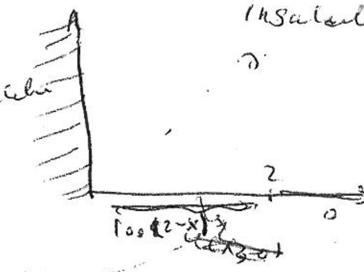


Steady State $\nabla^2 u = 0$

$$\cancel{u(x,y) = \int_0^\infty B(k) e^{-ky} \cos kx \, dk}$$

$$u(x,0) = \int_0^\infty$$

$\left.\frac{\partial u}{\partial x}\right|_{x=0} = 0$ insulated



x dir $A \sin kx + B \cos kx$

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} : \left. Ak\cos kx - Bk \sin x\right|_{x=0} = 0$$

$\Downarrow$

$A = 0$

we have only Cosine

y: e^{-ky} $\cancel{e^{ky}}$ $\Big\} \Rightarrow e^{-ky}$

$y \to \infty$ finite

$$u(x,y) = \int_0^\infty B(k) e^{-ky} \cos kx \, dk$$

$y = 0$

$$u(x,0) = \int_0^\infty B(k) \sin kx \, dk$$

$$B(k) = \sqrt{\frac{2}{\pi}}\, g_c(k) = \frac{2}{\pi}\int_0^\infty f_c(x) \cos kx = \frac{2}{\pi}\int_0^\infty u(x,0) \cos kx \, dx = \frac{2}{\pi}\int 100(2-x) \cos kx \, dx$$

$$B(k) = \frac{200}{\pi}\int_0^\infty (2-x) \cos kx \, dx = \frac{200}{\pi} k^{-2}(1-\cos 2k)$$

$$u(x,y) = \int_0^\infty B(k) e^{-ky} \cos kx \, dk$$

$$= \frac{200}{\pi}\int_0^\infty k^{-2}(1-\cos 2k)\, e^{-ky} \cos kx \, dk$$

p 15

9.3

$$\mathcal{L}(u) = U$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{\alpha^2}\frac{\partial u}{\partial t}$$

$$u_0 = u(x,0) = \frac{100x}{l}$$

$$\frac{\partial^2 U}{\partial x^2} = \frac{1}{\alpha^2}[pU - u_0] = \frac{1}{\alpha^2}pU - \frac{100x}{\alpha^2 l} \Rightarrow \underbrace{\frac{\partial^2 U}{\partial x^2} - \frac{1}{\alpha^2}pU = \frac{-100x}{\alpha^2 l}}$$

homogen. $\frac{\partial^2 U}{\partial x^2} - \frac{1}{\alpha^2}pU = 0$

$$U_h = \begin{cases} \sinh\left(x\sqrt{\frac{p}{\alpha^2}}\right) \\ \cosh\left(x\sqrt{\frac{p}{\alpha^2}}\right) \end{cases}$$

Partucular Solu.. $U = kx \Rightarrow \frac{\partial^2 u}{\partial x^2} = 0$

$$\frac{pU}{\alpha^2} = \frac{100x}{\alpha^2 l} \Rightarrow k = \frac{100}{pl}$$

$$U(x,p) = A\sinh(xp^{1/2}/\alpha) + B\cosh(xp^{1/2}/\alpha) + \frac{100x}{pl}$$

Temp $\big|_{x=0} = 0$ for all t $\quad u(0,t) = 0 \Rightarrow U(0,p) = 0 \Rightarrow B = 0$

$u(l,t) = 0 \Rightarrow U(l,p) = 0$

$$A\sinh(lp^{1/2}/\alpha) + \frac{100}{p} = 0 \Rightarrow A = \frac{-100}{p\sinh(lp^{1/2}/\alpha)}$$

$$U(x,p) = -\frac{100\sinh(xp^{1/2}/\alpha)}{p\sinh(lp^{1/2}/\alpha)} + \frac{100x}{pl}$$

assum i expansion given:

$$\frac{100\sinh(xp^{1/2}/\alpha)}{p\sinh(lp^{1/2}/\alpha)} = \frac{100x}{pl} - \frac{200}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\sin(n\pi x/l)}{n[p+(n\pi\alpha/l)^2]}$$

~~the inner laplace transf. using (L2) page 363~~

$$U(x,p) = \frac{200}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\sin(n\pi x/l)}{n\left[p+\left(\frac{n\pi\alpha}{l}\right)^2\right]}$$

(L2) Page 363

$\boxed{Y = \frac{1}{p+a} \to y = e^{-at}}$

$$u(x,t) = \frac{200}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1} e^{-\left(\frac{n\pi\alpha}{l}\right)^2 t}\sin\left(\frac{n\pi x}{l}\right)$$

chap 16

4.17 2 particle 4 box

M.B : $4^2 = 16$

F.D : $C(4,2) = \frac{4!}{(4-2)!\,2!} = \frac{4!}{2!\,2!} = \frac{\cancel{2}\times3\times4}{\cancel{2}\times2} = 6$

B.E : $C(5,2) = \frac{5!}{(5-2)!\,2!} = \frac{5!}{3!\,2!} = \frac{\cancel{2}\times\cancel{3}\times4\times5}{\cancel{2\times3}\times\cancel{2}} = 10$

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chap 16

$\boxed{9.3}$

$P_n = \frac{\mu^n}{n!} e^{-\mu}$

$N_{gp} = 6000$

$N_{int} = 3000$

$t = 50\times60$ min

$\mu = \bar{n} = \frac{6000}{(50)(60)} = \frac{100}{50} = 2$

$n = 0, 1, 2, 3, 4, 5$

$P_0 = \frac{2^0}{0!} e^{-2} = e^{-2}$, $P_1 = \frac{2^1}{1!} e^{-2} = 2e^{-2}$, $P_2 = \frac{2^2}{2!} e^{-2} = 2e^{-2}$, $P_3 = \frac{2^3}{3!} e^{-2} = \frac{4}{3} e^{-2}$

$P_4 = \frac{2^4}{4!} e^{-2} = \frac{2}{3} e^{-2}$, $P_5 = \frac{2^5}{5!} e^{-2} = \frac{4}{15} e^{-2}$

~~$N_0 = P_0 N$~~

$N_{int_0} = N_{int} P_0 = 3000 \times e^{-2} \cong 406$

$N_{int_1} = N_{int} P_1 \cong 812$

$N_{int_2} = N_{int} P_2 \cong 812$

$N_{int_3} = N_{int} P_3 \cong 541$

$N_{int_4} = N_{int} P_4 \cong 271$

$N_{int_5} = N_{int} P_5 \cong 163$