The instability of the classical atom

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I. Introduction and Background

Just over a hundred years ago the existence of atoms was far from established, in no small part due to the difficultly of observation at the atomic scale. Despite this, there were a variety of phenomena that had been observed or measured through experimentation that led some scientists to explore the idea further. One such phenomenon was that of spectral lines of electromagnetic radiation, i.e. light, which were emitted from certain materials after exposing them to incident light rays. These spectral lines were found to be equal to the wavelengths of light they had absorbed, leading to the idea that these spectral lines could be used to identify a given material [1]. Additionally, it was established by Maxwell's equations that electromagnetic radiation was emitted by charged particles as they experienced an acceleration, leading some to conclude that these small, characteristic building blocks of a material were in some way associated with electric charge [2]. Logically, if there were such a thing as atoms which form the matter of the external world, and these atoms involved electrical charge in some manner, then each atom would need to be composed of equal positive and negative charge if it was to be stable enough to form into macroscopic objects [1].

Following from the discovery of the electron outside of matter, J. J. Thomson first proposed the colloquially named "plum pudding" model of the atom. In this freely moving electrons were contained within a positive spherical charge distribution in which they would configure themselves such that all electrostatic forces would cancel out. When a ray of light was incident upon this atom, the electrons within would be accelerated, resulting in the emission of electromagnetic radiation [3]. For its time this was a compelling model, since it seemed to synthesize many established atomic characteristics. However, it could not explain why each type of atom seemed to only absorb and emit radiation at specific wavelengths. The Thomson model would be dealt a fatal blow in 1911, when a team of physicists led by Ernest Rutherford performed the famous "gold foil" experiment, where they measured the scattering angle of α particles from a cathode ray as they were passed through a piece of gold

foil. Using Thomson's model as their theoretical basis, they believed that the α particles would largely pass directly through with little to no scattering. What they found instead was that a significant number were scattered at much more extreme angles than predicted, leading to the conclusion that the positive charge distribution in the gold atoms was highly concentrated towards the atomic center [2]. By assuming the positively charge was located at the center, Rutherford's team was able to work out the deflection under an inverse square central force, in a very similar manner to orbits. In fact, by using many of the same formalisms developed to describe orbits, they were able to solve for the probability distribution of the scattering as a function of the scattering angle, and from that deduce the than dense positive charge distribution at the center of the gold atom must be no greater than 10^{-12} cm in radius, much smaller than the atomic radius of 10^{-8} cm [2]. This experiment marked the discovery of the nucleus.

From this experiment came the Rutherford model, which describes the atom as a tightly bound positive center, i.e. a nucleus, surrounded by electrons which orbit the center of mass. While this model certainly improved upon Thomson's model, it still failed to explain the presence of electromagnetic spectral lines. Even worse, the circular motion of the constituent electrons around the nucleus meant that they would experience an acceleration towards the center of mass everywhere along their path. Given the classical understanding of the day, this would result in the electrons emitting a continuous spectrum of electromagnetic radiation, radiating energy outwards, till they finally collapse into the nucleus. This was a catastrophic failure because if true, the existence of stable atoms could not be explained [4].

One question that arises immediately from this failure of the Rutherford model is roughly how much time would it take for an electron to collapse into its nucleus? After all, if it would take some astronomical amount of time, then maybe this isn't a failure, so much as it is instead a demonstration that what we know as stable matter is only stable relative to a human lifespan. In this paper we will try to see just how bad this was for the Rutherford model by attempting to determine the time it would take for an atomic collapse. What we will see is that this is not an issue that can be simply overlooked, it is a systemic failure of classical physics when trying to describe the behavior of atoms that requires a total new understanding of physics to fix [5].

In order to go about this, we must first gather a few important concepts from classical electromagnetism, namely the Poynting vector and the Larmor formula. From these, we will be able to find the energy emitted from an accelerating particle, as well as the change in that energy over some period of time. We can use solve for the time it takes for a hydrogen atom to collapse due to electron energy loss in terms of the change in its orbital radius. The hydrogen atom has long served as a sort of fundamental building block for the entire field

of atomic physics, due to its simplicity—being composed of only a single nuclear proton and a single orbiting electron—as well as the scalability of its atomic structure and behaviors to much larger and more complex atoms [5]. As we will see, the time for its collapse is such that the Rutherford model takes a massive blow to its viability. The instability of hydrogen here will spell doom for the classical Rutherford model.

II. The Poynting Vector

Since we are trying to solve for the time is takes for an electron to lose enough energy to fall from its orbit, a good first step would likely be to find some relationship between energy and time, i.e. an equation for power. Starting from classical physics, we can understand an atomic electron as a point charge, moving through an electromagnetic field generated by its nucleus. Intuitively, we may then want to reach for some equation that describes the force on a this electron particle due to an electromagnetic field. In order to derive a general equation, we will treat the electron as just some point charge, of an undetermined charge sign.

The net force on a point charge moving through an electromagnetic field is described via the Lorentz force. To find this, we simply add the forces due to the electric $\vec{E}$ and magnetic fields $\vec{B}$, where $\vec{v}$ and q are the velocity and charge magnitude of the charge.

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \qquad (1)$$

Following from another classical definition, we can easy find some amount of work done by this net force acting on the charge over some small interval of time dt.

$$dW = \vec{F} \cdot d\vec{l} = q(\vec{E} + \vec{v} \times \vec{B}) \cdot \vec{v}dt = q\vec{E} \cdot \vec{v}dt \qquad (2)$$

We can then divide over dt and substitute in a definition of the magnitude of the charge as the volume charge density times some differential volume $q = \rho d\tau$.

$$\frac{dW}{dt} = (\vec{E} \cdot \vec{v})\rho d\tau \qquad (3)$$

Following from the conservation of charge—found through the *continuity equation*, $\frac{\partial \rho}{\partial t} = -\vec{\nabla} \times \cdot \vec{J}$—we can then define the charge density times the velocity as the volume current density $p\vec{v} = \vec{J}$. We can then integrate over the total volume τ to find all the work done on the charged particle by the field forces.

$$\frac{dW}{dt} = \int_{\tau} (\vec{E} \cdot \vec{J}) d\tau \qquad (4)$$

This leaves us with the term $\vec{E} \cdot \vec{J}$, which can be seen as describing the work done per unit time, per unit volume, or alternatively, the power per unit volume. We can expand this expression to be only in terms of the electric and magnetic fields with the Ampere-Maxwell law (5).

$$\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right) \qquad (5)$$

$$\vec{J} = \frac{1}{\mu_0} \vec{\nabla} \times \vec{B} - \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{E} \cdot \vec{J} = \frac{1}{\mu_0} \vec{E} \cdot \vec{\nabla} \times \vec{B} - \epsilon_0 \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} \qquad (6)$$

Via the product rule [6]

$$\vec{\nabla} \cdot (\vec{E} \times \vec{B}) = \vec{B} \cdot (\vec{\nabla} \times \vec{E}) - \vec{E} \cdot (\vec{\nabla} \times \vec{B})$$

$$\vec{E} \cdot (\vec{\nabla} \times \vec{B}) = \vec{B} \cdot (\vec{\nabla} \times \vec{E}) - \vec{\nabla} \cdot (\vec{E} \times \vec{B}) \qquad (7).$$

We can then use the differential form of Faraday's law $\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}$ along with (7) and substitute both into (6) to get the following.

$$\vec{E} \cdot \vec{J} = -\frac{1}{\mu_0} \vec{B} \cdot \frac{\partial B}{\partial t} - \epsilon_0 \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} - \frac{1}{\mu_0} \vec{\nabla} \cdot (\vec{E} \times \vec{B}) \qquad (8)$$

$$\frac{1}{\mu_0} \vec{B} \cdot \frac{\partial B}{\partial t} = \frac{1}{2\mu_0} \frac{\partial}{\partial t} (B^2) \quad \epsilon_0 \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} = \frac{\epsilon_0}{2} \frac{\partial}{\partial t} (E^2) \qquad (9) \ \& \ (10)$$

$$\vec{E} \cdot \vec{J} = -\frac{1}{2} \frac{\partial}{\partial t} \left(\frac{1}{\mu_0} B^2 + \epsilon_0 E^2 \right) - \frac{1}{\mu_0} \vec{\nabla} \cdot (\vec{E} \times \vec{B}) \qquad (11)$$

Returning to (4) and applying the divergence theorem to the second term, we get

$$\frac{dW}{dt} = -\int_{\tau} \frac{1}{2} \frac{\partial}{\partial t} \left(\frac{1}{\mu_0} B^2 + \epsilon_0 E^2 \right) d\tau - \frac{1}{\mu_0} \oint_S (\vec{E} \times \vec{B}) \cdot d\vec{S} \qquad (12).$$

Where the first term is integrated over the volume τ, which is bound by the surface area S.

Equation (12) is called ***Poynting's theorem*** [7]. The first term on the right-hand side is the volume integral of the total energy density u stored in the fields, obviously giving us the total energy in the field when evaluated. The second term can be thought of as the energy flux through the surface of the volume which encloses our electromagnetic fields. Together, these two terms tell us that *the work done on the charges by the electromagnetic force is equal to the decrease in energy remaining in the fields, minus the energy that flowed out through the surface* [7].

When we take the second term out of the integral, we get the ***Poynting vector*** (13), which is simply the *the energy per unit time, per unit area*, i.e. the energy flux density [7].

$$\vec{S} \equiv \frac{1}{\mu_0}(\vec{E} \times \vec{B}) \qquad (13).$$

Using this, we are able to describe the flow of energy away from an electromagnetic source, which seems like it should be relevant for our purposes here. Following from the above, we can find the power from an electromagnetic field by integrating the Poynting vector over a given surface area. Since the power can be thought of as directly related to the change in energy over some unit time, this certainly seems like the relationship we stated we were interested in finding at the beginning of this section. We will continue on with this in the following section.

Something that has been taken for granted so far is the fact that electromagnetic energy is somehow located in the electromagnetic field, rather than the charges [7]. This may not seem immediately obvious, but this follows from the mathematical definitions given above, in particular from (12). This is much in the same way that gravitational energy is located in the gravitational field, rather than in the massive objects which are the source of the field. In that way, we can understand energy as being a medium through which objects interact, rather than something objects possess.

Interestingly, in the case where there is no work done on the charges in a volume, we get the

$$\int_\tau \frac{\partial u}{\partial t} d\tau = \oint_S \vec{S} \cdot d\vec{S} = \int_\tau (\vec{\nabla} \times \vec{S}) d\tau$$

$$\frac{\partial u}{\partial t} = \vec{\nabla} \times \vec{S} \qquad (14).$$

Equation (14) has no immediate bearing on the purpose of this paper, but it does establish an interesting relationship; the "continuity equation for energy" from which the *the local conservation of electromagnetic energy* is expressed [7]. Griffiths' chapter titled *Conservation Laws* has a more in-depth discussion about this, as does section 6.8 of Jackson's *Classical*

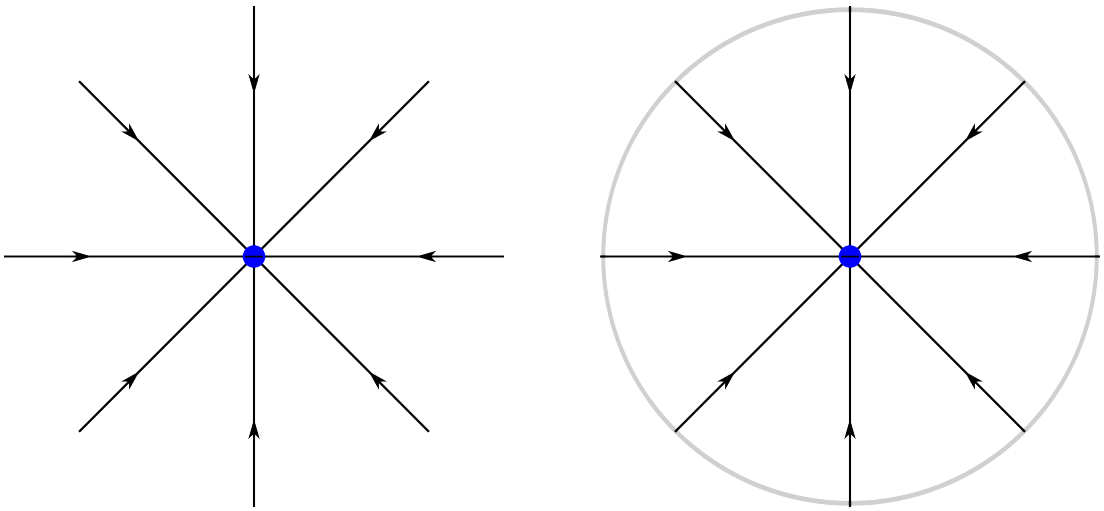
Electrodynamics [8]. All-in-all, the conservation laws that have been assumed so far in our derivation of the Poynting vector are incredibly important to consider when attempting to describe electromagnetic fields.

II. The Larmor Formula

With the the Poynting vector, we can find the power from an electromagnetic field. As we saw, all that is needed for us to do this is to determine some expressions for the fields. For a point charge, the expression for the power is well established and is called the ***Larmor formula***.

There are many derivations of the Larmor formula, with various levels of mathematical complexity. While some of these derivations, like those in Griffith or Jackson, make use of more *complete* definitions, involving discussions of potentials, fields, and gauges, the Larmor formula can be found in a relatively simple way through geometric and physical reasoning. This derivation was first written down by J. J. Thomson, and can found in Purcell's *Electricity and Magnetism* [9]. However, this derivation is slightly modified in order to accommodate an acceleration on an electron.

As we start, we can imagine the electric field emitted by an electron. This can be seen in Figure 1. When at rest, this field will be made up of straight electric field lines, flowing inwards towards the center of the charge.



(a) (b)

Figure 1: (a) Two dimensional representation of electric field lines emitted from a negative point charge, e.g. an electron. (b) Following from Gauss's law, we can see that the magnitude of the field only dependents on the radial distance from the source. Because of this, we can imagine an infinitesimally thin spherical shell forming at a specific radial distance. Anywhere we measure the electric field along this shell will have an equal magnitude.

The magnitude of the electric field depends only on the radial distance when the electron is at rest. In this way, we can imagine the electric field as forming an infinitesimally thin spherical shell of a given surface area determined by the radial distance. This is fundamentally what is being said by Gauss's law (15); the flux of an electric field through the surface of a volume enclosed by a region of space is equal to the magnitude of enclosed charge, divided by the constant for vacuum permittivity.

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{q_{enc}}{\epsilon_0} \text{ (15).}$$

With this, we can now consider a non-stationary charge. Imagine we have an electron that was stationary at the origin till $t = 0$. At this time, it suddenly experiences a small acceleration to some velocity Δv over a short interval of time Δt. We can visualize what the resulting electric field may look like in Figure 2. Specifically, the electric field will develop a "kink" or "pulse" due to this sudden acceleration, which will propagate away at the speed of light c.

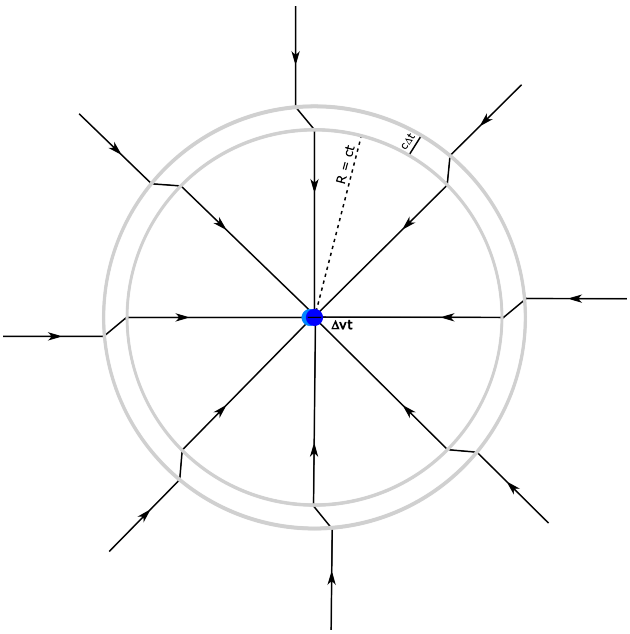


Figure 2: Electric field lines from an accelerated electron. The two spheres demonstrate the kink in the electric field that is visible when measured at some time t.

If we measure the electric field at some time t later, we can distinguish between the electric field line before and after the acceleration as being on the inside and outside of a sphere of radius ct with a thickness $c\Delta t$ centered at the electron's original location. Outside the sphere, the field lines have not yet *learned* that the charge has moved away from the origin due to the inability for information to travel faster than the speed of light. Therefore, they are still radially centered at the electron's original location. Inside the sphere, the field lines are radially centered at the electron's new location after moving some distance Δvt away from its original position. The shell separating the inner and outer regions, the two corresponding fields join together [9]. In the shell, there is an angular component of the electric field along the circumference. This component characterizes the "kink" in the electric field which is propagated away from the charge at the speed of light which makes up the emitted electromagnetic radiation [2]. Additionally, we can see how this component results in the electric field no longer being conservative, and thus there must be some loss of the field's energy.

Assuming the velocity Δv is small compared to c, we can approximate the field lines as radial at the origin at times $t = 0$ and t. We are also able to consider a small cone of field lines at some angle θ from the electron's path which it travels over the time interval. These two sets of field lines are joined in the shell as shown in Figure 3.

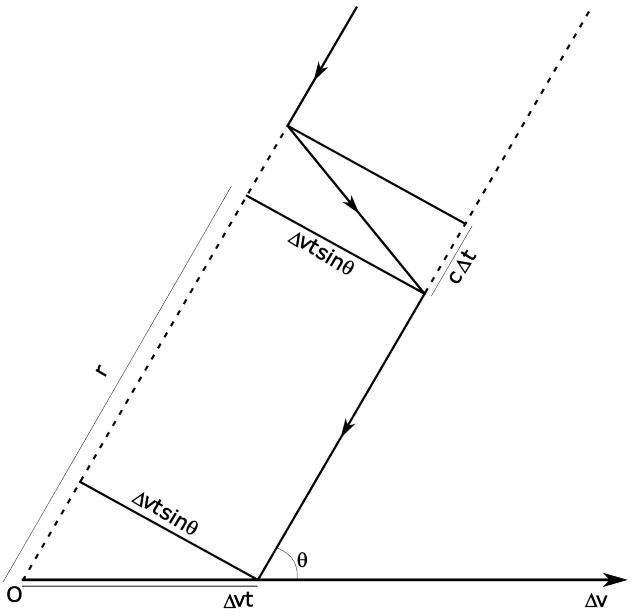


Figure 3: Diagram of electric field components due to acceleration. The electron accelerates till it reaches a constant velocity Δv.

The radial component of the electric field is given by Coulomb's law.

$$E_r = -\frac{1}{4\pi\epsilon_0}\frac{e}{r^2} \qquad (16).$$

Since $\Delta t << t$, we can ignore the contribution of the thickness to the radius and define $r \approx ct$. We then rewrite the radial electric field component as

$$E_r \approx -\frac{1}{4\pi\epsilon_0}\frac{e}{ctr} \qquad (16b).$$

Unfortunately, we cannot simply pull a definition of the azimuthal component from elsewhere. Fortunately, we can find the it by relating the geometry of the field line length components with those of the total electric field through the right triangle formed in the shell.

$$\frac{E_\theta}{E_r} = \frac{\Delta v t sin\theta}{c\Delta t} \qquad (17).$$

Solving for the E_θ gives us

$$E_\theta = -\frac{\Delta v t sin\theta}{c\Delta t}\frac{1}{4\pi\epsilon_0}\frac{e}{ctr} = \frac{e(\Delta v/\Delta t)sin\theta}{4\pi\epsilon_0 c^2 r} \qquad (18).$$

In (18), the $\frac{\Delta v}{\Delta t}$ term is just the acceleration of the charge. Since it was traveling in a

straight line from the center of our hypothetical sphere, we can say that this acceleration is purely radial, allowing us to substitute $\ddot{r}$ in the following form.

$$E_\theta = -\frac{e\ddot{r}sin\theta}{4\pi\epsilon_0 c^2 r} \qquad (18b).$$

As stated above, the E_θ component is responsible for the loss of energy in the electric field. Therefore, we take this component as the electric field in the Poynting vector.

Since the Poynting vector requires us to also have some expression for the magnetic field, we can use the following relationship between the electric and magnetic fields.

$$cB = E \rightarrow B = \frac{E}{c} \qquad (19).$$

Together with the fact that any magnetic field generated by the moving electron will be orthogonal to its electrical field, we can write the magnitude of the Poynting vector as

$$|\vec{S}| = \frac{1}{\mu_0}|\vec{E} \times \vec{B}| = \frac{1}{\mu_0}|\vec{E}||\vec{B}|sin(90^o)$$

$$= \frac{1}{\mu_0}|\vec{E}||\vec{B}| = \frac{1}{\mu_0}E\frac{E}{c} = \frac{1}{\mu_0}\frac{E^2}{c} \qquad (20).$$

Taking (20) and integrating over the spherical surface area of the shell, we will get

$$\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} S d\Omega = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\frac{1}{\mu_0}\frac{E^2}{c}d\Omega$$

$$P = \frac{dU}{dt} = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\frac{1}{\mu_0}\frac{E^2}{c}r^2 sin\theta d\theta d\phi$$

where we can substitute $\frac{1}{\mu_0} = c^2\epsilon_0$,

$$\frac{dU}{dt} = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} c^2\epsilon_0\frac{1}{c}\frac{e^2\ddot{r}^2 sin^2\theta}{4^2\pi^2\epsilon_0^2 c^4 r^2}r^2 sin\theta d\theta d\phi = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\frac{e^2\ddot{r}^2 sin^3\theta}{4^2\pi^2\epsilon_0 c^3}d\theta d\phi$$

$$= \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\frac{e^2\ddot{r}^2 sin^3\theta}{4^2\pi^2\epsilon_0 c^3}d\theta d\phi = \frac{e^2\ddot{r}^2}{4^2\pi^2\epsilon_0 c^3}(\frac{4}{3})(2\pi)$$

$$P = \frac{dU}{dt} = \frac{e^2\ddot{r}^2}{6\pi\epsilon_0 c^3} \qquad (21).$$

Finally, with equation (21) we have the Larmor formula without involving any messy potentials or fields [2] [9]. Notice that the only variable in the field's power is that of the radial acceleration. This seems to fit in with our previous claim that radiation comes from

accelerating charges.

Now with an expression for the total power dissipated in terms of a point charges acceleration, we can proceed to solve for the time of the hydrogen atoms collapse.

III. The Collapse of the Hydrogen Atom

Since the Rutherford model describes an atomic electron as moving in a circular orbit around a positive nucleus, we can describe its net force as follows.

$$|\vec{F}| = m_e\ddot{r} = m_e\frac{v^2}{r} = \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r^2} \qquad (22)$$

In (22), Z is he atomic number of the nucleus, i.e. the number of constituent protons and neutrons. Since we are considering a hydrogen atom, $Z = 1$. Using this and dividing over the electron's mass, we can solve for the angular acceleration of the electron.

$$a = \frac{v^2}{r} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{m_e r^2} \qquad (23)$$

Our next step will be to find the total, classical energy. This will of course involve both the kinetic and potential energy. In order to write this, we can solve for the magnitude of the velocity squared from the angular force. We drop the sign because we are looking at the square of the velocities magnitude and would like to avoid an imaginary number; I believe physical intuition allows for this.

$$v^2 = \frac{1}{4\pi\epsilon_0}\frac{e^2}{m_e r} \qquad (24)$$

This gives us the classical kinetic energy as

$$T = \frac{1}{2}m_e\left(\frac{1}{4\pi\epsilon_0}\frac{e^2}{m_e r}\right) = \frac{1}{8\pi\epsilon_0}\frac{e^2}{r} \qquad (25).$$

We can also find the electron's potential energy by taking the the negative integral of the force.

$$U = \frac{1}{4\pi\epsilon_0}\int_0^r \frac{e^2}{r^2}dr = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{r} \qquad (26)$$

With these, the total energy becomes

$$U_{tot} = T + U = \frac{1}{8\pi\epsilon_0}\frac{e^2}{r} - \frac{1}{4\pi\epsilon_0}\frac{e^2}{r} = -\frac{1}{8\pi\epsilon_0}\frac{e^2}{r} \qquad (27).$$

The power is then defined at $P = -\frac{dU_{tot}}{dt}$. So taking the negative derivative of the total energy with respect to time, we gain the following expression for the power.

$$P(t) = -\frac{dU_{tot}}{dt} = -\frac{1}{8\pi\epsilon_0}\frac{e^2}{r^2}\frac{dr}{dt} \qquad (28)$$

Since this same quantity is described by the Larmor formula (21), we can use this, as well as the electron's acceleration (23) to gain the following.

$$\frac{e^2\ddot{r}^2}{6\pi\epsilon_0 c^3} = -\frac{1}{8\pi\epsilon_0}\frac{e^2}{r^2}\frac{dr}{dt}$$

Moving the $\frac{dr}{dt}$ term to the left hand side gives us

$$\frac{dt}{dr} = -\frac{1}{8\pi\epsilon_0}\frac{e^2}{r^2}\frac{6\pi\epsilon_0 c^3}{e^2\ddot{r}^2} = -\frac{1}{8\pi\epsilon_0}\frac{e^2}{r^2}\frac{6\pi\epsilon_0 c^3}{e^2}\left(\frac{4\pi\epsilon_0 m_e r^2}{e^2}\right)^2$$

$$\frac{dt}{dr} = -\frac{1}{8\pi\epsilon_0}\frac{e^2}{r^2}\frac{6\pi\epsilon_0 c^3}{e^2}\frac{4^2\pi^2\epsilon_0^2 m_e^2 r^4}{e^4} = -\frac{12\pi^2\epsilon_0^2 m_e^2 r^2 c^3}{e^4} \qquad (29).$$

From here, we can set up and expression for the unit time as

$$dt = -\frac{12\pi^2\epsilon_0^2 m_e^2 r^2 c^3}{e^4}dr \qquad (30).$$

We use this to set up and integral expression for the time it would take for the electron to move between different orbital radii. We can set the electron's initial orbital at the value a_0, which is the Bohr radius. While this value was not known by the time the Rutherford model was considered the cutting edge of atomic physics, it corresponds to the orbital radius of the hydrogen atom's ground state, i.e. when the hydrogen atom possesses no additional energy other than that inherent in its own structure. We can then set the final radius to zero, where the electron has completely collapsed into the orbital center of mass.

$$t = -\int_{a_0}^{0}\frac{12\pi^2\epsilon_0^2 m_e^2 r^2 c^3}{e^4}dr = \int_0^{a_0}\frac{12\pi^2\epsilon_0^2 m_e^2 r^2 c^3}{e^4}dr \qquad (31)$$

Then evaluating the integral between the two bounds specified above,

$$t = \frac{4\pi^2\epsilon_0^2 m_e^2 a_0^3 c^3}{e^4} \qquad (32).$$

As a final step, we simply need to substitute the values for each constant in the expression. Evaluating (32) where $m_e \approx 9.11 \times 10^{-31}$ kg, $e \approx 1.602 \times 10^{-19}$ C, $c \approx 2.99 \times 10^8$ m/s, $\epsilon_0 \approx 8.85 \times 10^{-12}$ F/m, and $a_0 \approx 5.29 \times 10^{-11}$ m, we find that it would take a the electron of a hydrogen atom roughly

$$t \approx 1.5 \times 10^{-11} \text{ s}$$

to collapse into its nucleus.

This is clearly such a minuscule amount of time that it precludes the existence of stable hydrogen atoms, even on the scale of a human lifetime. Additionally, this is only the case of a hydrogen atom with a single proton in its nucleus. For atoms with greater number of protons, this collapse may even happen quicker due to the resulting greater Coulomb attractive force. The accuracy of this number could be improved in several ways, for instance, we didn't even consider the reduced electron mass that arises when we consider the electron orbit as a central body problem. However, any of these changes would be small, barely, if at all changing the order of magnitude. Nonetheless, this does show that there is a fatal flaw in the Rutherford model, which itself was a response to the failures of other classical models which had come before it.

The fix for this would ultimately come just a few years later in the form of the Bohr model. While preserving the the basic layout of the Rutherford model, the Bohr model describes the orbits of the atomic electrons as discrete, corresponding to a structurally determined angular momenta which define specific energy states. When in an orbit, an electron will not emit any electromagnetic radiation, only releasing energy as it "jumps" between predetermined energy states. This model not only fixed the problem laid out in the paper, but also provided the first theoretical explanation for the specific spectral lines touched upon much earlier. So while the Rutherford model does fail to describe atomic behavior in some very major ways, it nonetheless was an important step in atomic physics, as was every other model that came before.

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