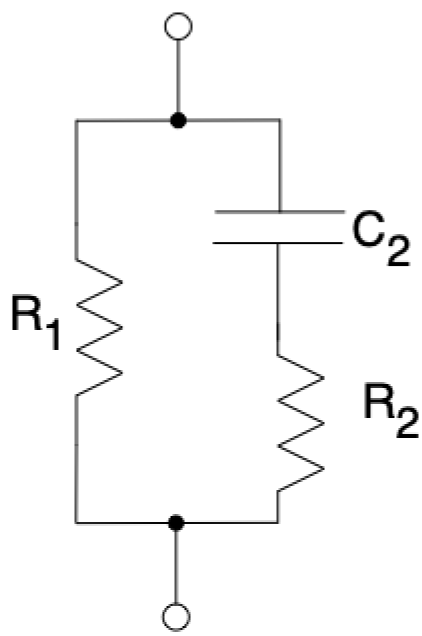
In [5]:
```
import os
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from scipy import stats
from scipy.optimize import curve_fit
%matplotlib inline
```

Impedance Spectroscopy Notebook

James Amidei, Zach Stedman, Max Markgraf

In this experiment, we tested two "systems under test" (i.e. s.u.t.). Or initial s.u.t was a circuit composed of a resistor and capacitor in series, together in parallel with another resistor. Our second was a russet potato.



(figure 1. Schematic of the inital s.u.t. used in this experiment.)

$$Z_{tot} = R_1 + \left(\frac{1}{R_2} + \frac{1}{j\omega C_2}\right)^{-1} = R_1 + \frac{j\omega C_2 R_2}{j\omega C_2 + R_2}$$

$$\frac{Z}{R_1} = \frac{1 + j\frac{\omega}{\omega_c}\frac{1}{1+\frac{R_1}{R_2}}}{1 + j\frac{\omega}{\omega_c}} = \frac{\left[1 + \left(\frac{\omega}{\omega_c}\right)^2 \frac{1}{1+\frac{R_1}{R_2}}\right] - j\left[\frac{\omega}{\omega_c}\frac{\frac{R1}{R2}}{1+\frac{R1}{R2}}\right]}{1 + \left(\frac{\omega}{\omega_c}\right)^2}$$

(Equation for the impedance of the circuit s.u.t in natural units. This is derived by adding the impedance of C_2 and the resistance of R_2 in series, and then adding the resistance of R_1 in parallel.)

To simplify the equation above, we took a note from the "NonlinearLeastSquaresFitImpedanceModel" file posted to canvas as defined both $\tau = \frac{1}{\omega_c}$, $f = \frac{\omega}{2\pi}$, and $r = \frac{R_1}{R_2}$. This gives us the following.

$$\frac{Z}{R_1} = \frac{1 + j2\pi f\tau \frac{1}{1+r}}{1 + j2\pi f\tau}$$

$$\frac{|Z|}{R_1} = \sqrt{\frac{1 + (2\pi f\tau)^2 \frac{1}{(1+r)^2}}{1 + (2\pi f\tau)^2}}$$

Which gives us the phase value of

$$\phi = arctan\left(-\frac{2\pi f\tau r}{1 + r + (2\pi f\tau)^2} \right)$$

These reworked values give us the benefit of being more computationally stable, since there are fewer multipplications and divisions of potentially very small and very larger numbers, as well as returns a normalized value of the impedance. This normalized value is independent of scale while preserving the behavior of the system, and thus makes it easier to see if any intrinsic characteristics arise in the math. In particular, we'll try to pay attention to the constants τ and r.

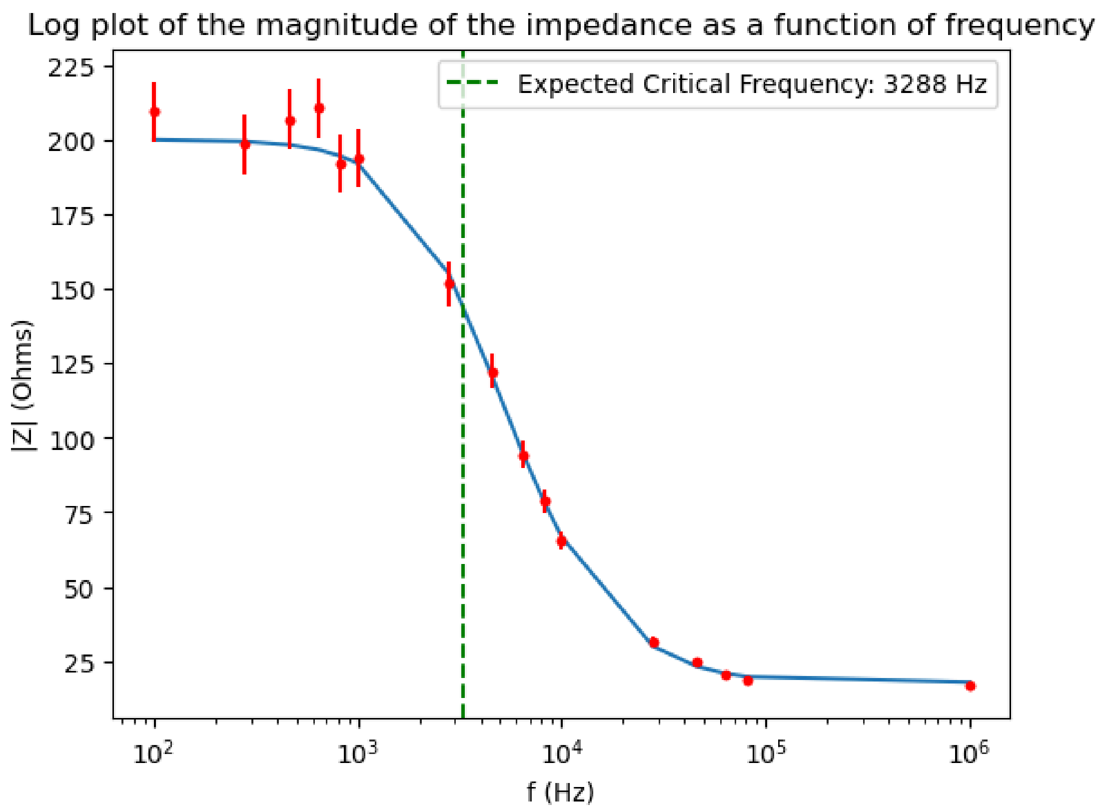
Calculated values

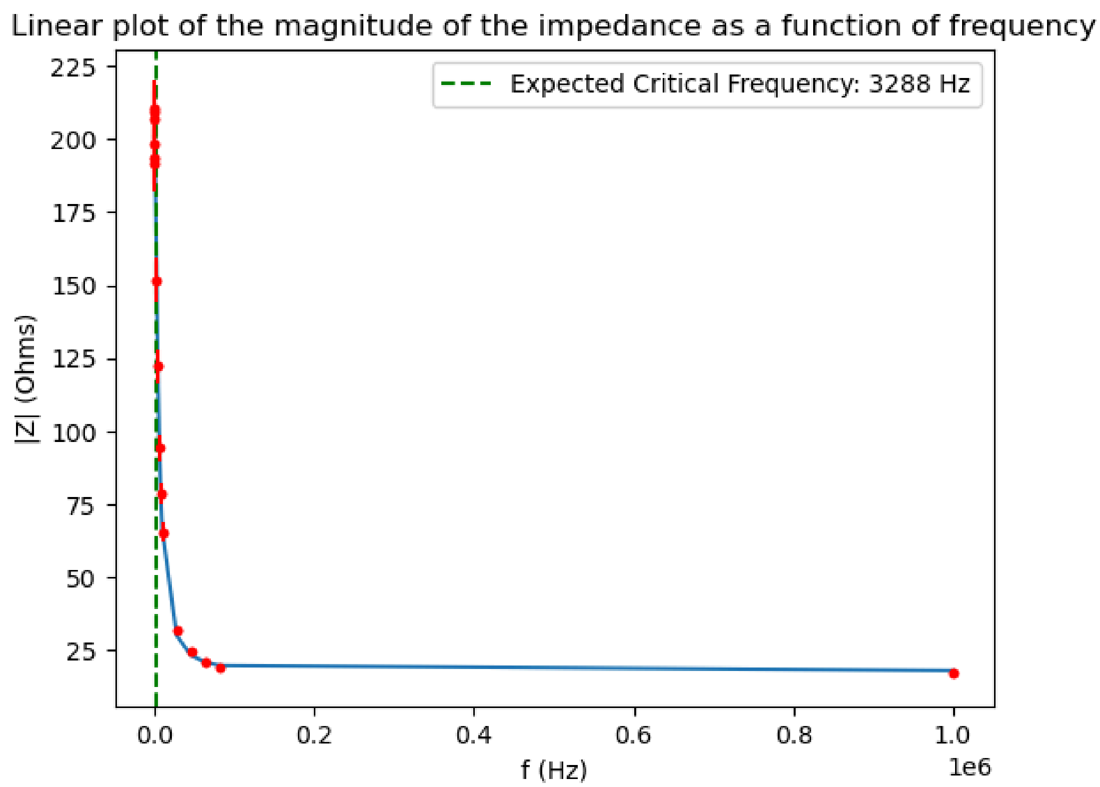
This is what we would expect to see given the expected values of our components. These are our initial calculation

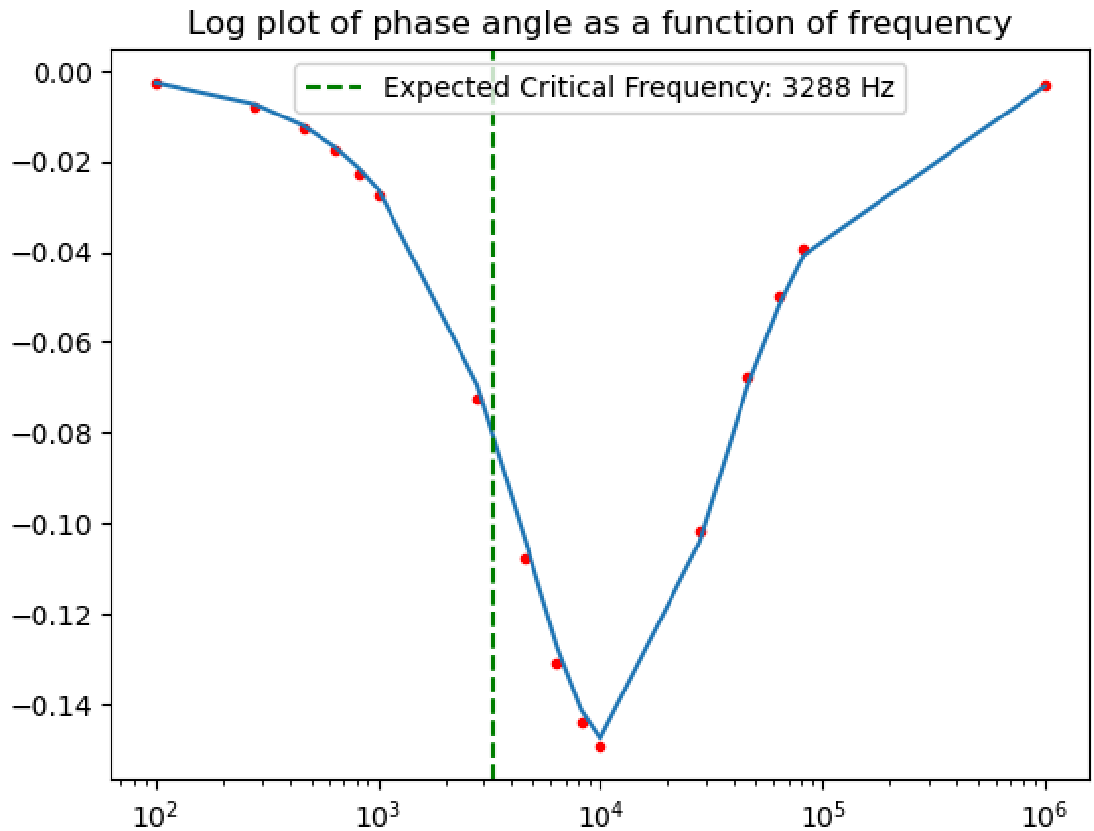
In [6]:
```
# These frequency values were chosen because they give us 5 even steps per
f = np.array([100, 280, 460, 640, 820, 1000, 2800, 4600, 6400, 8200, 10000

N_data_points = len(f)

def zfunc(ff, rr, tautau):
    ft2 = (2*np.pi*ff*tautau)**2
    zz = np.sqrt((1+ft2/(1+rr)**2)/(1+ft2))
    return zz

def phifunc(ff, rr, tautau):
    fftt = (2*np.pi*ff*tautau)
    phiphi = np.arctan(-fftt/(1+rr+fftt**2))
    return phiphi

R1 = 200 # Ohms
R2 = 20 # Ohms
C2 = 0.22e-6 # Farads

omega_c = 1/(C2*(R1+R2)) # critical angular frequency

f_c = omega_c/(2*np.pi) # critical frequnecy

r = R1/R2
tau = 1/(omega_c)

z_sigma_rel = 0.05 # relative uncertainty in impedance measurement
phase_sigma_rel = 0.2 # degrees

# normalized impedance data (impedance divided by DC resistance R1)
z_exact = zfunc(f, r, tau)
z_sigma = z_sigma_rel*z_exact
z = np.random.normal(z_exact, z_sigma, N_data_points) # create some random

phase_exact = phifunc(f, r, tau)
phase_sigma = phase_sigma_rel*phase_exact
#phase = np.random.normal(phase_exact, phase_sigma, N_data_points)
phase = phase_exact

r_guess = 10. # guess of ratio of DC resistance R1 to capacitive path resi
f_c_guess = 3500 # guess of critical frequency in Hz
tau_guess = 1/(2*np.pi*f_c_guess)

print('r_guess = {0:.1f}  tau_guess = {1:.3e} s'.format(r_guess,tau_guess)

popt, pcov = curve_fit(zfunc,f,z,p0=[r_guess,tau_guess])
r_fit = popt[0]
tau_fit = popt[1]

print('r_fit = {0:.3f}  tau_fit = {1:.3e} s   f_c_fit = {2:.3e} Hz'
      .format(r_fit, tau_fit, 1/(2*np.pi*tau_fit)))

print('R1 = {0:.1f} ohms    R2_fit = {1:.1f} ohms   C_2_fit = {2:.3e} F'
      .format(R1, R1/r_fit, tau_fit/(R1*(1+1/r_fit))))

zfit = zfunc(f, r_fit, tau_fit)
phi_fit = phifunc(f, r_fit, tau_fit)
```

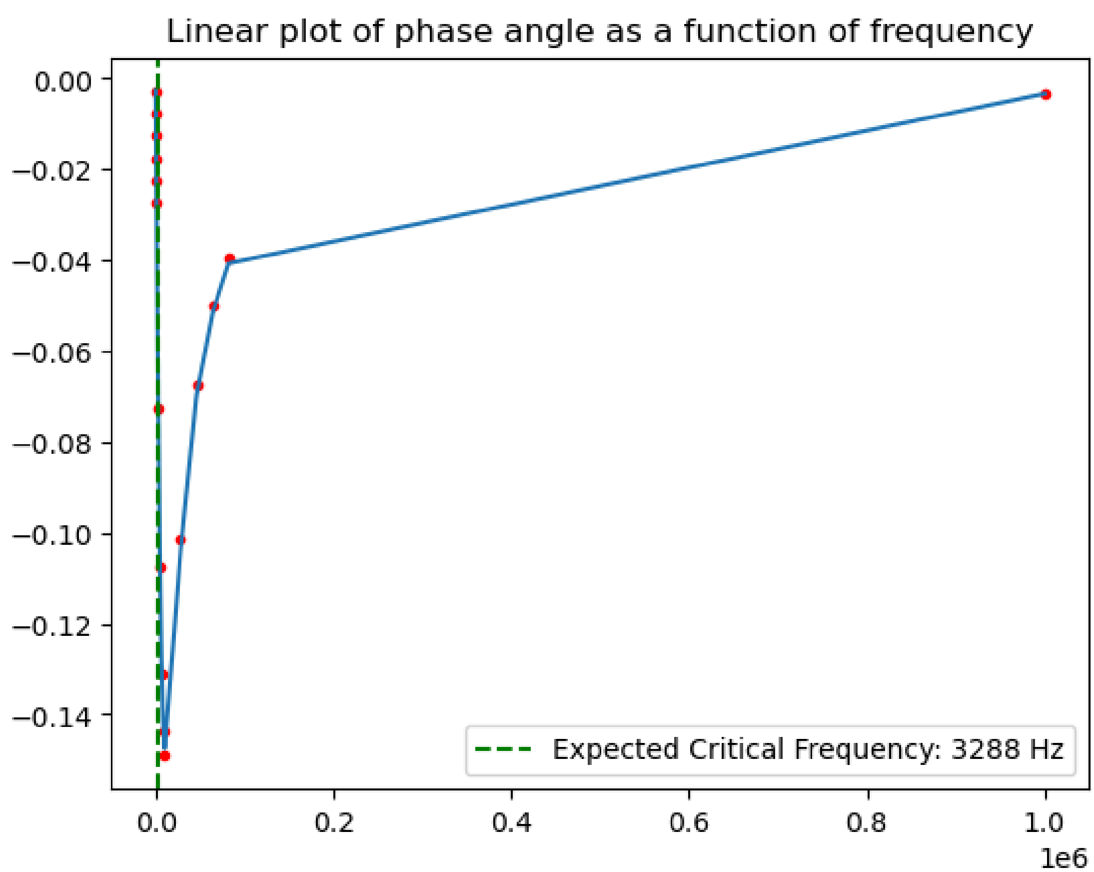
```
plt.figure(0)
plt.plot(f, z*R1,'r.', f, zfit*R1)
plt.axvline(x=f_c, color='g', linestyle='--', label=f'Expected Critical Fr
plt.xscale('log')
plt.errorbar(f, z*R1, yerr=z_sigma*R1, fmt='r.')
plt.xlabel('f (Hz)')
plt.ylabel('|Z| (Ohms)')
plt.title('Log plot of the magnitude of the impedance as a function of fre
plt.legend()

plt.figure(1)
plt.plot(f, z*R1,'r.', f, zfit*R1)
plt.axvline(x=f_c, color='g', linestyle='--', label=f'Expected Critical Fr
#plt.xscale('log')
plt.errorbar(f, z*R1, yerr=z_sigma*R1, fmt='r.')
plt.xlabel('f (Hz)')
plt.ylabel('|Z| (Ohms)')
plt.title('Linear plot of the magnitude of the impedance as a function of
plt.legend()

plt.figure(2)
plt.plot(f, phase, 'r.', f, phi_fit )
plt.axvline(x=f_c, color='g', linestyle='--', label=f'Expected Critical Fr
plt.xscale('log')
plt.title('Log plot of phase angle as a function of frequency')
plt.legend()

plt.figure(3)
plt.plot(f, phase, 'r.', f, phi_fit )
plt.axvline(x=f_c, color='g', linestyle='--', label=f'Expected Critical Fr
#plt.xscale('log')
plt.title('Linear plot of phase angle as a function of frequency')
plt.legend()

plt.show()
```

```
r_guess = 10.0  tau_guess = 4.547e-05 s
r_fit = 10.140  tau_fit = 4.671e-05 s   f_c_fit = 3.407e+03 Hz
R1 = 200.0 ohms    R2_fit = 19.7 ohms   C_2_fit = 2.126e-07 F
```


Log plot of the magnitude of the impedance as a function of frequency
Expected Critical Frequency: 3288 Hz
|Z| (Ohms)
f (Hz)
225
200
175
150
125
100
75
50
25
10^2
10^3
10^4
10^5
10^6


Linear plot of the magnitude of the impedance as a function of frequency
Expected Critical Frequency: 3288 Hz
|Z| (Ohms)
f (Hz)
225
200
175
150
125
100
75
50
25
0.0
0.2
0.4
0.6
0.8
1.0
1e6


Log plot of phase angle as a function of frequency
Expected Critical Frequency: 3288 Hz
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
10^2
10^3
10^4
10^5
10^6


Linear plot of phase angle as a function of frequency
Expected Critical Frequency: 3288 Hz
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
0.0
0.2
0.4
0.6
0.8
1.0
1e6

Collected Data

We performed three runs each with each of the three dominant resistors for each of the s.u.t. Data named "data_R0?_#" refers to data collect with the circuit used as the s.u.t. and data named "data_R0?p_#" refers to the data collected with the russet potato as the system under test. The "?" wildcard refers to the dominant resistor (i.e. a, b, or c) and "#" refers to the run number (e.g. 1, 2, or 3).

The average of the three runs was taken as used in the analysis to describe each setup. We then assign each column as a variable to work with later on in this notebook.

Note: Data was typed out by hand into .txt files. I got tried of trying to make a loop that ran through the files, so I just decided to do it by hand. Elegant? Absolutely not. Does it work?

In [3]:

```
# getting the data from our files
data_R0a_1 = np.genfromtxt('OrganizedData/ImpedanceSpecData3_18_24_R0a_Rur
data_R0a_2 = np.genfromtxt('OrganizedData/ImpedanceSpecData3_18_24_R0a_Rur
data_R0a_3 = np.genfromtxt('OrganizedData/ImpedanceSpecData3_18_24_R0a_Rur
data_R0b_1 = np.genfromtxt('OrganizedData/ImpedanceSpecData3_18_24_R0b_Rur
data_R0b_2 = np.genfromtxt('OrganizedData/ImpedanceSpecData3_18_24_R0b_Rur
data_R0b_3 = np.genfromtxt('OrganizedData/ImpedanceSpecData3_18_24_R0b_Rur
data_R0c_1 = np.genfromtxt('OrganizedData/ImpedanceSpecData3_21_24_R0c_Rur
data_R0c_2 = np.genfromtxt('OrganizedData/ImpedanceSpecData3_21_24_R0c_Rur
data_R0c_3 = np.genfromtxt('OrganizedData/ImpedanceSpecData3_21_24_R0c_Rur
data_R0ap_1 = np.genfromtxt('OrganizedData/ImpedanceSpecData3_21_24_R0a_wi
data_R0ap_2 = np.genfromtxt('OrganizedData/ImpedanceSpecData3_21_24_R0a_wi
data_R0ap_3 = np.genfromtxt('OrganizedData/ImpedanceSpecData3_21_24_R0a_wi
data_R0bp_1 = np.genfromtxt('OrganizedData/ImpedanceSpecData3_21_24_R0b_wi
data_R0bp_2 = np.genfromtxt('OrganizedData/ImpedanceSpecData3_21_24_R0b_wi
data_R0bp_3 = np.genfromtxt('OrganizedData/ImpedanceSpecData3_21_24_R0b_wi
data_R0cp_1 = np.genfromtxt('OrganizedData/ImpedanceSpecData3_21_24_R0c_wi
data_R0cp_2 = np.genfromtxt('OrganizedData/ImpedanceSpecData3_21_24_R0c_wi
data_R0cp_3 = np.genfromtxt('OrganizedData/ImpedanceSpecData3_21_24_R0c_wi
```

In [13]:
```
# averaging data
data_R0a_avg = (data_R0a_1 + data_R0a_2 + data_R0a_3)/3
data_R0b_avg = (data_R0b_1 + data_R0b_2 + data_R0b_3)/3
data_R0c_avg = (data_R0c_1 + data_R0c_2 + data_R0c_3)/3
data_R0ap_avg = (data_R0ap_1 + data_R0ap_2 + data_R0ap_3)/3
data_R0bp_avg = (data_R0bp_1 + data_R0bp_2 + data_R0bp_3)/3
data_R0cp_avg = (data_R0cp_1 + data_R0cp_2 + data_R0cp_3)/3

# defining variable from data
freq_a = data_R0a_avg[:,0]
sigma_freq_a = data_R0a_avg[:,1]
freq_b = data_R0b_avg[:,0]
sigma_freq_b = data_R0b_avg[:,1]
freq_c = data_R0c_avg[:,0]
sigma_freq_c = data_R0c_avg[:,1]
freq_ap = data_R0ap_avg[:,0]
sigma_freq_ap = data_R0ap_avg[:,1]
freq_bp = data_R0bp_avg[:,0]
sigma_freq_bp = data_R0bp_avg[:,1]
freq_cp = data_R0cp_avg[:,0]
sigma_freq_cp = data_R0cp_avg[:,1]

voltage_a = data_R0a_avg[:,2]
sigma_voltage_a = data_R0a_avg[:,3]
voltage_b = data_R0b_avg[:,2]
sigma_voltage_b = data_R0b_avg[:,3]
voltage_c = data_R0c_avg[:,2]
sigma_voltage_c = data_R0c_avg[:,3]
voltage_ap = data_R0ap_avg[:,2]
sigma_voltage_ap = data_R0ap_avg[:,3]
voltage_bp = data_R0bp_avg[:,2]
sigma_voltage_bp = data_R0bp_avg[:,3]
voltage_cp = data_R0cp_avg[:,2]
sigma_voltage_cp = data_R0cp_avg[:,3]

phase_a = data_R0a_avg[:,4]
sigma_phase_a = data_R0a_avg[:,5]
phase_b = data_R0b_avg[:,4]
sigma_phase_b = data_R0b_avg[:,5]
phase_c = data_R0c_avg[:,4]
sigma_phase_c = data_R0c_avg[:,5]
phase_ap = data_R0ap_avg[:,4]
sigma_phase_ap = data_R0ap_avg[:,5]
phase_bp = data_R0bp_avg[:,4]
sigma_phase_bp = data_R0bp_avg[:,5]
phase_cp = data_R0cp_avg[:,4]
sigma_phase_cp = data_R0cp_avg[:,5]
```

Analysis with collected data

This is what we see from our actaully measured data. Each component in the ciruit s.u.t. was measured using a LCR multimeter before performing any runs. These's values and their errors are what's used here.

Error propagation

$$\sigma_c = \sigma_a + \sigma_b \qquad \text{for addition and subtraction}$$
$$\sigma_c = \sum_i^N \left(\sigma_i \frac{\partial C}{\partial x_i}\right)^2 \qquad \text{for multiplication and division}$$

$$\omega_c = \frac{1}{C_2(R_1 + R_1)}$$

$$\frac{\partial \omega_c}{\partial C_2} = -\frac{1}{C_2^2(R_1 + R_2)} \qquad \frac{\partial = \omega_c}{\partial R_1} - \frac{1}{C_2(R_1 + R_2)^2}$$
$$\frac{\partial \omega_c}{\partial R_2} = -\frac{1}{C_2(R_1 + R_2)^2}$$

$$r = \frac{R_1}{R_2}$$

$$\frac{\partial r}{\partial R_1} = \frac{1}{R_2} \qquad \frac{\partial r}{\partial R_2} = -\frac{R_1}{R_2^2}$$

$$\tau = C_2(R_1 + R_2)$$

$$\frac{|Z|}{R_1} = \sqrt{\frac{1 + (2\pi f\tau)^2 \frac{1}{(1+r)^2}}{1 + (2\pi f\tau)^2}}$$

$$\frac{1}{R_1}\frac{\partial |Z|}{\partial f} = \frac{2^2 \pi^2 f \tau^2}{(r+1)^2 \sqrt{(2\pi f\tau)^2 + 1}\sqrt{\frac{(2\pi f\tau)^2}{(r+1)^2} + 1}} - \frac{2^2 \pi^2 f \tau^2 \sqrt{\frac{(2\pi f\tau)^2}{(r+1)^2} + 1}}{((2\pi f\tau)^2 + 1)^{3/2}}$$

$$\frac{1}{R_1}\frac{\partial |Z|}{\partial \tau} = \frac{2^2 \pi^2 f^2 \tau}{(r+1)^2 \sqrt{(2\pi f\tau)^2 + 1}\sqrt{\frac{(2\pi f\tau)^2}{(r+1)^2} + 1}} - \frac{2^2 \pi^2 f^2 \tau \sqrt{\frac{(2\pi f\tau)^2}{(r+1)^2} + 1}}{((2\pi f\tau)^2 + 1)^{3/2}}$$

$$\frac{1}{R_1}\frac{\partial |Z|}{\partial r} = -\frac{(2\pi f\tau)^2}{(r+1)^3 \sqrt{(2\pi f\tau)^2 + 1}\sqrt{\frac{(2\pi f\tau)^2}{(r+1)^2} + 1}}$$

In [8]:
```
R1_real = 197.4 # Ohms
sigma_R1_real = 0.1 # Ohms

R2_real = 19.7 # Ohms
sigma_R2_real = 0.1 # Ohms

C2_real = 0.239e-6 # Farads
sigma_C2_real = 0.0005e-6 # Farads

omega_c_real = 1/(C2_real*(R1_real + R2_real))
del_o_del_c = -1/((R1_real+R2_real)*C2_real**2)
del_o_del_r1 = -1/(C2_real*(R1_real+R2_real)**2)
del_o_del_r2 = -1/(C2_real*(R1_real+R2_real)**2)
sigma_omega_c_real = np.sqrt((sigma_C2_real*del_o_del_c)**2 + (sigma_R1_re

f_c_real = omega_c_real/(2*np.pi)
sigma_f_c_real = sigma_omega_c_real/(2*np.pi)

tau_real = 1/omega_c_real

sigma_tau_real = np.sqrt( (sigma_C2_real*(R1_real+R2_real))**2 + ((sigma_R

r_real = R1_real/R2_real
del_r_del_r1 = 1/R2_real
del_r_del_r2 = -R1_real/R2_real**2
sigma_r_real = np.sqrt((sigma_R1_real*del_r_del_r1)**2 + (sigma_R2_real*de

print(f'f_c_real = {f_c_real:.2f} +/- {sigma_f_c_real:.2f}')
print()
print(f'r_real = {r_real:.2f} +/- {sigma_r_real:.2f}')
```

```
f_c_real = 3067.34 +/- 6.72

r_real = 10.02 +/- 0.05
```

R0a

In [9]:

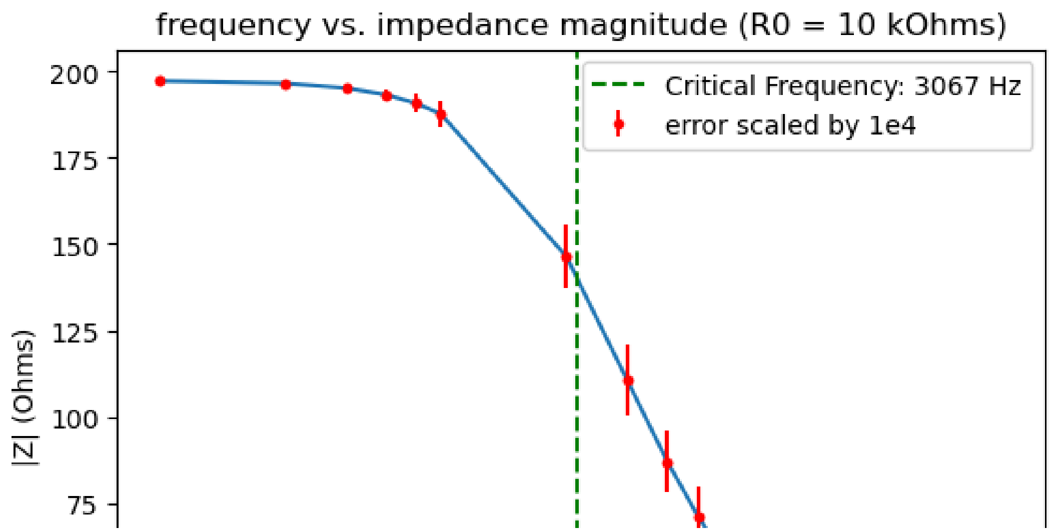
```
# calculation for error of the impedance

alpha = 2*np.pi*freq_a*tau_real
rho = r+1

del_z_del_fa = ((freq_a*(2*np.pi*tau_real)**2)/((rho**2) * np.sqrt((alpha*
del_z_del_tau = ((tau_real*(2*np.pi*freq_a)**2)/( (rho**2)*np.sqrt((alpha*
del_z_del_r = - ((alpha**2)/((rho**3)*np.sqrt(alpha**2 + 1)*np.sqrt((alpha

# Note: This value of the error for the impedance alreay assumes the R1 ho
# So there is no need to multiply it back in later.
# This would result in massively overblown error bars.
sigma_z_a = np.sqrt((sigma_freq_a*del_z_del_fa)**2 + (sigma_tau_real*del_z
```

In [10]:

```
z_exact_a = zfunc(freq_a, r_real, tau_real)
z_a = z_exact_a

phase_exact_a = phifunc(freq_a, r_real, tau_real)
phase_a = phase_exact_a

r_guess = 10.0 # guess of ratio of DC resistance R1 to capacitive path res
f_c_guess = 3000 # guess of critical frequency in Hz
tau_guess = 1/(2*np.pi*f_c_guess)

print('r_guess = {0:.1f}  tau_guess = {1:.3e} s'.format(r_guess, tau_guess

popt, pcov = curve_fit(zfunc, freq_a, z_exact_a, p0=[r_guess,tau_guess])
r_fit_a = popt[0]
tau_fit_a = popt[1]

print('r_fit = {0:.3f}  tau_fit = {1:.3e} s   f_c_fit = {2:.3e} Hz'
      .format(r_fit, tau_fit,1/(2*np.pi*tau_fit)))

print('R1 = {0:.1f} ohms    R2_fit = {1:.1f} ohms   C_2_fit = {2:.3e} F'
      .format(R1_real, R1_real/r_fit, tau_fit/(R1*(1+1/r_fit))))

zfit_a = zfunc(freq_a, r_fit_a, tau_fit_a)
phi_fit_a = phifunc(freq_a, r_fit_a, tau_fit_a)

plt.figure(0)
plt.plot(freq_a, z_a*R1_real,'r.', freq_a, zfit_a*R1_real)
plt.axvline(x=f_c_real, color='g', linestyle='--', label=f'Critical Freque
plt.xscale('log')
plt.errorbar(freq_a, z_a*R1_real, yerr=1e4*sigma_z_a, fmt='r.', label = 'e
plt.xlabel('f (Hz)')
plt.ylabel('|Z| (Ohms)')
plt.title('frequency vs. impedance magnitude (R0 = 10 kOhms)')
plt.legend()

plt.figure(1)
plt.plot(freq_a, z_a*R1_real,'r.', freq_a, zfit_a*R1_real)
plt.axvline(x=f_c_real, color='g', linestyle='--', label=f'Critical Freque
#plt.xscale('log')
plt.errorbar(freq_a, z_a*R1_real, yerr=1e4*sigma_z_a, fmt='r.', label = 'e
plt.xlabel('f (Hz)')
plt.ylabel('|Z| (Ohms)')
plt.title('Linear plot of the magnitude of the impedance as a function of
plt.legend()

plt.figure(2)
plt.plot(freq_a, phase_a, 'r.', freq_a, phi_fit_a )
plt.axvline(x=f_c_real, color='g', linestyle='--', label=f'Expected Critic
plt.errorbar(freq_a, phase_a, yerr=10*sigma_z_a, fmt='r.', label = 'error
plt.xscale('log')
plt.xlabel('f (Hz)')
plt.ylabel('radians')
plt.title('frequency v.s. phase angle (R0 = 10 kOhms)')
plt.legend()

plt.figure(3)
plt.plot(freq_a, phase_a, 'r.', freq_a, phi_fit_a )
```

```
plt.axvline(x=f_c_real, color='g', linestyle='--', label=f'Expected Critic
plt.errorbar(freq_a, phase_a, yerr=10*sigma_z_a, fmt='r.', label = 'error 
#plt.xscale('log')
plt.title('Linear plot of phase angle as a function of frequency (R0 = 10 
plt.legend()

plt.show()
```

```
r_guess = 10.0  tau_guess = 5.305e-05 s
r_fit = 10.140  tau_fit = 4.671e-05 s   f_c_fit = 3.407e+03 Hz
R1 = 197.4 ohms    R2_fit = 19.5 ohms   C_2_fit = 2.126e-07 F
```



R0b

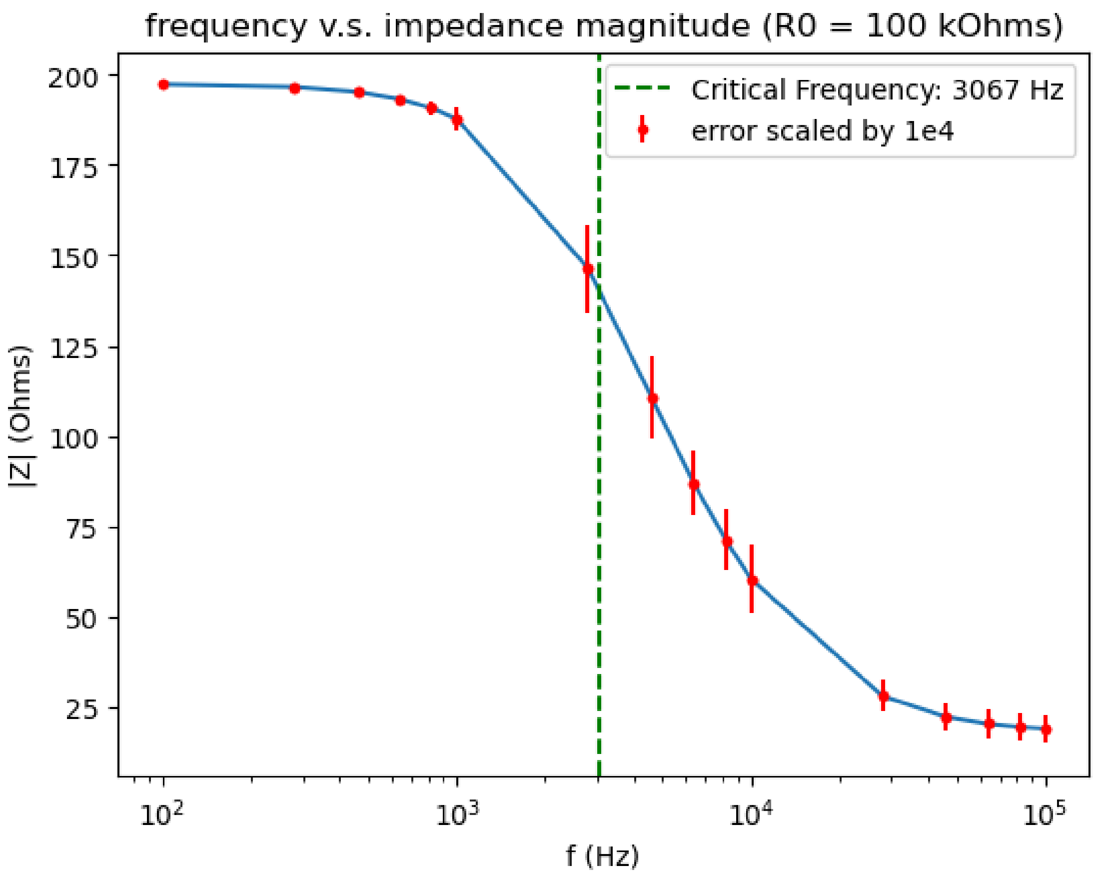
In [11]:

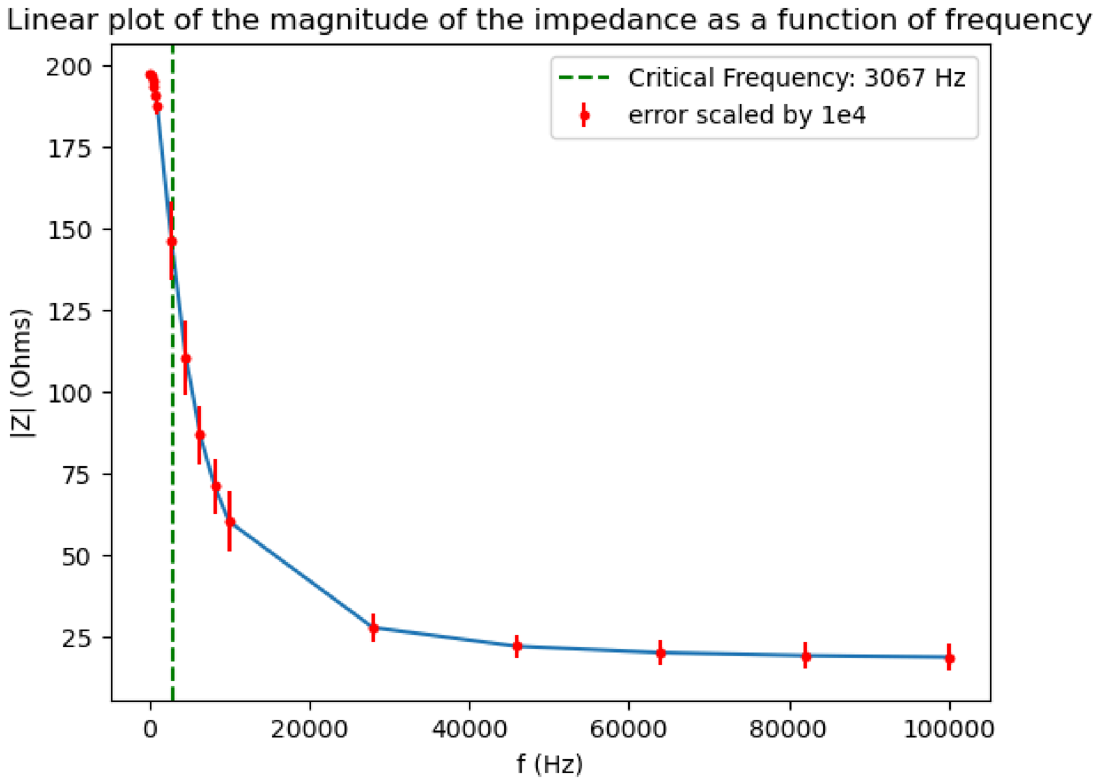
```
alpha = 2*np.pi*freq_b*tau_real
rho = r+1

del_z_del_fb = ((freq_b*(2*np.pi*tau_real)**2)/((rho**2) * np.sqrt((alpha*
del_z_del_tau = ((tau_real*(2*np.pi*freq_b)**2)/( (rho**2)*np.sqrt((alpha*
del_z_del_r = - ((alpha**2)/((rho**3)*np.sqrt(alpha**2 + 1)*np.sqrt((alpha

# Note: This value of the error for the impedance alreay assumes the R1 ho
# So there is no need to multiply it back in later.
# This would result in massively overblown error bars.
sigma_z_b = np.sqrt((sigma_freq_b*del_z_del_fa)**2 + (sigma_tau_real*del_z
```

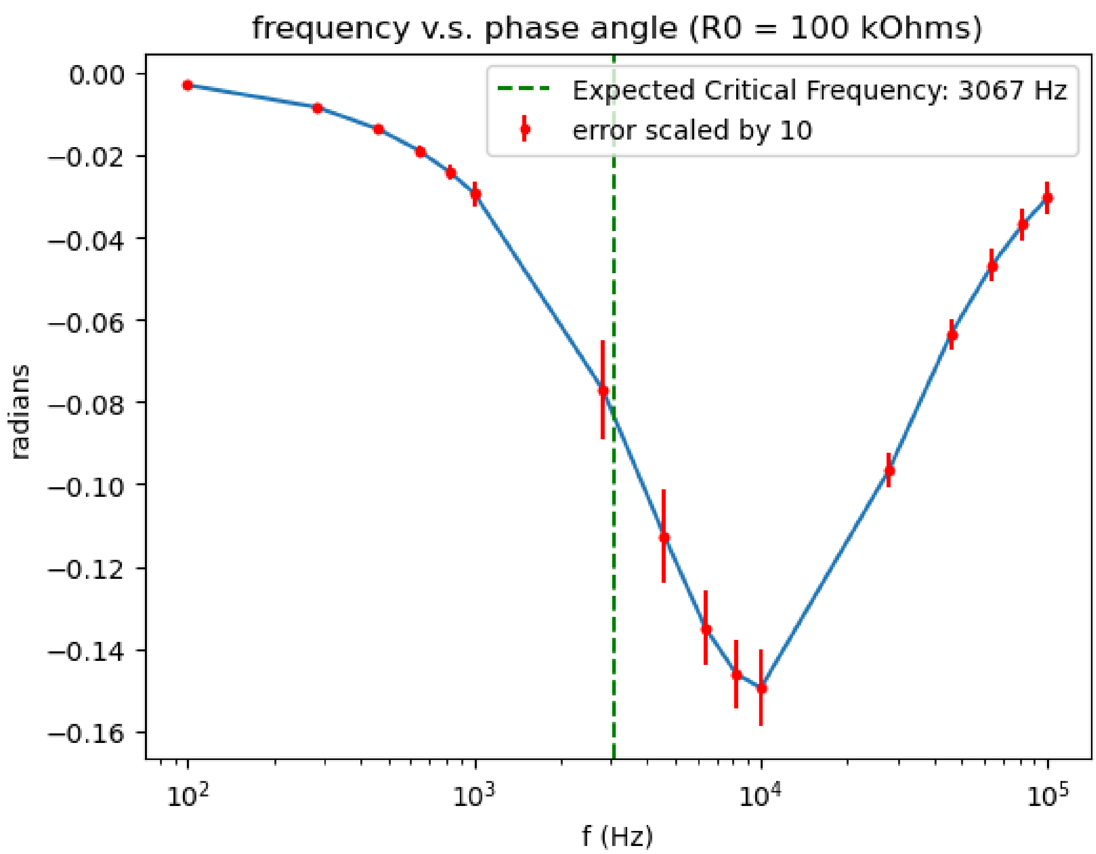
In [12]:
```
z_exact_b = zfunc(freq_b, r_real, tau_real)
z_b = z_exact_b

phase_exact_b = phifunc(freq_b, r_real, tau_real)
phase_b = phase_exact_b

r_guess = 10.0 # guess of ratio of DC resistance R1 to capacitive path res
f_c_guess = 3000 # guess of critical frequency in Hz
tau_guess = 1/(2*np.pi*f_c_guess)

print('r_guess = {0:.1f}  tau_guess = {1:.3e} s'.format(r_guess, tau_guess

popt, pcov = curve_fit(zfunc, freq_b, z_exact_b, p0=[r_guess,tau_guess])
r_fit_b = popt[0]
tau_fit_b = popt[1]

print('r_fit = {0:.3f}  tau_fit = {1:.3e} s   f_c_fit = {2:.3e} Hz'
      .format(r_fit, tau_fit,1/(2*np.pi*tau_fit)))

print('R1 = {0:.1f} ohms    R2_fit = {1:.1f} ohms   C_2_fit = {2:.3e} F'
      .format(R1_real, R1_real/r_fit, tau_fit/(R1*(1+1/r_fit))))

zfit_b = zfunc(freq_b, r_fit_b, tau_fit_b)
phi_fit_b = phifunc(freq_b, r_fit_b, tau_fit_b)

plt.figure(0)
plt.plot(freq_b, z_b*R1_real,'r.', freq_b, zfit_b*R1_real)
plt.axvline(x=f_c_real, color='g', linestyle='--', label=f'Critical Freque
plt.xscale('log')
plt.errorbar(freq_b, z_b*R1_real, yerr=1e4*sigma_z_b, fmt='r.',label = 'er
plt.xlabel('f (Hz)')
plt.ylabel('|Z| (Ohms)')
plt.title('frequency v.s. impedance magnitude (R0 = 100 kOhms)')
plt.legend()

plt.figure(1)
plt.plot(freq_b, z_b*R1_real,'r.', freq_b, zfit_b*R1_real)
plt.axvline(x=f_c_real, color='g', linestyle='--', label=f'Critical Freque
#plt.xscale('log')
plt.errorbar(freq_b, z_b*R1_real, yerr=1e4*sigma_z_b, fmt='r.', label = 'e
plt.xlabel('f (Hz)')
plt.ylabel('|Z| (Ohms)')
plt.title('Linear plot of the magnitude of the impedance as a function of 
plt.legend()

plt.figure(2)
plt.plot(freq_b, phase_b, 'r.', freq_b, phi_fit_b)
plt.axvline(x=f_c_real, color='g', linestyle='--', label=f'Expected Critic
plt.errorbar(freq_b, phase_b, yerr=10*sigma_z_b, fmt='r.',label = 'error s
plt.xscale('log')
plt.xlabel('f (Hz)')
plt.ylabel('radians')
plt.title('frequency v.s. phase angle (R0 = 100 kOhms)')
plt.legend()

plt.figure(3)
plt.plot(freq_b, phase_b, 'r.', freq_b, phi_fit_b)
```

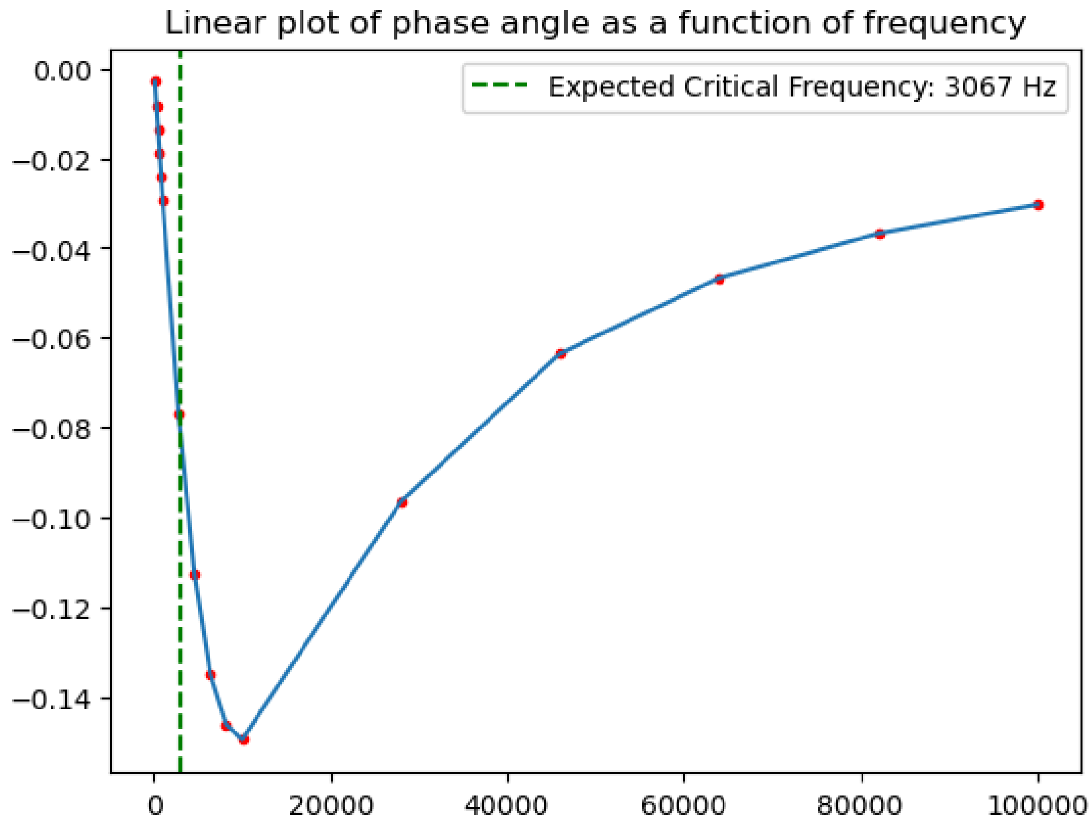
```
plt.axvline(x=f_c_real, color='g', linestyle='--', label=f'Expected Critic
#plt.xscale('log')
plt.title('Linear plot of phase angle as a function of frequency')
plt.legend()

plt.show()
```

```
r_guess = 10.0  tau_guess = 5.305e-05 s
r_fit = 10.140  tau_fit = 4.671e-05 s   f_c_fit = 3.407e+03 Hz
R1 = 197.4 ohms    R2_fit = 19.5 ohms   C_2_fit = 2.126e-07 F
```




Linear plot of the magnitude of the impedance as a function of frequency
Critical Frequency: 3067 Hz
error scaled by 1e4
|Z| (Ohms)
f (Hz)
200
175
150
125
100
75
50
25
0
20000
40000
60000
80000
100000


frequency v.s. phase angle (R0 = 100 kOhms)
Expected Critical Frequency: 3067 Hz
error scaled by 10
radians
f (Hz)
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
10^2
10^3
10^4
10^5



R0c

In [47]:

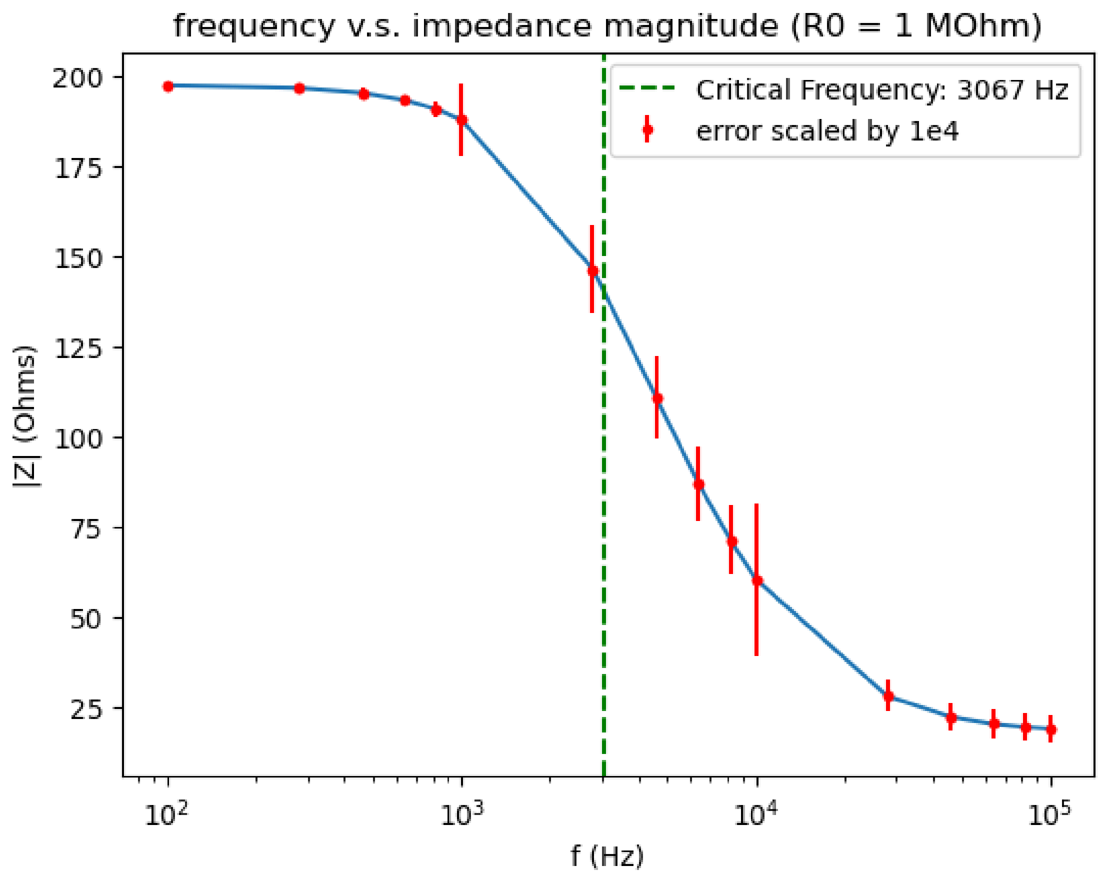
```
alpha = 2*np.pi*freq_c*tau_real
rho = r+1

del_z_del_fb = ((freq_c*(2*np.pi*tau_real)**2)/((rho**2) * np.sqrt((alpha*
del_z_del_tau = ((tau_real*(2*np.pi*freq_c)**2)/( (rho**2)*np.sqrt((alpha*
del_z_del_r = - ((alpha**2)/((rho**3)*np.sqrt(alpha**2 + 1)*np.sqrt((alpha

# Note: This value of the error for the impedance alreay assumes the R1 ho
# So there is no need to multiply it back in later.
# This would result in massively overblown error bars.
sigma_z_c = np.sqrt((sigma_freq_c*del_z_del_fa)**2 + (sigma_tau_real*del_z
```

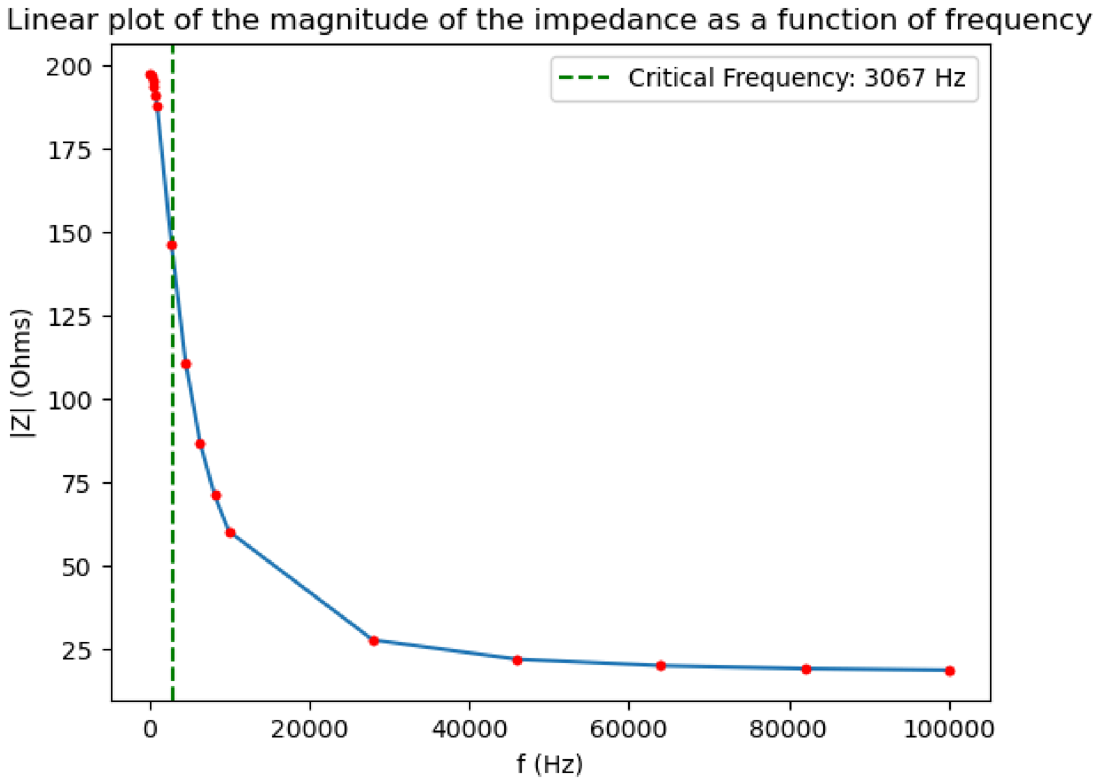
In [51]:

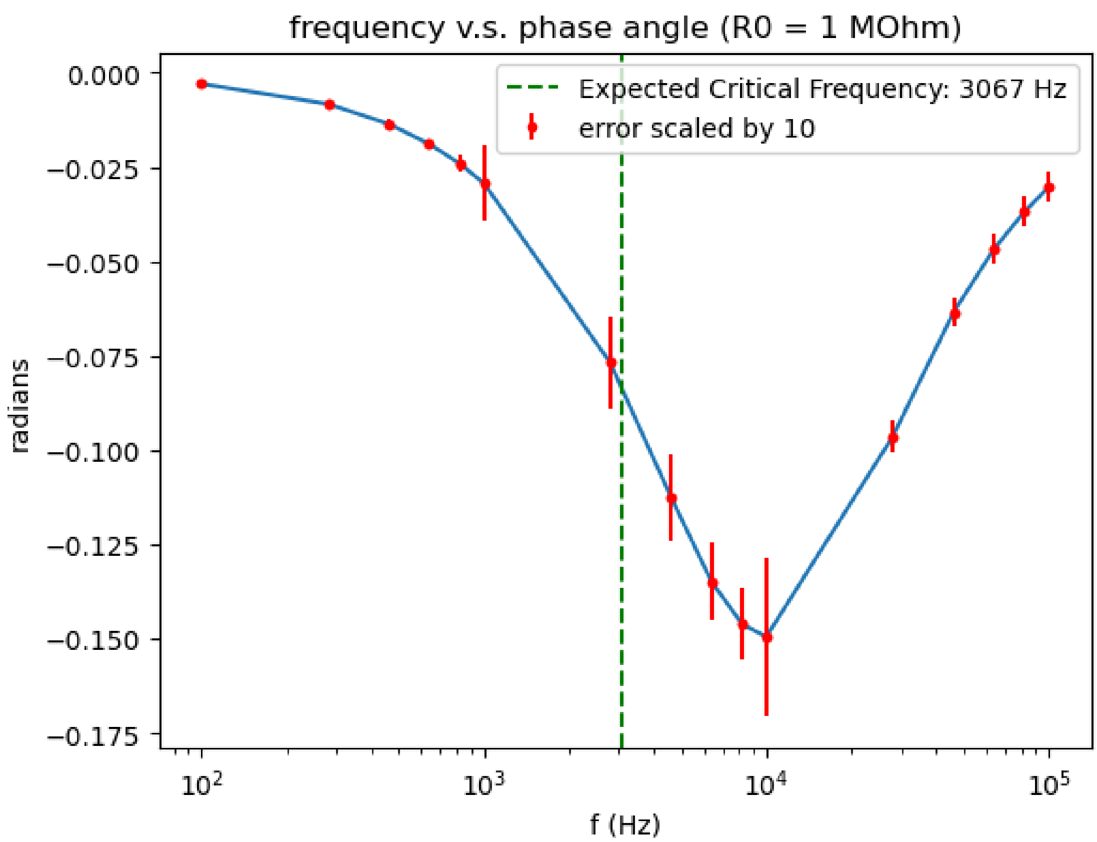
```
z_exact_c = zfunc(freq_c, r_real, tau_real)
z_c = z_exact_c

phase_exact_c = phifunc(freq_c, r_real, tau_real)
phase_c = phase_exact_c

r_guess = 10.0 # guess of ratio of DC resistance R1 to capacitive path res
f_c_guess = 3000 # guess of critical frequency in Hz
tau_guess = 1/(2*np.pi*f_c_guess)

print('r_guess = {0:.1f}  tau_guess = {1:.3e} s'.format(r_guess, tau_guess

popt, pcov = curve_fit(zfunc, freq_c, z_exact_c, p0=[r_guess,tau_guess])
r_fit_c = popt[0]
tau_fit_c = popt[1]

print('r_fit = {0:.3f}  tau_fit = {1:.3e} s   f_c_fit = {2:.3e} Hz'
      .format(r_fit, tau_fit,1/(2*np.pi*tau_fit)))

print('R1 = {0:.1f} ohms    R2_fit = {1:.1f} ohms   C_2_fit = {2:.3e} F'
      .format(R1_real, R1_real/r_fit, tau_fit/(R1*(1+1/r_fit))))

zfit_c = zfunc(freq_c, r_fit_b, tau_fit_b)
phi_fit_c = phifunc(freq_c, r_fit_c, tau_fit_c)

plt.figure(0)
plt.plot(freq_c, z_c*R1_real,'r.', freq_c, zfit_c*R1_real)
plt.axvline(x=f_c_real, color='g', linestyle='--', label=f'Critical Freque
plt.xscale('log')
plt.errorbar(freq_c, z_c*R1_real, yerr=1e4*sigma_z_c, fmt='r.', label='err
plt.xlabel('f (Hz)')
plt.ylabel('|Z| (Ohms)')
plt.title('frequency v.s. impedance magnitude (R0 = 1 MOhm)')
plt.legend()

plt.figure(1)
plt.plot(freq_c, z_c*R1_real,'r.', freq_c, zfit_c*R1_real)
plt.axvline(x=f_c_real, color='g', linestyle='--', label=f'Critical Freque
#plt.xscale('log')
plt.errorbar(freq_c, z_c*R1_real, yerr=sigma_z_c, fmt='r.')
plt.xlabel('f (Hz)')
plt.ylabel('|Z| (Ohms)')
plt.title('Linear plot of the magnitude of the impedance as a function of
plt.legend()

plt.figure(2)
plt.plot(freq_c, phase_c, 'r.', freq_c, phi_fit_c)
plt.axvline(x=f_c_real, color='g', linestyle='--', label=f'Expected Critic
plt.errorbar(freq_c, phase_c, yerr=10*sigma_z_c, fmt='r.', label='error sc
plt.xscale('log')
plt.xlabel('f (Hz)')
plt.ylabel('radians')
plt.title('frequency v.s. phase angle (R0 = 1 MOhm)')
plt.legend()

plt.figure(3)
plt.plot(freq_c, phase_c, 'r.', freq_c, phi_fit_c)
```

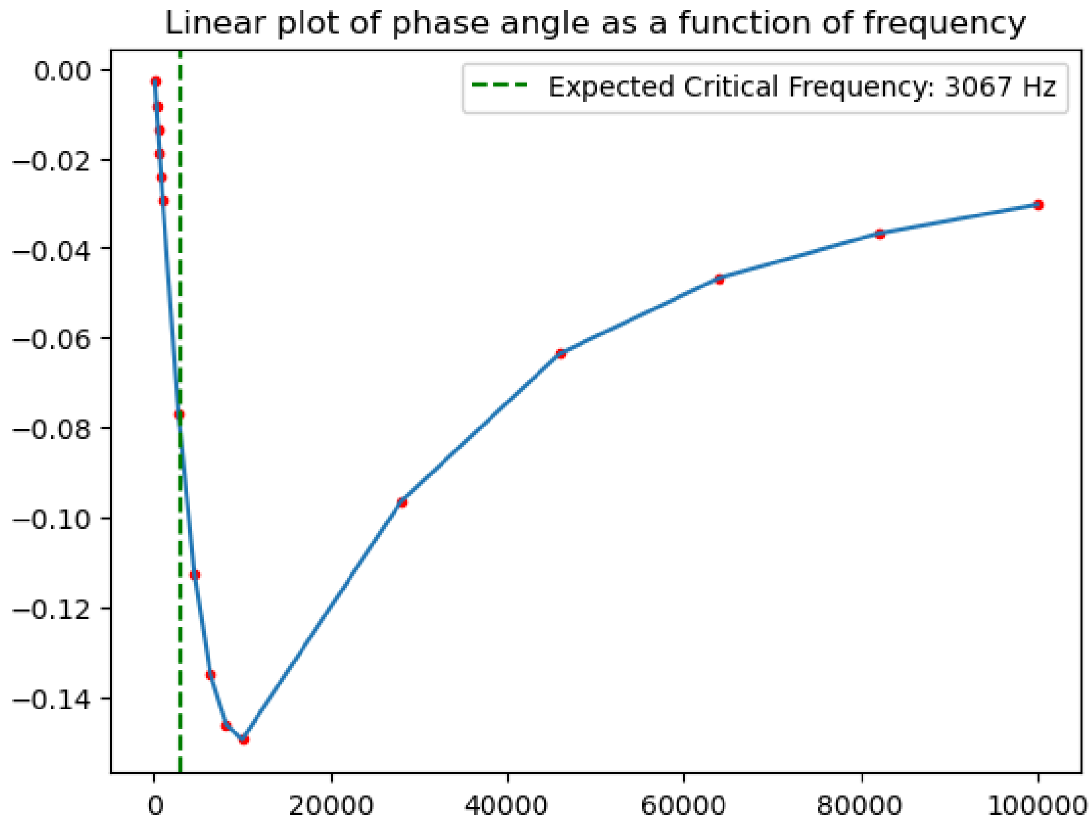
```
plt.axvline(x=f_c_real, color='g', linestyle='--', label=f'Expected Critic
#plt.xscale('log')
plt.title('Linear plot of phase angle as a function of frequency')
plt.legend()

plt.show()
```

```
r_guess = 10.0  tau_guess = 5.305e-05 s
r_fit = 10.456  tau_fit = 4.663e-05 s   f_c_fit = 3.413e+03 Hz
R1 = 197.4 ohms    R2_fit = 18.9 ohms   C_2_fit = 2.128e-07 F
```




Linear plot of the magnitude of the impedance as a function of frequency
Critical Frequency: 3067 Hz
|Z| (Ohms)
f (Hz)
200
175
150
125
100
75
50
25
0
20000
40000
60000
80000
100000


frequency v.s. phase angle (R0 = 1 MOhm)
Expected Critical Frequency: 3067 Hz
error scaled by 10
radians
f (Hz)
0.000
−0.025
−0.050
−0.075
−0.100
−0.125
−0.150
−0.175



R0a with potato (incorrect)

In [52]:

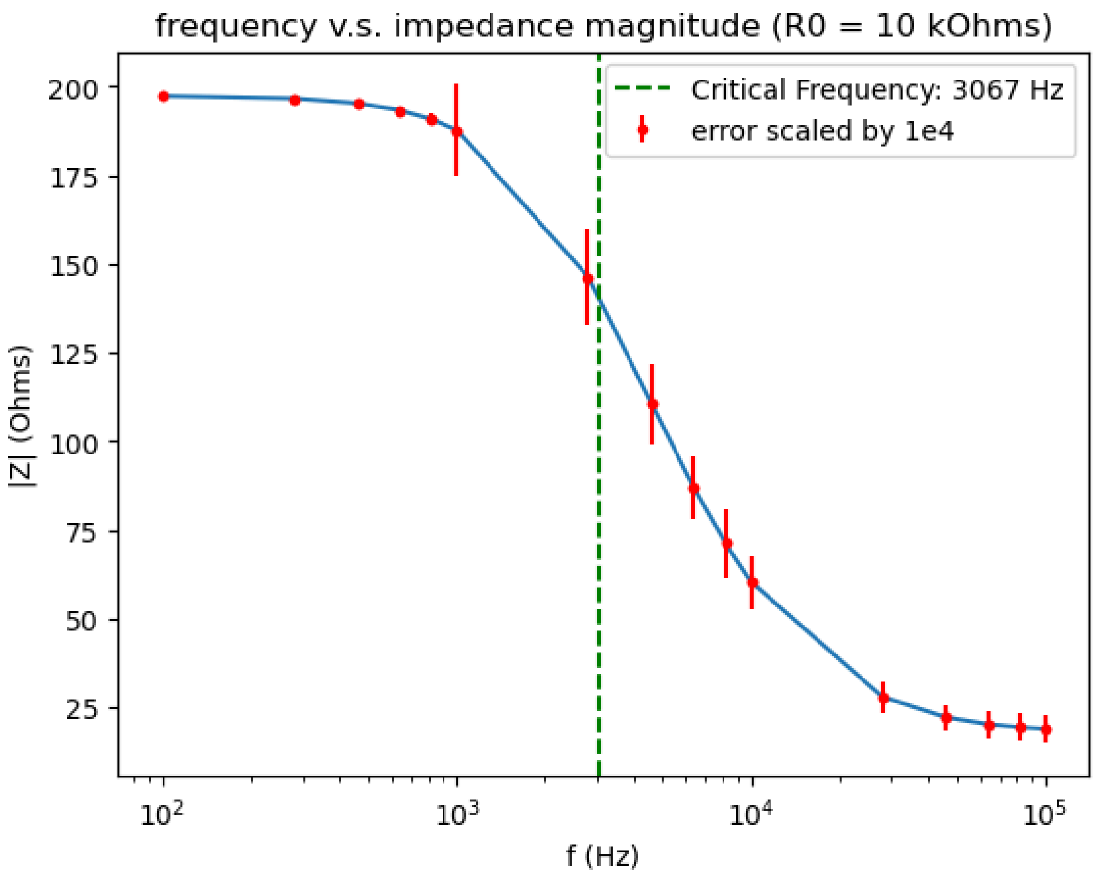
```
alpha = 2*np.pi*freq_ap*tau_real
rho = r+1

del_z_del_fap = ((freq_ap*(2*np.pi*tau_real)**2)/((rho**2) * np.sqrt((alph
del_z_del_tau = ((tau_real*(2*np.pi*freq_ap)**2)/( (rho**2)*np.sqrt((alpha
del_z_del_r = - ((alpha**2)/((rho**3)*np.sqrt(alpha**2 + 1)*np.sqrt((alpha

# Note: This value of the error for the impedance alreay assumes the R1 ha
# So there is no need to multiply it back in later.
# This would result in massively overblown error bars.
sigma_z_ap = np.sqrt((sigma_freq_ap*del_z_del_fap)**2 + (sigma_tau_real*de
```

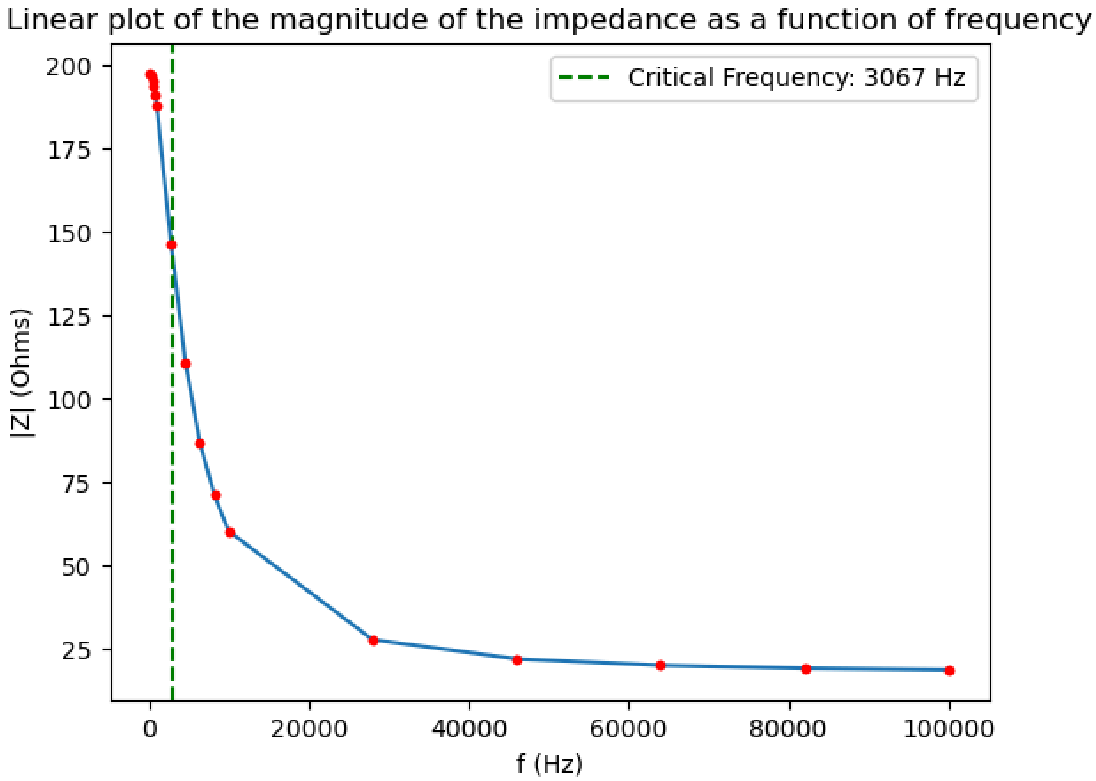
In [57]:

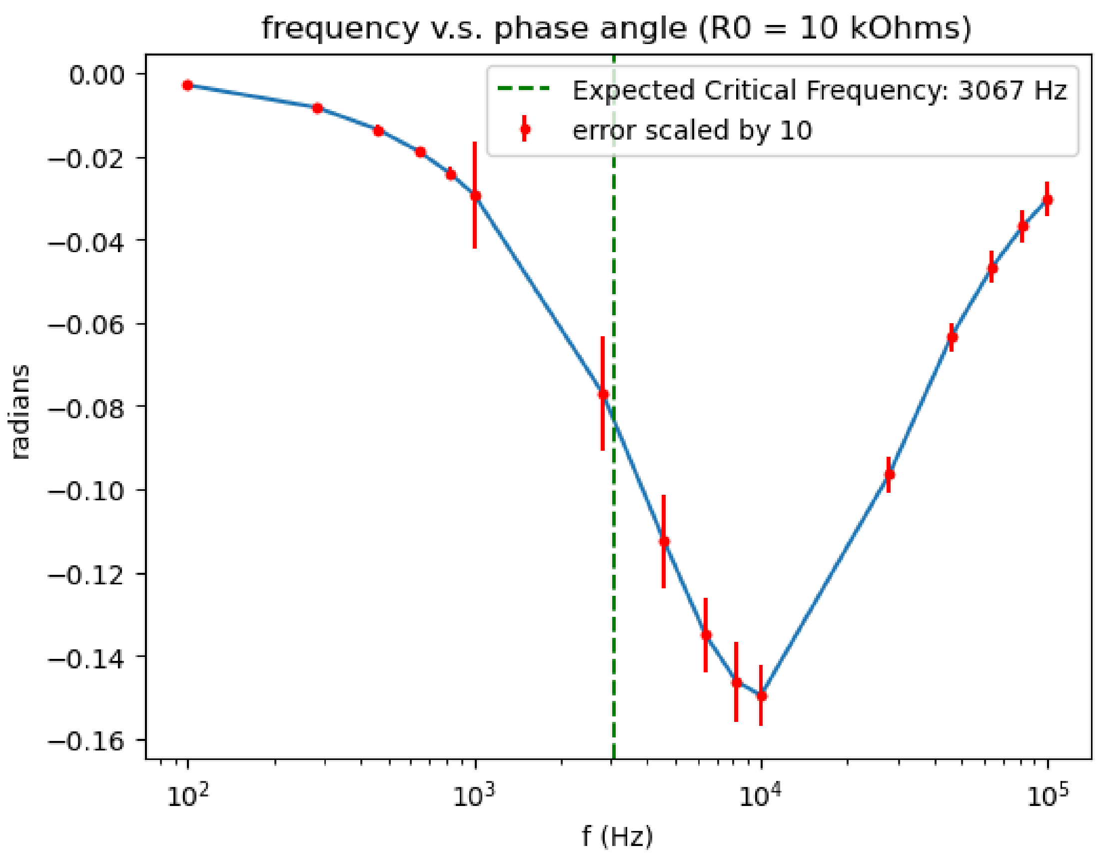
```
z_exact_ap = zfunc(freq_ap, r_real, tau_real)
z_ap = z_exact_ap

phase_exact_ap = phifunc(freq_ap, r_real, tau_real)
phase_ap = phase_exact_ap

r_guess = 10.0 # guess of ratio of DC resistance R1 to capacitive path res
f_c_guess = 3000 # guess of critical frequency in Hz
tau_guess = 1/(2*np.pi*f_c_guess)

print('r_guess = {0:.1f}  tau_guess = {1:.3e} s'.format(r_guess, tau_guess

popt, pcov = curve_fit(zfunc, freq_ap, z_exact_ap, p0=[r_guess,tau_guess])
r_fit_ap = popt[0]
tau_fit_ap = popt[1]

print('r_fit = {0:.3f}  tau_fit = {1:.3e} s   f_c_fit = {2:.3e} Hz'
      .format(r_fit, tau_fit,1/(2*np.pi*tau_fit)))

print('R1 = {0:.1f} ohms    R2_fit = {1:.1f} ohms   C_2_fit = {2:.3e} F'
      .format(R1_real, R1_real/r_fit, tau_fit/(R1*(1+1/r_fit))))

zfit_ap = zfunc(freq_ap, r_fit_ap, tau_fit_ap)
phi_fit_ap = phifunc(freq_ap, r_fit_ap, tau_fit_ap)

plt.figure(0)
plt.plot(freq_ap, z_ap*R1_real,'r.', freq_ap, zfit_ap*R1_real)
plt.axvline(x=f_c_real, color='g', linestyle='--', label=f'Critical Freque
plt.xscale('log')
plt.errorbar(freq_ap, z_ap*R1_real, yerr=1e4*sigma_z_ap, fmt='r.', label='
plt.xlabel('f (Hz)')
plt.ylabel('|Z| (Ohms)')
plt.title('frequency v.s. impedance magnitude (R0 = 10 kOhms)')
plt.legend()

plt.figure(1)
plt.plot(freq_ap, z_ap*R1_real,'r.', freq_ap, zfit_ap*R1_real)
plt.axvline(x=f_c_real, color='g', linestyle='--', label=f'Critical Freque
#plt.xscale('log')
plt.errorbar(freq_ap, z_ap*R1_real, yerr=sigma_z_ap, fmt='r.')
plt.xlabel('f (Hz)')
plt.ylabel('|Z| (Ohms)')
plt.title('Linear plot of the magnitude of the impedance as a function of
plt.legend()

plt.figure(2)
plt.plot(freq_ap, phase_ap, 'r.', freq_ap, phi_fit_ap)
plt.axvline(x=f_c_real, color='g', linestyle='--', label=f'Expected Critic
plt.xscale('log')
plt.errorbar(freq_ap, phase_ap, yerr=10*sigma_z_ap, fmt='r.', label='error
plt.xlabel('f (Hz)')
plt.ylabel('radians')
plt.title('frequency v.s. phase angle (R0 = 10 kOhms)')
plt.legend()

plt.figure(3)
plt.plot(freq_ap, phase_ap, 'r.', freq_ap, phi_fit_ap)
```

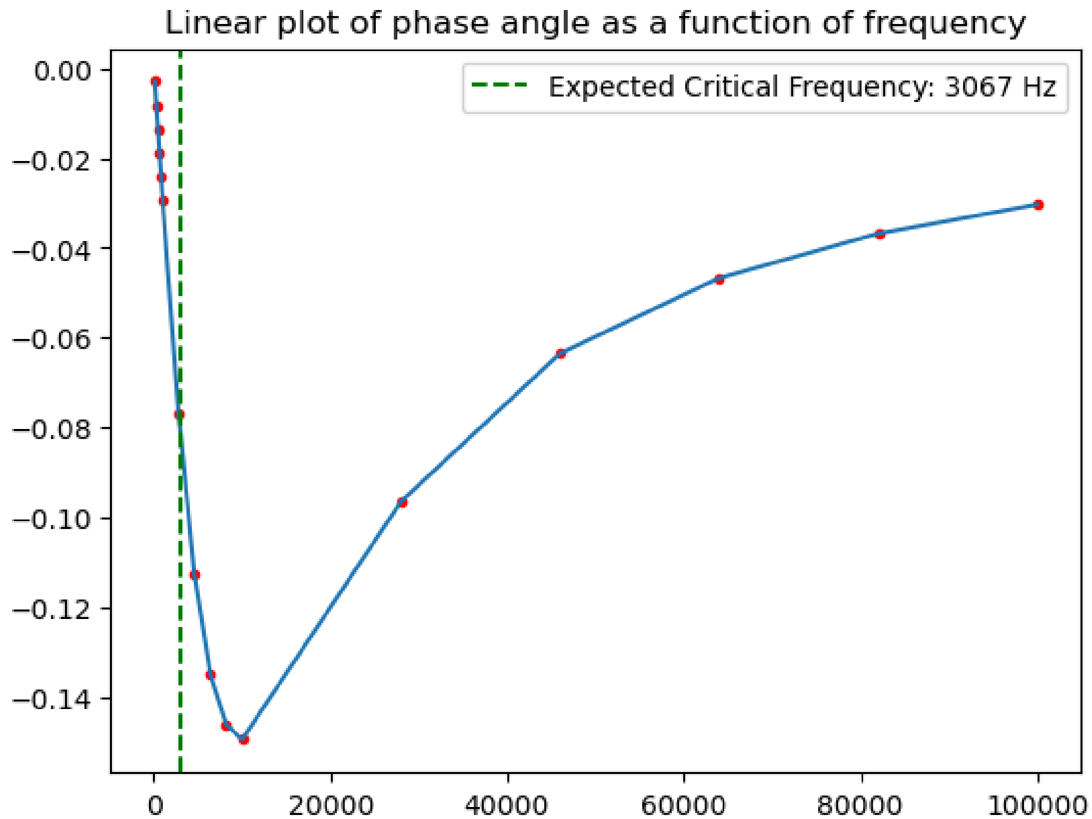
```
plt.axvline(x=f_c_real, color='g', linestyle='--', label=f'Expected Critic
#plt.xscale('log')
plt.title('Linear plot of phase angle as a function of frequency')
plt.legend()

plt.show()
```

```
r_guess = 10.0  tau_guess = 5.305e-05 s
r_fit = 10.456  tau_fit = 4.663e-05 s   f_c_fit = 3.413e+03 Hz
R1 = 197.4 ohms    R2_fit = 18.9 ohms   C_2_fit = 2.128e-07 F
```




Linear plot of the magnitude of the impedance as a function of frequency
Critical Frequency: 3067 Hz
|Z| (Ohms)
f (Hz)
200
175
150
125
100
75
50
25
0
20000
40000
60000
80000
100000


frequency v.s. phase angle (R0 = 10 kOhms)
Expected Critical Frequency: 3067 Hz
error scaled by 10
radians
f (Hz)
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
10^2
10^3
10^4
10^5



R0b with potato (incorrect)

In [55]:

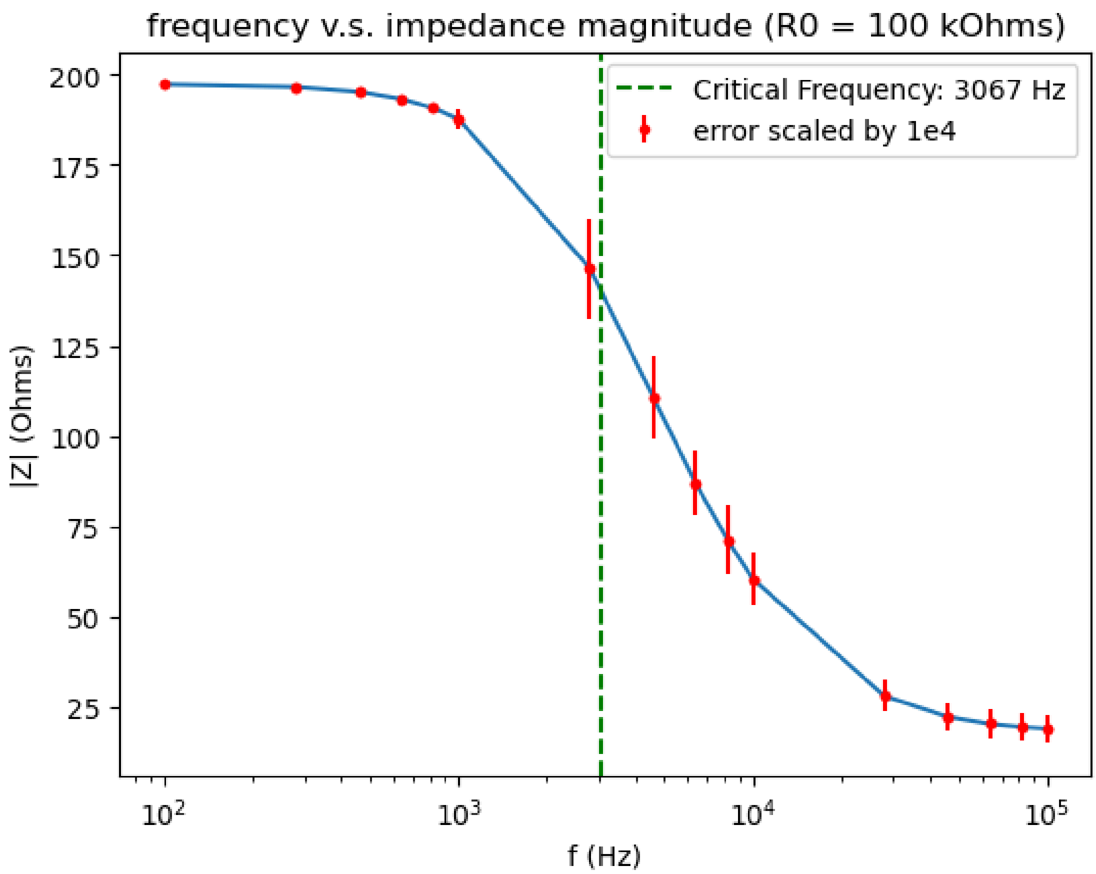
```
alpha = 2*np.pi*freq_bp*tau_real
rho = r+1

del_z_del_fbp = ((freq_bp*(2*np.pi*tau_real)**2)/((rho**2) * np.sqrt((alph
del_z_del_tau = ((tau_real*(2*np.pi*freq_bp)**2)/( (rho**2)*np.sqrt((alpha
del_z_del_r = - ((alpha**2)/((rho**3)*np.sqrt(alpha**2 + 1)*np.sqrt((alpha

# Note: This value of the error for the impedance alreay assumes the R1 ho
# So there is no need to multiply it back in later.
# This would result in massively overblown error bars.
sigma_z_bp = np.sqrt((sigma_freq_bp*del_z_del_fap)**2 + (sigma_tau_real*de
```

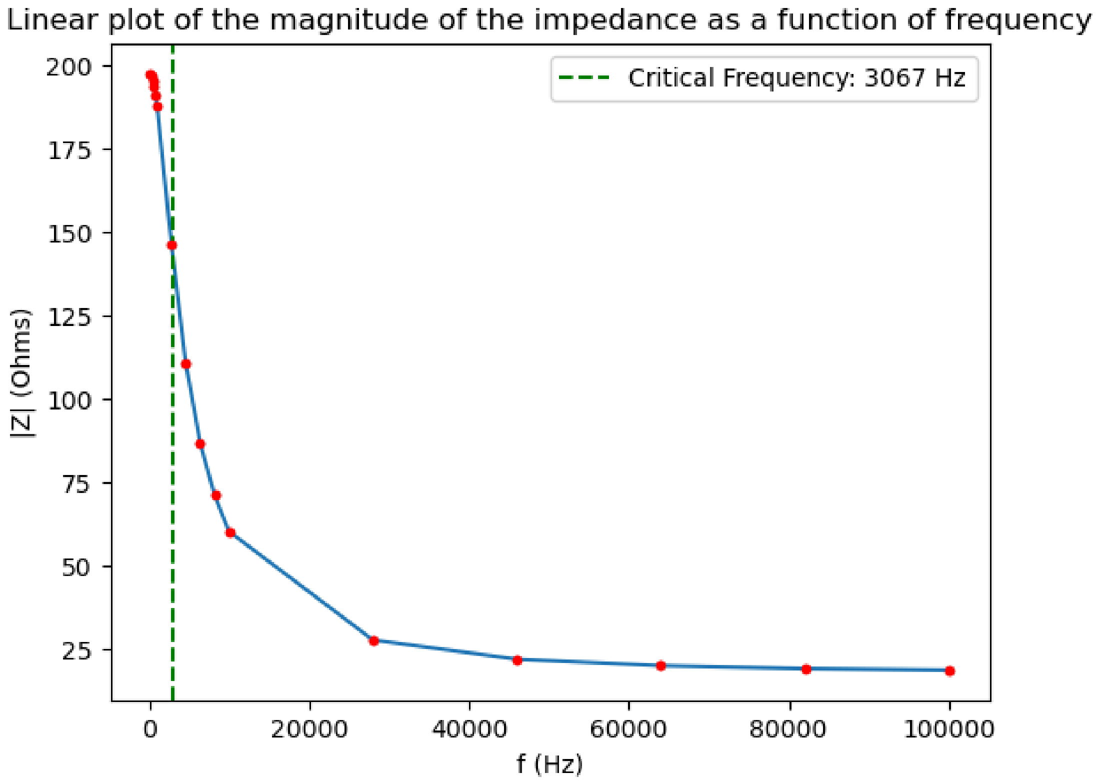
In [56]:

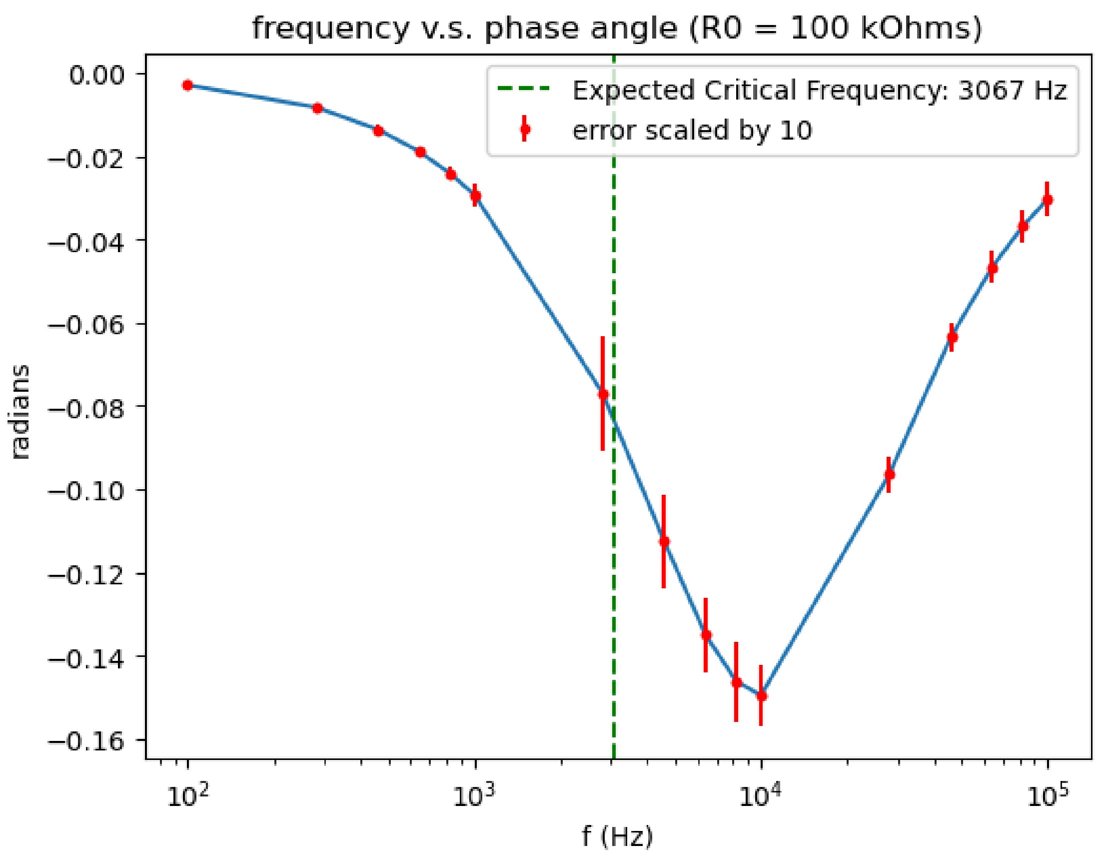
```
z_exact_bp = zfunc(freq_bp, r_real, tau_real)
z_bp = z_exact_bp

phase_exact_bp = phifunc(freq_bp, r_real, tau_real)
phase_bp = phase_exact_bp

r_guess = 10.0 # guess of ratio of DC resistance R1 to capacitive path res
f_c_guess = 3000 # guess of critical frequency in Hz
tau_guess = 1/(2*np.pi*f_c_guess)

print('r_guess = {0:.1f}  tau_guess = {1:.3e} s'.format(r_guess, tau_guess

popt, pcov = curve_fit(zfunc, freq_bp, z_exact_bp, p0=[r_guess, tau_guess]
r_fit_bp = popt[0]
tau_fit_bp = popt[1]

print('r_fit = {0:.3f}  tau_fit = {1:.3e} s   f_c_fit = {2:.3e} Hz'
      .format(r_fit, tau_fit,1/(2*np.pi*tau_fit)))

print('R1 = {0:.1f} ohms    R2_fit = {1:.1f} ohms   C_2_fit = {2:.3e} F'
      .format(R1_real, R1_real/r_fit, tau_fit/(R1*(1+1/r_fit))))

zfit_bp = zfunc(freq_bp, r_fit_bp, tau_fit_bp)
phi_fit_bp = phifunc(freq_bp, r_fit_bp, tau_fit_bp)

plt.figure(0)
plt.plot(freq_bp, z_bp*R1_real,'r.', freq_bp, zfit_bp*R1_real)
plt.axvline(x=f_c_real, color='g', linestyle='--', label=f'Critical Freque
plt.xscale('log')
plt.errorbar(freq_bp, z_bp*R1_real, yerr=1e4*sigma_z_bp, fmt='r.', label='
plt.xlabel('f (Hz)')
plt.ylabel('|Z| (Ohms)')
plt.title('frequency v.s. impedance magnitude (R0 = 100 kOhms)')
plt.legend()

plt.figure(1)
plt.plot(freq_bp, z_bp*R1_real,'r.', freq_bp, zfit_bp*R1_real)
plt.axvline(x=f_c_real, color='g', linestyle='--', label=f'Critical Freque
#plt.xscale('log')
plt.errorbar(freq_bp, z_bp*R1_real, yerr=sigma_z_bp, fmt='r.')
plt.xlabel('f (Hz)')
plt.ylabel('|Z| (Ohms)')
plt.title('Linear plot of the magnitude of the impedance as a function of
plt.legend()

plt.figure(2)
plt.plot(freq_bp, phase_bp, 'r.', freq_bp, phi_fit_bp)
plt.axvline(x=f_c_real, color='g', linestyle='--', label=f'Expected Critic
plt.errorbar(freq_bp, phase_bp, yerr=10*sigma_z_bp, fmt='r.', label='error
plt.xscale('log')
plt.xlabel('f (Hz)')
plt.ylabel('radians')
plt.title('frequency v.s. phase angle (R0 = 100 kOhms)')
plt.legend()

plt.figure(3)
plt.plot(freq_bp, phase_bp, 'r.', freq_bp, phi_fit_bp)
```

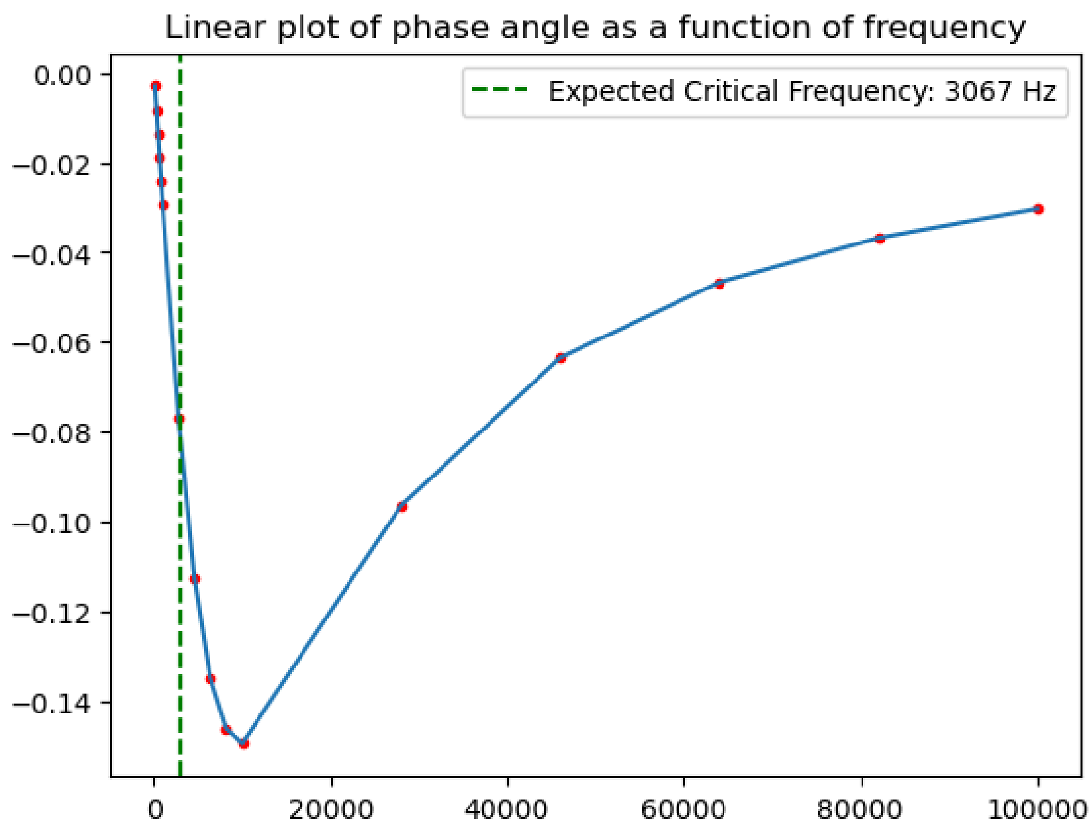
```
plt.axvline(x=f_c_real, color='g', linestyle='--', label=f'Expected Critic
#plt.xscale('log')
plt.title('Linear plot of phase angle as a function of frequency')
plt.legend()

plt.show()
```

```
r_guess = 10.0  tau_guess = 5.305e-05 s
r_fit = 10.456  tau_fit = 4.663e-05 s   f_c_fit = 3.413e+03 Hz
R1 = 197.4 ohms    R2_fit = 18.9 ohms   C_2_fit = 2.128e-07 F
```




Linear plot of the magnitude of the impedance as a function of frequency
Critical Frequency: 3067 Hz
|Z| (Ohms)
f (Hz)
0
20000
40000
60000
80000
100000
25
50
75
100
125
150
175
200


frequency v.s. phase angle (R0 = 100 kOhms)
Expected Critical Frequency: 3067 Hz
error scaled by 10
radians
f (Hz)
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
10^2
10^3
10^4
10^5



R0c with potato (incorrect)

In [58]:

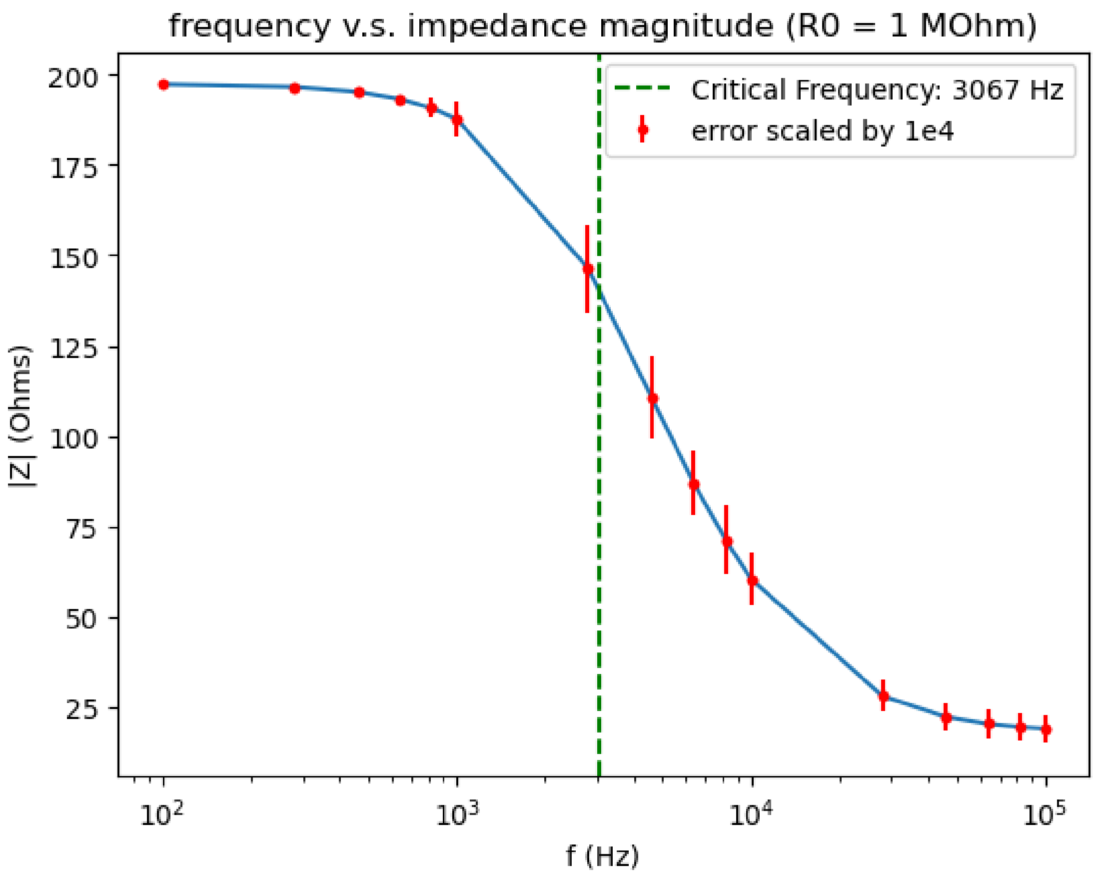
```
alpha = 2*np.pi*freq_cp*tau_real
rho = r+1

del_z_del_fcp = ((freq_cp*(2*np.pi*tau_real)**2)/((rho**2) * np.sqrt((alph
del_z_del_tau = ((tau_real*(2*np.pi*freq_cp)**2)/( (rho**2)*np.sqrt((alpha
del_z_del_r = - ((alpha**2)/((rho**3)*np.sqrt(alpha**2 + 1)*np.sqrt((alpha

# Note: This value of the error for the impedance alreay assumes the R1 ha
# So there is no need to multiply it back in later.
# This would result in massively overblown error bars.
sigma_z_cp = np.sqrt((sigma_freq_cp*del_z_del_fap)**2 + (sigma_tau_real*de
```

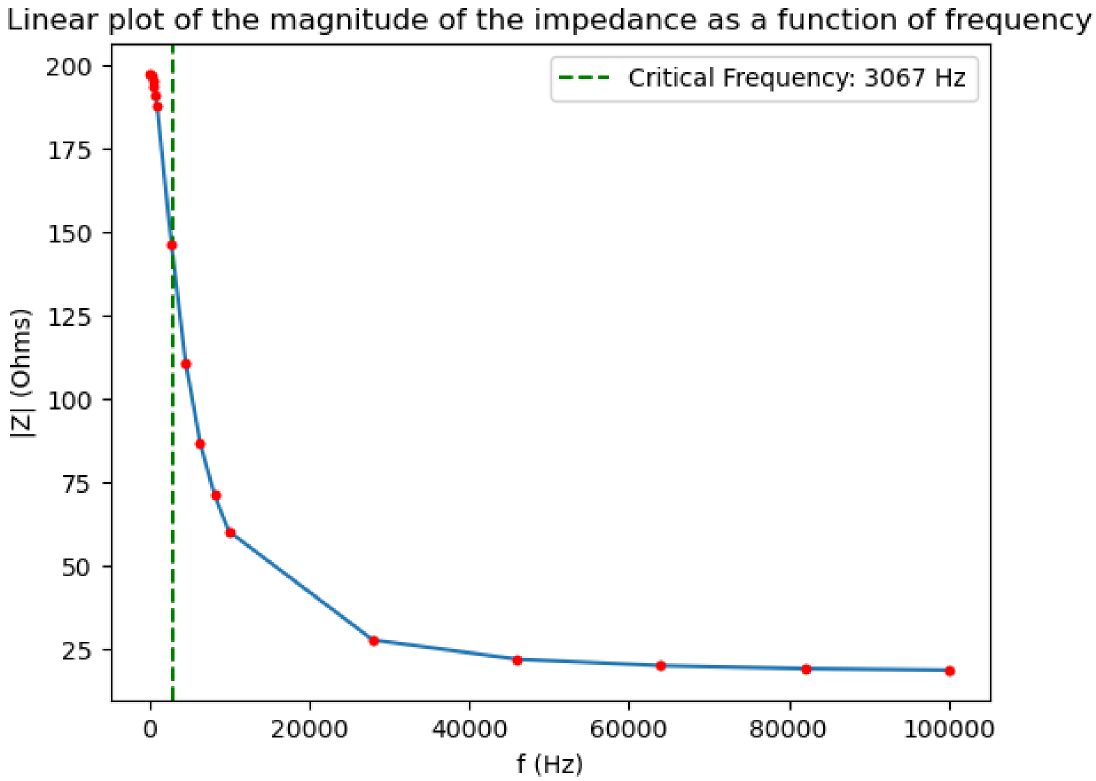
In [59]:

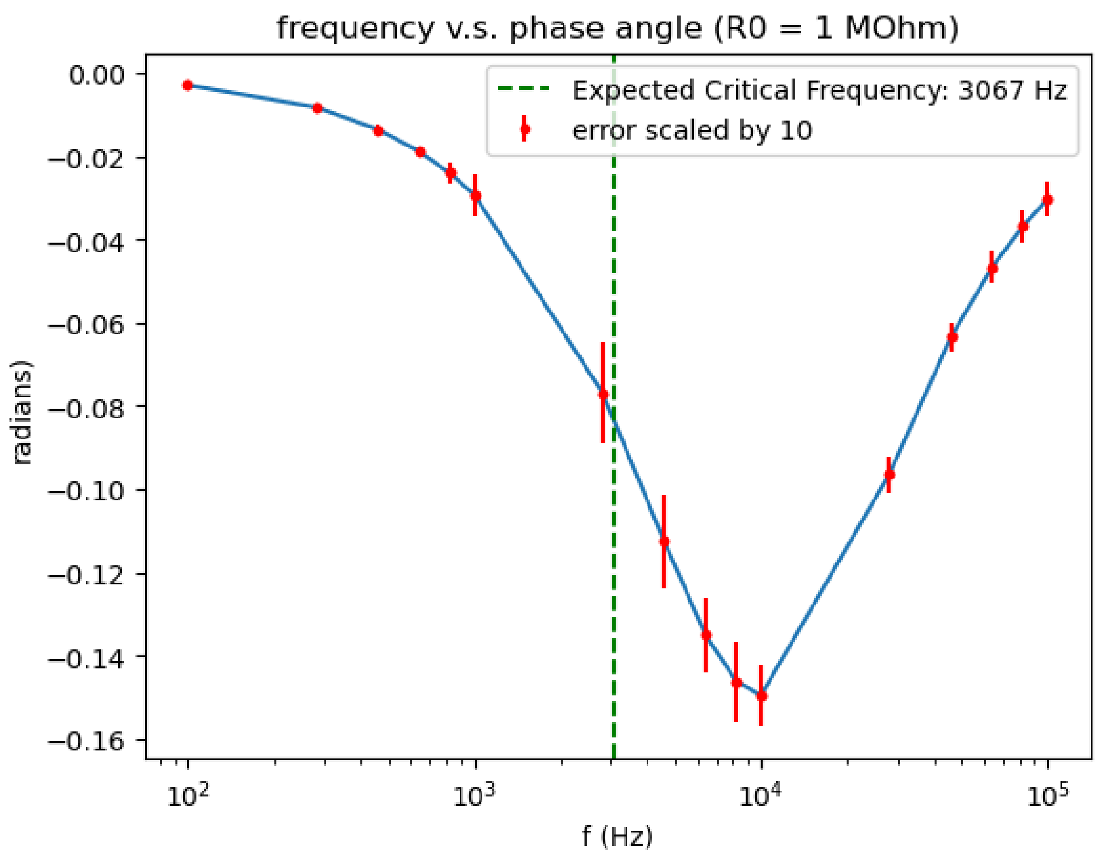
```
z_exact_cp = zfunc(freq_cp, r_real, tau_real)
z_cp = z_exact_cp

phase_exact_cp = phifunc(freq_cp, r_real, tau_real)
phase_cp = phase_exact_cp

r_guess = 10.0 # guess of ratio of DC resistance R1 to capacitive path res
f_c_guess = 3000 # guess of critical frequency in Hz
tau_guess = 1/(2*np.pi*f_c_guess)

print('r_guess = {0:.1f}  tau_guess = {1:.3e} s'.format(r_guess, tau_guess

popt, pcov = curve_fit(zfunc, freq_cp, z_exact_cp, p0=[r_guess, tau_guess]
r_fit_cp = popt[0]
tau_fit_cp = popt[1]

print('r_fit = {0:.3f}  tau_fit = {1:.3e} s   f_c_fit = {2:.3e} Hz'
      .format(r_fit, tau_fit,1/(2*np.pi*tau_fit)))

print('R1 = {0:.1f} ohms    R2_fit = {1:.1f} ohms   C_2_fit = {2:.3e} F'
      .format(R1_real, R1_real/r_fit, tau_fit/(R1*(1+1/r_fit))))

zfit_cp = zfunc(freq_cp, r_fit_cp, tau_fit_cp)
phi_fit_cp = phifunc(freq_cp, r_fit_cp, tau_fit_cp)

plt.figure(0)
plt.plot(freq_cp, z_cp*R1_real,'r.', freq_cp, zfit_cp*R1_real)
plt.axvline(x=f_c_real, color='g', linestyle='--', label=f'Critical Freque
plt.xscale('log')
plt.errorbar(freq_cp, z_cp*R1_real, yerr=1e4*sigma_z_cp, fmt='r.', label='
plt.xlabel('f (Hz)')
plt.ylabel('|Z| (Ohms)')
plt.title('frequency v.s. impedance magnitude (R0 = 1 MOhm)')
plt.legend()

plt.figure(1)
plt.plot(freq_cp, z_cp*R1_real,'r.', freq_cp, zfit_cp*R1_real)
plt.axvline(x=f_c_real, color='g', linestyle='--', label=f'Critical Freque
#plt.xscale('log')
plt.errorbar(freq_cp, z_cp*R1_real, yerr=sigma_z_cp, fmt='r.')
plt.xlabel('f (Hz)')
plt.ylabel('|Z| (Ohms)')
plt.title('Linear plot of the magnitude of the impedance as a function of
plt.legend()

plt.figure(2)
plt.plot(freq_cp, phase_cp, 'r.', freq_cp, phi_fit_cp)
plt.axvline(x=f_c_real, color='g', linestyle='--', label=f'Expected Critic
plt.xscale('log')
plt.errorbar(freq_cp, phase_cp, yerr=10*sigma_z_cp, fmt='r.', label='error
plt.xlabel('f (Hz)')
plt.ylabel('radians)')
plt.title('frequency v.s. phase angle (R0 = 1 MOhm)')
plt.legend()

plt.figure(3)
plt.plot(freq_cp, phase_cp, 'r.', freq_cp, phi_fit_cp)
```

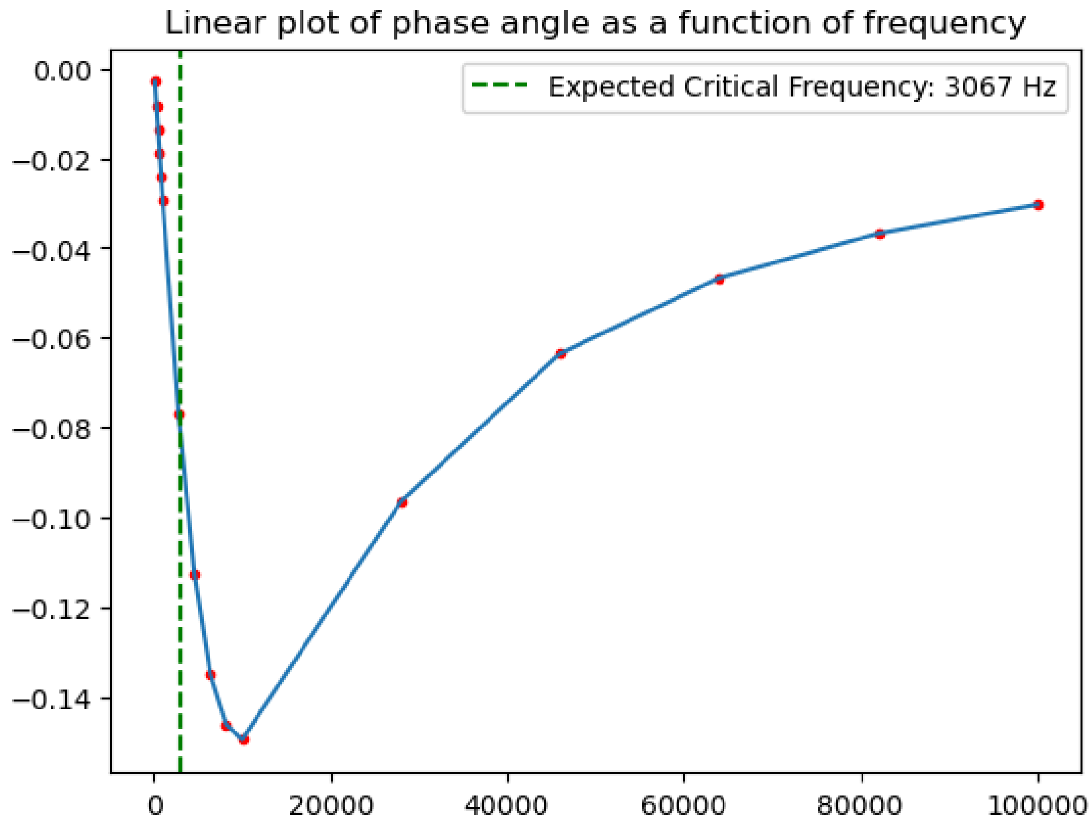
```
plt.axvline(x=f_c_real, color='g', linestyle='--', label=f'Expected Critic
#plt.xscale('log')
plt.title('Linear plot of phase angle as a function of frequency')
plt.legend()

plt.show()
```

```
r_guess = 10.0  tau_guess = 5.305e-05 s
r_fit = 10.456  tau_fit = 4.663e-05 s   f_c_fit = 3.413e+03 Hz
R1 = 197.4 ohms    R2_fit = 18.9 ohms   C_2_fit = 2.128e-07 F
```




Linear plot of the magnitude of the impedance as a function of frequency
Critical Frequency: 3067 Hz
|Z| (Ohms)
f (Hz)
200
175
150
125
100
75
50
25
0
20000
40000
60000
80000
100000

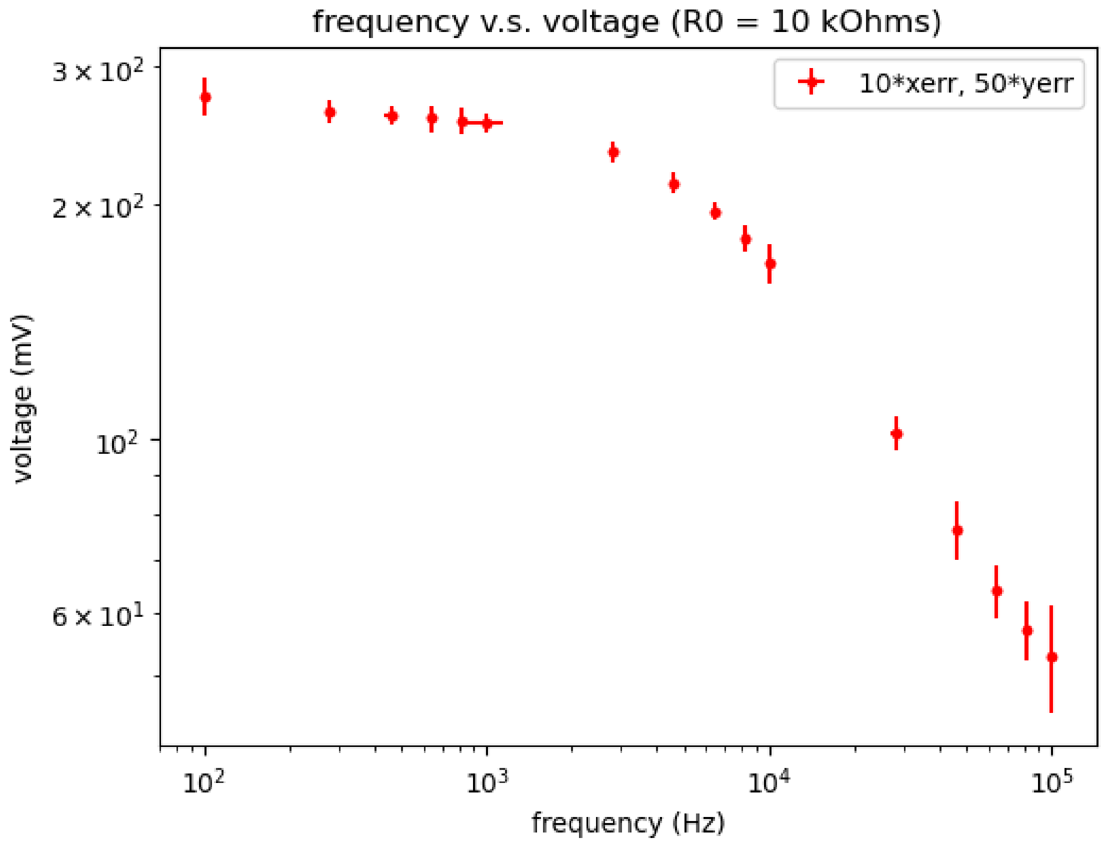

frequency v.s. phase angle (R0 = 1 MOhm)
Expected Critical Frequency: 3067 Hz
error scaled by 10
radians)
f (Hz)
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
10^2
10^3
10^4
10^5

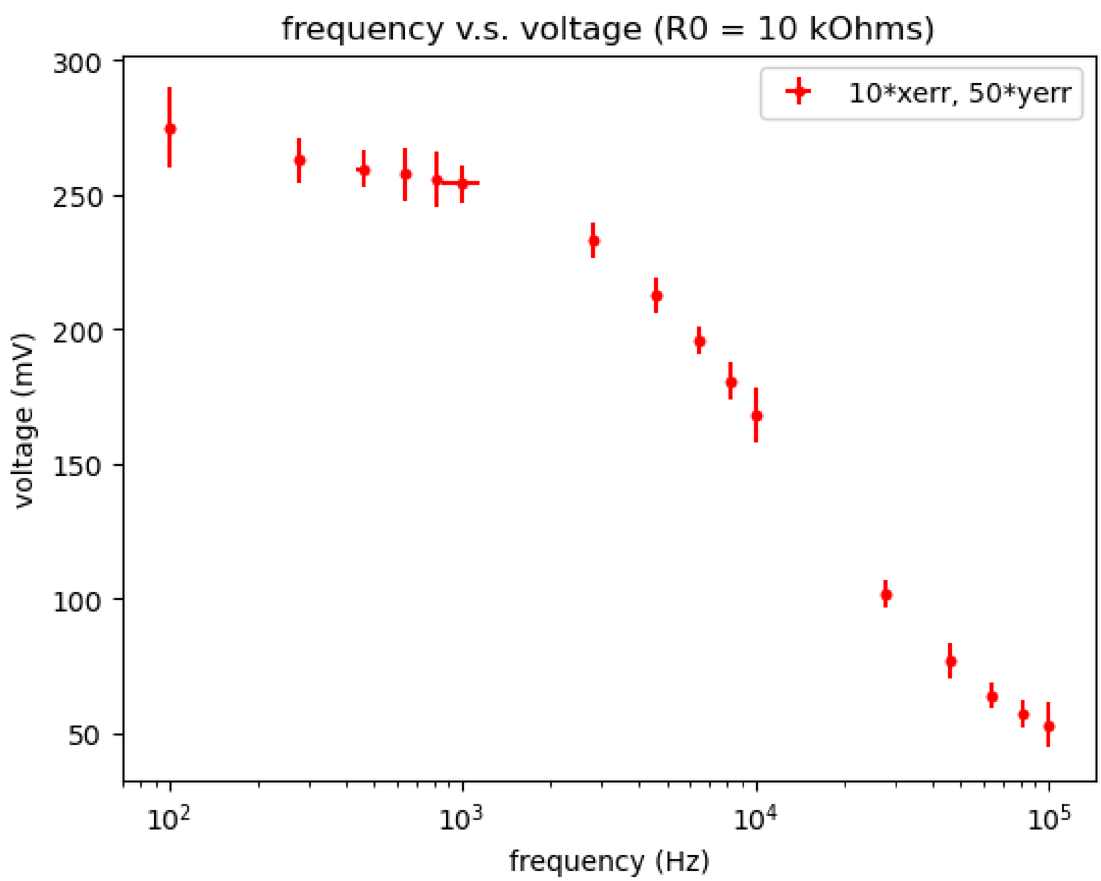

Linear plot of phase angle as a function of frequency
Expected Critical Frequency: 3067 Hz
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
0
20000
40000
60000
80000
100000

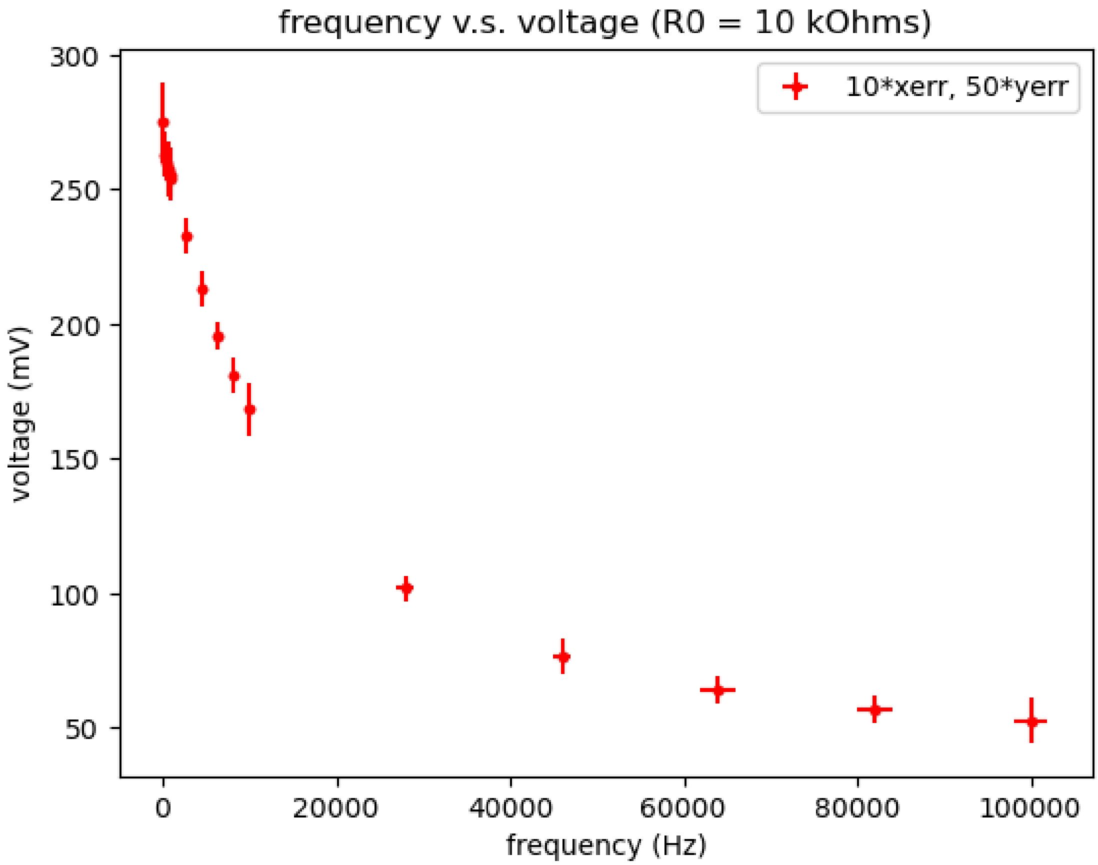
R0a with potato (attempt to correct)

In [78]:

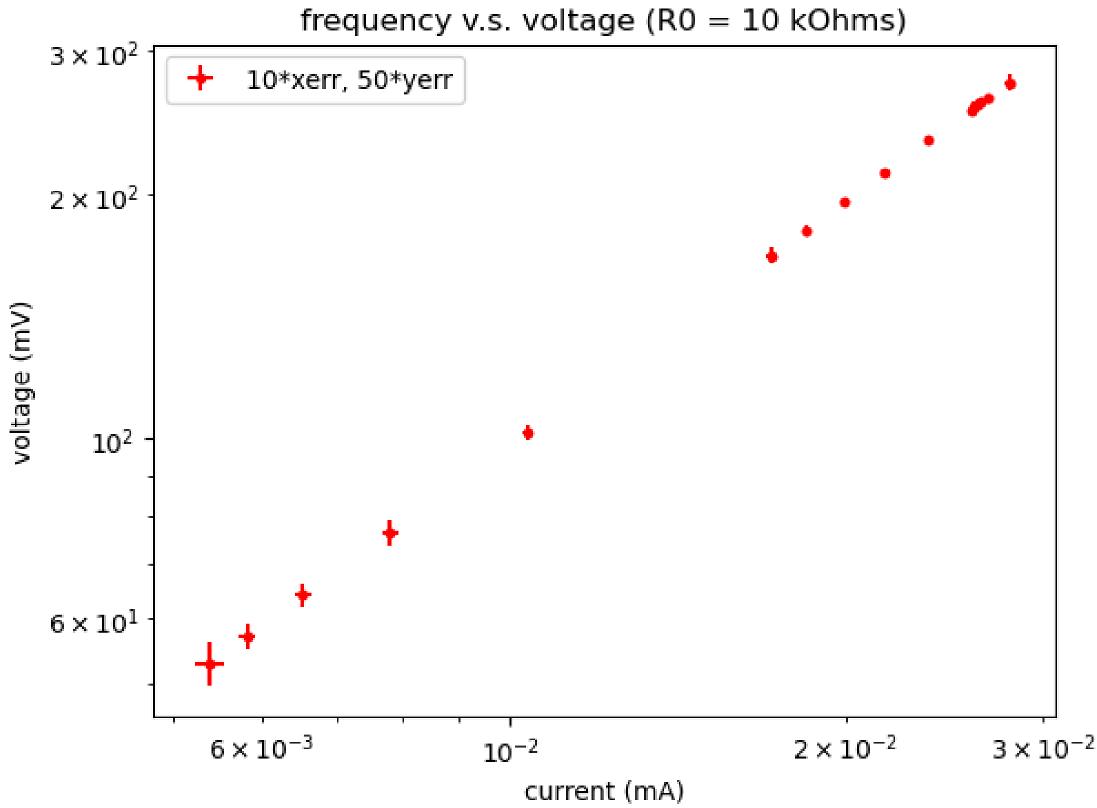
```
# freq v.s. voltage

plt.figure(0)
plt.plot(freq_ap, voltage_ap, 'r.')
plt.errorbar(freq_ap, voltage_ap, xerr=10*sigma_freq_ap ,yerr=50*sigma_vol
plt.xscale('log')
plt.yscale('log')
plt.xlabel('frequency (Hz)')
plt.ylabel('voltage (mV)')
plt.title('frequency v.s. voltage (R0 = 10 kOhms)')
plt.legend()
plt.show()

plt.figure(1)
plt.plot(freq_ap, voltage_ap, 'r.')
plt.errorbar(freq_ap, voltage_ap, xerr=10*sigma_freq_ap, yerr=50*sigma_vol
plt.xscale('log')
#plt.yscale('log')
plt.xlabel('frequency (Hz)')
plt.ylabel('voltage (mV)')
plt.title('frequency v.s. voltage (R0 = 10 kOhms)')
plt.legend()
plt.show()

plt.figure(2)
plt.plot(freq_ap, voltage_ap, 'r.')
plt.errorbar(freq_ap, voltage_ap, xerr=10*sigma_freq_ap, yerr=50*sigma_vol
#plt.xscale('log')
#plt.yscale('log')
plt.xlabel('frequency (Hz)')
plt.ylabel('voltage (mV)')
plt.title('frequency v.s. voltage (R0 = 10 kOhms)')
plt.legend()
plt.show()
```

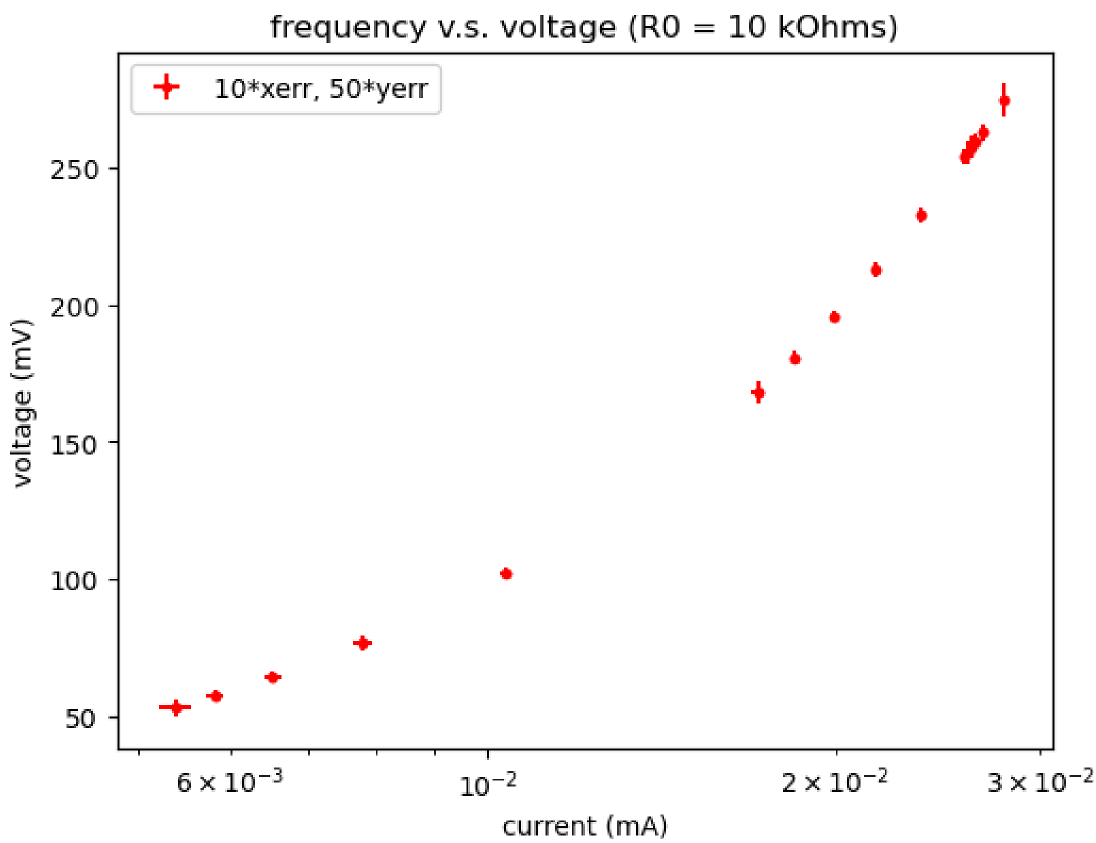

frequency v.s. voltage (R0 = 10 kOhms)
10*xerr, 50*yerr
voltage (mV)
frequency (Hz)
3×10^2
2×10^2
10^2
6×10^1
10^2
10^3
10^4
10^5

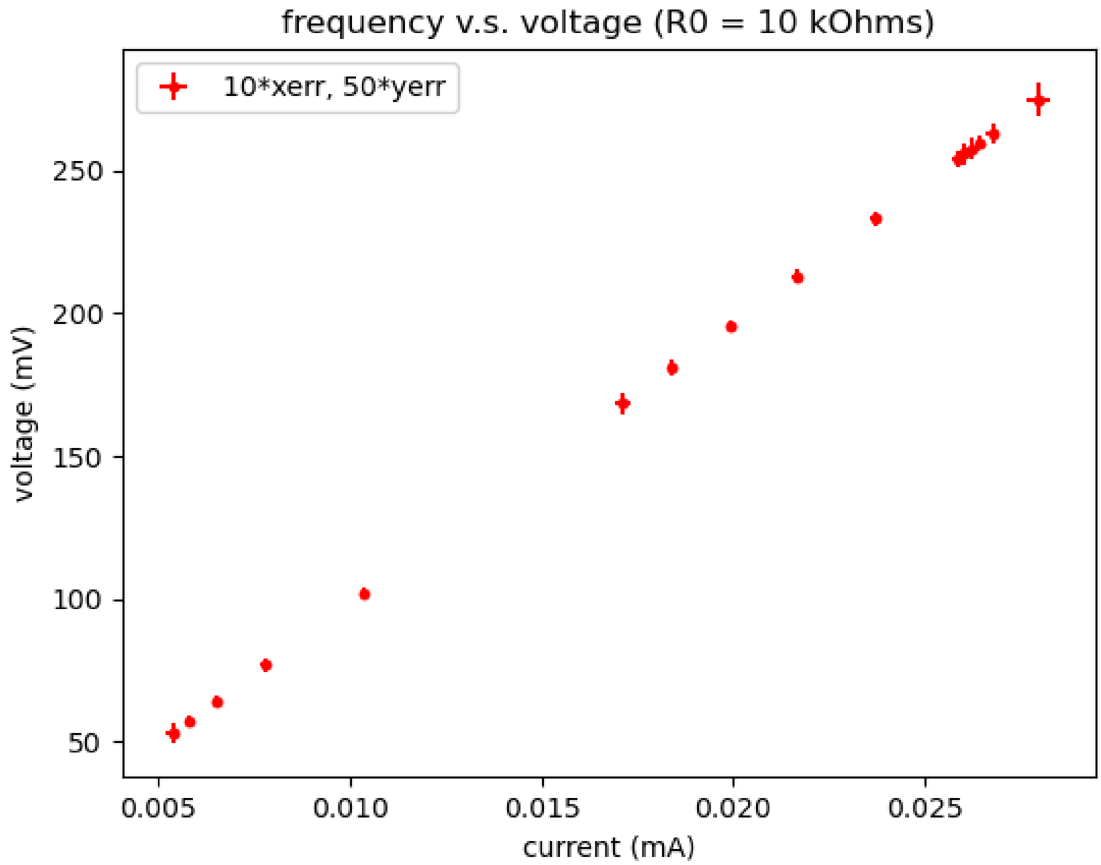

frequency v.s. voltage (R0 = 10 kOhms)
10*xerr, 50*yerr
voltage (mV)
frequency (Hz)
300
250
200
150
100
50
10^2
10^3
10^4
10^5


frequency v.s. voltage (R0 = 10 kOhms)
10*xerr, 50*yerr
voltage (mV)
300
250
200
150
100
50
0
20000
40000
60000
80000
100000
frequency (Hz)

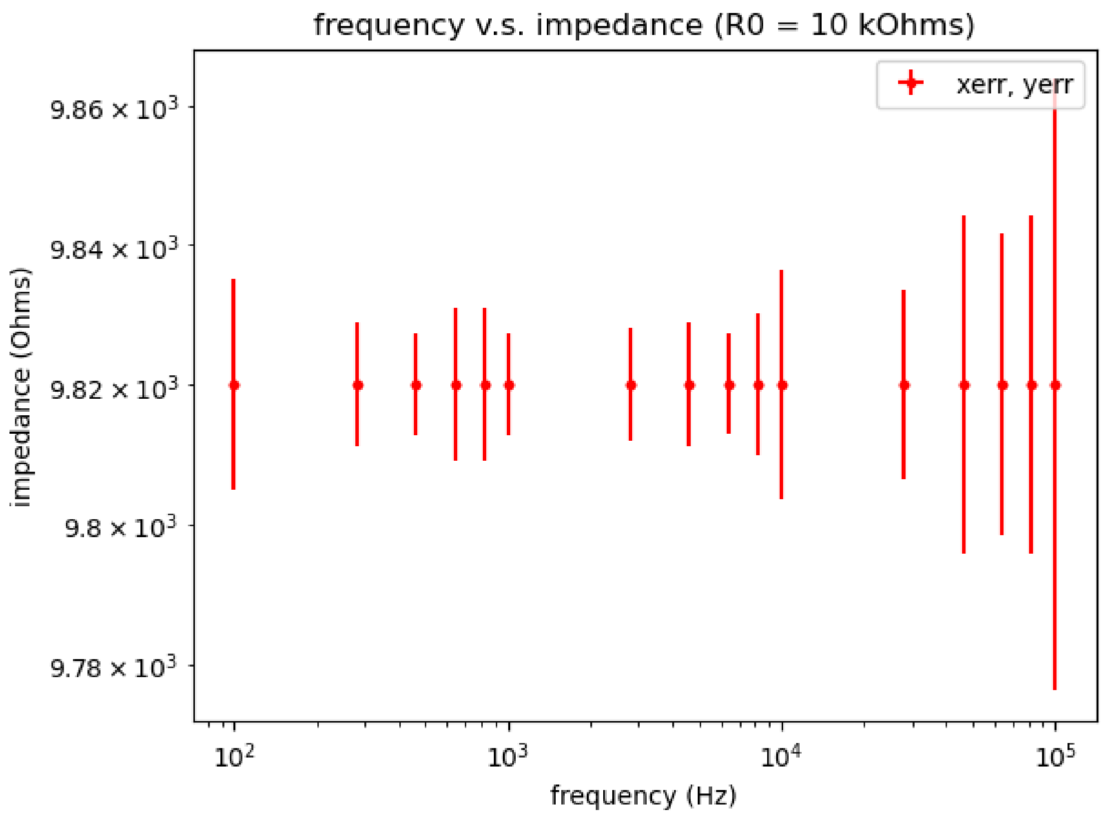
In [62]:
```
# current v.s. voltage
R0a = 9.82e3 # Ohms
sigma_R0a = 0.001e3 #Ohms
I_ap = voltage_ap/R0a
sigma_I_ap = np.sqrt((sigma_R0a*(-voltage_ap/R0a**2))**2 + (sigma_voltage_

plt.figure(0)
plt.plot(I_ap, voltage_ap, 'r.')
plt.errorbar(I_ap, voltage_ap, xerr=10*sigma_I_ap ,yerr=20*sigma_voltage_a
plt.xscale('log')
plt.yscale('log')
plt.xlabel('current (mA)')
plt.ylabel('voltage (mV)')
plt.title('frequency v.s. voltage (R0 = 10 kOhms)')
plt.legend()
plt.show()

plt.figure(1)
plt.plot(I_ap, voltage_ap, 'r.')
plt.errorbar(I_ap, voltage_ap, xerr=10*sigma_I_ap ,yerr=20*sigma_voltage_a
plt.xscale('log')
#plt.yscale('log')
plt.xlabel('current (mA)')
plt.ylabel('voltage (mV)')
plt.title('frequency v.s. voltage (R0 = 10 kOhms)')
plt.legend()
plt.show()

plt.figure(2)
plt.plot(I_ap, voltage_ap, 'r.')
plt.errorbar(I_ap, voltage_ap, xerr=10*sigma_I_ap ,yerr=20*sigma_voltage_a
#plt.xscale('log')
#plt.yscale('log')
plt.xlabel('current (mA)')
plt.ylabel('voltage (mV)')
plt.title('frequency v.s. voltage (R0 = 10 kOhms)')
plt.legend()
plt.show()
```

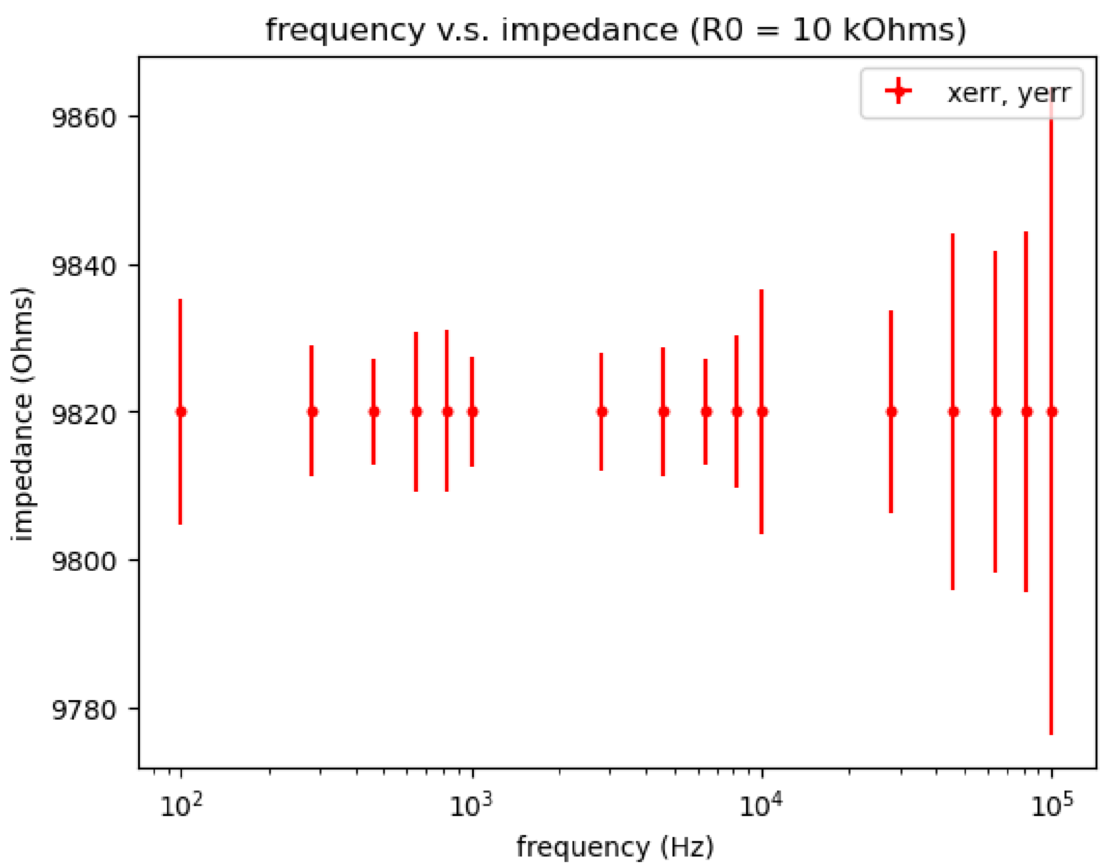

frequency v.s. voltage (R0 = 10 kOhms)
10*xerr, 50*yerr
voltage (mV)
current (mA)
3×10^2
2×10^2
10^2
6×10^1
6×10^{-3}
10^{-2}
2×10^{-2}
3×10^{-2}

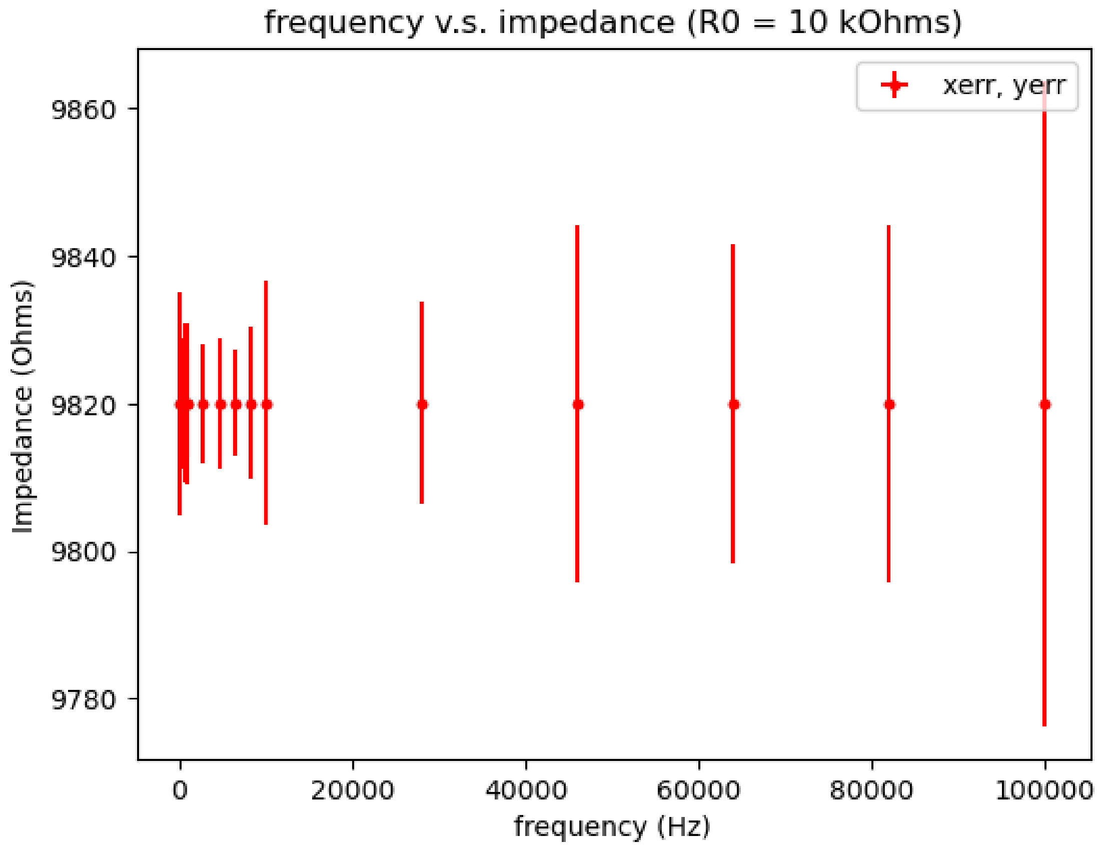

frequency v.s. voltage (R0 = 10 kOhms)
10*xerr, 50*yerr
voltage (mV)
current (mA)
250
200
150
100
50
6×10^{-3}
10^{-2}
2×10^{-2}
3×10^{-2}


frequency v.s. voltage (R0 = 10 kOhms)
10*xerr, 50*yerr
voltage (mV)
current (mA)
50
100
150
200
250
0.005
0.010
0.015
0.020
0.025

In [80]:
```
# frequncy v.s. impedance
R0a = 9.82e3 # Ohms
sigma_R0a = 0.001e3 #Ohms

I_ap = voltage_ap/R0a

sigma_I_ap = np.sqrt((sigma_R0a*(-voltage_ap/R0a**2))**2 + (sigma_voltage_

Z_ap = voltage_ap/I_ap

sigma_Z_ap = np.sqrt((sigma_I_ap*(-voltage_ap/I_ap**2))**2 + (sigma_voltag


plt.figure(0)
plt.plot(freq_ap, Z_ap, 'r.')
plt.errorbar(freq_ap, Z_ap, xerr=sigma_freq_ap ,yerr=sigma_Z_ap, fmt='r.',
plt.xscale('log')
plt.yscale('log')
plt.xlabel('frequency (Hz)')
plt.ylabel('impedance (Ohms)')
plt.title('frequency v.s. impedance (R0 = 10 kOhms)')
plt.legend()
plt.show()

plt.figure(1)
plt.plot(freq_ap, Z_ap, 'r.')
plt.errorbar(freq_ap, Z_ap, xerr=sigma_freq_ap ,yerr=sigma_Z_ap, fmt='r.',
plt.xscale('log')
#plt.yscale('log')
plt.xlabel('frequency (Hz)')
plt.ylabel('impedance (Ohms)')
plt.title('frequency v.s. impedance (R0 = 10 kOhms)')
plt.legend()
plt.show()

plt.figure(2)
plt.plot(freq_ap, Z_ap, 'r.')
plt.errorbar(freq_ap, Z_ap, xerr=sigma_freq_ap ,yerr=sigma_Z_ap, fmt='r.',
#plt.xscale('log')
#plt.yscale('log')
plt.xlabel('frequency (Hz)')
plt.ylabel('Impedance (Ohms)')
plt.title('frequency v.s. impedance (R0 = 10 kOhms)')
plt.legend()
plt.show()
```


frequency v.s. impedance (R0 = 10 kOhms)
xerr, yerr
9.86×10^3
9.84×10^3
9.82×10^3
9.8×10^3
9.78×10^3
impedance (Ohms)
10^2
10^3
10^4
10^5
frequency (Hz)


frequency v.s. impedance (R0 = 10 kOhms)
xerr, yerr
9860
9840
9820
9800
9780
impedance (Ohms)
10^2
10^3
10^4
10^5
frequency (Hz)


frequency v.s. impedance (R0 = 10 kOhms)
xerr, yerr
Impedance (Ohms)
9860
9840
9820
9800
9780
0
20000
40000
60000
80000
100000
frequency (Hz)

In [ ]: