Physics 4211/5211 Fall 2024
Homework 01

Total Points: 35.5 points
All questions are graded for **reasoning** and correctness. Providing *only* the correct answer without clearly showing your work and/or explaining your reasoning is generally worth no more than 25% of the possible credit. Submitting solutions to problem sets is about growing your skills both in problem solving and in formal, scientific communication.

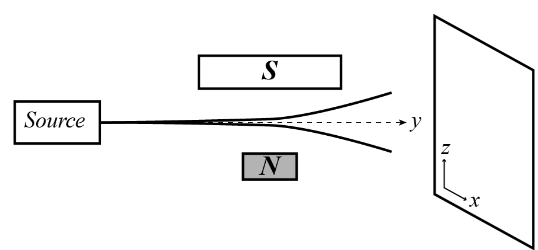
The first three questions are meant as a review of some of the linear algebra principles we'll be drawing on in the first few weeks but will not review heavily in class.

1. **[1.5 points]** Consider the complex number $z = -1 + \sqrt{3}\, i$.
   a) **[0.5 points]** Plot z in the complex plane, where the horizontal (vertical) axis is the real (imaginary) part.
   b) **[1 point]** Rewrite z in standard *polar* form $z = re^{i\theta}$

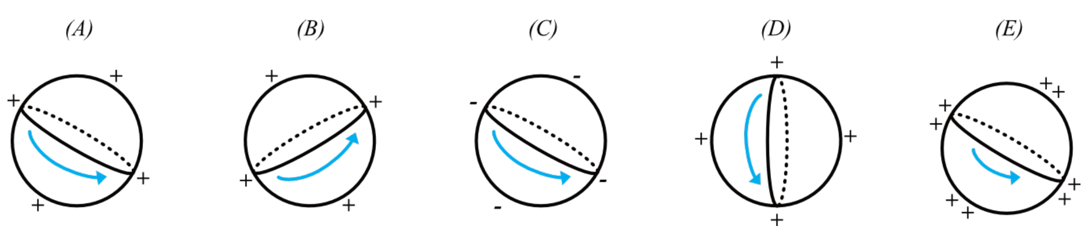
2. **[2.5 points]** Consider the complex number $z = ie^{-i\frac{\pi}{2}}$. Note that this is **not** in standard polar form.
   a) **[1 point]** Rewrite z in standard *polar* form.
   b) **[0.5 points]** Plot z in the complex plane.
   c) **[1 point]** Describe what the number $e^{-i\frac{\pi}{2}}$ looks like in the complex plane. Then write a brief description of the effect of multiplying a complex number by *i*.

3. **[4.5 points]** Consider the matrices $\boldsymbol{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} 2i & 0 \\ 0 & -i \end{pmatrix}$, and $\boldsymbol{C} = \begin{pmatrix} 5/13 \\ 12/13 \end{pmatrix}$.
   a) **[0.5 points]** Determine the product $\boldsymbol{AB}$.
   b) **[0.5 points]** Determine the product $\boldsymbol{BA}$. Is this the same as your answer in a)? Should it be?
   c) **[1 point]** Determine the eigenvalues of $\boldsymbol{A}$.
   d) **[1.5 points]** What does matrix **A** "do" to the basis vector $\binom{0}{1}$? What about the basis vector $\binom{1}{0}$? Describe how you could determine this *without* doing any matrix multiplication.
   e) **[1 point]** Can you determine the product $\boldsymbol{CB}$? What about $\boldsymbol{BC}$? Compute any products that can be computed. For any product(s) that can't be computed, explain why.

4. **[4 points total]** Consider a Stern-Gerlach beam experiment where the gradient of the magnetic field is in the -*z* direction, and the particle beam travels at speed v in the *y* direction, as shown in the figure at right. We are going to send through a beam of spinning, charged, *marbles* (classical particles).



   a) **[2.5 points]** Rank the five marbles (A-E) shown below based on the "splitting" force they would experience in this SG device. Explain your reasoning. *Note:* Each marble has the same magnitude of angular momentum $|\vec{S}|$ and charge $|q|$, except case E which has double the charge and half the spin.
   b) **[1.5 point]** Make a sketch of the screen that indicates where each of these marbles would hit. *Hint:* Consider the *Lorentz Force* as well as the splitting force!



5. **[6 points]** Consider a *classical* bar magnet with a dipole moment $\vec{\mu}$, that is free to point in any direction defined by spherical coordinates (θ, ϕ).

a) **[1.5 points]** Justify (explain!) why we can determine the angular probability distribution (*i.e* the differentially small probability dP that the bar magnet is pointing in the small range $d\theta$) as a ratio involving the differentially small area dA of a "ring."

b) **[2 points]** Make a sketch that clearly shows the dipole moment μ and indicates the dimensions of the ring in terms of relevant coordinates. Use this sketch to determine an expression for the area dA in terms of $d\theta$.

c) **[1 point]** Show that the probability distribution is $dP = \frac{\sin\theta}{2} d\theta$.

d) **[2.5 points]** Justify that this expression is reasonable by considering (1) the most likely orientation(s) of the bar magnet and (2) the probability that the dipole moment points anywhere in the region $0 \leq \theta \leq \pi$.

For the next two problems you will need to use Postulate 4 (pg. 15 of McIntyre). Another way of writing this (more similar to our discussion in class) is: For a system in state $|\psi\rangle = a|+\rangle + b|-\rangle$, the probability of measuring the particle to be "spin-up" in the *z* direction (*i.e.* $+\hbar/2$ for a measurement of S_z) is $|a|^2$ and the probability of measuring "spin-down" in the *z* direction is $|b|^2$.

This applies to *each* component of the spin. That is, for $|\psi\rangle = c|+\rangle_x + d|-\rangle_x$ the probability of measuring $S_x = +\hbar/2$ is $|c|^2$ and the probability of measuring $S_x = -\hbar/2$ is $|d|^2$.

6. **[6 points]** A beam of spin-1/2 particles is prepared in the state $|\psi\rangle = \frac{5}{13}|+\rangle + i\frac{12}{13}|-\rangle$.

a) **[2 points]** What are the possible results of a measurement of the spin component S_z, and with what probabilities would they occur?

b) **[1.5 points]** You take all the particles that are measured to have a value of $S_z = -\hbar/2$ and send them through a second SG analyzer that measures S_x. What are the possible results of this measurement, and with what probabilities would they occur?

c) **[2.5 points]** What is the probability that a particle from the initial beam will be measured to have $S_x = +\hbar/2$ by the second SG analyzer? As part of your explanation, draw a schematic diagram depicting the successive measurement in parts a and b (similar to Fig. 1.4 in the textbook) given an input of 100 particles.

7. **[8 points]** A beam of spin-1/2 particles is prepared in the state $|\psi\rangle = \frac{2}{\sqrt{5}}|+\rangle_x - \frac{1}{\sqrt{5}}|-\rangle_x$.

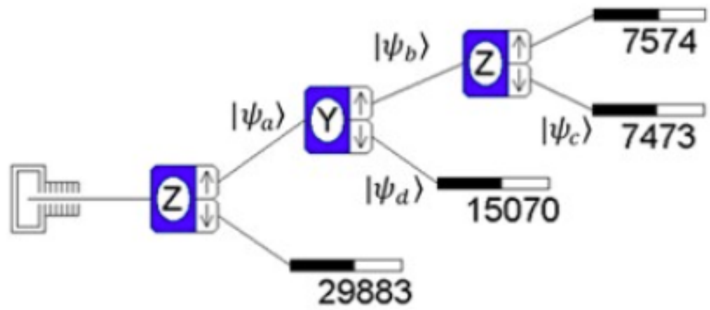
a) **[2 points]** What are the possible results of a measurement of the spin component S_x, and with what probabilities would they occur?

b) **[3 points]** Use the definition of the $|\pm\rangle_x$ kets *in the z-basis* to determine the possible results of a measurement of S_z and the corresponding probabilities.

c) **[1.5 points]** Say that we take a *single* particle in state $|\psi\rangle$ and measure $S_x = +\frac{\hbar}{2}$. We *then* measure S_z for this same particle. What are the possible results? With what probabilities?

d) **[1.5 points]** Say that the second measurement in part c were of S_x instead of S_z. What are the possible results? With what probabilities?

8. **[3 points]** A beam of spin-1/2 particles leaves a thermal (random) source, and is sent through the series of SG analyzers shown at right. The kets $|\psi_n\rangle$ indicate the states of the particles following the various measurements.



a) **[1.5 points]** How many particles were released from the oven? Calculate it exactly, but also explain a quick way to determine the approximate number based *only* on the first analyzer. Explain your reasoning for both cases.

b) **[1.5 points]** Write the states $|\psi_a\rangle, |\psi_b\rangle, |\psi_c\rangle$, and $|\psi_d\rangle$ in terms of $|\pm\rangle$ or $|\pm\rangle_y$. Briefly explain.