Written HW 1

1 Problem 1

(Excercise 1.1-1) Find the second order Taylor polynomial for $f(x) = e^x \sin x$ about $x_0 = 0$.

a) Compute $P_2(0.4)$ to approximate $f(0.4)$. Use the remainder term $R_2(0.4)$ to find an upper bound for the error $|P_2(0.4) - f(0.4)|$. Compare the upper bound with the actual error.

b) (**MATH 5660 ONLY**) Compute $\int_0^1 P_2(x)dx$ to approximate $\int_0^1 f(x)dx$. Find an upper bound for the error using $\int_0^1 R_2(x)dx$, and compare it to the actual error.

2 Problem 2

(Exercise 1.3-1) Consider the following toy model for a normalized floating-point representation in base 2: $x = (-1)^s(1.a_2a_3)_2 \times 2^e$ where $-1 \le e \le 1$. Find all positive machine numbers (there are 12 of them) that can be represented in this model. Convert the numbers to base 10, and then carefully plot them on the number line, by hand, and comment on how the numbers are spaced.

3 Problem 3

(Exercise 1.3-2) The x-intercept of the line passing through the points (x_1, y_1) and (x_2, y_2) can be computed using either one of the following formulas:

$$x = \frac{x_1y_2 - x_2y_1}{y_2 - y_1}$$

or,

$$x = x_1 - \frac{(x_2 - x_1)y_1}{y_2 - y_1}$$

with the assumption $y_1 \neq y_2$.

a) Show that the formulas are equivalent to each other.

b) Compute the x-intercept using each formula when $(x_1, y_1) = (1.02, 3.32)$ and $(x_2, y_2) = (1.31, 4.31)$. Use three-digit rounding arithmetic.

c) Use Python (or a calculator) to compute the x-intercept using the full-precision of the device (you can use either one of the formulas). Using this result, compute the relative and absolute errors of the answers you gave in part (b). Discuss which formula is better and why.

4 Problem 4

(Exercise 1.3-4) Polynomials can be evaluated in a nested form (also called Horner's method) that has two advantages: the nested form has significantly less computation, and it can reduce roundoff error. For

$$p(x) = a_0 + a_1 x + a_2 x^2 + ... + a_{n-1} x^{n-1} + a_n x^n$$

its nested form is

$$p(x) = a_0 + x(a_1 + x(a_2 + ... + x(a_{n-1} + x(a_n))...)).$$

Consider the polynomial $p(x) = x^2 + 1.1x - 2.8$.

a) Compute $p(3.5)$ using three-digit rounding, and three-digit chopping arithmetic. What are the absolute errors? (Note that the exact value of $p(3.5)$ is 13.3.)

b) Write $x^2 + 1.1x - 2.8$ in nested form by these simple steps:

$$x^2 + 1.1x - 2.8 = (x^2 + 1.1x) - 2.8 = (x + 1.1)x - 2.8.$$

Then compute $p(3.5)$ using three-digit rounding and chopping using the nested form. What are the absolute errors? Compare the errors with the ones you found in (a).

5 Problem 5

(Exercise 1.3-5) Consider the polynomial written in standard form: $5x^4 + 3x^3 + 4x^2 + 7x - 5$.

a) Write the polynomial in its nested form.

b) (**MATH 5660 ONLY**) How many multiplications does the nested form require when we evaluate the polynomial at a real number? How many multiplications does the standard form require? Can you generalize your answer to any nth degree polynomial?