Impedance Spectroscopy

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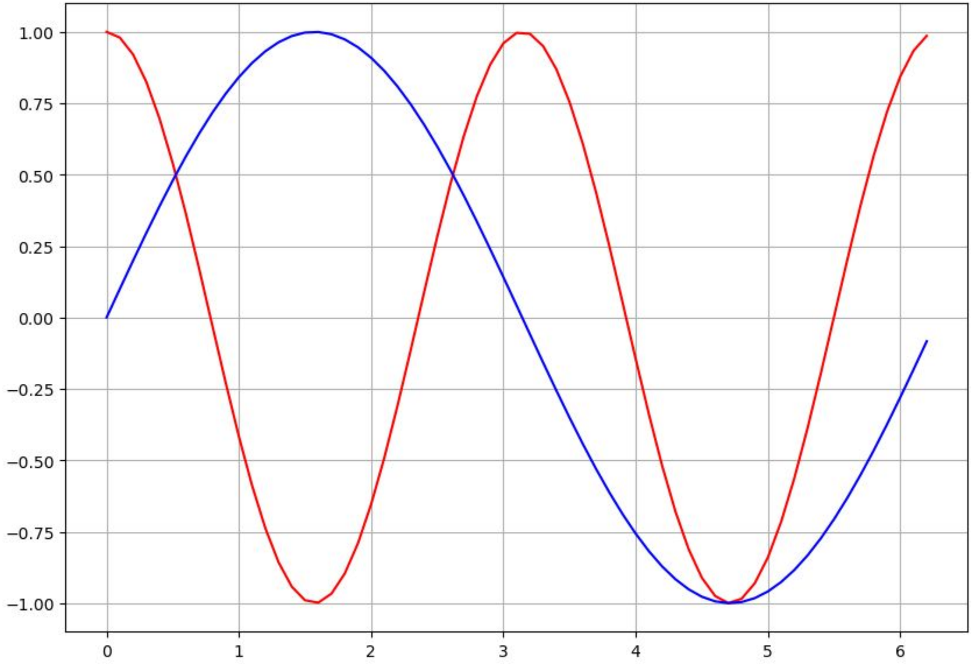


Figure 1.

Main Idea

- Impedance is the total response of a system to an AC current, given by the combination of resistances and reactances of components in a circuit.

- Impedance spectroscopy involves the testing of a system over a range of frequencies in order to measure its magnitude.

- This can give us valuable information about the properties and composition of a system.

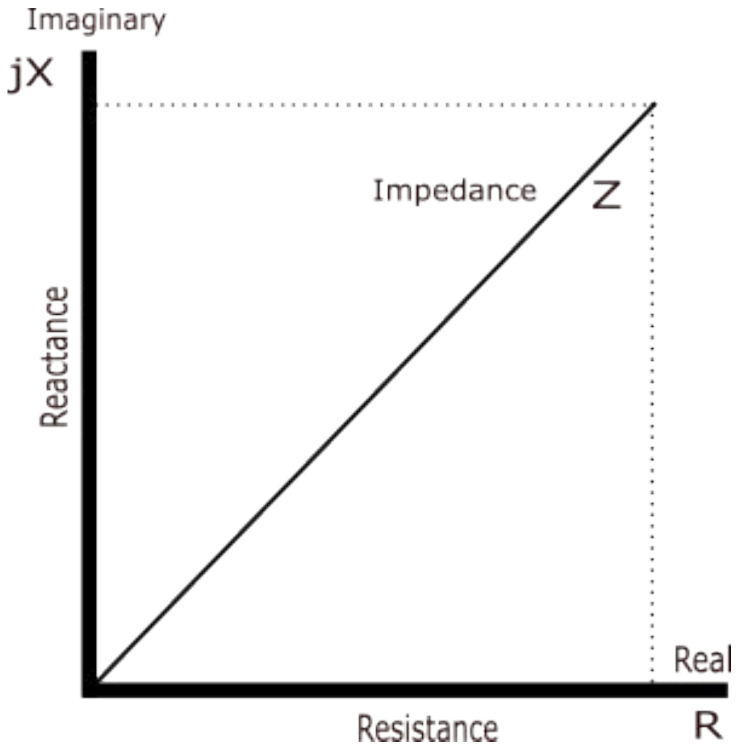


Figure 2.

Instrumentation: Block Diagram

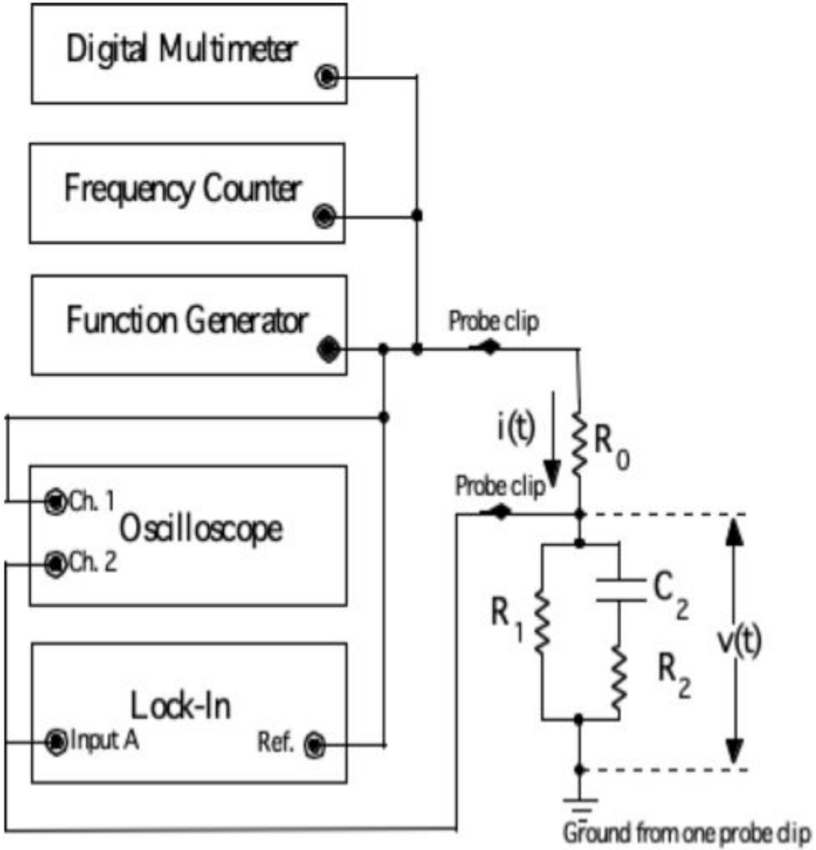


Figure 3: Block diagram of experimental setup.

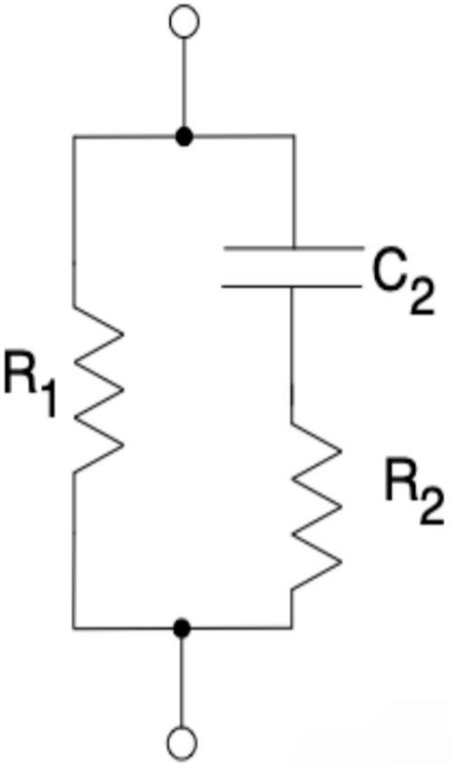
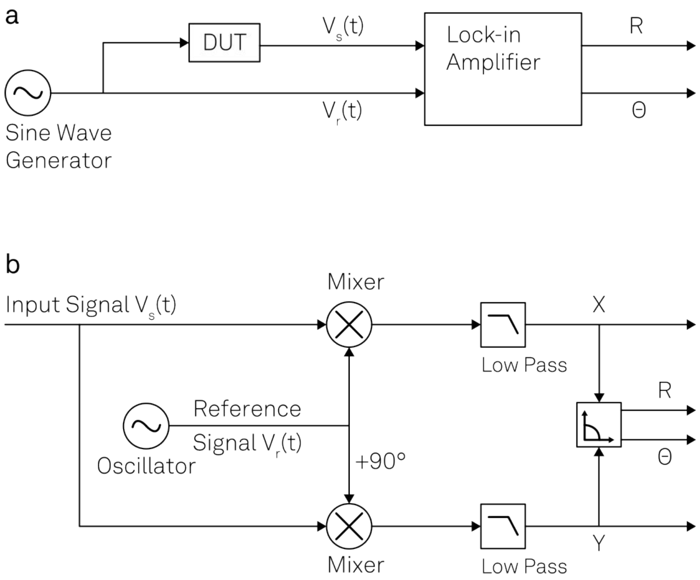


Figure 4: Schematic of circuit s.u.t.

Instrumentation Focus: Lock-In Amplifier

- Lock-in amplifiers are used to extract signal information from noisy inputs.
- A reference signal is generated at a specific frequency and multiplied by the input in a frequency mixer (phase-sensitive detector).
- The output of this signal multiplication will return two signals; one with a frequency that is the difference of the two frequencies and the other their sum.
- The output is passed through a low-pass filter which attenuates frequencies higher than the difference frequency.
- If there is a phase between the two signals, a second, orthogonal reference signal is multiplied by the input, returning two outputs.
- The two outputs are orthogonal and are used to find the amplitude and phase.

Figure 5.) Lock-in amplifier schematic



$$V_i(t) = V_i sin(2\pi f_i + \phi)$$

$$V_{r1}(t) = sin(2\pi f_r)$$

$$V_{r2}(t) = sin(2\pi f_r + 90^o) = cos(2\pi f_r)$$

$$V_{psd1} = \frac{1}{2}V_i cos(2\pi(f_i - f_r) + \phi) = \frac{1}{2}V_i cos(\phi)$$

$$V_{psd2} = \frac{1}{2}V_i sin(2\pi(f_i - f_r) + \phi) = \frac{1}{2}V_i sin(\phi)$$

$$\sqrt{\left(\frac{1}{2}V_i cos(\phi)\right)^2 + \left(\frac{1}{2}V_i sin(\phi)\right)^2}$$

$$\frac{1}{2}V_i\sqrt{\left(cos(\phi)\right)^2 + \left(sin(\phi)\right)^2} = \frac{1}{2}V_i$$

$$\frac{\frac{1}{2}V_i sin(\phi)}{\frac{1}{2}V_i cos(\phi)} = tan(\phi)$$

Data Analysis

- The data were collected in two stages.
  - The first used a circuit (figure 5) as the system under test (s.u.t.) with three different dominant resistors (10 kOhms, 100 kOhms, and 1 MOhm) placed before it in series.
  - The second used a russet potato as the s.u.t. with the same three dominant resistors.
- The impedance was defined with the formulas off to the side, derived from the circuit s.u.t.
- This equation was simplified by defining three constant terms that were used in the final equation.
- The equation for the impedance magnitude was normalized in order to describe the behavior of the system's impedance independent of scale.
- Both the impedance and the phase angle were plotted as functions of the frequency.

$$Z_{tot} = R_1 + \left(\frac{1}{R_2} + \frac{1}{j\omega C_2}\right)^{-1} = R_1 + \frac{j\omega C_2 R_2}{j\omega C_2 + R_2}$$

(total impedance)

$$\tau \equiv \frac{1}{\omega_c} \qquad f = \frac{\omega}{2\pi}$$

$$r \equiv \frac{R_1}{R_2}$$

(constants in impedance)

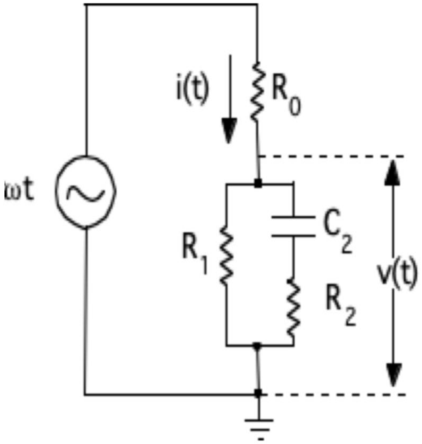


Figure 4: Schematic of circuit s.u.t. R0 is the dominant resistor.

$$\frac{Z}{R_1} = \frac{1 + j2\pi f\tau \frac{1}{1+r}}{1 + j2\pi f\tau}$$

$$\frac{|Z|}{R_1} = \sqrt{\frac{1 + (2\pi f\tau)^2 \frac{1}{(1+r)^2}}{1 + (2\pi f\tau)^2}}$$

$$\phi = arctan\left(-\frac{2\pi f\tau r}{1 + r + (2\pi f\tau)^2}\right)$$

(simplified version of the normalized impedance and phase angle)

Circuit s.u.t.

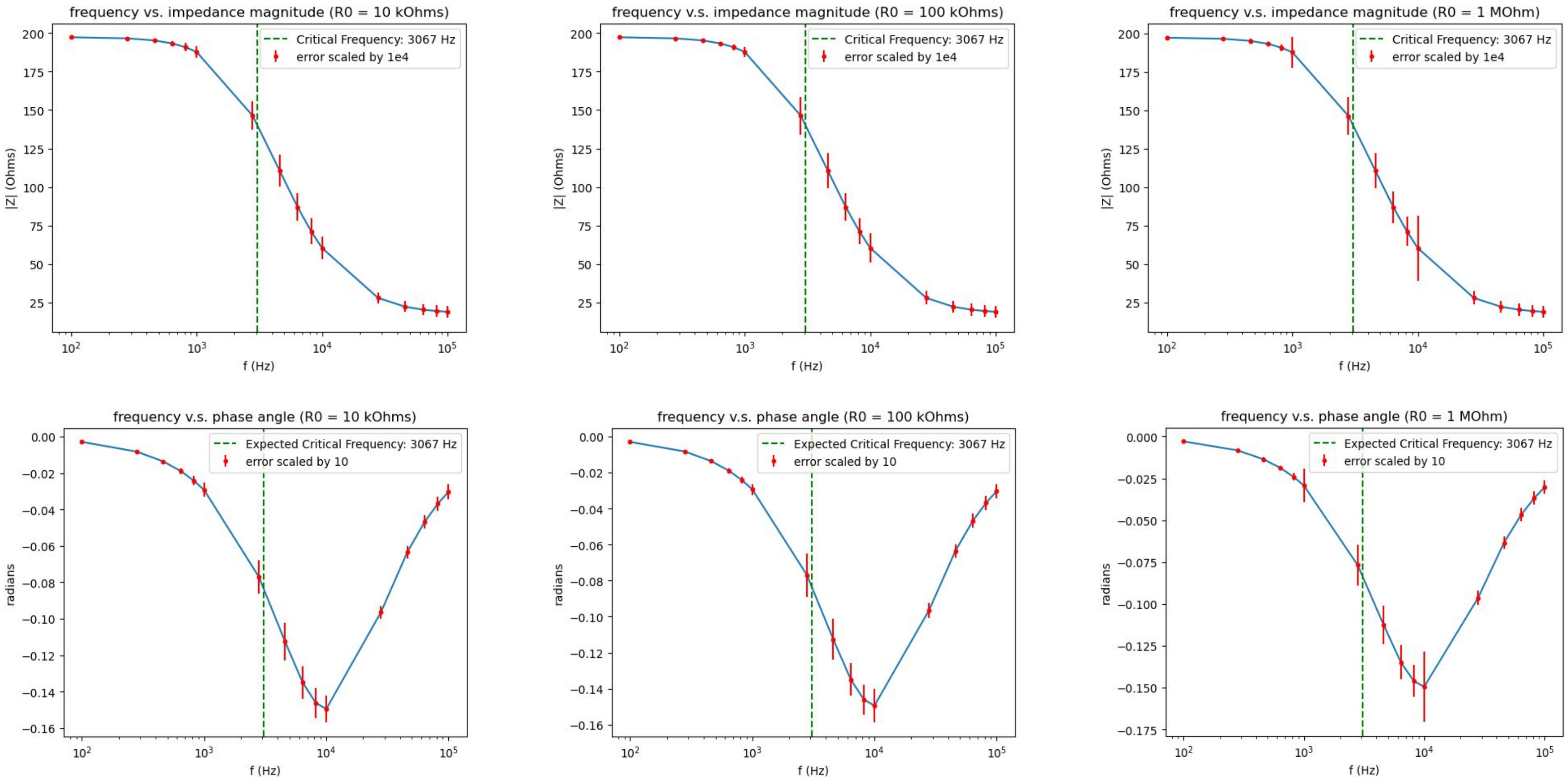


Figure 6a, 6b, 6c: Impedance as a function of frequency. When the frequency is low, the impedance is dominated by R1. As the frequency increase, it falls to the magnitude of R2. Figure 6d, 6e, 6f: Phase angle as a function of frequency. The increase at higher frequencies seems odd and may be explainable by inductance behavior of the circuit at higher frequencies.

Special Topic: Bioimpedance Analysis

- Bioimpedance analysis is a noninvasive approach used in body composition measurements and healthcare assessment systems.
- A weak current is sent through the body and the voltage is measured in order to determine the impedance.
- Muscles hold more water than fat, meaning that muscle will have a lower impedance.
- Determining the impedance using bioimpedance analysis can help to estimate total body water as well as the fat free mass of the human body.

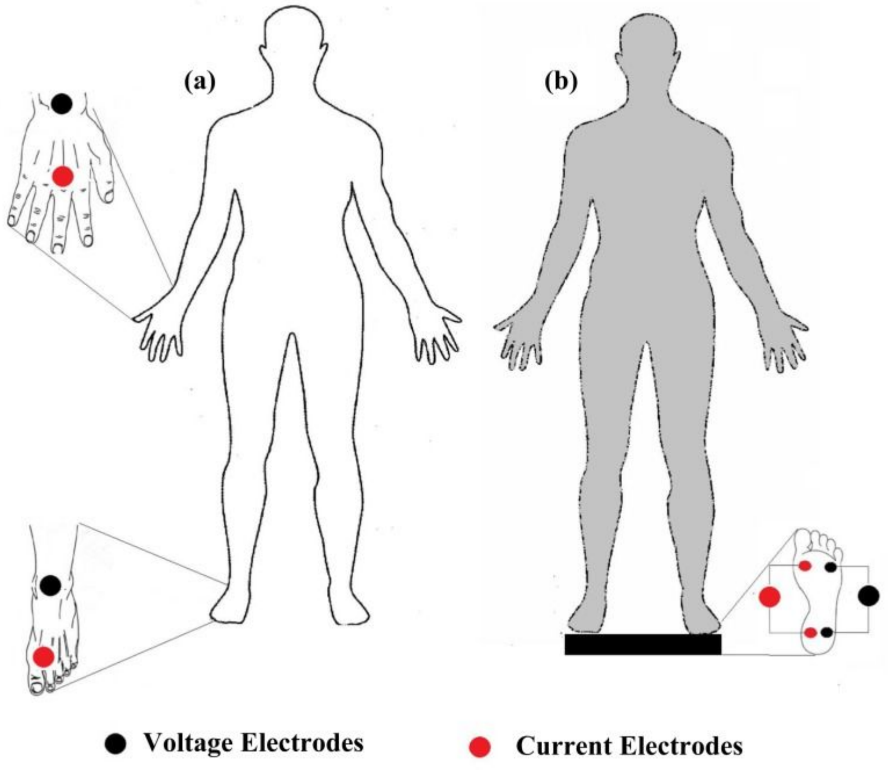


Figure 7: electrode placement for whole body bioimpedance analysis.

References

Figures 1, 2, 6a, 6b,6c, 6d, 6e, 6f made by author.

Figure 3 and 4 taken from lab procedure and ideas handouts.

Figure 5 taken from https://www.zhinst.com/americas/en/resources/principles-of-lock-in-detection

Figure 7 taken from https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4118362/

Lock-in amplifier manual:
https://www.thinksrs.com/downloads/pdfs/applicationnotes/Lock-In%20Basics.pdf

Bioimpedance analysis source: https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4118362/