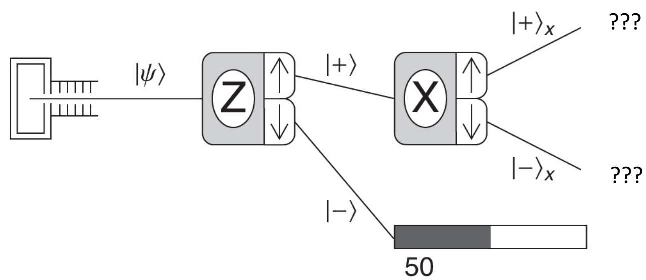
Stern Gerlach Results



Day 2:
Stern Gerlach Experiments
Kets
Activity I: Repeated Spin Measurements

Up Next:
HW01 and Activity 1 (+ Reading 1) due Tuesday midnight
Quantum States
Bras and Kets, Hilbert Space, Inner Products
Superposition States

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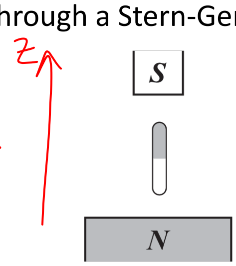
Force on neutral silver atoms: $\vec{F}_z \approx \mu_z \frac{\partial B}{\partial z}\hat{z} = \cancel{g}\,\tfrac{1}{2}\frac{-e}{m_e} S_z \frac{\partial B}{\partial z}\hat{z}$

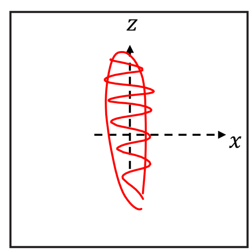
Rotating

non-uniform

What pattern would you expect to see (**classically**) for a thin beam of silver atoms (bar magnets) passing through a Stern-Gerlach device?

RANDOM





~~A)~~ A **continuous** (uniform) smear – same number of atoms hit everywhere

B) A **continuous** (non-uniform) smear – more atoms hit in some locations

~~C)~~ A single **discrete** spot

~~D)~~ Multiple **discrete** separated spots

E) None of these

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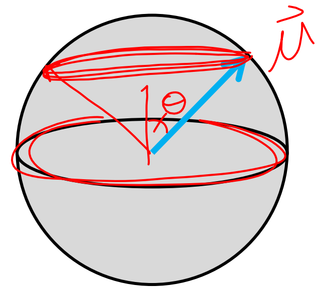
Probability is a **ratio**:

$$P = \frac{\text{want}}{\text{total}} = \frac{\text{Area ring}}{\text{TOTAL area}}$$

$$dP(\theta) = \frac{dA_{ring}}{A_{tot}}$$



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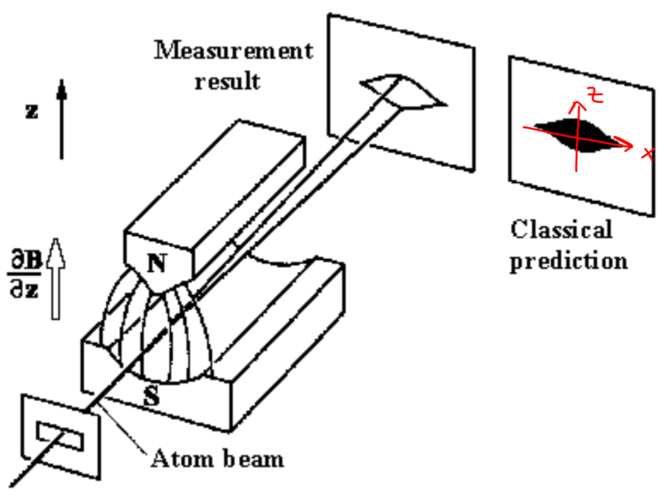
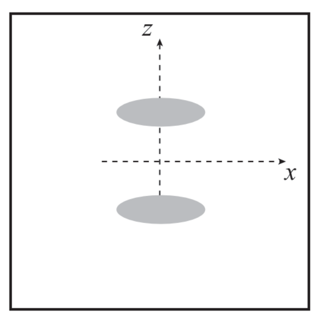
Experimental Result

$$F_z = g\,\frac{-e}{2m_e}\,S_z\,\frac{\partial B_z}{\partial z}$$





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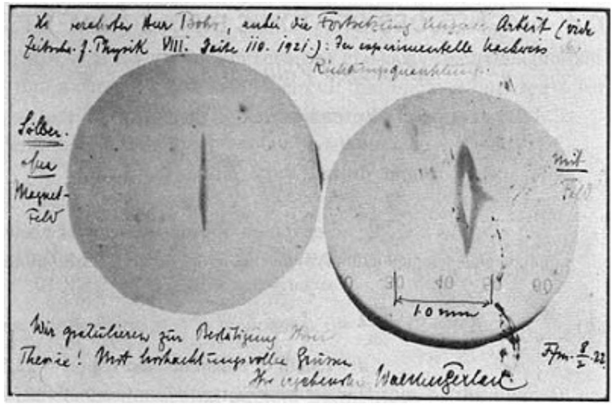
B-field **off** B-field **on**



"My esteemed Herr Bohr, attached is the continuation of our work: the experimental proof of directional quantization. We congratulate you on the confirmation of your theory! With respectful greetings. Your most humble Walther Gerlach."

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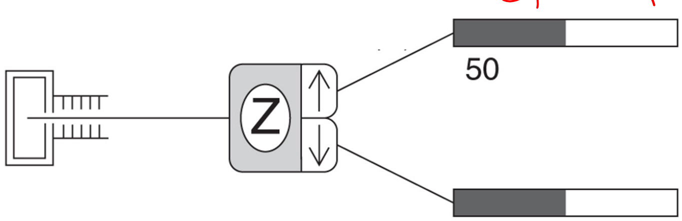
A "spin ½ system"

$S_{\uparrow} = +\frac{\hbar}{2}$

Spin-up



ΔS

Spin-down

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Postulate 1 of QM

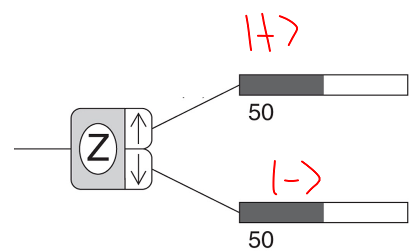
The state of a quantum system, *encapsulating all the information that can in principle be known about it,* is represented by... a normalized ket

|ket⟩ is a vector

$|Spin - up\rangle = |+\rangle = |\uparrow\rangle_{z} = |\uparrow_z\rangle_z$

Basis

$|Spin - down\ x\rangle = |-_x\rangle = |\downarrow_x\rangle$



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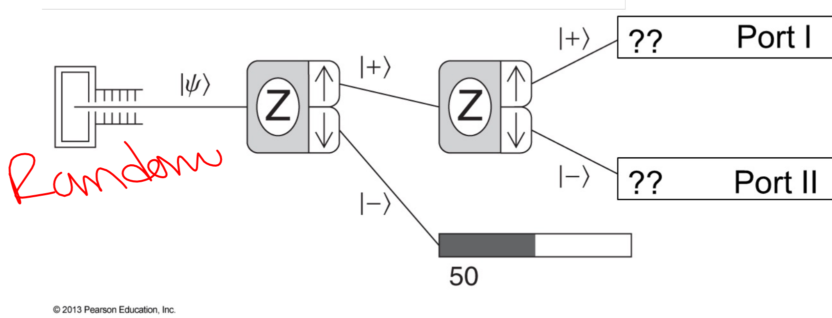
What is the result of:



A) ~50 atoms exit port I (none out port II) — if $|\uparrow\rangle$ to begin, stays
B) ~50 atoms exit port II (none out of port I)
C) ~25 atoms exit port I, ~25 exit Port II — Doesn't matter what it was
D) Something else, the numbers above aren't right!
E) Really not sure

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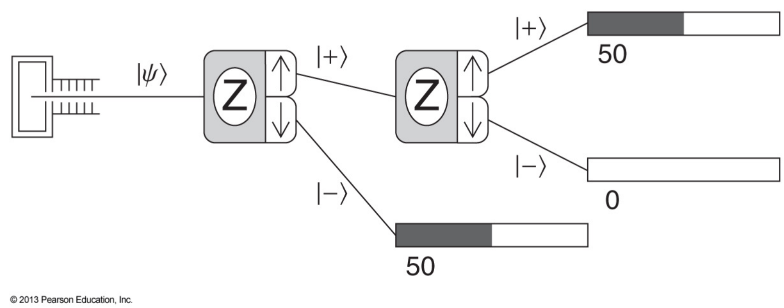
Experiment 1: **Results**



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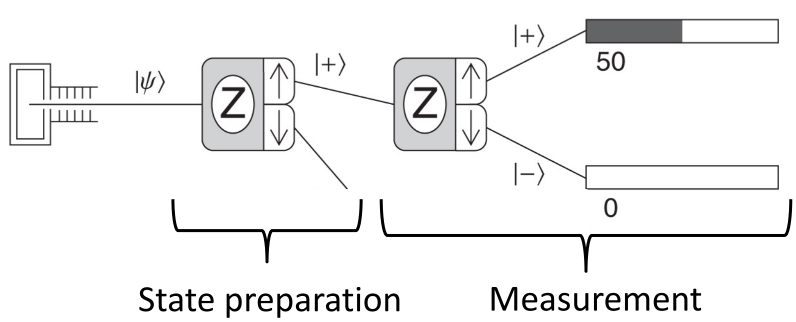
Experiment 1: **Interpretation**



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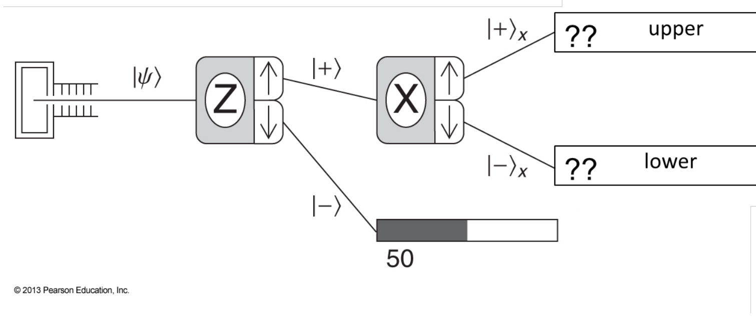
What is the result of this "Chained Stern-Gerlach"?

Note: This is Experiment #2 from McIntyre



A) (Roughly) 50 counts in each of the 2 counters makes atoms

B) (Roughly) 25 counts in each of the 2 counters if $|\uparrow_z\rangle$ don't know x

C) (Roughly) 50 counts in the upper counter only if $\uparrow_z$ also $\uparrow_x$

D) NO counts in either one!

E) Other/not sure….

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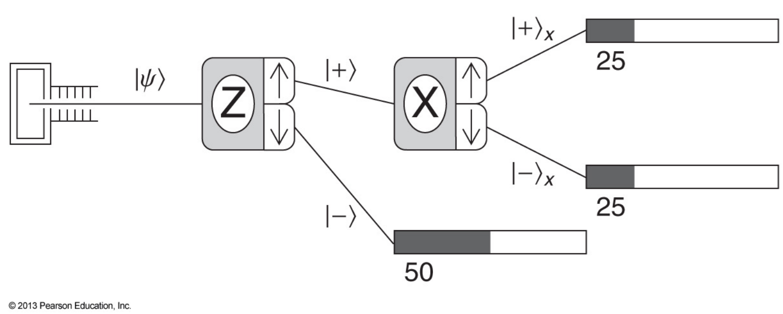
Experiment 2: **Results**



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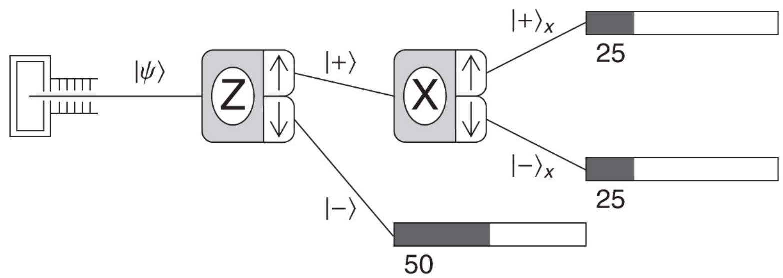
Experiment 2: **Interpretation**

QM is **fundamentally** probabilistic



Could there be a "hidden variable" that tells you which way the atom will go?

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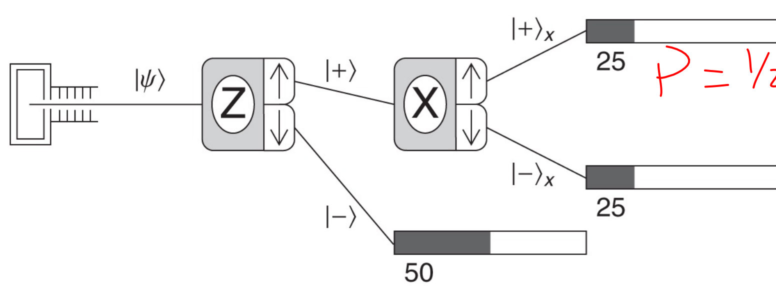
Quantum state vectors



$P = 1/2$

Coefficient of a quantum state is related to the **measurement probability**

Given this experimental result, how could I write $|+\rangle$ in terms of the *x-kets* ($|+\rangle_x$ and $|-\rangle_x$)

$$|+_z\rangle = \frac{1}{2}|+_x\rangle + \frac{1}{2}|-_x\rangle$$

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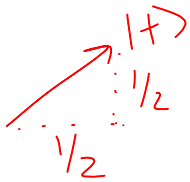
Quantum state vectors - **Normalization**

1st postulate says state is given by a **normalized** ket. This means a ket of magnitude one.



$$\text{mag} = \sqrt{(1/2)^2 + (1/2)^2}$$
$$= \sqrt{1/2}$$

$$\text{norm} = \frac{|\text{ket}\rangle}{\text{mag of ket}} = \frac{\frac{1}{2}\left(|+_x\rangle + |-_x\rangle\right)}{1/\sqrt{2}} = \frac{\sqrt{2}}{2}\left(\text{state}\right)$$
$$\frac{1}{\sqrt{2}}|+_x\rangle + \frac{1}{\sqrt{2}}|-_x\rangle$$

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4^{th} Postulate (version 1)

Often called the "measurement postulate"

We'll write this a few different ways. To start, we'll say that the probability of measuring a given outcome is the **magnitude squared** of the associated coefficient of a normalized quantum state.

generic

So if $|\psi\rangle = a|+\rangle + b|-\rangle$

- The probability that this particle is measured to be "spin-up" (in z) is $\mathcal{P}_{+z} = |a|^2$

$$|+_z\rangle = \frac{1}{\sqrt{2}}\left(|+_x\rangle + |-_x\rangle\right) \quad \text{So } P_{+x} = \left|\frac{1}{\sqrt{2}}\right|^2 = 1/2$$

- The probability that this particle is measured to be "spin-down" (in z) is $\mathcal{P}_{-z} = |b|^2$

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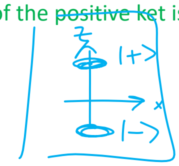
Hilbert Space

Convention: The coefficient of the ~~positive ket~~ is **real** and **positive**

$|\psi\rangle = a|+_x\rangle + b|-_x\rangle$

Quantum states "live in" Hilbert Space

- Complex vector space with some specific rules $P_{+x} = |a|^2$
- w/i The basis kets of the Hilbert space are the quantized *observable* states



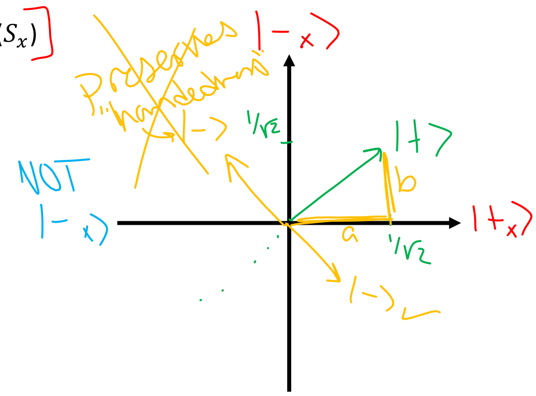
Say the axes at right represent the $\boxed{\text{2D Hilbert space for } x\text{-spin } (S_x)}$

- Where are $|+\rangle_x$ and $|-\rangle_x$ in this Hilbert Space?

Orthogonal

- Where is $|+\rangle_z$? $= \frac{1}{\sqrt{2}}|+_x\rangle + \frac{1}{\sqrt{2}}|-\rangle_x$

- Where is $|-\rangle_z$? $= \frac{1}{\sqrt{2}}|+_x\rangle - \frac{1}{\sqrt{2}}|-\rangle_x$ Convention



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Quantum state vectors – Relating bases

$|+\rangle_z\,(in\ x-basis) = \frac{1}{\sqrt{2}}\left(|+_x\rangle + |-_x\rangle\right)$

$|-\rangle_z\,(in\ x-basis) = \frac{1}{\sqrt{2}}\left(|+_x\rangle - |-_x\rangle\right)$

$|+\rangle_x(in\ z-basis) = \frac{1}{\sqrt{2}}\left(|+\rangle \overset{e^{i\theta}}{+} |-\rangle\right)$

$|-\rangle_x(in\ z-basis) = \frac{1}{\sqrt{2}}\left(|+\rangle - |-\rangle\right)$

$|\pm\rangle_y(in\ z-basis) = \frac{1}{\sqrt{2}}\left(|+\rangle \pm i|-\rangle\right)$

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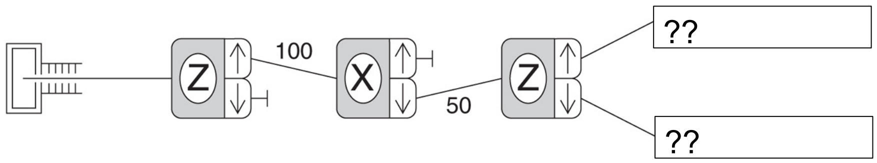
What is the result of Experiment 3?



A) (Roughly) 25 counts in each of the 2 counters
~~B)~~ (Roughly) 50 counts in each of the 2 counters
C) (Roughly) 50 counts in the upper counter only
D) (Roughly) 50 counts in the lower counter only
E) Other….

No memory
Created particles
Have memory
Flip

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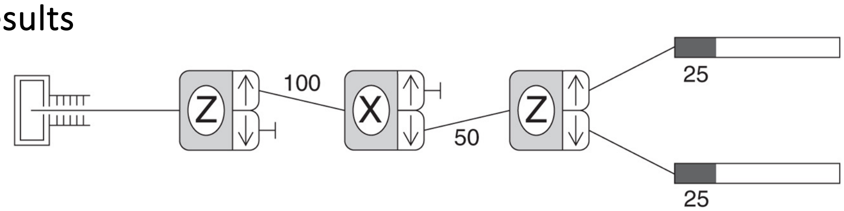
Experiment 3: **Results**



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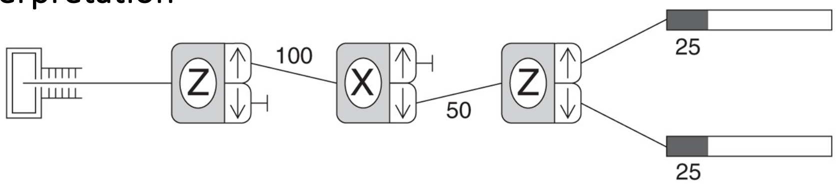
Experiment 3: **Interpretation**



States have no memory.

You know what you just measured (usually) nothing else

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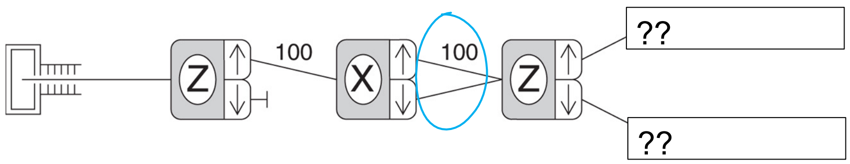
Experiment 4: **Interference**



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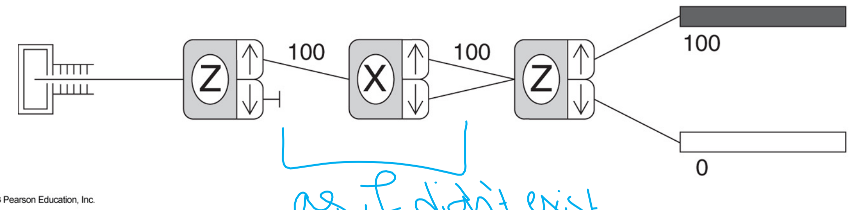
Experiment 4: **Results & Interpretation**



as if didn't exist.

Like double-slit:

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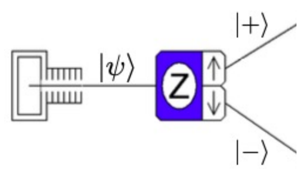
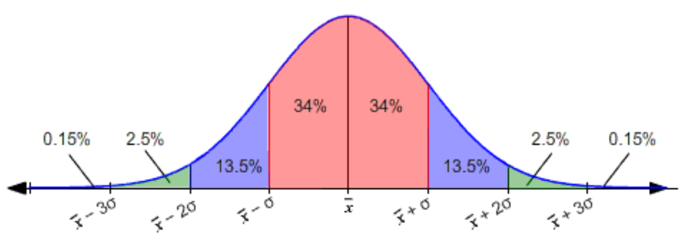
$\sigma \propto \sqrt{N}$

Activity I – Repeated Spin Measurements

$\bar{x} = \frac{N}{2}$





Curve shows the fraction of data that falls in each range. *e.g.* There is a 68% probability of experimental outcomes falling within **one** standard deviation of the expected average. Note that this is specific to a "normal" distribution, which is valid for Binomial (two state) situations.

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