Algorithms, 2235, Homework 4
due Tuesday, April 2, 2024, 11:59pm

Problem 1

Given is a sequence $a_1, a_2, \ldots, a_n$ of numbers. We say that a subsequence $a_{j_1}, a_{j_2}, \ldots, a_{j_k}$, where $j_1 < j_2 < \ldots < j_k$, is *convex* if $a_{j_{i-1}} + a_{j_{i+1}} \geq 2a_{j_i}$ for every $i \in \{2, 3, \ldots, k-1\}$. Give an $O(n^3)$ algorithm that finds the longest convex subsequence.

Hint: Use a 2D dynamic programming array.

Problem 2

Consider a 2-backpack version of Knapsack. Given are two backpacks of capacities W_1 and W_2. Given are also n items $(w_1, c_1), (w_2, c_2), \ldots, (w_n, c_n)$, where w_i is the weight and c_i the cost of the i-th item. We want to find a set of items that can be split into two parts: one that fits in the first backpack and the other in the second backpack, and the sum of their costs is the largest possible. All the numbers are positive and W_1, W_2, and all the w_i's are integers. Give an $O(nW_1W_2)$ algorithm that finds the largest possible cost.

Problem 3

Given is an $n \times n$ grid where each cell is either colored black or white. Give an $O(n^2)$ algorithm that finds the largest k such that there is an all-white $k \times k$ square in the grid.

Problem 4

Recall the Matrix Chain Multiplication problem: we are given $a_0, a_1, \ldots, a_n$ that denote the dimensions of n matrices - the i-th matrix A_i is of dimensions $a_{i-1} \times a_i$, and we want to find a parenthesizing that minimizes the number of operations needed to multiply $A_1 A_2 \ldots A_n$. We assume that the number of operations needed to multiply two matrices of dimensions $p \times q$ and $q \times r$ is pqr. In class, we will soon discuss how to compute the minimal cost. Given an $O(n^3)$ algorithm that finds the corresponding parenthesizing (more precisely, use $O(n^3)$ time to find the optimal cost, then $O(n)$ time to find the parenthesizing).