Stat Learning 1: Information Theory

Overview of the summer

- https://github.com/JJBannister/StatisticalLearningGroup
- Bi-weekly meetings (mostly)
- Friday 12-1
- Presentations by grad students and postdocs
- No textbook, lots of presenter freedom
- High level exploration of different topics in Statistics and Machine Learning

Lecure (David Mumford - Pattern Theory Chapter 1)

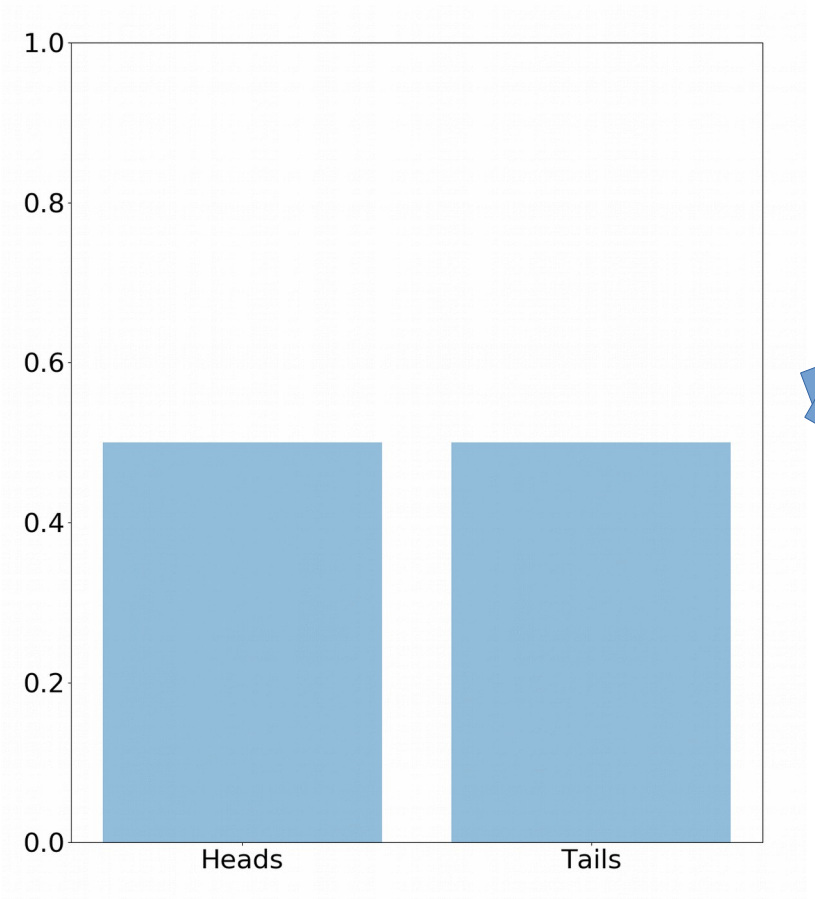
- Entropy
- Conditional Entropy
- Mutual Information
- Relative Entropy
- Differential Entropy
- eg. English Text

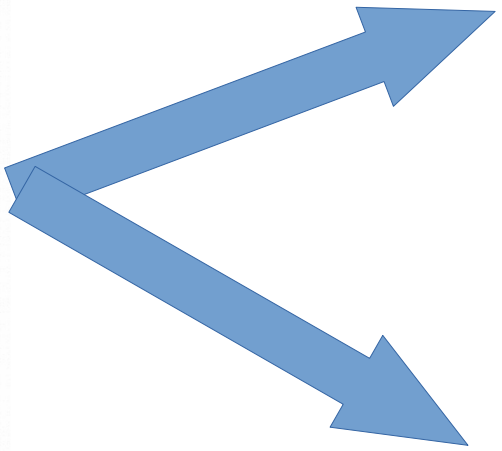
Thanks to the GSA Quality Money Fund, BMEG, MTC, and especially the presenters!!

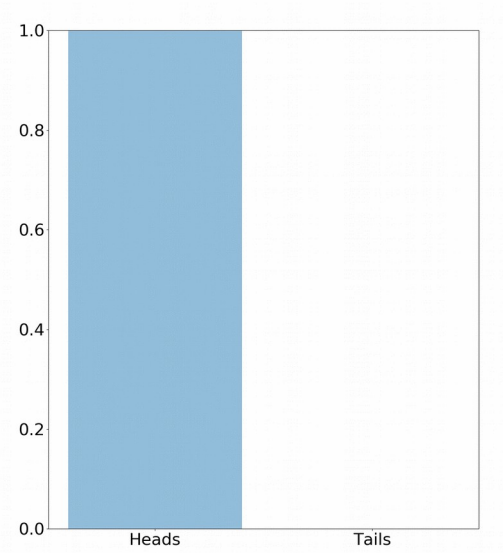
Entropy

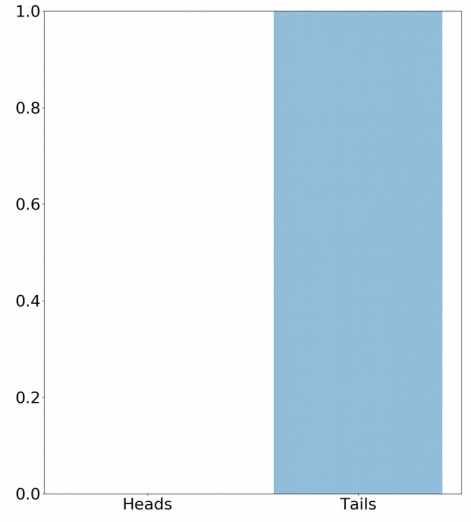
- Physics: **Disorder** (Boltzman 1877)
  - Statistical mechanics (entropy+energy), thermodynamics, quantum computing, black holes!
- Communication Systems: **Information Capacity** (Shannon 1948)
  - Channel capacity, coding, compression...
- Statistics: **Uncertainty** (Kullback 1959)
  - Model training, model comparisson, hypothesis testing, experimental design...

The Fair Coin

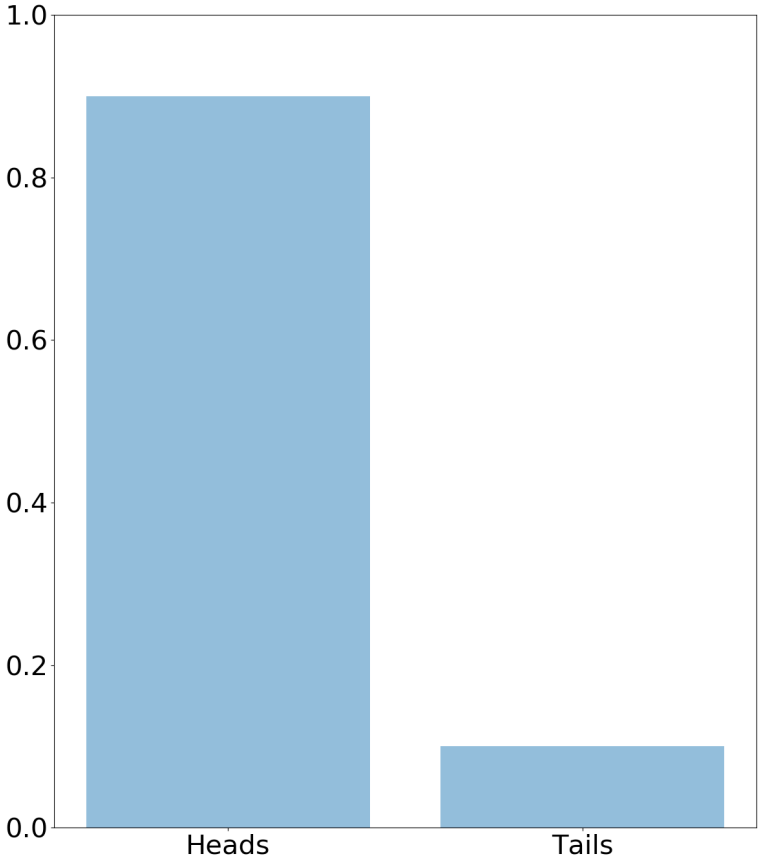


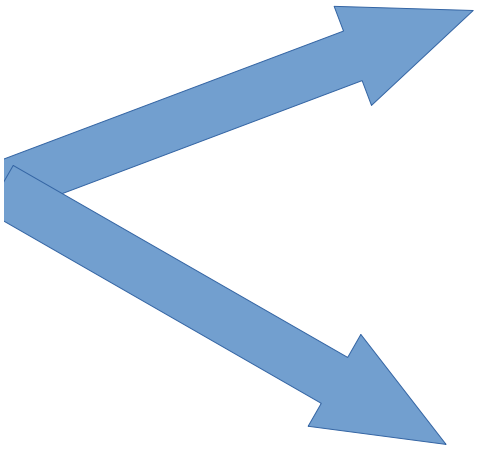


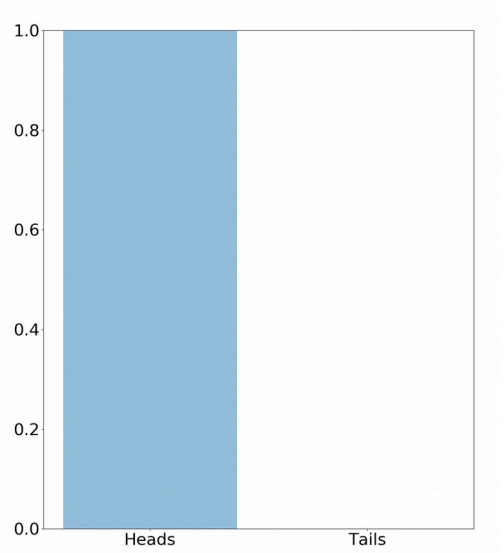


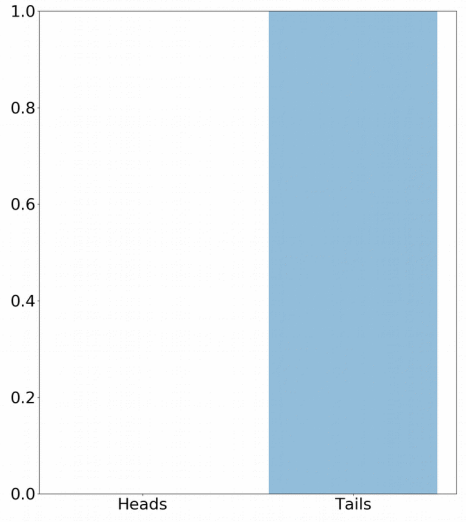


The Loaded Coin









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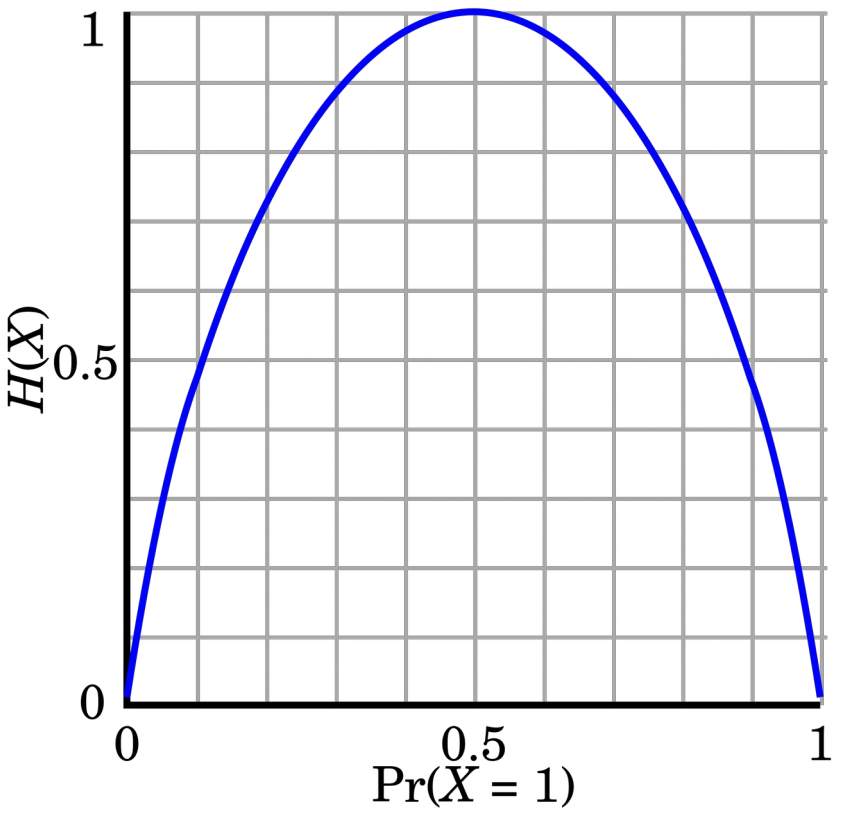
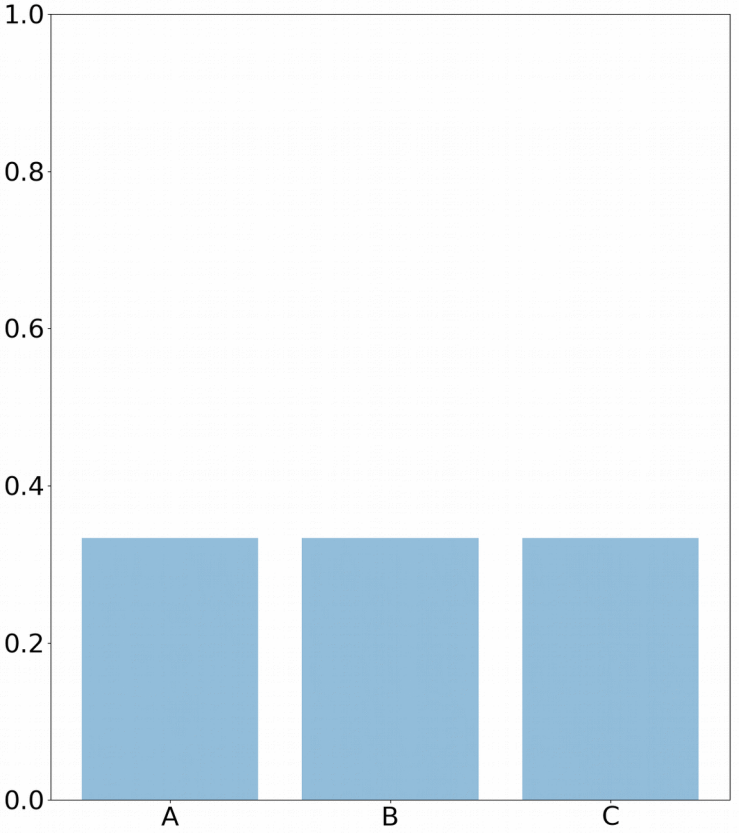
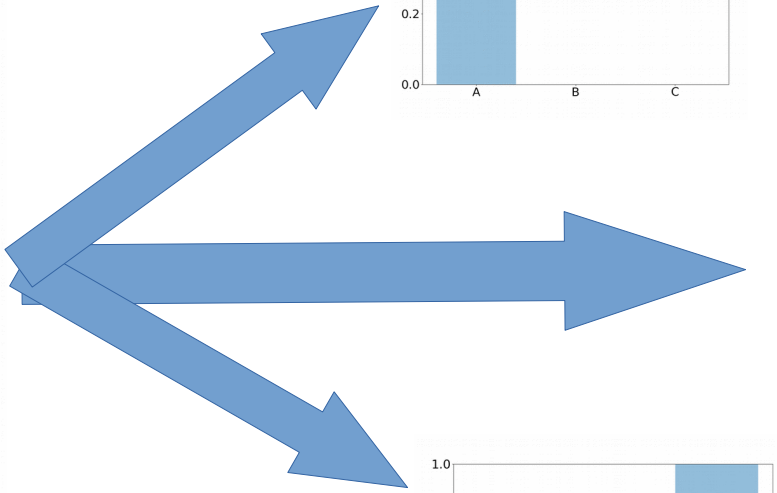
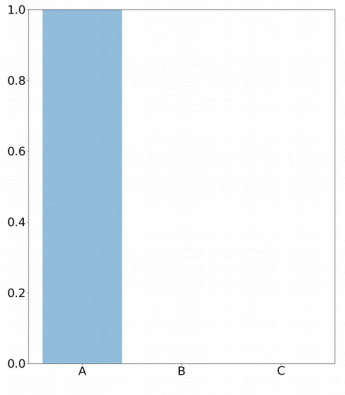
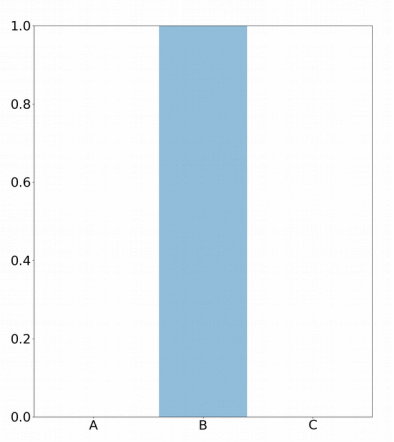


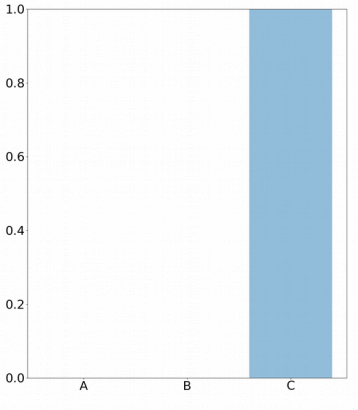
The 3-sided Die









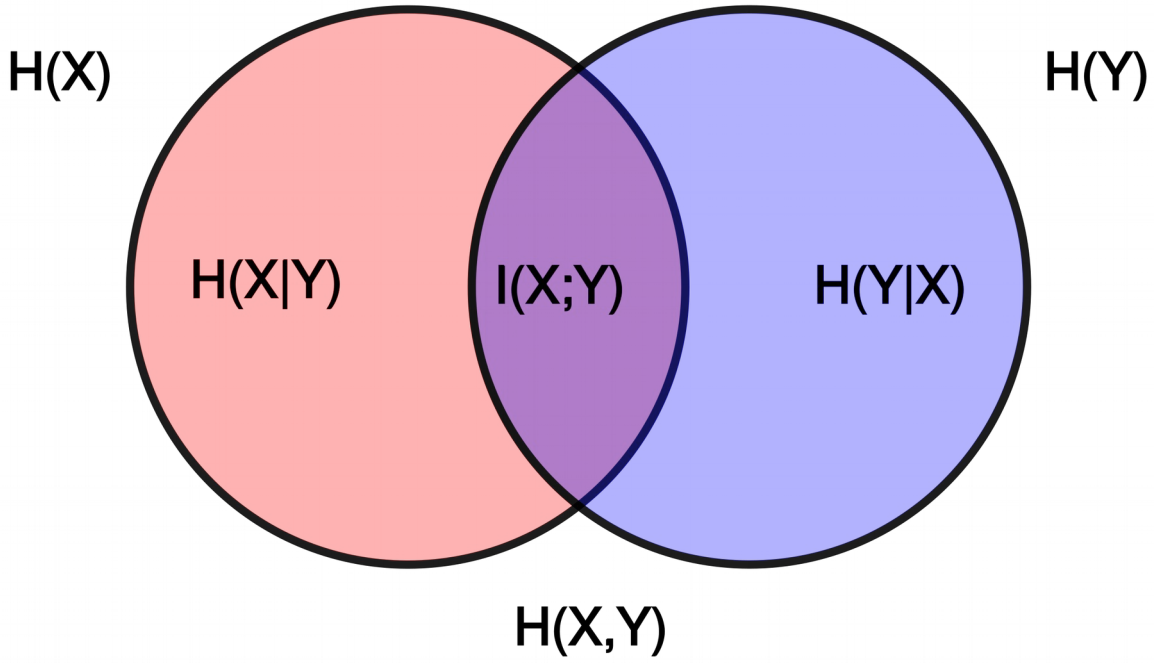


What is the entropy of english text?

- **English text** {a,b,c,d,...}
- Morse Code {*, -, ‘ ‘}
- DNA {G,A,T,C}
- Anything digital! {1,0}

Example

Mutual Information



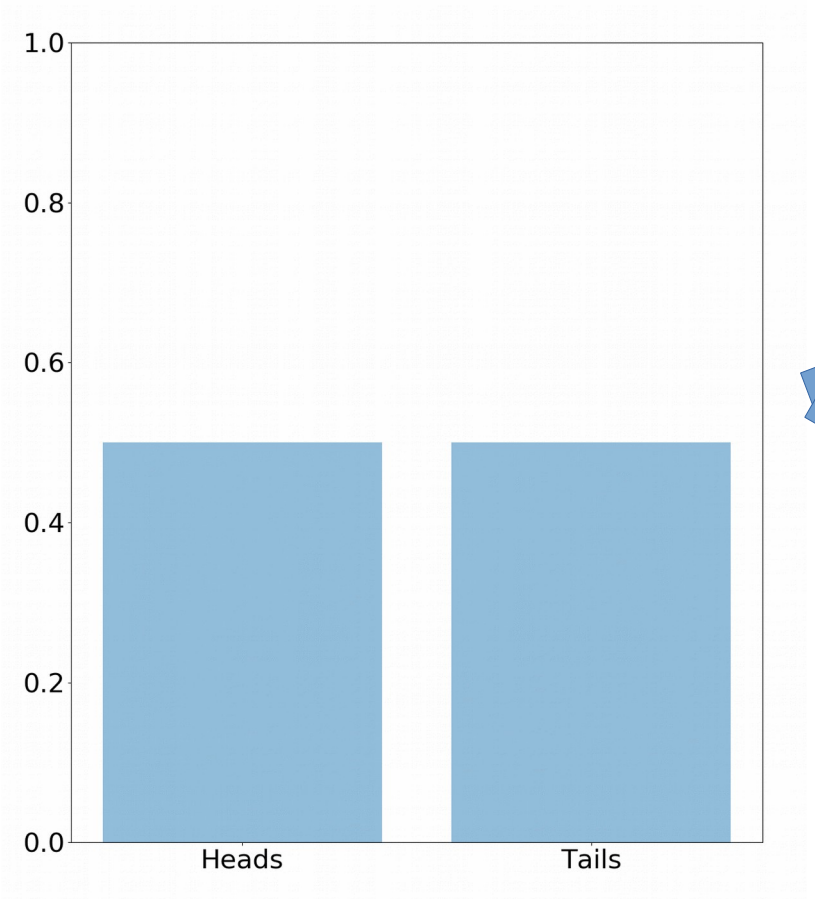
How much information does one letter tell us about the next letter?

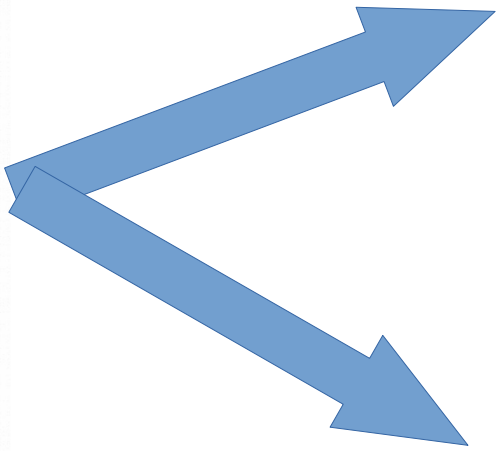
How much information did our model learn?

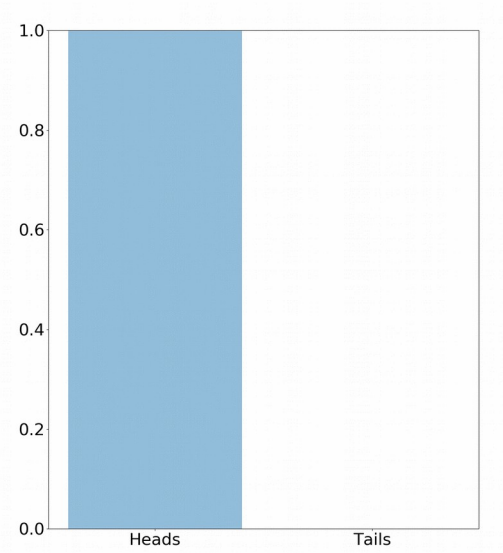
- What is the expected difference in information between the informed model, and the naive model? **KL Divergence**

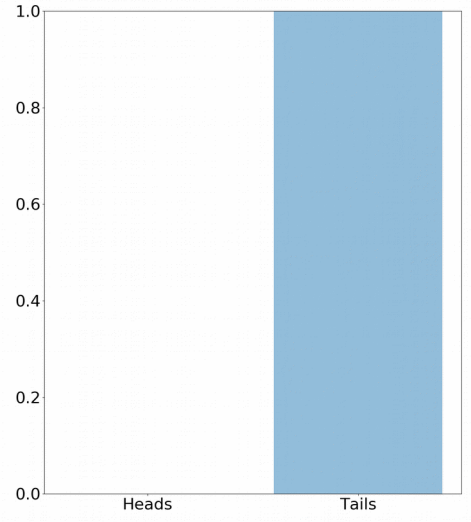
- Not symmetric, not a distance metric.

The Fair Coin

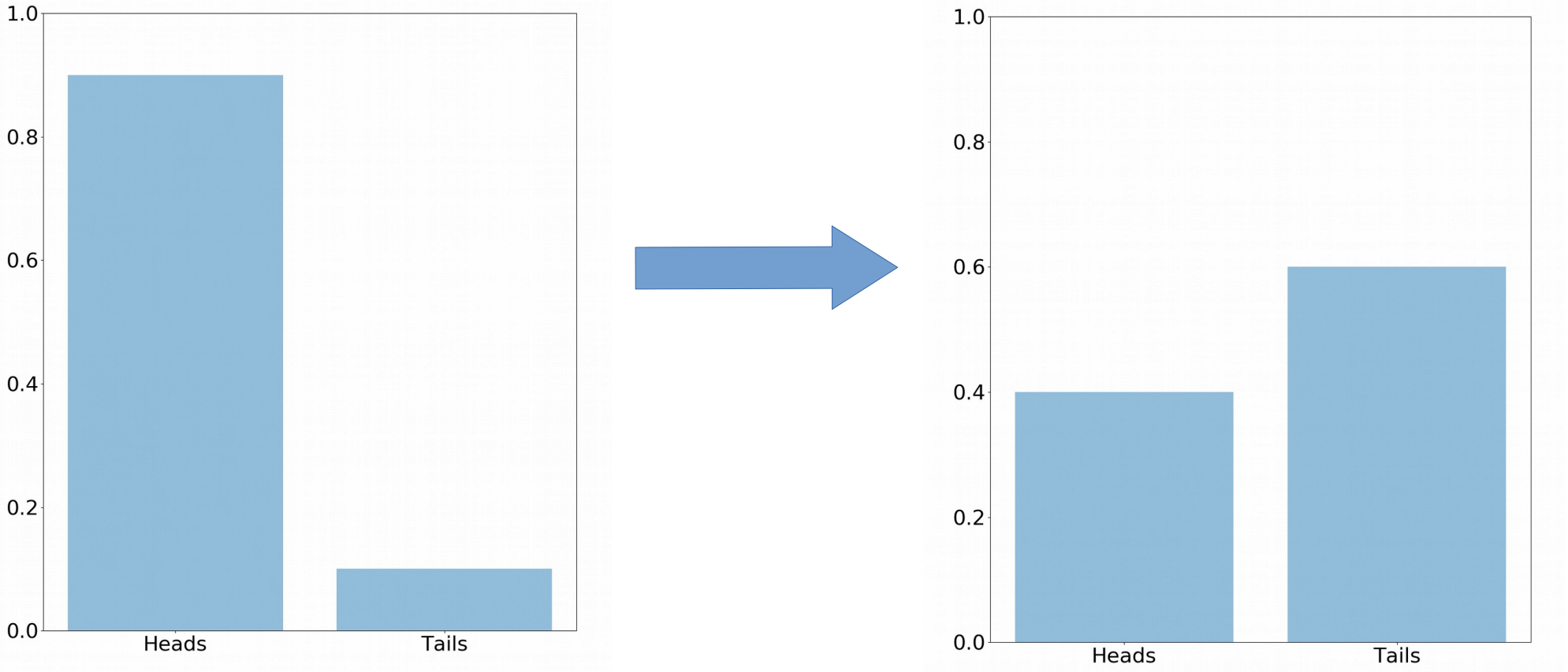








Incremental Learning



Aplications

- Model Fitting (MLE, VAE, ICA, InfoGAN...)
- Model validation (cross validated KL Divergence)
- Model comparisson
  - AIC, WAIC, GIC, TIC, Bayes Factor
  - Goodness of fit + complexity penalty
  - Entropy + energy

What about Continuous Variables?

- Differential Entropy is not exactly entropy...
  - The true entropy of a continuous function is infinity
  - Differential entropy must be relative to a reference measure and so:
    - Units matter (mm vs m)
    - Can be negative