Homework 3

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Problem 1 - Section A

System 1

A

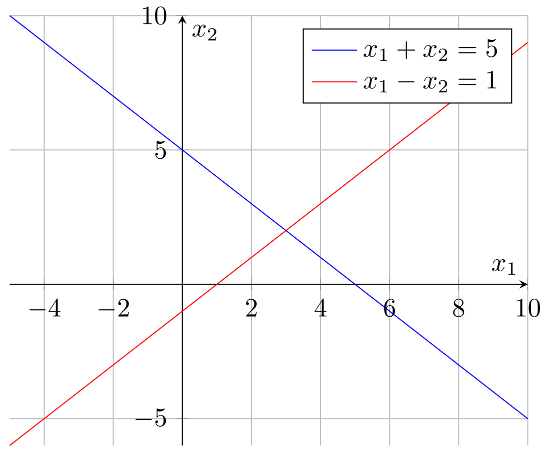


Figure 1: Plot of $x_1 + x_2 = 5$ and $x_1 - x_2 = 1$

B

A unique solution since there is one intersection point between the lines.

C

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

D

$$\begin{bmatrix} 1 & 1 & 5 \\ 1 & -1 & 1 \end{bmatrix}$$

E

$$\begin{bmatrix} 1 & 1 & 5 \\ 1 & -1 & 1 \end{bmatrix} \xrightarrow{R_2 = R_2 - R_1} \begin{bmatrix} 1 & 1 & 5 \\ 0 & -2 & -4 \end{bmatrix}$$

F

$$\begin{bmatrix} 1 & 1 & 5 \\ 0 & -2 & -4 \end{bmatrix} \xrightarrow{R_2=-\frac{1}{2}R_2} \begin{bmatrix} 1 & 1 & 5 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{R_1=R_1-R_2} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \end{bmatrix}$$

Therefore, given this RREF, it follows that

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

System 2

A

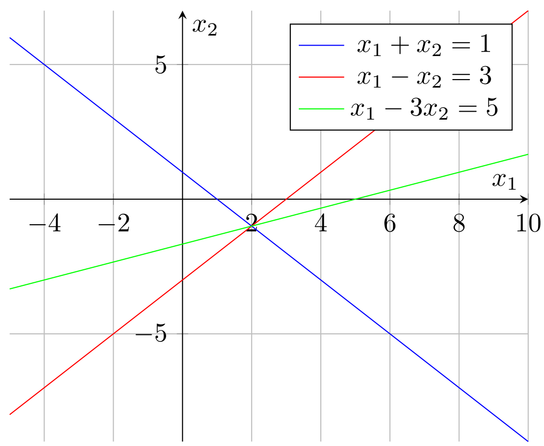


Figure 2: Plot of $x_1 + x_2 = 1$, $x_1 - x_2 = 3$, and $x_1 - 3x_2 = 5$

B

The SLE will have one solution since all lines intersect at the same point.

C

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & -3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

D

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 3 \\ 1 & -3 & 5 \end{bmatrix}$$

E

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 3 \\ 1 & -3 & 5 \end{bmatrix} \xrightarrow{R_2=R_2-R_1, R_3=R_3-R_1} \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 2 \\ 0 & -4 & 4 \end{bmatrix} \xrightarrow{R_3=R_3-2R_2} \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 2 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2=-\frac{1}{2}R_2} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1=R_1-R_2} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Therefore, given this RREF, it follows that

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

System 3

A

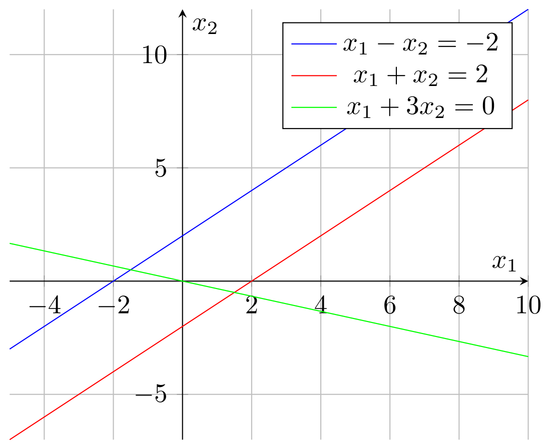


Figure 3: Plot of $x_1 - x_2 = -2$, $x_1 + x_2 = 2$, and $x_1 + 3x_2 = 0$

B

No solution to SLE since the lines do not all have common intersection point.

C

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}$$

D

$$\begin{bmatrix} 1 & -1 & -2 \\ 1 & 1 & 2 \\ 1 & 3 & 0 \end{bmatrix}$$

E

$$\begin{bmatrix} 1 & -1 & -2 \\ 1 & 1 & 2 \\ 1 & 3 & 0 \end{bmatrix} \xrightarrow{R_2 = R_2 - R_1, R_3 = R_3 - R_1} \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 4 \\ 0 & 4 & 2 \end{bmatrix} \xrightarrow{R_3 = R_3 - 2R_2} \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 4 \\ 0 & 0 & -6 \end{bmatrix}$$

F

The last column of the REF effectively shows that $x_1 \cdot 0 = -6$, which is sufficient to determine that the SLE is not consistent.

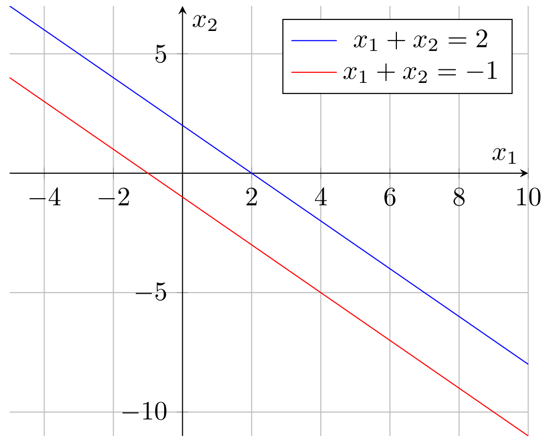


Figure 4: Plot of $x_1 + x_2 = 2$ and $x_1 + x_2 = -1$

Problem 1 - Section B

System 4

B

The SLE will have no solutions since the lines are parallel and therefore will never intersect.

C

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & -2 \end{bmatrix} \xrightarrow{R_2 = R_2 - R_1} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix} \xrightarrow{R_2 = -\frac{1}{4}R_2} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 = R_1 - 2R_2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

D

```
A = [1,1;1,1];
disp(rref(A))
```

```
     1     1
     0     0
```

E

```
A = [1,1,2;1,1,-2];
disp(rref(A))
```

```
     1     1     0
     0     0     1
```

F

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Column 1: Pivot Column 2: Free variable Column 3: Pivot

G

Since there is a pivot in the last column, from which is follows that $x_2 \cdot 0 = 1$, the SLE is in fact not consistent.

System 5

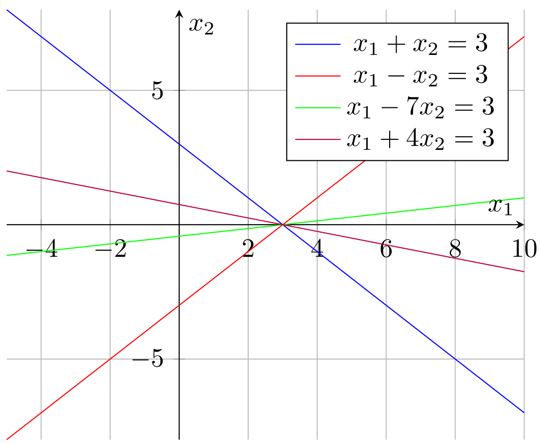


Figure 5: Plot of $x_1 + x_2 = 3$, $x_1 - x_2 = 3$, $x_1 - 7x_2 = 3$, and $x_1 + 4x_2 = 3$

B

Unique solution since there exists a common intersection point of the lines.

C

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & -1 & 3 \\ 1 & -7 & 3 \\ 1 & 4 & 3 \end{bmatrix} \xrightarrow{R_2=R_2-R_1, R_3=R_3-R_1, R_4=R_4-R_1} \begin{bmatrix} 1 & 1 & 3 \\ 0 & -2 & 0 \\ 0 & -8 & 0 \\ 0 & 3 & 0 \end{bmatrix} \xrightarrow{R_4=2R_4} \begin{bmatrix} 1 & 1 & 3 \\ 0 & -2 & 0 \\ 0 & -8 & 0 \\ 0 & 6 & 0 \end{bmatrix}$$

$$\xrightarrow{R_4=R_4+3R_3, R_3=R_3-4R_2} \begin{bmatrix} 1 & 1 & 3 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2=-frac12R_2} \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1=R_1-R_2} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

D

```
A = [1,1;1,-1;1,3;1,4];
disp(rref(A))
```

```
1    0
0    1
0    0
0    0
```

E

```
A = [1,1,3;1,-1,3;1,-7,3;1,4,3];
disp(rref(A))
```

```
1   0   3
0   1   0
0   0   0
0   0   0
```

F

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Column 1: Pivot Column 2: Pivot

G

Yes, the SLE is indeed consistent and unique as predicted. By expanding the augmented matrix back to two equations we are left with $x_1 = 3, x_2 = 0$, which is indeed the same point shown in Fig.5.

System 6

A

-

B

Since there are 3 equations in 5 unknowns, if the system is consistent it must have infinitely many solutions (with two free variables).

C

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 4 \\ -1 & 1 & -1 & 1 & 1 & 2 \\ 2 & 2 & 0 & -1 & 5 & 4 \end{bmatrix} \xrightarrow{R_2=R_2+R_1, \quad R_3=R_3-2R_1} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 4 \\ 0 & 2 & 0 & 2 & 2 & 6 \\ 0 & 0 & -2 & -3 & 3 & -4 \end{bmatrix}$$

$$\xrightarrow{R_2=\frac{1}{2}R_2, \quad R_3=-\frac{1}{2}R_3} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 4 \\ 0 & 1 & 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & \frac{3}{2} & -\frac{3}{2} & 2 \end{bmatrix}$$

$$\xrightarrow{R_1=R_1-R_2} \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & \frac{3}{2} & -\frac{3}{2} & 2 \end{bmatrix}$$

$$\xrightarrow{R_1=R_1-R_3} \begin{bmatrix} 1 & 0 & 0 & -\frac{3}{2} & \frac{3}{2} & -1 \\ 0 & 1 & 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & \frac{3}{2} & -\frac{3}{2} & 2 \end{bmatrix}$$

D

```
A = [1,1,1,1,1;-1,1,-1,1,1;2,2,0,-1,5];
disp(rref(A))
```

```
1.0000       0          0   -1.5000    1.5000
0       1.0000          0    1.0000    1.0000
0            0     1.0000    1.5000   -1.5000
```

E

```
A = [1,1,1,1,1,4;-1,1,-1,1,1,2;2,2,0,-1,5,4];
disp(rref(A))
```

```
1.0000       0          0   -1.5000    1.5000   -1.0000
0       1.0000          0    1.0000    1.0000    3.0000
0            0     1.0000    1.5000   -1.5000    2.0000
```

F

Column 1: Pivot Column 2: Pivot Column 3: Pivot Column 4: Free Variable Column 5: Free Variable

G

Given the RREF, and asssuming $x_4 = t_4, x_5 = t_5$, it can be deduced that

$$x_1 = -1 + \frac{3}{2}t_4 - \frac{3}{2}t_5,$$
$$x_2 = 3 - t_4 - t_5,$$
$$x_3 = 2 - \frac{3}{2}t_4 + \frac{3}{2}t_5.$$

Therefore, demonstrating the aforementioned proposition system is consistent and not unique, but has infinitely many solutions.

Problem 2

0.1 A

$A\vec{v} = \vec{b}$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -3 & 0 \\ -1 & 2 & 6 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$$

$[A|\vec{v}]$

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & -3 & 0 & 1 \\ -1 & 2 & 6 & 5 \end{bmatrix}$$

B

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & -3 & 0 & 1 \\ -1 & 2 & 6 & 5 \end{bmatrix} \xrightarrow{R_2=R_2-R_1,\ R_3=R_3+R_1} \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -4 & -1 & -1 \\ 0 & 3 & 7 & 7 \end{bmatrix} \xrightarrow{R_3=R_3+\frac{3}{4}R_2} \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -4 & -1 & -1 \\ 0 & 0 & \frac{25}{4} & \frac{25}{4} \end{bmatrix}$$

C

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -4 & -1 & -1 \\ 0 & 0 & \frac{25}{4} & \frac{25}{4} \end{bmatrix}$$

D

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -4 & -1 & -1 \\ 0 & 0 & \frac{25}{4} & \frac{25}{4} \end{bmatrix} \xrightarrow{R_2=-\frac{1}{4}R_2,\ R_3=\frac{4}{25}R_3} \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{R_2=R_2-\frac{1}{4}R_3} \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\xrightarrow{R_1=R_1-R_3} \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{R_1=R_1-R_2} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

E

```
A = [1,1,1; 1,-3,0; -1,2,6];
b= [2;1;5]
x = A \ b
```

```
b =

     2
     1
     5


x =

     1
     0
     1
```

G

```
% \\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\
%     Problem 2:  Plotting 3 intersecting planes
%
% \\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\


% -- Suppose we have an Ax = b equation, where A = (3 x 3), and it
%    representes these 3 equations:
```

```
%   Plane #1:        x1  +  x2  +  x3  = 2
%   Plane #2:        x1  -  x2  + 2x3  = 3
%   Plane #3:       -x1  + 2x2  + 4x3  = 3

% -- Define matrix A

A = [ 1  1  1 ;       % -- row vector r1
      1 -3  0 ;       % -- row vector r2
     -1  2  6 ]       % -- row vector r3

%    a1 a2 a3         % -- Column vectors a1, a2, a3


% -- Define target vector b
b = [ 2 1 5 ]'


% -- We can solve for x by using the \  command:

disp('The solution to our Ax = b problem is');
x = A\b


% \\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\
%
%   Task 1:    Drawing the 3 planes of interest
%
%   We will learn how to do this later when we talk about nullspace
%   solutions.  For now, just trust the code here =)
%
%
%   Sub-tasks:
% ---------------------------------------------------
%
% -- 1.  Extract the row vectors r1, r2, r3 from matrix A
%
% -- 2.  Then, we will find the nullspace vectors to the equation
%                r' * xn1  = 0
%                r' * xn2  = 0
%
% -- 3.  Then, will draw the plane spanned by the 2 nullspace vectors
%        xn1 and xn2
%
% \\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\


% -- Define my own color pallet

my_colormap = [ 1  0.6  0     ;   % --- orange
                0   1   0     ;   % --  green
                0   0   1 ];      % --  blue


% -- Create a new figure
```

```
figure;


% -- Forcing the new figure to have a certain size on your screen
%    and at a certain position
set(gcf, 'Position',  [100, 100, 700, 600])


% --  Iterate over each row vector r1, r2, r3

for count = 1:1:3

    r1 = A(count,:);    % -- Extract the row vector (r1, r2, r3)

    % -- First, we will auto-calculate the:
    %    a)  Particular solution:  my_xp
    %    b)  Nullspace vector #1:  my_xn1
    %    c)  Nullspace vector #2:  my_xn2


    xp  = [ b(count)       0  0 ]' / r1(1);
    xn1 = [ -r1(2)/r1(1)   1  0 ]' ;
    xn2 = [ -r1(3)/r1(1)   0  1 ]' ;


    % -- Then, we will plot the plane spanned by my_xn1
    %                                           my_xn2
    %    with the centroid of the plane at:      xp

    % -- Now, we will use the "patch" command to draw a 2D plane that
    %    represents the span{xn1, xn2}
    %
    %    These are the coordinates for the 4 corners of the plane
    %
    %
    %    Point Q                                                Point M
    %
    %   2*(-xn1+xn2)       ----------------------------- 2*(xn1 + xn2)
    %                     |            |              |
    %                     |            |              |
    %                      <--------   xp --------------->
    %                     |            |              |
    %    Point P          |            |              |         Point N
    %                     |            |              |
    %  2*(-xn1-xn2)      -----------------------------2*(xn1 - xn22)
    %


    % -- We can change the size + scale of our plane by using this
    %    magnification factor

    my_scale = 2;
```

```
    % -- Compute the cooridnates for the 4 corners.  The data structure
    %    here looks like this:
    %
    %                 Points: [   M       N        P         Q   ]  + xp
    % -------------------------------------------------------------------
    my_corners = my_scale .* [ xn1+xn2  xn1-xn2 -xn1-xn2  -xn1+xn2 ] + xp;

    % -- Plot the plane using the patch command  (# of vertices = 4)
    %
    %
    % Syntax:  my_plothandle = patch( 'XData', [ xcoords ],
    %                                 'YData', [ ycoords ],
    %                                 'ZData', [ zcoords ] );

    my_plane_plothandle(count) = patch('XData', my_corners(1,:), 'YData', my_corners(2,:), 'ZData',
    set(my_plane_plothandle(count), 'FaceColor', my_colormap(count,:), 'FaceAlpha', 0.2)

    hold on;

end


% \\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\
%
%   Task #2:  Plot the solution vector x  (the answer) as a
%             big black dot
%
%   ** You will have to change the "my_x" vector to have it
%      plotted correctly !!
%
% \\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\

% -- Define solution vector
x_solution = [ 1  0  1 ]';   %  <---  you need to change this !!


% -- Plot it as a black dot using the "plot3" command:

x_solution_plothandle = plot3( x_solution(1), x_solution(2), x_solution(3), '.');
set(x_solution_plothandle, 'Color', 'black', 'Markersize', 30);


% -- cosmetics for the plot
xmin = -5;
xmax = 5;
ymin = -5;
ymax = 5;
zmin = -5;
zmax = 5;

% -- Using "axis square" will ensure equally-spaced grid lines
%    across all 3 axes x, y, z !!

grid on;
```

```
axis([xmin xmax ymin ymax zmin zmax]);
axis square;

xlabel('x1-axis');
ylabel('x2-axis');
zlabel('x3-axis');
title('Problem 1:  3 intersecting planes (unique solution)');


% \\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\
% -- Create legend strings
%
% -- We will extract the row vector numbers from matrix A using the
%    num2str( ) command !!
%
% \\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\

my_legend_strings = { };  % -- Initialize string matrix

for count = 1:1:3

    % -- Creat the legend strings for each plan
    %    Note:  "32"  =  space bar (in ASCII code)

    my_legend_strings{count} = strcat( 'Plane #', num2str(count), ':', 32, num2str(A(count,1)), 'x_

end

% -- Then, add the 4th legend (the x = solution black dot)
 my_legend_strings{count+1}  = strcat( 'x_{solution} = [ ', 32, num2str(x_solution'), 32, ']^T'  );

% -- Plot the legend itself
%
%                                 plothandles         , string matrix
%                       -------------------------------------------------
my_legend_plothandle = legend( [ my_plane_plothandle   x_solution_plothandle  ], my_legend_strings)
set(my_legend_plothandle, 'FontName', 'Arial', 'Fontsize', 9);

% -- Release the hold on the plot
hold off;


% -- 3D camera position (so that you can view the 3D planes at a
%    convenient perpsective)

campos([66.5824   49.0335   25.7390]);
```

```
A =

     1     1     1
     1    -3     0
    -1     2     6
```

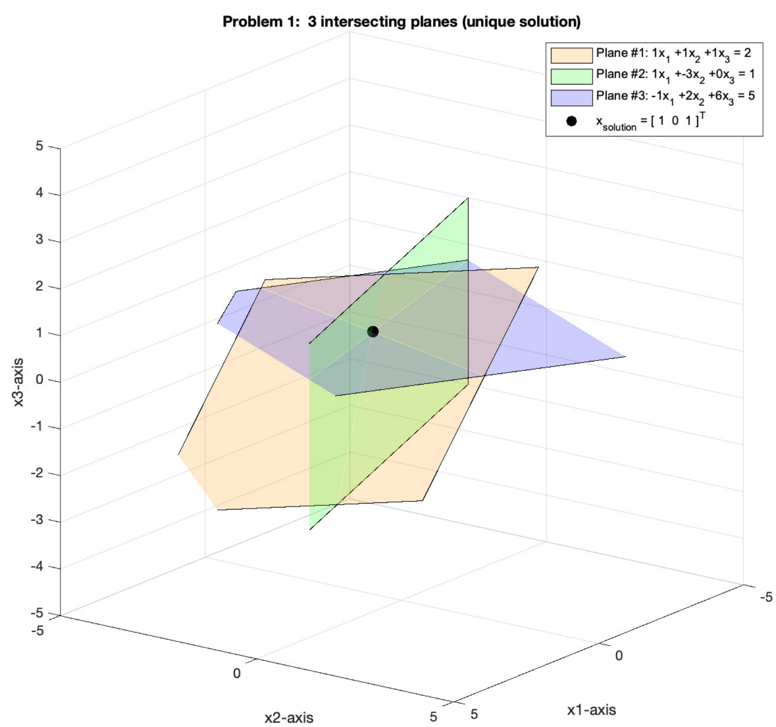
```
b =

     2
     1
     5

The solution to our Ax = b problem is

x =

     1
     0
     1
```



Problem 3

A

$A\vec{x} = \vec{b}$

$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 1 & -2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$[A|\vec{b}]$

$$\begin{bmatrix} 2 & -1 & 1 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

B

$$\begin{bmatrix} 2 & -1 & 1 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix} \xrightarrow{R_2 = R_2 - 2R_1} \begin{bmatrix} 2 & -1 & 1 & 2 \\ -3 & 3 & -4 & -4 \end{bmatrix} \xrightarrow{R_1 = \frac{1}{2}R_1,\ R_2 = -\frac{1}{3}R_2} \begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & 1 \\ 1 & -1 & \frac{4}{3} & \frac{4}{3} \end{bmatrix}$$

$$\xrightarrow{R_2 = R_2 - R_1} \begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & 1 \\ 0 & -\frac{1}{2} & \frac{5}{6} & \frac{1}{3} \end{bmatrix} \xrightarrow{R_2 = -2R_2} \begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & 1 \\ 0 & 1 & -\frac{5}{3} & -\frac{2}{3} \end{bmatrix} \xrightarrow{R_1 = R_1 + \frac{1}{2}R_2} \begin{bmatrix} 1 & 0 & -\frac{1}{3} & \frac{2}{3} \\ 0 & 1 & -\frac{5}{3} & -\frac{2}{3} \end{bmatrix}$$

C

```
A = [2,-1,1,2; 1,1,-2,0];
disp(rref(A))
```

```
1.0000   0        -0.3333    0.6667
0        1.0000   -1.6667   -0.6667
```

D

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} & \frac{2}{3} \\ 0 & 1 & -\frac{5}{3} & -\frac{2}{3} \end{bmatrix}$$

Column 1: Pivot Column 2: Pivot Column 3: Free Variable

E

Given the RREF matrix, setting the free variable $x_3 = t_3$, we can then formulate the solutio to the SLE As

$$x_1 - \frac{1}{3}t_3 = \frac{2}{3}$$
$$x_2 - \frac{5}{3}t_3 = -\frac{2}{3}x_3 = t_3$$

Which solved for x_1, x_2 gives

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \\ -\frac{2}{3} \\ 0 \end{bmatrix} + t_3 \begin{bmatrix} \frac{1}{3} \\ \frac{5}{3} \\ 1 \end{bmatrix}$$

Which is the parametric form of a line, which is indeed the intersection between two planes, and therefore has an infinite number of points lying on it.

F

Since the intersection between the planes is a line (they are not parallel or identical) the we can parametrise this line and iterate over it with a scalar parameter (t_3) to get an infinite number of points which satisfies the SLE.

G

$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} \frac{2}{3} \\ -\frac{2}{3} \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \cdot \frac{2}{3} + (-1) \cdot \left(-\frac{2}{3}\right) + 1 \cdot 0 \\ 1 \cdot \frac{2}{3} + 1 \cdot \left(-\frac{2}{3}\right) + (-2) \cdot 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{3} + \frac{2}{3} \\ \frac{2}{3} - \frac{2}{3} \end{bmatrix} = \begin{bmatrix} \frac{6}{3} \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}.$$

Problem 4

A

$A\vec{x} = \vec{b}$

$$\begin{bmatrix} 1 & 0 & 10 \\ 2 & 0 & 2 \\ 2 & 0 & -2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 2 \end{bmatrix}$$

$[A|\vec{b}]$

$$\begin{bmatrix} 1 & 0 & 10 & 4 \\ 2 & 0 & 2 & -2 \\ 2 & 0 & -2 & 2 \end{bmatrix}$$

B

$$\begin{bmatrix} 1 & 0 & 10 & 4 \\ 2 & 0 & 2 & -2 \\ 2 & 0 & -2 & 2 \end{bmatrix} \xrightarrow{R_2 = R_2 - 2R_1, R_3 = R_3 - 2R_1} \begin{bmatrix} 1 & 0 & 10 & 4 \\ 0 & 0 & -18 & -10 \\ 0 & 0 & -22 & -6 \end{bmatrix} \xrightarrow{R_3 = R_3 - \frac{11}{9}R_2} \begin{bmatrix} 1 & 0 & 10 & 4 \\ 0 & 0 & -18 & -10 \\ 0 & 0 & 0 & \frac{56}{9} \end{bmatrix}$$

C

By expanding the last row of the REF matrix, we are presented with $0 \cdot x_3 = \frac{56}{9}$, which is not a valid equation and therefore proves that the SLE is not consistent as for any value $k \in \mathbb{R}$ for x_3 would yield $0 = \frac{56}{9}$ which is false.

D

```
A = [1,0,10,4; 2,0,2,-2;2,0,-2,2];
disp(rref(A))
```

```
1    0    0    0
0    0    1    0
0    0    0    1
```

E

Once again, this RREF matrix is not consistent, as the last row, expanded, gives $0 \cdot x_3 = 1$, which deems the SLE as not consistent as for any value $k \in \mathbb{R}$ for x_3 would yield $0 = 1$ which is false.