Homework 6

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Problem 1:

1.a)

Solution:

$$v_{\_} = v_{+} = 0V$$

$$v_p = v_n = 4V$$

At node A between $2k\Omega$, $1mA$ and $v_{\_}$ KCL

$$i_s + i_f = i_n = i_p = 0$$

$$1mA + \frac{v_o - v_A}{2k\Omega} = 0$$

$$v_a = v_n = 4V$$

$$v_0 = -1mA \times 2k\Omega + 4V = -2V + 4V = 2V$$

1.b)

Solution:

$$v_{+} = 3V, v_{\_} = v_o + 1V$$

$$v_{\_} = v_{+} \Rightarrow v_o + 1 = 3 \Rightarrow v_o = 2V$$

Problem 2:

Solution:

$$v_{+} = 1V \times \frac{90k\Omega}{10k\Omega + 90k\Omega} = 0.9V$$

$$v_{\_} = v_{+} = 0.9V$$

$$v_{\_} = v_o \times \frac{50k\Omega}{50k\Omega + 100k\Omega} = \frac{v_o}{3}$$

$$v_o = 3 \times 0.9V = 2.7V$$

$$i_o = \frac{v_o}{10k\Omega} + \frac{v_o - v_{\_}}{100k\Omega} = \frac{2.7}{10k\Omega} + \frac{2.7 - 0.9}{100k\Omega} = 0.27mA + 0.018mA = 0.288mA$$

Problem 3:

3.a)

Solution:

$$v_+ = 0,$$
$$v_- = 0$$
$$i_s = \frac{0 - v_b}{R_1} \Rightarrow v_b = -R_1 i_s$$

Let B be the node between R_1, R_2 and R_3

$$\frac{v_b}{R_1} + \frac{v_b}{R_2} + \frac{v_b - v_o}{R_3} = 0$$
$$v_o = -i_s R_1 + R_3 + \frac{R_1 R_3}{R_2}$$
$$\frac{v_o}{i_s} = -R_1 + R_3 + \frac{R_1 R_3}{R_2}$$

3.b)

Solution:

$$\frac{v_o}{i_s} = -R_1 + R_3 + \frac{R_1 R_3}{R_2} = -20k\Omega + 40k\Omega + \frac{20k\Omega \times 40k\Omega}{25k\Omega} = -20k\Omega + 40k\Omega + 32k\Omega = 52k\Omega$$

Problem 4:

Solution:

$$v_- = v_+ \Rightarrow v_o = v_b$$
$$\frac{v_b - v_a}{6k\Omega} + \frac{v_b - v_a}{12k\Omega} + \frac{v_b}{6k\Omega} = 0$$
$$\Rightarrow$$
$$5v_b - 3v_a = 0 \Rightarrow v_b = \frac{3}{5}v_a$$
$$4mA = \frac{v_a}{3k\Omega} + \frac{v_a - v_b}{6k\Omega} + \frac{v_a - v_b}{12k\Omega}$$
$$\Rightarrow$$
$$7v_a - 3v_b = 48 \Rightarrow 7v_a - 3\left(\frac{3}{5}v_a\right) = 48 \Rightarrow v_a = \frac{120}{13}V$$
$$v_b = v_o = \frac{3}{5}v_a = \frac{72}{13}V$$
$$i_x = \frac{v_b}{6k\Omega} = \frac{72}{13}/6k\Omega = \frac{12}{13}mA \approx 0.923mA$$

Problem 5:

Solution:

Let node voltages be (x_1, x_2, x_3, x_4) from left to right on the top rail.

First op-amp inverting:

$$\Rightarrow x_2 = -\frac{50\Omega}{25\Omega} \times V_{s1} \qquad [1]$$
$$\Rightarrow x_2 = -2V_{s1} \qquad [2]$$

Second op-amp inverting summer, inputs through $100k\Omega$ from x_2 and $50k\Omega$ from V_{s2} feedback $100k\Omega$ to output x_3:

$$\Rightarrow x_3 = -\frac{100\Omega}{100\Omega}x_2 - \frac{100\Omega}{50\Omega}V_{s2} \qquad [3]$$
$$\Rightarrow x_3 = -x_2 - 2V_{s2} \qquad [4]$$

Penultimate op-amp no input current to right op-amp (+) node => no current in series $100k\Omega$:

$$x_4 = x_3 \quad [5]$$

Right op-amp non-inverting gain $\left(1 + \frac{100k\Omega}{50k\Omega} = 3\right)$:

$$v_o = 3x_4 = 3x_3 = 3(-x_2 - 2V_{s2}) = 3(2V_{s1} - 2V_{s2}) \quad [6]$$

$$v_o = 6V_{s1} - 6V_{s2} \quad [7]$$

Problem 6:

Solution:

since $v_L(t) = L\frac{di_L(t)}{dt}$ and $i_L(t) = 0$ for $t < 0$ and $t > 12\mu s$, we have

$$v_L(t) = \begin{cases} 10 \times 10^{-3} \times 4000 = 40\text{V} & 0 \leq t \leq 5\mu s \\ 10 \times 10^{-3} \times (-4000) = -40\text{V} & 5\mu s < t \leq 10\mu s \\ 0 & t > 10\mu s \end{cases} \quad [8]$$

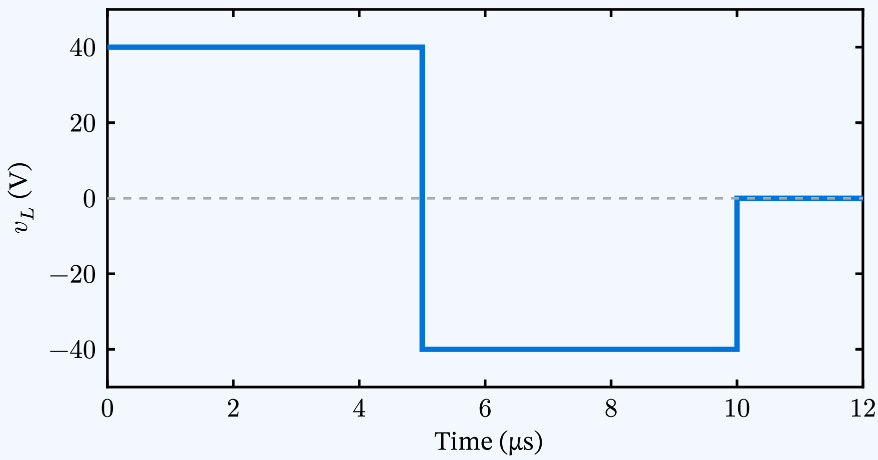

Figure 1: $v_L(t)$

since $p_L(t) = v_L(t) \times i(t)$ we have

$$p_L(t) = \begin{cases} p_L = 40 \times 4t \times 10^{-3} = 0.16t\text{W} & 0 \leq t \leq 5\mu s \\ p_L = -40 \times (-4t + 40) \times 10^{-3} = 0.16t - 1.6\text{W} & 5\mu s < t \leq 10\mu s \\ 0 & t > 10\mu s \end{cases} \quad [9]$$

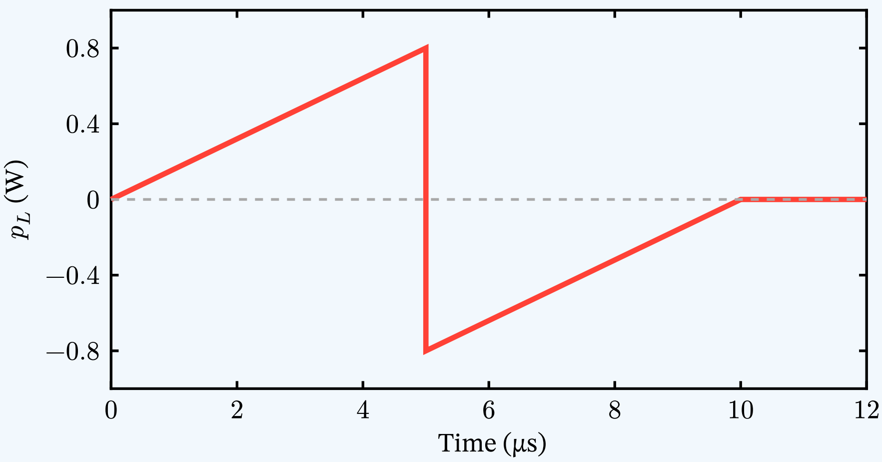

Figure 2: $p_L(t)$

since $w_L(t) = \frac{1}{2}Li^2(t)$ we have

$$w_L(t) = \begin{cases} \frac{1}{2} \times 10 \times 10^{-3} \times (4t \times 10^{-3})^2 = 8 \times 10^{-8} t^2 \text{J} & 0 \leq t \leq 5\mu s \\ \frac{1}{2} \times 10 \times 10^{-3} \times ((-4t + 40) \times 10^{-3})^2 = 8 \times 10^{-8} (t-10)^2 \text{J} 5\mu s < t \leq 10\mu s \\ 0 & t > 10\mu s \end{cases} \quad [10]$$

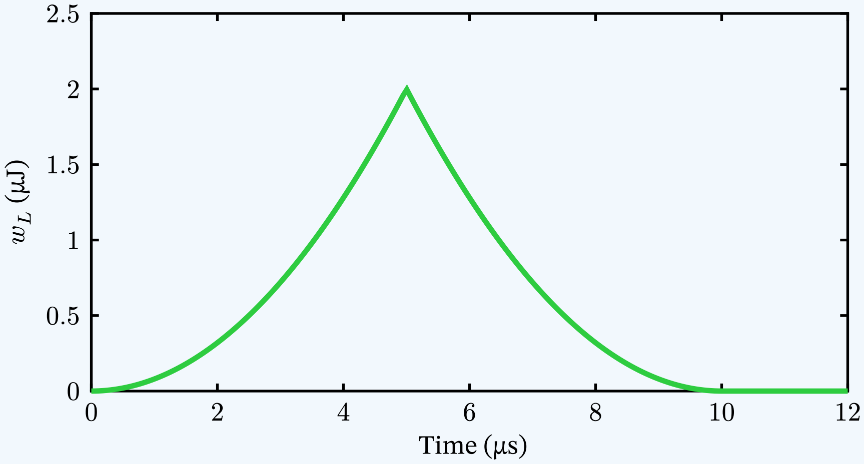

Figure 3: $w_L(t)$

Problem 7:

Solution:

$$v_L(t) = L\frac{di_L(t)}{dt} = 0.1 \times (-1)e^{-10t} = -0.1e^{-10t} V$$
$$p_L(t) = v_L(t) \times i_L(t) = (-0.1e^{-10t})(0.1e^{-10t}) = -0.01e^{-20t} W$$
$$w_L(t) = \frac{1}{2} L i_L^2(t) = \frac{1}{2} \times 0.1 \times (0.1e^{-10t})^2 = 5 \times 10^{-4} e^{-20t} J \quad [11]$$

Since $p_L(t) < 0$ for all $t > 0$, the inductor is delivering power.

Problem 8:

Solution:

since in parallel, same voltage: $v_R(t) = v_L(t) = L\frac{di_L(t)}{dt} = 0.1 \times (-1000) \times 0.02e^{-1000t} = -2e^{-1000t} V$

$$i_R(t) = \frac{v_R(t)}{R} = \frac{-2e^{-1000t}}{33k\Omega} \approx -60.6\mu A \times e^{-1000t} \quad [12]$$

Problem 9:

Solution:

$$i(t) = C\frac{dv_C(t)}{dt} = 3.3\mu F \times (-10 \times 2000) \sin(2000t) = -0.066 \sin(2000t) A$$
$$v_R(t) = i(t)R = (-0.066) \times 1k\Omega \times \sin(2000t) = -66 \sin(2000t) V \quad [13]$$
$$= 66 \cos(2000t - \tfrac{\pi}{2}) V$$

Problem 10:

Solution:

$$(10 + 3.3) \parallel (1 + 2.2)\mu F = \frac{13.3 \times 3.2}{13.3 + 3.2}\mu F \approx 2.58\mu F$$

Solution:

$$150 + (25 + 50 \parallel (100 \parallel 100))\mu F = 150 + (25 + 50 \parallel 50)\mu F = 150 + \left(\frac{75 \times 50}{75 + 50}\right)\mu F = 150 + 30 = 180\mu F$$

Problem 11:

Solution:

$$\text{Left Capacitors: } 10 + (6 \parallel 3)\mu F \Rightarrow C_{EQ} = (10 + (6 \parallel 3)) = 10 + 2 = 12\mu F$$

$$\text{Right inductors: } 0.5 + (3 \parallel 3)\mu H \Rightarrow L_{EQ} = (0.5 + (3 \parallel 3)) = 0.5 + 1.5 = 2\mu H$$

Problem 12:

12.a)

Solution:

$$\text{since we have an inverting summer } V_o = -R_f \times \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \frac{V_4}{R_4}\right)$$

12.b)

Solution:

We want to have $2V$ at 1111_2. pick $1V$ reference voltage.

Weight should be $MSB - 8 - 4 - 2 - 1 - LSB$

$$2 = -R_f \times \left(\frac{1+1+1+1}{8+4+2+1+R_n}\right)$$

$$\frac{R_f}{R_n} = \frac{2}{15}$$

Pick $R_4(\text{LSB}) = 10k\Omega$ so $R_f = \frac{2}{15} \times 10k\Omega = 1.333k\Omega$

Then

Resistor	Value
R_1	$1.25k\Omega$
R_2	$2.5k\Omega$
R_3	$5k\Omega$
R_4	$10k\Omega$
R_f	$(\frac{2}{15})k\Omega$