ENG EK 103:   Computational Linear Algebra: Problem set 8

** Remember: You can use matlab to check all of your handwritten answers !!!   =)

Matlab commands

- To check your eigenvalues / eigenvectors :   [V, LAMBDA] = eig(A)

★★ This commands is uber-useful for your engineering career !!   =) ★★

Note: The purpose of problems 1 thru 4

Each of those 4 matrices A, B, C, and D   exhibit different, subtle "flavors," such the # of repeated eigenvalues present in the problem, whether you need synthetic division to solve for them, the different kinds of reduced-row echelon forms you might see when you're solving for the eigenvectors, etc…

It may seem like a lot of busy-work, but we would like yall to get used to solving for the eigenvalues / eigenvectors such that these kinds of problems will become "second nature" for you when you're taking both midterm #2 and the final exam !   =)

Problem 1: Suppose you were given matrix A:

$$A = \begin{bmatrix} 2 & -1 & -2 \\ 0 & 1 & -2 \\ 0 & -2 & 1 \end{bmatrix}$$

(a) Find the eigenvalues and their corresponding eigenvectors for this matrix.

Hint: Depending on how you're evaluating the determinant $[A - \lambda I]$, you might have to use synthetic division to find the eigenvalues. If that's the case, here is a website that will teach you how to do this !

https://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut37_syndiv.htm ★★

(b) Hand-calculate the determinant of A.

(c) On paper, verify this really cool property (it is true for any square matrix A of any size):

$$det(A) = The\ product\ of\ the\ eigenvalues\ of\ A$$

(d) If you had used matlab's `[V, D] = eig(A)` command to calculate the eigenvalues and eigenvectors of matrix A, matlab will give you this answer:

$$V = \begin{bmatrix} 1. & 0.5774 & 0.5774 \\ 0 & 0.5774 & 0.5774 \\ 0 & -0.5774 & 0.5774 \end{bmatrix} \quad , \quad D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

- Do the columns within the eigenvector matrix V match your hand-calculated eigenvectors ?
- Using 1 or 2 sentences, explain why matlab reported each eigenvectors in a decimal format like this (as opposed to giving you whole number entries).

Problem 2: Suppose you were given matrix B:

$$B = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 1 \\ 2 & -1 & 0 \end{bmatrix}$$

(a) Find the eigenvalues and their corresponding eigenvectors for this matrix.

(b) If you had used matlab's `[V, D] = eig(B)` command to calculate the eigenvalues and eigenvectors of matrix B, matlab will give you this answer:

$$V = \begin{bmatrix} -0.5774 & -0.1348 & 0.2487 \\ -0.5774 & -0.8224 & -0.3894 \\ -0.5774 & -0.5527 & -0.8868 \end{bmatrix} \quad , \quad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

- Does the columns within the eigenvector matrix V match your hand-calculated eigenvectors ? If there are eigenvectors do match your hand-written answers, identify those vectors (and tell me which eigenvalue(s) are associated with those eigenvectors).

- What about the eigenvectors that *do not match* your hand-written answers ? Which eigenvalue(s) are associated with those eigenvectors ?

** *Moral of the story*: *1) When you have repeated eigenvalues*
2) You want to be careful on trusting matlab's answers for those eigenvectors !! = ★★

Problem 3: Suppose you were given matrix C:

$$C = \begin{bmatrix} -2 & 7 & 1 \\ 2 & 0 & -2 \\ -8 & 11 & 7 \end{bmatrix}$$

Find the eigenvalues and their corresponding eigenvectors for this matrix.

Hint #1: Again, if you were unlucky and picked a difficult row or column to perform cofactor expansions, you might need synthetic division to evaluate the eigenvalues.

Hint #2: For this homework, you can also use matlab's `roots( )` function to find the roots of the cubic polynomial.

Problem 4: Suppose you were given matrix D:

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

(a) Find the eigenvalues and their corresponding eigenvectors for this matrix.

(b) If you had used matlab's `[V, D] = eig(D)` function to calculate the eigenvalues and eigenvectors of matrix D, matlab will give you this answer:

$$V = \begin{bmatrix} 1.0000 & 0 & 0 \\ 0 & 1.0000 & -1.0000 \\ 0 & 0 & 0.0000 \end{bmatrix} \quad , \quad D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

- So apparently, matlab says matrix D possess 3 eigenvectors. Does the number of eigenvectors matches what you got from your hand-derived calculations ?

** <u>*Moral of the story*</u>*:*

1) *Matlab'e* `eig()` *command will sometimes give you super-weird + unexpected results !! =(=(*

2) <u>*Trivia*</u>*: You don't have to know this, but matlab is giving you 3 "eigenvectors" because it is calculating the eigenvectors of a cousin of matrix D…. sometimes called the "Jordan form" of matrix D. Google this if you want to know more about it =)*

3) *You REALLY need to be careful when you're using the* `eig( )` *function !! Never trust it 100% without first thinking about the computer-derived answers.*

Problem 5: Eigenvalues, eigenvectors, and geometrical transformations

Download the matlab file called `Problem5_transformation.m` . This script will generate a 3D plot of the following:

<u>Given</u>: Transformation matrix A and an unit cube X, where X is a matrix containing the x, y, z-coordinates of the 8 vertices of the cube (with 3 of the 6 faces shaded in *opaque* yellow, green, and blue)

<u>Output</u>: The script will plot the original cube X and the post-transformed image $Y = AX$ (with the 3 corresponding faces shaded in *semi-transparent* yellow, blue, and green).

(a) Run the code with the default transformation:

$$A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

and observe the shape of the post-transformed object $Y = AX$. Feel free to explore by zooming in and rotate the object in the output figure ! You don't have to publish the results from part (a), but you *will have to publish the results from part (b)* below.

(b) Then, in the code, change matrix A (Lines 8 thru 10 in the code) into this one:

$$A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

Rotate the object such that it is in a presentable 3D orientation. Publish the plot, the matlab code, and the command window outputs and turn it in with your homework.

(c) In 2 or 3 sentences, tell me: Why does the post-transformed object looks like a "flattened 2D pancake" in 3D space?

Hint: In your matlab command window, examine the reported eigenvalues + the eigenvectors of A carefully… =)