PS9

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1

A

$$D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

B

$$D = \begin{bmatrix} 5 & 0 \\ 0 & -2 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 1 \\ 1 & -6 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 6/7 & 1/7 \\ 1/7 & -1/7 \end{bmatrix}$$

C

$$D = \begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

D

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{bmatrix}$$

E

Not Diagonalizable:

$(1-\lambda)^2 = 0$, so $\lambda = 1$

$$(A - \lambda I) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$(A - \lambda I)\mathbf{v} = \mathbf{0}$$

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This yields $y = 0$. The eigenspace is spanned by

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

The dimension of the nullspace of $(A - \lambda I)$ is 1. The algebraic multiplicity of $\lambda = 1$ is $k = 2$. Since $1 < 2$, the matrix is not diagonalizable.

F

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

G

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 1/2 & 0 & -1/2 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$$

2

A

$\lambda_1 = 2, \lambda_2 = 4$

$\mathbf{v_1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$P = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}, P^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$D^5 = \begin{bmatrix} 2^5 & 0 \\ 0 & 4^5 \end{bmatrix} = \begin{bmatrix} 32 & 0 \\ 0 & 1024 \end{bmatrix}$$

$$A^5 = PDP^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 32 & 0 \\ 0 & 1024 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 32 & 992 \\ 0 & 1024 \end{bmatrix}$$

B

$\lambda_1 = 2, \lambda_2 = 4, \lambda_3 = 5$

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \mathbf{v_3} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}, P^{-1} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & -1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$D^2 = \begin{bmatrix} 2^2 & 0 & 0 \\ 0 & 4^2 & 0 \\ 0 & 0 & 5^2 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 25 \end{bmatrix}$$

$$A^2 = PDP^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 25 \end{bmatrix} \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & -1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 10 & -6 & 0 \\ -6 & 10 & 0 \\ 0 & 0 & 25 \end{bmatrix}$$

3

A

Since the principal stresses are the eigenvalues of T, then

$$(\lambda - 15)^2 = 25$$

$$\lambda - 15 = \pm 5$$

$$\sigma_1 = 10, \sigma_2 = 20$$

B

The eigenvector for σ_1 is

$$\begin{bmatrix} 5 & -5 \\ -5 & 5 \end{bmatrix} \to \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \to \left\{ t \begin{bmatrix} 1 \\ 1 \end{bmatrix} : t \in \mathbb{R} \right\}$$

$$\vec{v_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

and for σ_2 is

$$\begin{bmatrix} -5 & -5 \\ -5 & -5 \end{bmatrix} \to \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \to \left\{ t \begin{bmatrix} 1 \\ -1 \end{bmatrix} : t \in \mathbb{R} \right\}$$

$$\vec{v_2} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

C

$$\sigma_{vM} = \frac{\sqrt{2}}{2} \sqrt{(10-20)^2 + 10^2 + 20^2} = 10\sqrt{3} \approx 17.32$$

$$17.32 > 17 \therefore \text{ Breaks}$$