Note: $\text{span}\left\{\begin{bmatrix}1\\7\\3\end{bmatrix}\right\}$ can be described geometrically as a line in $\mathcal{R}^3$ passing through the origin, while $\text{span}\left\{\begin{bmatrix}1\\7\\0\\0\end{bmatrix}, \begin{bmatrix}3\\2\\0\\0\end{bmatrix}, \begin{bmatrix}4\\9\\1\\5\end{bmatrix}\right\}$ is a volume in $\mathcal{R}^4$ containing the origin.

1. Consider the matrix

$$A = \begin{bmatrix}-3 & 6\\ 4 & 8\\ 5 & -10\end{bmatrix}.$$

   (a) Find the column space of A, aka Col A. Express Col A as the span of a set of vectors.

   (b) Find a basis for Col A. Find the dimension of Col A. Describe the geometry of Col A.

   (c) Provide a nonzero vector $\mathbf{b}_1$ for which the system of linear equations corresponding to $A\mathbf{x} = \mathbf{b}_1$ is consistent. How are $\mathbf{b}_1$ and Col A related, geometrically?

   (d) Provide a nonzero vector $\mathbf{b}_2$ for which the system of linear equations corresponding to $A\mathbf{x} = \mathbf{b}_2$ is inconsistent. How are $\mathbf{b}_2$ and Col A related (or not related), geometrically?

2. Consider the matrix

$$B = \begin{bmatrix}3 & 2 & 5 & -5\\ 0 & 4 & -2 & 8\\ 2 & 0 & 4 & -6\end{bmatrix}.$$

   (a) Find the null space of B, aka Nul B. Express Nul B as the span of a set of vectors.

   (b) Find a basis for Nul B. Find the dimension of Nul B. Describe the geometry of Nul B.

   (c) Write $\mathbf{x}_h$, the homogeneous part of the complete solution to $B\mathbf{x} = \mathbf{0}$. Compare $\mathbf{x}_h$ to Nul B - how are they related?

   (d) Provide a nontrivial solution to $B\mathbf{x} = \mathbf{0}$.

3. Referring to matrix B above:

   (a) Find the row space of B, aka Row B, aka Col B^T. Express Row B as the span of a set of vectors.

   (b) Describe the geometry of Row B.

   (c) How many pivots does B have? How many vectors are in Row B?

4. This is a MATLAB exercise. Publish your MATLAB output with the commands and resulting output as directed below.
Consider the matrix
$$C = \begin{bmatrix} 3 & 5 & -2 & -1 & 1 \\ 2 & 0 & 2 & 4 & 2 \\ -2 & 7 & -9 & -5 & 4 \end{bmatrix}.$$

   (a) From a blank MATLAB script, code in matrix C.

   (b) Use the rref() function to find the RREF of C. You will see 2 free columns - that means we expect 2 nullspace solution vectors to $C\mathbf{x} = \mathbf{0}$.

   (c) Find Nul C using the command: "null_vec = null(sym(C))"

   (d) Verify that each vector in the nullspace is a nontrivial solution to $C\mathbf{x} = \mathbf{0}$ via matrix multiplication. Your work **must involve a for loop** in MATLAB.
   It may be useful to know that one can extract and store the first nullspace vector using the command: "xn1 = null_vec(:,1)".