Homework 3

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Problem 1 - Section A

System 1

\mathbf{A}

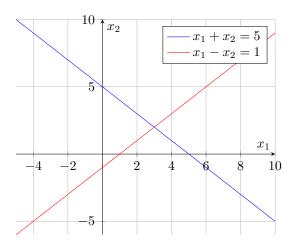


Figure 1: Plot of $x_1 + x_2 = 5$ and $x_1 - x_2 = 1$

\mathbf{B}

A unique solution since there is one intersection point between the lines.

 \mathbf{C}

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

 \mathbf{D}

$$\begin{bmatrix} 1 & 1 & 5 \\ 1 & -1 & 1 \end{bmatrix}$$

 ${f E}$

$$\begin{bmatrix} 1 & 1 & 5 \\ 1 & -1 & 1 \end{bmatrix} \xrightarrow{R_2 = R_2 - R_1} \begin{bmatrix} 1 & 1 & 5 \\ 0 & -2 & -4 \end{bmatrix}$$

 \mathbf{F}

$$\begin{bmatrix} 1 & 1 & 5 \\ 0 & -2 & -4 \end{bmatrix} \xrightarrow{R_2 = -\frac{1}{2}R_2} \begin{bmatrix} 1 & 1 & 5 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{R_1 = R_1 - R_2} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \end{bmatrix}$$

Therefore, given this RREF, it follows that

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

System 2

 \mathbf{A}

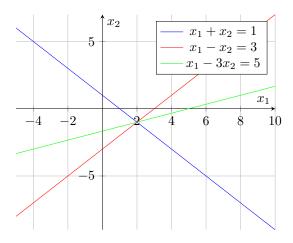


Figure 2: Plot of $x_1 + x_2 = 1$, $x_1 - x_2 = 3$, and $x_1 - 3x_2 = 5$

 \mathbf{B}

The SLE will have one solution since all lines intersect at the same point.

 \mathbf{C}

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & -3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

D

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 3 \\ 1 & -3 & 5 \end{bmatrix}$$

 \mathbf{E}

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 3 \\ 1 & -3 & 5 \end{bmatrix} \xrightarrow{R_2 = R_2 - R_1, R_3 = R_3 - R_1} \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 2 \\ 0 & -4 & 4 \end{bmatrix} \xrightarrow{R_3 = R_3 - 2R_2} \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 2 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2 = -\frac{1}{2}R_2} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 = R_1 - R_2} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Therefore, given this RREF, it follows that

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

System 3

 \mathbf{A}

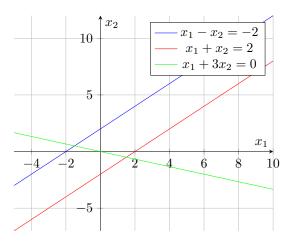


Figure 3: Plot of $x_1 - x_2 = -2$, $x_1 + x_2 = 2$, and $x_1 + 3x_2 = 0$

 \mathbf{B}

No solution to SLE since the lines do not all have common intersection point.

 \mathbf{C}

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}$$

 \mathbf{D}

$$\begin{bmatrix} 1 & -1 & -2 \\ 1 & 1 & 2 \\ 1 & 3 & 0 \end{bmatrix}$$

 \mathbf{E}

$$\begin{bmatrix} 1 & -1 & -2 \\ 1 & 1 & 2 \\ 1 & 3 & 0 \end{bmatrix} \xrightarrow{R_2 = R_2 - R_1, R_3 = R_3 - R_1} \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 4 \\ 0 & 4 & 2 \end{bmatrix} \xrightarrow{R_3 = R_3 - 2R_2} \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 4 \\ 0 & 0 & -6 \end{bmatrix}$$

F

The last column of the REF effectively shows that $x_1 \cdot 0 = -6$, which is sufficient to determine that the SLE is not consistent.

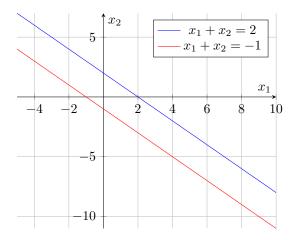


Figure 4: Plot of $x_1 + x_2 = 2$ and $x_1 + x_2 = -1$

Problem 1 - Section B

System 4

 \mathbf{B}

The SLE will have no solutions since the lines are parallel and therefore will never intersect.

 \mathbf{C}

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & -2 \end{bmatrix} \xrightarrow{R_2 = R_2 - R_1} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix} \xrightarrow{R_2 = -\frac{1}{4}R_2} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 = R_1 - 2R_2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

 \mathbf{D}

 \mathbf{E}

 \mathbf{F}

$$\begin{bmatrix}
1 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}$$

Column 1: Pivot Column 2: Free variable Column 3: Pivot

 \mathbf{G}

Since there is a pivot in the last column, from which is follows that $x_2 \cdot 0 = 1$, the SLE is in fact not consistent.

System 5

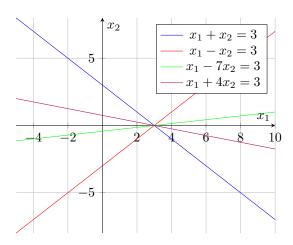


Figure 5: Plot of $x_1 + x_2 = 3$, $x_1 - x_2 = 3$, $x_1 - 7x_2 = 3$, and $x_1 + 4x_2 = 3$

 \mathbf{B}

Unique solution since there exists a common intersection point of the lines.

 \mathbf{C}

$$\begin{bmatrix}
1 & 1 & 3 \\
1 & -1 & 3 \\
1 & -7 & 3 \\
1 & 4 & 3
\end{bmatrix}
\xrightarrow{R_2 = R_2 - R_1, R_3 = R_3 - R_1, R_4 = R_4 - R_1}
\begin{bmatrix}
1 & 1 & 3 \\
0 & -2 & 0 \\
0 & -8 & 0 \\
0 & 3 & 0
\end{bmatrix}
\xrightarrow{R_4 = 2R_4}
\begin{bmatrix}
1 & 1 & 3 \\
0 & -2 & 0 \\
0 & -8 & 0 \\
0 & 6 & 0
\end{bmatrix}$$

$$\xrightarrow{R_4 = R_4 + 3R_3, R_3 = R_3 - 4R_2}
\begin{bmatrix}
1 & 1 & 3 \\
0 & -2 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\xrightarrow{R_2 = -frac12R_2}
\begin{bmatrix}
1 & 1 & 3 \\
0 & 1 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\xrightarrow{R_1 = R_1 - R_2}
\begin{bmatrix}
1 & 0 & 3 \\
0 & 1 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}$$

 \mathbf{D}

 \mathbf{E}

$$A = [1,1,3;1,-1,3;1,-7,3;1,4,3];$$

disp(rref(A))

 \mathbf{F}

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Column 1: Pivot Column 2: Pivot

 \mathbf{G}

Yes, the SLE is indeed consistent and unique as predicted. By expanding the augmented matrix back to two equations we are left with $x_1 = 3$, $x_2 = 0$, which is indeed the same point shown in Fig.5.

System 6

 \mathbf{A}

-

 \mathbf{B}

Since there are 3 equations in 5 unknowns, if the system is consistent it must have infinitely many solutions (with two free variables).

 \mathbf{C}

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 4 \\ -1 & 1 & -1 & 1 & 1 & 2 \\ 2 & 2 & 0 & -1 & 5 & 4 \end{bmatrix} \xrightarrow{R_2 = R_2 + R_1, \quad R_3 = R_3 - 2R_1} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 4 \\ 0 & 2 & 0 & 2 & 2 & 6 \\ 0 & 0 & -2 & -3 & 3 & -4 \end{bmatrix}$$

$$\xrightarrow{R_2 = \frac{1}{2}R_2, \quad R_3 = -\frac{1}{2}R_3} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 4 \\ 0 & 1 & 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & \frac{3}{2} & -\frac{3}{2} & 2 \end{bmatrix}$$

$$\xrightarrow{R_1 = R_1 - R_2} \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & \frac{3}{2} & -\frac{3}{2} & 2 \end{bmatrix}$$

$$\xrightarrow{R_1 = R_1 - R_3} \begin{bmatrix} 1 & 0 & 0 & -\frac{3}{2} & \frac{3}{2} & -1 \\ 0 & 1 & 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & \frac{3}{2} & -\frac{3}{2} & 2 \end{bmatrix}$$

 \mathbf{D}

$$A = [1,1,1,1,1;-1,1,-1,1,1;2,2,0,-1,5];$$

disp(rref(A))

 \mathbf{E}

$$A = [1,1,1,1,1,4;-1,1,-1,1,1,2;2,2,0,-1,5,4];$$

disp(rref(A))

 \mathbf{F}

Column 1: Pivot Column 2: Pivot Column 3: Pivot Column 4: Free Variable Column 5: Free Variable

 \mathbf{G}

Given the RREF, and asssuming $x_4 = t_4, x_5 = t_5$, it can be deduced that

$$x_1 = -1 + \frac{3}{2}t_4 - \frac{3}{2}t_5,$$

$$x_2 = 3 - t_4 - t_5,$$

$$x_3 = 2 - \frac{3}{2}t_4 + \frac{3}{2}t_5.$$

Therefore, demonstrating the aforementioned proposition system is consistent and not unique, but has infinitely many solutions.

Problem 2

0.1 A

$$A\vec{v} = \vec{b}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -3 & 0 \\ -1 & 2 & 6 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$$

 $[A|\vec{v}]$

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & -3 & 0 & 1 \\ -1 & 2 & 6 & 5 \end{bmatrix}$$

$$\mathbf{B}$$

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & -3 & 0 & 1 \\ -1 & 2 & 6 & 5 \end{bmatrix} \xrightarrow{R_2 = R_2 - R_1, \ R_3 = R_3 + R_1} \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -4 & -1 & -1 \\ 0 & 3 & 7 & 7 \end{bmatrix} \xrightarrow{R_3 = R_3 + \frac{3}{4}R_2} \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -4 & -1 & -1 \\ 0 & 0 & \frac{25}{4} & \frac{25}{4} \end{bmatrix}$$

 \mathbf{C}

$$\begin{bmatrix}
1 & 1 & 1 & 2 \\
0 & (-4) & -1 & -1 \\
0 & 0 & (\frac{25}{4}) & \frac{25}{4}
\end{bmatrix}$$

D

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -4 & -1 & -1 \\ 0 & 0 & \frac{25}{4} & \frac{25}{4} \end{bmatrix} \xrightarrow{R_2 = -\frac{1}{4}R_2, R_3 = \frac{4}{25}R_3} \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{R_2 = R_2 - \frac{1}{4}R_3} \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$
$$\xrightarrow{R_1 = R_1 - R_3} \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{R_1 = R_1 - R_2} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

 \mathbf{E}

$$A = [1,1,1; 1,-3,0; -1,2,6];$$

 $b = [2;1;5]$
 $x = A \setminus b$

b =

2

x =

1

 \mathbf{G}

Problem 2: Plotting 3 intersecting planes

% -- Suppose we have an Ax = b equation, where A = (3 x 3), and it representes these 3 equations:

```
Plane #1:
             x1 + x2 + x3 = 2
%
  Plane #2:
               x1 - x2 + 2x3 = 3
% Plane #3:
               -x1 + 2x2 + 4x3 = 3
% -- Define matrix A
A = [1 \ 1 \ 1 \ ;
                % -- row vector r1
                % -- row vector r2
    1 -3 0;
    -1 2 6]
                % -- row vector r3
% a1 a2 a3
                % -- Column vectors a1, a2, a3
% -- Define target vector b
b = [ 2 1 5 ]'
% -- We can solve for x by using the \ command:
disp('The solution to our Ax = b problem is');
x = A b
%
   Task 1:
           Drawing the 3 planes of interest
%
%
  We will learn how to do this later when we talk about nullspace
%
   solutions. For now, just trust the code here =)
%
%
%
   Sub-tasks:
%
% -- 1. Extract the row vectors r1, r2, r3 from matrix A
% -- 2. Then, we will find the nullspace vectors to the equation
%
              r' * xn1 = 0
%
              r' * xn2 = 0
% -- 3. Then, will draw the plane spanned by the 2 nullspace vectors
%
      xn1 and xn2
% -- Define my own color pallet
```

% -- Create a new figure

```
\mbox{\ensuremath{\mbox{\%}}} -- Forcing the new figure to have a certain size on your screen
    and at a certain position
set(gcf, 'Position', [100, 100, 700, 600])
% -- Iterate over each row vector r1, r2, r3
for count = 1:1:3
   r1 = A(count,:); % -- Extract the row vector (r1, r2, r3)
   % -- First, we will auto-calculate the:
        a) Particular solution: my_xp
        b) Nullspace vector #1: my_xn1
        c) Nullspace vector #2: my_xn2
   xp = [b(count)]
                       0 0]'/r1(1);
   xn1 = [-r1(2)/r1(1) 1 0];
   xn2 = [-r1(3)/r1(1) 0 1];
   % -- Then, we will plot the plane spanned by my_xn1
                                                my_xn2
        with the centroid of the plane at:
                                                хp
   \% -- Now, we will use the "patch" command to draw a 2D plane that
        represents the span{xn1, xn2}
   %
   %
        These are the coordinates for the 4 corners of the plane
   %
   %
   %
        Point Q
                                                              Point M
   %
   %
       2*(-xn1+xn2)
                                     -----2*(xn1 + xn2)
   %
                                      -
   %
                                      %
                                      хр --
   %
                                      %
        Point P
                                                              Point N
   %
   % 2*(-xn1-xn2)
                                            ----2*(xn1 - xn22)
   \% -- We can change the size + scale of our plane by using this
      magnification factor
   my_scale = 2;
```

figure;

```
%
        here looks like this:
   %
   %
                   Points: [ M
                                     N
                                                        Q ] + xp
   my_corners = my_scale .* [ xn1+xn2 xn1-xn2 -xn1-xn2 -xn1+xn2 ] + xp;
   % -- Plot the plane using the patch command (# of vertices = 4)
   %
   %
   % Syntax: my_plothandle = patch( 'XData', [ xcoords ],
                                  'YData', [ ycoords ],
   %
                                  'ZData', [ zcoords ] );
   my_plane_plothandle(count) = patch('XData', my_corners(1,:), 'YData', my_corners(2,:), 'ZData',
   set(my_plane_plothandle(count), 'FaceColor', my_colormap(count,:), 'FaceAlpha', 0.2)
   hold on;
end
%
   Task #2: Plot the solution vector x (the answer) as a
%
            big black dot
%
%
   ** You will have to change the "my_x" vector to have it
%
      plotted correctly !!
%
% -- Define solution vector
x_solution = [ 1 0 1 ]';
                         % <--- you need to change this !!
\% -- Plot it as a black dot using the "plot3" command:
x_solution_plothandle = plot3( x_solution(1), x_solution(2), x_solution(3), '.');
set(x_solution_plothandle, 'Color', 'black', 'Markersize', 30);
% -- cosmetics for the plot
xmin = -5;
xmax = 5;
ymin = -5;
ymax = 5;
zmin = -5;
zmax = 5;
\% -- Using "axis square" will ensure equally-spaced grid lines
    across all 3 axes x, y, z !!
grid on;
```

% -- Compute the cooridnates for the 4 corners. The data structure

```
axis([xmin xmax ymin ymax zmin zmax]);
axis square;
xlabel('x1-axis');
ylabel('x2-axis');
zlabel('x3-axis');
title('Problem 1: 3 intersecting planes (unique solution)');
% -- Create legend strings
\% -- We will extract the row vector numbers from matrix A using the
   num2str( ) command !!
my_legend_strings = { };  % -- Initialize string matrix
for count = 1:1:3
   % -- Creat the legend strings for each plan
      Note: "32" = space bar (in ASCII code)
   my_legend_strings{count} = strcat( 'Plane #', num2str(count), ':', 32, num2str(A(count,1)), 'x_
end
% -- Then, add the 4th legend (the x = solution black dot)
my_legend_strings{count+1} = strcat('x_{solution}'), 32, ']^T' );
% -- Plot the legend itself
%
%
                          plothandles , string matrix
set(my_legend_plothandle, 'FontName', 'Arial', 'Fontsize', 9);
% -- Release the hold on the plot
hold off;
\% -- 3D camera position (so that you can view the 3D planes at a
% convenient perpsective)
campos([66.5824 49.0335 25.7390]);
A =
   1 1 1
       -3
   1
   -1
       2
```

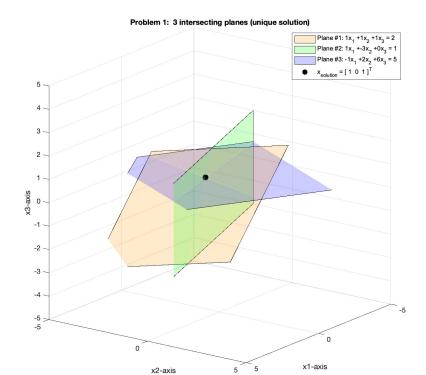
b =

2

1

The solution to our Ax = b problem is

1



Problem 3

 \mathbf{A}

 $A\vec{x}=\vec{b}$

$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 1 & -2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

 $[A|\vec{b}]$

$$\begin{bmatrix} 2 & -1 & 1 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

В

$$\begin{bmatrix} 2 & -1 & 1 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix} \xrightarrow{R_2 = R_2 - 2R_1} \begin{bmatrix} 2 & -1 & 1 & 2 \\ -3 & 3 & -4 & -4 \end{bmatrix} \xrightarrow{R_1 = \frac{1}{2}R_1, R_2 = -\frac{1}{3}R_2} \begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & 1 \\ 1 & -1 & \frac{4}{3} & \frac{4}{3} \end{bmatrix}$$

$$\xrightarrow{R_2 = R_2 - R_1} \begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & 1 \\ 0 & -\frac{1}{2} & \frac{5}{6} & \frac{1}{3} \end{bmatrix} \xrightarrow{R_2 = -2R_2} \begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & 1 \\ 0 & 1 & -\frac{5}{3} & -\frac{2}{3} \end{bmatrix} \xrightarrow{R_1 = R_1 + \frac{1}{2}R_2} \begin{bmatrix} 1 & 0 & -\frac{1}{3} & \frac{2}{3} \\ 0 & 1 & -\frac{5}{3} & -\frac{2}{3} \end{bmatrix}$$

 \mathbf{C}

$$A = [2,-1,1,2; 1,1,-2,0];$$

disp(rref(A))

 \mathbf{D}

$$\begin{bmatrix}
1 & 0 & -\frac{1}{3} & \frac{2}{3} \\
0 & 1 & -\frac{5}{3} & -\frac{2}{3}
\end{bmatrix}$$

Column 1: Pivot Column 2: Pivot Column 3: Free Variable

 \mathbf{E}

Given the RREF matrix, setting the free variable $x_3 = t_3$, we can then formulate the solutio to the SLE As

$$x_1 - \frac{1}{3}t_3 = \frac{2}{3}$$
$$x_2 - \frac{5}{3}t_3 = -\frac{2}{3}x_3 = t_3$$

Which solved for x_1, x_2 gives

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \\ -\frac{2}{3} \\ 0 \end{bmatrix} + t_3 \begin{bmatrix} \frac{1}{3} \\ \frac{5}{3} \\ 1 \end{bmatrix}$$

Which is the parametric form of a line, which is indeed the intersection between two planes, and therefore has an infinite number of points lying on it.

 \mathbf{F}

Since the intersection between the planes is a line (they are not parallel or identical) the we can parametrise this line and iterate over it with a scalar parameter (t_3) to get an infinite number of points which satisfies the SLE.

 \mathbf{G}

$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} \frac{2}{3} \\ -\frac{2}{3} \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \cdot \frac{2}{3} + (-1) \cdot \left(-\frac{2}{3} \right) + 1 \cdot 0 \\ 1 \cdot \frac{2}{3} + 1 \cdot \left(-\frac{2}{3} \right) + (-2) \cdot 0 \end{bmatrix}$$
$$= \begin{bmatrix} \frac{4}{3} + \frac{2}{3} \\ \frac{2}{3} - \frac{2}{3} \end{bmatrix} = \begin{bmatrix} \frac{6}{3} \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}.$$

Problem 4

 \mathbf{A}

$$A\vec{x}=\vec{b}$$

$$\begin{bmatrix} 1 & 0 & 10 \\ 2 & 0 & 2 \\ 2 & 0 & -2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 2 \end{bmatrix}$$

 $[A|\vec{b}]$

$$\begin{bmatrix} 1 & 0 & 10 & 4 \\ 2 & 0 & 2 & -2 \\ 2 & 0 & -2 & 2 \end{bmatrix}$$

 \mathbf{B}