HW3 - 9/24 - GIACOMO CAPPONETTO

1) LEFT LOOP KVL:

$5mA = \frac{V_0}{10k\Omega} \Rightarrow 5\cdot 10^{-3} A = \frac{V_0}{10\cdot 10^3 \Omega} \Rightarrow V_0 = 5\cdot 10 = 50 V$

RIGHT LOOP:

$i_s = 0.01 V_0 = 0.01 \cdot 50 = 0.5 A$

$R_{EQ} = \frac{5\cdot 20}{5+20}\cdot 10^3 \Omega = \frac{100}{25} = 4k\Omega \rightarrow V = i_s R_{eq} = 0.5A \cdot 4k\Omega = \boxed{2000V}$

$\frac{i_{20}}{i_s} = \frac{4k\Omega}{20k\Omega} \Rightarrow i_{20} = 0.5A \cdot \frac{4k\Omega}{20k\Omega} = \boxed{0.1A} \Rightarrow P = V i_{20} = 2000 \cdot 0.1 = \boxed{200W}$

2) a) BY INSPECTION

$$\begin{bmatrix} \left(\frac{1}{20}+\frac{1}{8}+\frac{1}{10}\right) & -\frac{1}{8} \\ -\frac{1}{8} & \left(\frac{1}{8}+\frac{1}{4}\right) \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 3A \\ 1A-3A \end{bmatrix}$$

$$\begin{bmatrix} \frac{11}{40} & -\frac{1}{8} \\ -\frac{1}{8} & \frac{3}{8} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix} \quad [A|b] \Rightarrow \left[\begin{array}{cc|c} \frac{11}{40} & -\frac{5}{40} & 3 \\ -\frac{5}{40} & \frac{15}{40} & -2 \end{array}\right]$$

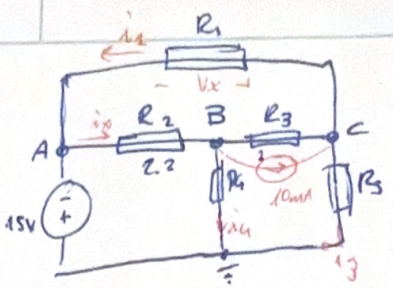
$$\Rightarrow \begin{bmatrix} 11 & -5 & 120 \\ -1 & 3 & -16 \end{bmatrix} \Rightarrow \begin{bmatrix} 11 & -5 & 120 \\ 1 & -3 & 16 \end{bmatrix} \xrightarrow{R_1 = R_1 - 11R_2} \Rightarrow$$

$$\begin{bmatrix} 0 & 28 & -56 \\ 1 & -3 & 16 \end{bmatrix} \xrightarrow{R_1 = R_1 \cdot \frac{1}{28}} \begin{bmatrix} 0 & 1 & -2 \\ 1 & -3 & 16 \end{bmatrix} \xrightarrow{R_2 = R_2 + 3R_1} \Rightarrow \begin{bmatrix} 0 & 1 & -2 \\ 1 & 0 & 10 \end{bmatrix}$$

$\Rightarrow \therefore V_B = -2 \;,\; V_A = 10$

$\Rightarrow V_x = V_B - 0$ (KVL) $= \boxed{-2V} \rightarrow \frac{V_A}{20\Omega} = i_{20} = \frac{10}{20} = \boxed{0.5A}$

3)



(A) WE KNOW $V_A = -15V$ BY SOURCE

(B) $i_x + i_4 + i_2 + 10mA = 0 \Rightarrow \frac{V_B - V_A}{R_2} + \frac{V_B}{R_4} + \frac{V_B - V_C}{R_3} + 10mA = 0$

(C) $i_5 + i_1 + i_2 - 10mA = 0 \Rightarrow \frac{V_C}{R_5} + \frac{V_x}{R_1} + \frac{V_C - V_B}{R_3} = 10mA$

$\Rightarrow$ BY INSPECTION, GIVEN $V_A = -15V$, $Ax = b$ FOR $x = [V_B, V_C]^T$, $\left(G_n = \frac{1}{R_n}\right)$

$$\begin{bmatrix} (G_2 + G_3 + G_4) & -G_3 \\ -G_3 & (G_1 + G_5 + G_3) \end{bmatrix} \begin{bmatrix} V_B \\ V_C \end{bmatrix} = \begin{bmatrix} G_2 V_A - 10mA \\ G_3 \cdot V_A + 10mA \end{bmatrix} = \begin{bmatrix} -(G_2 \cdot 15 + 10mA) \\ (G_3 \cdot 15 + 10mA) \end{bmatrix}$$

SUBSTITUTE $\Rightarrow$ G_n

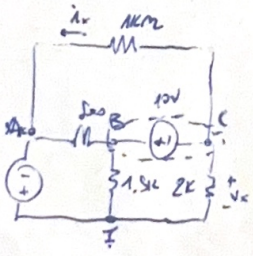
$$\begin{bmatrix} \frac{1}{2.2k} + \frac{1}{1k} + \frac{1}{1k} & -\frac{1}{1k} \\ -\frac{1}{1k} & \frac{1}{1k} + \frac{1}{1k} + \frac{1}{3.3k} \end{bmatrix} \begin{bmatrix} V_A \\ V_3 \end{bmatrix} = \begin{bmatrix} -\left(\frac{15}{2.2k}\right) - 10mA \\ -\frac{15}{1.2k} + 10mA \end{bmatrix}$$

$\Rightarrow$ WITH MATLAB $\Rightarrow$ $\begin{bmatrix} V_B \\ V_C \end{bmatrix} = \begin{bmatrix} -9.344053 \\ -6.252220 \end{bmatrix} \Rightarrow V_B \approx -9.4$, $V_C \approx -6.25$.

$$\frac{V_A - V_B}{2.2k\Omega} = i_x = \frac{-15 + 9.4}{2.2 \cdot 10^3} = \boxed{-2.545\ mA}$$

$$V_x = V_C - V_A = -6.25 + 15 = \boxed{8.75\ V}$$

4)



(A) $V_A = -15V \rightarrow$ ~~$\frac{V_B - V_A}{1k\Omega} + \frac{V_A - V_B}{500\Omega}$~~

(B+C) $\frac{V_B - V_A}{500} + \frac{V_B}{1.5k\Omega} + \frac{V_C}{2k\Omega} + \frac{V_C - V_A}{1k\Omega} = 0$

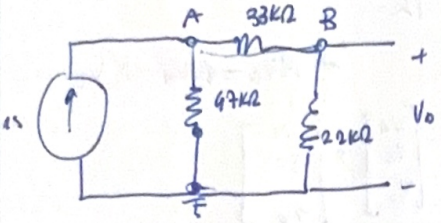
↳ IN (B+C) $V_B - V_C = 10V$ FROM SOURCE

$$\Rightarrow \begin{bmatrix} \frac{1}{500} + \frac{1}{1000} & \frac{1}{2000} + \frac{1}{1000} \\ 1 & -1 \end{bmatrix} \begin{bmatrix} V_B \\ V_C \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{500} + \frac{1}{1000}\right)(-15) \\ 10 \end{bmatrix} \Rightarrow V_B = -7.2V \quad V_C = -17.2V$$

CLEARLY $V_x = V_C = \boxed{-17.2V}$

$$i_x = \frac{V_C - V_A}{1000\Omega} = \frac{17.2 - (-15)\ V}{1000} = \boxed{-2.2mA}$$

5)



(A) $-i_s + \frac{V_A - V_B}{33k\Omega} + \frac{V_A}{47k\Omega} = 0$

(B) $V_B = V_o$, $\frac{V_B - V_A}{33k\Omega} + \frac{V_B}{22k\Omega} = 0$

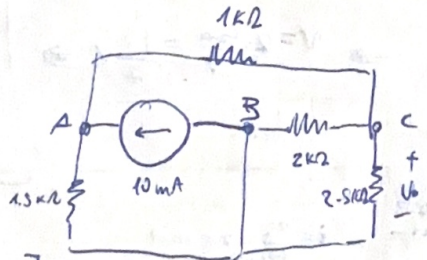
(B).66 $\Rightarrow 2V_B - 2V_A + 3V_B = 0 \Rightarrow V_A = \frac{5}{2}V_B \Rightarrow V_A = \frac{5}{2}V_o$

(A) $i_s = \frac{\frac{5}{2}V_o}{47k\Omega} + \frac{\frac{5}{2}V_o - V_o}{33k\Omega} \Rightarrow i_s = \frac{5V_o}{94k\Omega} + \frac{3V_o}{66k\Omega} \Rightarrow i_s = \frac{51}{517k\Omega}V_o$

$\Rightarrow \frac{V_o}{i_s} = \frac{517 \cdot 10^3}{51} \approx 10.137\ k\Omega = k$

- If $i_s = 1mA$, $V_o = \frac{517 \cdot 10^3}{51} \cdot 1 \cdot 10^{-3} \approx 10.1\ V$
- if $i_s = 15mA$, $V_o = \frac{517 \cdot 15}{51} \approx 152.1\ V$

6) 10mA on :



(A) $\frac{V_A}{1.5k\Omega} + \frac{V_A - V_C}{1k\Omega} - 10mA = 0$

(B) $\frac{V_B - V_C}{2k\Omega} + 10mA = 0 \Rightarrow V_B - V_C = -20$

(C) $\frac{V_C - V_A}{1k\Omega} + \frac{V_C - V_B}{2k\Omega} + \frac{V_C}{2.5k\Omega} = 0$

→ (B) → (C) $\frac{V_C}{2500} + \frac{V_C - (V_C - 20)}{2000} + \frac{V_C - V_A}{1000} = 0$

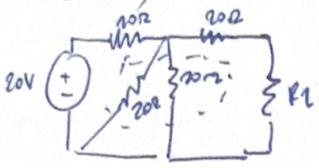
$\frac{V_C}{2500} + 0.01 + \frac{V_C - V_A}{1000} = 0 \Rightarrow 7V_C - 5V_A = -50$

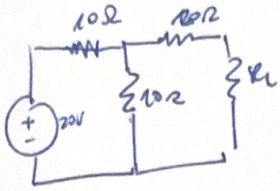
(A) $2V_A + 3(V_A - V_C) = 50 \Rightarrow 5V_A - 3V_C = 30$

$V_A = \frac{30 + (V_C \cdot 3)}{5} \Rightarrow 7V_C - 30 + 3V_C = -50$

$\Rightarrow 4V_C - 30 = -30 \Rightarrow 4V_C = 20 \Rightarrow V_C = V_o = -5\ V$

a) $(20 \| 20) = 10\Omega$

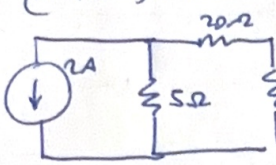


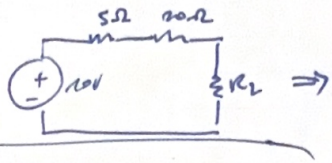


$i_1 = \frac{20}{10} = 2A$

$\Rightarrow \quad (10 \| 10) = 5\Omega$

$V_1 = 2A \cdot 5\Omega = 10V$





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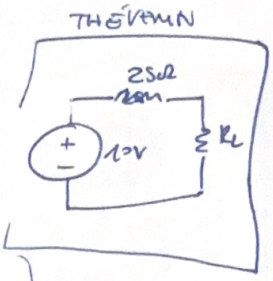

$$\Rightarrow \quad V_L = 10V \, \frac{R_L}{R_L + 25}$$

$R_L = 5\Omega \Rightarrow V_L = 10 \cdot \frac{5}{30} = \frac{5}{3} V$

$R_L = 10\Omega \Rightarrow V_L = 10 \cdot \frac{10}{35} = \frac{20}{7} V$

$R_L = 20\Omega \Rightarrow V_L = 10 \cdot \frac{20}{85} = \frac{40}{11} V$