EK307: Circuits

Lecture notes for Circuits (EK307)

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Published: September 05, 2025
Last updated: September 05, 2025

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Chapter 1: Current, Voltage, Charge and Power

Variables and Fundamental Quantities

Electric Charge — Definition 1.1

Charge is a fundamental property of matter that determines electromagnetic interaction. It comes in two types (positive and negative) and is conserved in all physical processes. Important facts:

- Unit: coulomb (C). The elementary charge carried by an electron has magnitude $e = 1.602 \times 10^{-19} C$.
- Conservation of charge: In any isolated system, the algebraic sum of charge remains constant.

Electric Current — Definition 1.2

Current measures the rate at which charge flows past a reference point in a circuit:

$$i(t) = \frac{dq(t)}{dt} \quad [1]$$

where $q(t)$ is the algebraic charge that has crossed the reference. Key points:

- Unit: ampere (A) with $1A = 1\ \mathrm{C/s}$.
- Current direction follows the *conventional* positive-charge flow from higher to lower potential; electron flow is opposite.
- If a reference direction is chosen, a negative value of $i(t)$ indicates actual flow opposite to that reference.

Transferred Charge over an Interval — Definition 1.3

The algebraic charge transferred between t_0 and t is

$$q(t) - q(t_0) = \int_{t_0}^{t} i(\tau) d\tau \quad [2]$$

and, equivalently, $i(t) = \frac{dq(t)}{dt}$.

DC vs AC Current **Note 1.1**

DC (direct current) means the current maintains one direction over time (its sign does not change). AC (alternating current) changes direction periodically.

From q(t) to i(t) **Example 1.1**

Suppose the transferred charge is piecewise linear (in μC)

$$q(t) = \begin{cases} 0 & \text{if } t < 0 \\ 30t & \text{if } 0 \le t < 1 \\ 30 - 30(t-1) & \text{if } 1 \le t < 2 \\ -30 + 15(t-2) & \text{if } 2 \le t < 4 \\ 0 & \text{if } t \ge 4 \end{cases} \quad [3]$$

with t in seconds. Find $i(t)$ and comment on current direction.

Solution: Differentiate $q(t)$ on each interval (and convert to amperes by μC/s = μA):

$$i(t) = \begin{cases} 0 & \text{if } t < 0 \\ 30\mu A & \text{if } 0 \le t < 1 \\ -30\mu A & \text{if } 1 \le t < 2 \\ 15\mu A & \text{if } 2 \le t < 4 \\ 0 & \text{if } t \ge 4 \end{cases} \quad [4]$$

Intervals with negative slope give negative current, meaning actual flow opposite to the chosen reference direction during $1 \le t < 2$.

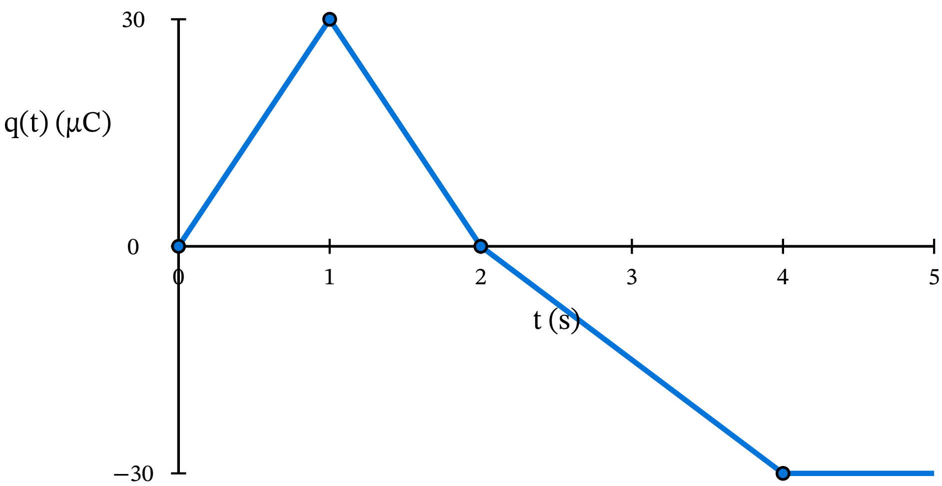

Figure 1: Piecewise linear charge function q(t)

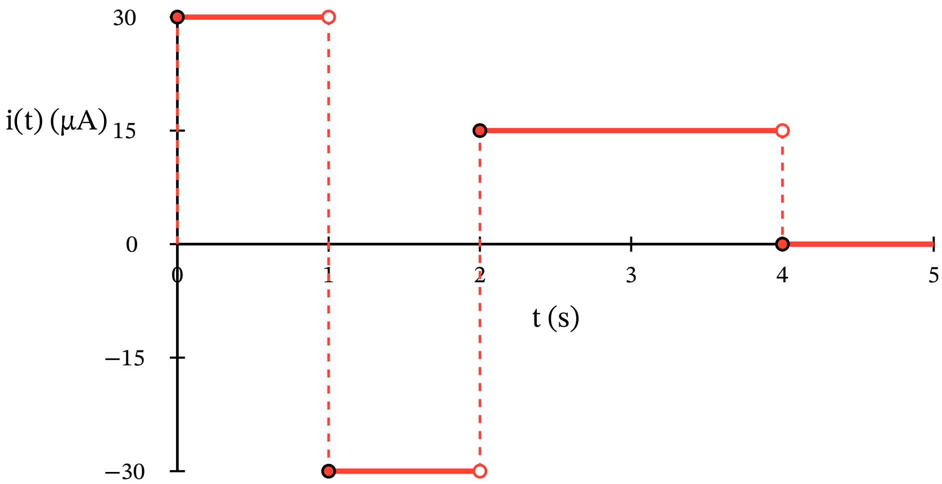

Figure 2: Piecewise constant current function i(t) = dq/dt

Voltage (Potential Difference)

Voltage — Definition 1.4

Voltage is the change in potential energy per unit charge between two points:

$$v(t) = \frac{dw}{dq}, \quad 1V = 1\ \mathrm{J/C} \tag{5}$$

Properties and usage:

- Voltage is always measured *between* two points and is a relative quantity; a reference point ("ground") is often chosen to report node voltages.
- A "voltage drop" is the potential decrease across an element following a specified reference polarity.

Resistance and Conductance

Resistance and Ohm's Law — Definition 1.5

Resistance models opposition to the flow of charge. For an ohmic element,

$$v = iR \quad \text{or} \quad i = Gv \tag{6}$$

where R is resistance in ohms (Ω) and $G = \frac{1}{R}$ is conductance in siemens (S). In the i–v plane the slope is $\frac{di}{dv} = G$ (a straight line through the origin for an ideal resistor).

Power and Energy

Instantaneous Power — Definition 1.6

Electrical power is the rate of change of energy with respect to time:

$$p(t) = \frac{dw}{dt} = \frac{dw}{dq}\frac{dq}{dt} = v(t)i(t) \tag{7}$$

For a resistor using Ohm's law,

$$\boxed{p = vi = i^2R = \frac{v^2}{R}} \tag{8}$$

Under the passive sign convention, $p > 0$ indicates the element absorbs power, while $p < 0$ indicates it delivers power.

Unit Checks — Note 1.2

Combine unit checks frequently: V·A = W. For example, a 2 kΩ resistor carrying 5 mA absorbs $p = i^2R = (5\ \mathrm{mA})^2 \cdot 2\ \mathrm{k\Omega} = 50\ \mathrm{mW}$.