Homework 2

Giacomo Cappelletto

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1 Linear Transformations

A)

I)

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

II)

$$ScalarMatrix$$

B)

I

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

II)

$$Horizontalshear$$

C)

I)

$$\begin{bmatrix} \cos(270^\circ) & -\sin(270^\circ) \\ \sin(270^\circ) & \cos(270^\circ) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

II)

$$RotationMatrix$$

1.1 E)

I)

$$\begin{bmatrix} \cos(180^\circ) & -\sin(180^\circ) \\ \sin(180^\circ) & \cos(180^\circ) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

II)

$$ReflectionMatrix$$

1.2 F)

I)

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

II)

$$Projection(matrix)$$

2 Rotations

A)

$$\cos(\theta + \phi) = \frac{v_1}{r} \Rightarrow v_1 = r \cdot \cos(\theta + \phi)$$

$$v = \begin{bmatrix} r \cdot \cos(\theta + \phi) \\ r \cdot \sin(\theta + \phi) \end{bmatrix}$$

$$\cos(\theta) = \frac{z_1}{r} \Rightarrow z_1 = r \cdot \cos(\theta)$$

$$z = \begin{bmatrix} r \cdot \cos(\phi) \\ r \cdot \sin(\phi) \end{bmatrix}$$

B)

Since

$$\cos(\theta + \phi) = \cos(\theta)\cos(\phi) - \sin(\theta)\sin(\phi)$$

Then

$$v_1 = r \cdot \cos(\theta + \phi) = r \cdot \cos(\theta)\cos(\phi) - r \cdot \sin(\theta)\sin(\phi)$$

C)

Given

$$z_1 = r \cdot \cos(\theta), z_2 = \cdot \sin(\theta)$$

Then

$$v_1 = z_1 \cdot \cos(\theta) - z_2 \cdot \sin(\theta)$$

D)

$$v_1 = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \end{bmatrix} \cdot \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = z_1 \cdot \cos(\theta) - z_2 \cdot \sin(\theta)$$

E)

Since

$$\sin(\theta + \phi) = \sin(\theta)\cos(\phi) + \cos(\theta)\sin(\phi)$$

Then

$$v_2 = r \cdot \sin(\theta + \phi) = r \cdot \sin(\theta)\cos(\phi) + r \cdot \cos(\theta)\sin(\phi)$$

F)

Given

$$z_1 = r \cdot \cos(\theta), z_2 = \cdot \sin(\theta)$$

Then

$$v_2 = z_1 \cdot \sin(\theta) + z_2 \cdot \cos(\theta)$$

G)

$$v_2 = \begin{bmatrix} \sin(\theta) & \cos(\theta) \end{bmatrix} \cdot \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = z_1 \cdot \sin(\theta) + z_2 \cdot \cos(\theta)$$

H)

$$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \cdot \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$