PS8

Giacomo Cappelletto

April 2, 2025

1

A

$$A - \lambda I = \begin{bmatrix} 2-\lambda & -1 & -2 \\ 0 & 1-\lambda & -2 \\ 0 & -2 & 1-\lambda \end{bmatrix}$$

$$\begin{aligned} \det(A-\lambda I) &= (2-\lambda)\det\begin{pmatrix} 1-\lambda & -2 \\ -2 & 1-\lambda \end{pmatrix} - 0 + 0 \\ &= (2-\lambda)\left[(1-\lambda)(1-\lambda) - (-2)(-2)\right] \\ &= (2-\lambda)\left[(1-\lambda)^2 - 4\right] \\ &= (2-\lambda)\left[1 - 2\lambda + \lambda^2 - 4\right] \\ &= (2-\lambda)(\lambda^2 - 2\lambda - 3) \\ &(2-\lambda)(\lambda-3)(\lambda+1) = 0 \end{aligned}$$

The eigenvalues are $\boldsymbol{\lambda_1 = 2}$, $\boldsymbol{\lambda_2 = 3}$, and $\boldsymbol{\lambda_3 = -1}$.

For $\lambda_1 = 2$:

$$\begin{bmatrix} 0 & -1 & -2 \\ 0 & -1 & -2 \\ 0 & -2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & -2 \\ 0 & -1 & -2 \\ 0 & -2 & -1 \end{bmatrix} \xrightarrow[R_3=R_3-2R_1]{R_2=R_2-R_1} \begin{bmatrix} 0 & -1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix} \xrightarrow{R_1\leftrightarrow R_3, R_3\to R_2} \begin{bmatrix} 0 & 0 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow[R_2=-R_2]{R_1=\frac{1}{3}R_1} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2=R_2-2R_1} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \text{A basis eigenvector for} \lambda_2 = 3 \text{is } \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

For $\lambda_2 = 3$:

$$\begin{bmatrix} -1 & -1 & -2 \\ 0 & -2 & -2 \\ 0 & -2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 & -2 \\ 0 & -2 & -2 \\ 0 & -2 & -2 \end{bmatrix} \xrightarrow{R_3=R_3-R_2} \begin{bmatrix} -1 & -1 & -2 \\ 0 & -2 & -2 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2=-\frac{1}{2}R_2} \begin{bmatrix} -1 & -1 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1=R_1+R_2} \begin{bmatrix} -1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1=-R_1} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \text{A basis eigenvector for} \lambda_2 = 3 \text{is } \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

For $\lambda_3 = -1$:

$$\begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -2 \\ 0 & -2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix}3 & -1 & -2\\0 & 2 & -2\\0 & -2 & 2\end{bmatrix} \xrightarrow{R_3=R_3+R_2} \begin{bmatrix}3 & -1 & -2\\0 & 2 & -2\\0 & 0 & 0\end{bmatrix} \xrightarrow{R_2=\frac{1}{2}R_2} \begin{bmatrix}3 & -1 & -2\\0 & 1 & -1\\0 & 0 & 0\end{bmatrix} \xrightarrow{R_1=R_1+R_2} \begin{bmatrix}3 & 0 & -3\\0 & 1 & -1\\0 & 0 & 0\end{bmatrix} \xrightarrow{R_1=\frac{1}{3}R_1} \begin{bmatrix}1 & 0 & -1\\0 & 1 & -1\\0 & 0 & 0\end{bmatrix}$$

$$\therefore \text{A basis eigenvector for} \lambda_3 = -1 \text{is } \begin{bmatrix}1\\1\\1\end{bmatrix}$$

B

$$A = \begin{bmatrix}2 & -1 & -2\\0 & 1 & -2\\0 & -2 & 1\end{bmatrix}$$

$$\begin{aligned}\det(A) &= 2 \cdot \det\begin{pmatrix}1 & -2\\-2 & 1\end{pmatrix} - 0 \cdot \det\begin{pmatrix}-1 & -2\\-2 & 1\end{pmatrix} + 0 \cdot \det\begin{pmatrix}-1 & -2\\1 & -2\end{pmatrix}\\
&= 2\left[(1)(1) - (-2)(-2)\right] - 0 + 0\\
&= 2[1-4]\\
&= 2[-3]\\
&= \mathbf{-6}\end{aligned}$$

$$\det(A) = -6$$

C

$$\lambda_1 \times \lambda_2 \times \lambda_3 = (2) \times (3) \times (-1) = \mathbf{-6}$$

D

Matlab standardizes the eigenvectors by normalizing them to have a length (norm) of 1, ensuring a unique representative for each eigenvector direction. The calculated eigenvectors are simply multiples of the ones given by Matlab

2

A

$$B - \lambda I = \begin{bmatrix}1-\lambda & -1 & 1\\2 & -2-\lambda & 1\\2 & -1 & -\lambda\end{bmatrix}$$

$$\begin{aligned}\det(B-\lambda I) &= (1-\lambda)\det\begin{pmatrix}-2-\lambda & 1\\-1 & -\lambda\end{pmatrix} - (-1)\det\begin{pmatrix}2 & 1\\2 & -\lambda\end{pmatrix} + 1\det\begin{pmatrix}2 & -2-\lambda\\2 & -1\end{pmatrix}\\
&= (1-\lambda)[(-\lambda)(-2-\lambda) - (1)(-1)] + 1[(2)(-\lambda) - (1)(2)] + 1[(2)(-1) - (-2-\lambda)(2)]\\
&= (1-\lambda)[2\lambda + \lambda^2 + 1] + [-2\lambda - 2] + [-2 - (-4 - 2\lambda)]\\
&= (1-\lambda)(\lambda+1)^2 - 2\lambda - 2 + [-2 + 4 + 2\lambda]\\
&= (1-\lambda)(\lambda+1)^2 - 2\lambda - 2 + 2 + 2\lambda\\
&= (1-\lambda)(\lambda+1)^2\\
&= -(\lambda-1)(\lambda+1)^2\end{aligned}$$

Setting $\det(B-\lambda I) = 0$, the eigenvalues are $\lambda_{\mathbf{1}} = \mathbf{1}$ $(k=1)$ and $\lambda_{\mathbf{2}} = \mathbf{-1}$ $(k=2)$.

For $\lambda_1 = 1$:

$$\begin{bmatrix}0 & -1 & 1\\2 & -3 & 1\\2 & -1 & -1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}0 & -1 & 1\\ 2 & -3 & 1\\ 2 & -1 & -1\end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix}2 & -3 & 1\\ 0 & -1 & 1\\ 2 & -1 & -1\end{bmatrix} \xrightarrow{R_3 = R_3 - R_1} \begin{bmatrix}2 & -3 & 1\\ 0 & -1 & 1\\ 0 & 2 & -2\end{bmatrix} \xrightarrow{R_3 = R_3 + 2R_2} \begin{bmatrix}2 & -3 & 1\\ 0 & -1 & 1\\ 0 & 0 & 0\end{bmatrix}$$

$$\xrightarrow{R_2 = -R_2} \begin{bmatrix}2 & -3 & 1\\ 0 & 1 & -1\\ 0 & 0 & 0\end{bmatrix} \xrightarrow{R_1 = R_1 + 3R_2} \begin{bmatrix}2 & 0 & -2\\ 0 & 1 & -1\\ 0 & 0 & 0\end{bmatrix} \xrightarrow{R_1 = \frac{1}{2}R_1} \begin{bmatrix}1 & 0 & -1\\ 0 & 1 & -1\\ 0 & 0 & 0\end{bmatrix}$$

The eigenvectors are $t\begin{bmatrix}1\\1\\1\end{bmatrix}$.

$$\therefore \text{A basis eigenvector for } \lambda_1 = 1 \text{ is } \begin{bmatrix}1\\1\\1\end{bmatrix}$$

For $\lambda_2 = -1$:

$$\begin{bmatrix}2 & -1 & 1\\ 2 & -1 & 1\\ 2 & -1 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}2 & -1 & 1\\ 2 & -1 & 1\\ 2 & -1 & 1\end{bmatrix} \xrightarrow[R_3 = R_3 - R_1]{R_2 = R_2 - R_1} \begin{bmatrix}2 & -1 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

The eigenvectors are $\begin{bmatrix}\frac{1}{2}s - \frac{1}{2}t\\ s\\ t\end{bmatrix} = s\begin{bmatrix}1/2\\1\\0\end{bmatrix} + t\begin{bmatrix}-1/2\\0\\1\end{bmatrix}$. We can choose integer basis vectors by setting $(s = 2, t = 0)$ and $(s = 0, t = 2)$.

$$\therefore \text{Basis eigenvectors for } \lambda_2 = -1 \text{ are } \begin{bmatrix}1\\2\\0\end{bmatrix} \text{ and } \begin{bmatrix}-1\\0\\2\end{bmatrix}$$

B

Yes, the first column of V, matches our hand-calculated eigenvector $\mathbf{v_1} = \begin{bmatrix}1\\1\\1\end{bmatrix}$ associated with the eigenvalue $\lambda_1 = 1$.

The second and third columns of V do not directly match our hand-calculated basis vectors $\mathbf{v_2} = \begin{bmatrix}1\\2\\0\end{bmatrix}$ or $\mathbf{v_3} = \begin{bmatrix}-1\\0\\2\end{bmatrix}$. Both sets of vectors form a basis for the eigenspace associated with the eigenvalue $\lambda_2 = -1$. Matlab provides a different basis than the one we found by hand.

3

$$\begin{aligned}
\det(C-\lambda I) &= -2\det\begin{pmatrix} 7 & 1 \\ 11 & 7-\lambda \end{pmatrix} + (-\lambda)\det\begin{pmatrix} -2-\lambda & 1 \\ -8 & 7-\lambda \end{pmatrix} - (-2)\det\begin{pmatrix} -2-\lambda & 7 \\ -8 & 11 \end{pmatrix} \\
&= -2[7(7-\lambda)-(1)(11)] - \lambda[(-2-\lambda)(7-\lambda)-(1)(-8)] + 2[(-2-\lambda)(11)-(7)(-8)] \\
&= -2[49-7\lambda-11] - \lambda[-14+2\lambda-7\lambda+\lambda^2+8] + 2[-22-11\lambda+56] \\
&= -2[38-7\lambda] - \lambda[\lambda^2-5\lambda-6] + 2[34-11\lambda] \\
&= -76+14\lambda-\lambda^3+5\lambda^2+6\lambda+68-22\lambda \\
&= -\lambda^3+5\lambda^2+(14+6-22)\lambda+(-76+68) \\
&= -\lambda^3+5\lambda^2-2\lambda-8
\end{aligned}$$

The characteristic equation is $-\lambda^3+5\lambda^2-2\lambda-8=0$, or $\lambda^3-5\lambda^2+2\lambda+8=0$. Let $P(\lambda)=\lambda^3-5\lambda^2+2\lambda+8$. Possible rational roots: $\pm1,\pm2,\pm4,\pm8$. $P(-1)=-1-5-2+8=0$. $P(2)=8-5(4)+2(2)+8=8-20+4+8=0$. $P(4)=64-5(16)+2(4)+8=64-80+8+8=0$. The eigenvalues are $\boldsymbol{\lambda_1=-1}$, $\boldsymbol{\lambda_2=2}$, $\boldsymbol{\lambda_3=4}$.

For $\lambda_1=-1$:

$$\begin{bmatrix} -1 & 7 & 1 \\ 2 & 1 & -2 \\ -8 & 11 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 7 & 1 \\ 2 & 1 & -2 \\ -8 & 11 & 8 \end{bmatrix} \xrightarrow[R_3=R_3-8R_1]{R_2=R_2+2R_1} \begin{bmatrix} -1 & 7 & 1 \\ 0 & 15 & 0 \\ 0 & -45 & 0 \end{bmatrix} \xrightarrow{R_2=\frac{1}{15}R_2} \begin{bmatrix} -1 & 7 & 1 \\ 0 & 1 & 0 \\ 0 & -45 & 0 \end{bmatrix}$$

$$\xrightarrow[R_3=R_3+45R_2]{R_1=R_1-7R_2} \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1=-R_1} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \text{ A basis eigenvector for } \lambda_1=-1 \text{ is } \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

For $\lambda_2=2$:

$$\begin{bmatrix} -4 & 7 & 1 \\ 2 & -2 & -2 \\ -8 & 11 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 7 & 1 \\ 2 & -2 & -2 \\ -8 & 11 & 5 \end{bmatrix} \xrightarrow{R_1\leftrightarrow R_2} \begin{bmatrix} 2 & -2 & -2 \\ -4 & 7 & 1 \\ -8 & 11 & 5 \end{bmatrix} \xrightarrow{R_1=\frac{1}{2}R_1} \begin{bmatrix} 1 & -1 & -1 \\ -4 & 7 & 1 \\ -8 & 11 & 5 \end{bmatrix}$$

$$\xrightarrow[R_3=R_3+8R_1]{R_2=R_2+4R_1} \begin{bmatrix} 1 & -1 & -1 \\ 0 & 3 & -3 \\ 0 & 3 & -3 \end{bmatrix} \xrightarrow{R_2=\frac{1}{3}R_2} \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & -1 \\ 0 & 3 & -3 \end{bmatrix}$$

$$\xrightarrow[R_3=R_3-3R_2]{R_1=R_1+R_2} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \text{ A basis eigenvector for } \lambda_2=2 \text{ is } \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

For $\lambda_3=4$:

$$\begin{bmatrix} -6 & 7 & 1 \\ 2 & -4 & -2 \\ -8 & 11 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -6 & 7 & 1 \\ 2 & -4 & -2 \\ -8 & 11 & 3 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 2 & -4 & -2 \\ -6 & 7 & 1 \\ -8 & 11 & 3 \end{bmatrix} \xrightarrow{R_1 = \frac{1}{2} R_1} \begin{bmatrix} 1 & -2 & -1 \\ -6 & 7 & 1 \\ -8 & 11 & 3 \end{bmatrix}$$

$$\xrightarrow[R_3 = R_3 + 8R_1]{R_2 = R_2 + 6R_1} \begin{bmatrix} 1 & -2 & -1 \\ 0 & -5 & -5 \\ 0 & -5 & -5 \end{bmatrix} \xrightarrow{R_2 = -\frac{1}{5} R_2} \begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & 1 \\ 0 & -5 & -5 \end{bmatrix}$$

$$\xrightarrow[R_3 = R_3 + 5R_2]{R_1 = R_1 + 2R_2} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \text{A basis eigenvector for } \lambda_3 = 4 \text{ is } \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

4

A

$$D - \lambda I = \begin{bmatrix} 2-\lambda & 0 & 0 \\ 0 & 2-\lambda & 1 \\ 0 & 0 & 2-\lambda \end{bmatrix}$$

$$\det(D - \lambda I) = (2-\lambda)(2-\lambda)(2-\lambda) = (2-\lambda)^3$$

Setting $\det(D - \lambda I) = 0$, we get $(2-\lambda)^3 = 0$. The only eigenvalue is $\lambda = \mathbf{2}$ with $k = 3$

For $\lambda = 2$**:**

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The eigenvectors are of the form $t_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + t_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$. The eigenspace is 2-dimensional ($k = 2$).

$$\therefore \text{Basis eigenvectors for } \lambda = 2 \text{ are } \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

B

No, the number of eigenvectors reported by Matlab (3 columns in V) does not match the number of linearly independent eigenvectors found in the hand-derived calculations (2 basis vectors). The hand calculation found that the eigenspace for $\lambda = 2$ is only 2-dimensional, meaning a basis consists of exactly two linearly independent eigenvectors.

5

B

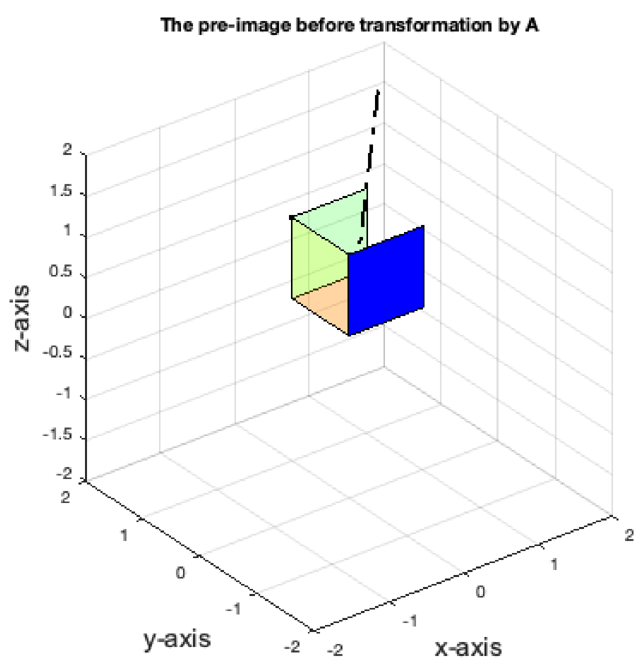
```
clear all;
close all;


% -- Default matrix A  (you can change this matrix A !!)
```

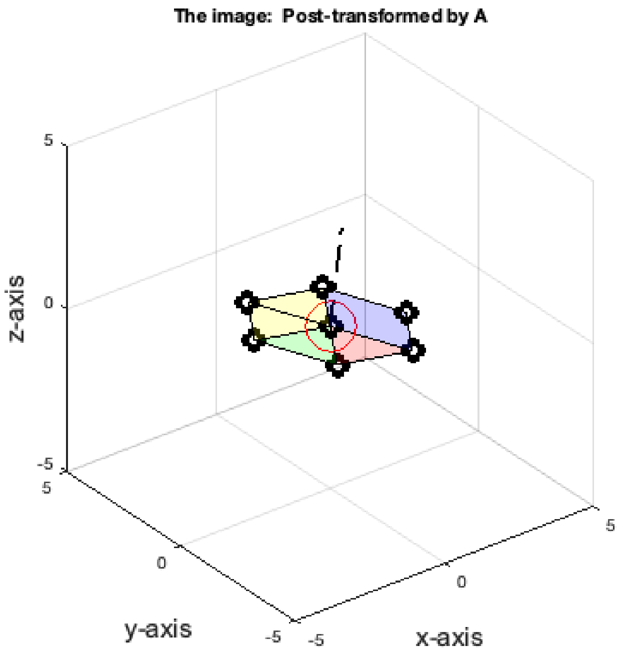
```
A  =  [  2,-1,-1;
   -1,2,-1;
   -1,-1,2]


% -- Select the eigenvector to plot

eigvec_to_plot = 1;   % -- This is associated with:  lambda1 = 0
%eigvec_to_plot = 2;   % -- This is associated with:  lambda2 = 3
%eigvec_to_plot = 3;   % -- This is associated with:  lambda3 = 3


disp('The eigenvalues and eigenvectors of matrix A are:')
[V,D] = eig(A)

% -- Define cube

shift_x = 0;
shift_y = 0;
shift_z = 0;

x1 = [0 1 1 0 0 1 1 0] + shift_x;
x2 = [0 0 0 0 1 1 1 1] + shift_y;
x3 = [0 0 1 1 0 0 1 1] + shift_z;

% -- Define x
x = [x1;  x2;  x3];

% -- Calculate transformation
y = A*x;

% -- Housekeeping duties
xmin = -2;
xmax = 2;
ymin = -2;
ymax = 2;
zmin = -2;
zmax = 2;

% -- Plot data points (preimage and image)

figure

for count = 1:1:length(x1)
 preimage_handle(count) = plot3(x(1,count), x(2,count), x(3,count), '.');



 if count == 1
  set(preimage_handle(count), 'Color', 'red', 'MarkerSize', 50, 'Linewidth', 5);
 else

  set(preimage_handle(count), 'Color', 'black', 'MarkerSize', 15, 'Linewidth', 5);
```

```
 end
end


% --  Plot one face of the preimage and its corresponding image

preimageFace_handle(1) = patch(x(1, 1:4), x(2, 1:4), x(3, 1:4),  4);
preimageFace_handle(2) = patch(x(1, 5:8), x(2, 5:8), x(3, 5:8),  4);
preimageFace_handle(3) = patch(x(1, [1 4 8 5] ), x(2, [1 4 8 5]), x(3, [1 4 8 5]),  4);
preimageFace_handle(4) = patch(x(1, [1 2 6 5]), x(2, [1 2 6 5]), x(3, [1 2 6 5]),  4);

set(preimageFace_handle(1), 'FaceColor', 'blue', 'FaceAlpha', 1);
set(preimageFace_handle(2), 'FaceColor', 'green', 'FaceAlpha', 0.2);
set(preimageFace_handle(3), 'FaceColor', 'yellow', 'FaceAlpha', 0.2);
set(preimageFace_handle(4), 'FaceColor', 'red', 'FaceAlpha', 0.2);


% -- Emphasize the x, y, z-axexs
%line([0 xmax], [0 0], [0 0], 'Color', 'black', 'Linestyle', '-.', 'Linewidth', 2);  % -- x-axis
%line([0 0], [0 ymax], [0 0], 'Color', 'black', 'Linestyle', '-', 'Linewidth', 2);  % -- y-axis
%line([0 0], [0 0], [0 zmax], 'Color', 'black', 'Linestyle', 'x', 'Linewidth', 2);  % -- z-axis

eig_scale = 3;
% -- Plot eigenvector #2
line('XData', eig_scale.*[0 V(1,eigvec_to_plot)], 'YData', eig_scale.*[0 V(2,eigvec_to_plot)], 'ZData'

axis square;
grid on;
xlabel('x-axis', 'FontName', 'Arial', 'FontSize', 15);
ylabel('y-axis', 'FontName', 'Arial', 'FontSize', 15);
zlabel('z-axis', 'FontName', 'Arial', 'FontSize', 15);
axis([xmin  xmax  ymin  ymax  zmin  zmax]   );
hold off
title('The pre-image before transformation by A')



% \\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\
%   Figure 2:  Plot the transformed image
% \\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\

xmin2 = -5;
xmax2 = 5;
ymin2 = -5;
ymax2 = 5;
zmin2 = -5;
zmax2 = 5;


figure

for count = 1:1:length(x1)
 %preimage_handle(count) = plot3(x(1,count), x(2,count), x(3,count), '.');
 %hold on;
```

```
 image_handle(count) = plot3(y(1,count), y(2,count), y(3,count), 'o');
 hold on;

 if count == 1
   %set(preimage_handle(count), 'Color', 'red', 'MarkerSize', 50, 'Linewidth', 5);
   set(image_handle(count), 'Color', 'red', 'MarkerSize', 30, 'Linewidth', 1);
 else

  %set(preimage_handle(count), 'Color', 'black', 'MarkerSize', 15, 'Linewidth', 5);
  set(image_handle(count), 'Color', 'black', 'MarkerSize', 10, 'Linewidth', 5);
 end
end


% --  Plot one face of the preimage and its corresponding image

% preimageFace_handle(1) = patch(x(1, 1:4), x(2, 1:4), x(3, 1:4),  4);
% preimageFace_handle(2) = patch(x(1, 5:8), x(2, 5:8), x(3, 5:8),  4);
% preimageFace_handle(3) = patch(x(1, [1 4 8 5] ), x(2, [1 4 8 5]), x(3, [1 4 8 5]),  4);
% preimageFace_handle(4) = patch(x(1, [1 2 6 5]), x(2, [1 2 6 5]), x(3, [1 2 6 5]),  4);
%
% set(preimageFace_handle(1), 'FaceColor', 'blue', 'FaceAlpha', 1);
% set(preimageFace_handle(2), 'FaceColor', 'green', 'FaceAlpha', 0.2);
% set(preimageFace_handle(3), 'FaceColor', 'yellow', 'FaceAlpha', 0.2);
% set(preimageFace_handle(4), 'FaceColor', 'red', 'FaceAlpha', 0.2);

imageFace_handle(1) = patch(y(1, 1:4), y(2, 1:4), y(3, 1:4),  4);
imageFace_handle(2) = patch(y(1, 5:8), y(2, 5:8), y(3, 5:8),  4);
imageFace_handle(3) = patch(y(1, [1 4 8 5]), y(2, [1 4 8 5]), y(3, [1 4 8 5]),  4);
imageFace_handle(4) = patch(y(1, [1 2 6 5]), y(2, [1 2 6 5]), y(3, [1 2 6 5]),  4);

set(imageFace_handle(1), 'FaceColor', 'blue', 'FaceAlpha', 0.2);
set(imageFace_handle(2), 'FaceColor', 'green', 'FaceAlpha', 0.2);
set(imageFace_handle(3), 'FaceColor', 'yellow', 'FaceAlpha', 0.2);
set(imageFace_handle(4), 'FaceColor', 'red', 'FaceAlpha', 0.2);


% -- Emphasize the x, y, z-axexs
%line([0 xmax], [0 0], [0 0], 'Color', 'black', 'Linestyle', '-.', 'Linewidth', 2);  % -- x-axis
%line([0 0], [0 ymax], [0 0], 'Color', 'black', 'Linestyle', '-', 'Linewidth', 2);  % -- y-axis
%line([0 0], [0 0], [0 zmax], 'Color', 'black', 'Linestyle', 'x', 'Linewidth', 2);  % -- z-axis

eig_scale = 3;
% -- Plot eigenvector #2
line('XData', eig_scale.*[0 V(1,eigvec_to_plot)], 'YData', eig_scale.*[0 V(2,eigvec_to_plot)], 'ZData'

axis square;
grid on;
xlabel('x-axis', 'FontName', 'Arial', 'FontSize', 15);
ylabel('y-axis', 'FontName', 'Arial', 'FontSize', 15);
zlabel('z-axis', 'FontName', 'Arial', 'FontSize', 15);
axis([xmin2  xmax2  ymin2  ymax2  zmin2  zmax2]   );
hold off
title('The image:  Post-transformed by A')
```

```
A =

  2    -1    -1
 -1     2    -1
 -1    -1     2

The eigenvalues and eigenvectors of matrix A are:

V =

 0.5774    0.3000    0.7594
 0.5774   -0.8076   -0.1199
 0.5774    0.5077   -0.6395


D =

  0     0     0
  0     3     0
  0     0     3
```





C

If you take the cross product of any two columns of the transformation matrix and normalize it (make it unit length), the output vectors ($\vec{n_n}$) will be identical (or in opposite direction $-\vec{n_n}$). This demonstrates that they are all in the same plane. Since the matrix we are transforming is a unit cube (or identity matrix I) then the output of the multiplication is guaranteed to have entries which are all in the plane whose normal is $\vec{n_n}$, and is proven by the known identity $A \times I = A$. Calculations below

```
A_1=[2;-1;-1];
A_2=[-1;2;-1];
A_3=[-1;-1;2];

C_12 = cross(A_1,A_2);
C_31 = cross(A_3,A_1);
C_23 = cross(A_2, A_3);

C_12 = C_12/norm(C_12)
C_31 = C_31/norm(C_31)
C_23 = C_23/norm(C_23)

figure;
hold on;
grid on;

quiver3(0, 0, 0, A_1(1), A_1(2), A_1(3), 'r', 'LineWidth', 2, 'MaxHeadSize', 0.5);
quiver3(0, 0, 0, A_2(1), A_2(2), A_2(3), 'g', 'LineWidth', 2, 'MaxHeadSize', 0.5);
quiver3(0, 0, 0, A_3(1), A_3(2), A_3(3), 'b', 'LineWidth', 2, 'MaxHeadSize', 0.5);

quiver3(0, 0, 0, C_12(1), C_12(2), C_12(3), 'k', 'LineWidth', 2, 'MaxHeadSize', 0.5);

[X, Y] = meshgrid(-2:0.5:2, -2:0.5:2);
```

```
Z = (-C_12(1)*X - C_12(2)*Y) / C_12(3);

surf(X, Y, Z, 'FaceAlpha', 0.3, 'EdgeColor', 'none', 'FaceColor', 'cyan');

xlabel('X'); ylabel('Y'); zlabel('Z');
title('Vectors A1, A2, A3 and their Normal Vector');
legend('A1', 'A2', 'A3', 'Normal (C_{12})', 'Plane');

hold off;
axis equal;
view(3);


C_12 =

 0.5774
 0.5774
 0.5774


C_31 =

 0.5774
 0.5774
 0.5774


C_23 =

 0.5774
 0.5774
 0.5774
```