Homework 9

EK307

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Problem 1:

1.a)

Solution:

$v(t) = 20e^{-i60°}$

$v(t) = 20\cos(-60°) + i20\sin(-60°) = 10 - 10\sqrt{3}i$

$$\boxed{10 - 10\sqrt{3}i}$$

1.b)

Solution:

$$v(t) = 10e^{i180°}$$

$v(t) = 10\cos(180°) + i10\sin(180°) = -10$

$$\boxed{-10}$$

1.c)

Solution:

$$i(t) = -4e^{i0°} + 3e^{-i90°}$$

$i(t) = -4\cos(0°) - i4\sin(0°) + 3\cos(-90°) + i3\sin(-90°) = -4 - 3i$

$$\boxed{-4 - 3i}$$

Problem 2:

2.a)

Solution:

$6 - 8i \Rightarrow \arctan\left(\frac{-8}{6}\right) = -53.13° \Rightarrow \theta = -53.13°, \sqrt{6^2 + 8^2} = 10 \Rightarrow = 10e^{-i53.13°}$

$2 + i \Rightarrow \arctan\left(\frac{1}{2}\right) = 26.56° \Rightarrow \theta = 26.56°, \sqrt{2^2 + 1^2} = \sqrt{5} \Rightarrow \sqrt{5}e^{i26.56°}$

$\frac{3\angle 60}{\sqrt{5}\angle 26.56°} = \frac{3e^{i60°}}{\sqrt{5}e^{i26.56°}} = \frac{3}{\sqrt{5}}e^{i(60°-26.56°)} = \frac{3}{\sqrt{5}}e^{i33.44°} = \frac{3}{\sqrt{5}}\angle 33.44°$

$\frac{3}{\sqrt{5}}\angle 33.44° \cdot 5\angle 30° = \frac{3}{\sqrt{5}} \cdot 5e^{i(33.44°+30°)} = \frac{15}{\sqrt{5}}e^{i63.44°} = \frac{15}{\sqrt{5}}\angle 63.44°$

$10\angle -53.13° \cdot 5\angle 30° = 50e^{i(-53.13°+30°)} = 50e^{-i23.13°} = 50\angle -23.13°$

$$50\angle -23.13° + \frac{15}{\sqrt{5}}\angle 63.44°$$

$$50\angle -23.13° = 50\cos(-23.13°) + i \cdot 50\sin(-23.13°) = 45.98 - 19.64i$$

$$\frac{15}{\sqrt{5}}\angle 63.44^\circ = 6.708\cos(63.44^\circ) + i\cdot 6.708\sin(63.44^\circ) = 3.0 + 6.0i$$

$$(45.98 + 3.0) + i(-19.64 + 6.0) = 48.98 - 13.64i$$

$$\sqrt{48.98^2 + 13.64^2} = 50.84, \quad \arctan\left(\frac{-13.64}{48.98}\right) = -15.56^\circ$$

$$\boxed{50.84\angle - 15.56^\circ}$$

2.b)

Solution:

$$(2 + i6) - (5 + i) = (2 - 5) + i(6 - 1) = -3 + i5$$

$\sqrt{(-3)^2 + 5^2} = \sqrt{9 + 25} = \sqrt{34} \approx 5.831$

Angle: Since real part is negative and imaginary is positive, we're in Q2:

$$\theta = 180^\circ - \arctan\left(\frac{5}{3}\right) = 180^\circ - 59.04^\circ = 120.96^\circ$$

$$\therefore -3 + i5 = \sqrt{34}\angle 120.96^\circ$$

$$(10\angle 60^\circ)(35\angle - 50^\circ) = (10\cdot 35)\angle(60^\circ + (-50^\circ)) = 350\angle 10^\circ$$

$$\frac{350\angle 10^\circ}{\sqrt{34}\angle 120.96^\circ} = \frac{350}{\sqrt{34}}\angle(10^\circ - 120.96^\circ) = \frac{350}{\sqrt{34}}\angle - 110.96^\circ$$

$$\frac{350}{\sqrt{34}} = \frac{350}{5.831} \approx 60.02$$

$$\boxed{60.02\angle - 110.96^\circ}$$

$$60.02\angle - 110.96^\circ = 60.02\cos(-110.96^\circ) + i\cdot 60.02\sin(-110.96^\circ)$$

$$= 60.02(-0.358) + i\cdot 60.02(-0.934) = \boxed{-21.49 - i56.06}$$

Problem 3:

3.a)

Solution:

$[3\cos(10^\circ) - 5\cos(-30^\circ)] + [3\sin(10^\circ) - 5\sin(-30^\circ)]i$

$= (3\cdot 0.985 - 5\cdot 0.866) + (3\cdot 0.174 - 5\cdot(-0.5))i$

$= (2.955 - 4.33) + (0.522 + 2.5)i$

$= -1.375 + 3.022i$

$\theta = 180^\circ + \arctan(\frac{3.022}{-1.375}) = 180^\circ - 65.5^\circ = 114.5^\circ$

$$\sqrt{(-1.375)^2 + (3.022)^2} = \sqrt{1.890625 + 9.132484} = \sqrt{11.023109} \approx 3.32$$

$$= \boxed{3.32\cos(20t + 114.5^\circ)}$$

3.b)

Solution:

$$40\sin(50t) = 40\cos(50t - 90^\circ)$$

$$40\angle-90^\circ+30\angle-45^\circ$$
$$[40\cos(-90^\circ)+30\cos(-45^\circ)]+[40\sin(-90^\circ)+30\sin(-45^\circ)]i$$
$$=(0+21.21)+(-40-21.21)i$$
$$=21.21-61.21i$$
$$\sqrt{21.21^2+61.21^2}=\sqrt{450.06+3746.66}=\sqrt{4196.72}\approx 64.78$$
$$\theta=\arctan\left(\frac{-61.21}{21.21}\right)=-70.9^\circ$$
$$=\boxed{64.78\cos\left(50t-70.9^\circ\right)}$$

3.c)

Solution:

$$20\sin(400t)=20\cos(400t-90^\circ)$$
$$-5\sin(400t-20^\circ)=-5\cos(400t-110^\circ)=5\cos(400t+70^\circ)$$
$$20\angle-90^\circ+10\angle60^\circ+5\angle70^\circ$$
$$[20\cos(-90^\circ)+10\cos(60^\circ)+5\cos(70^\circ)]+[20\sin(-90^\circ)+10\sin(60^\circ)+5\sin(70^\circ)]i$$
$$=(0+5+1.71)+(-20+8.66+4.70)i$$
$$=6.71-6.64i$$
$$\sqrt{6.71^2+6.64^2}=\sqrt{45.02+44.09}=\sqrt{89.11}\approx 9.44$$
$$\theta=\arctan\left(\frac{-6.64}{6.71}\right)=-44.7^\circ$$
$$=\boxed{9.44\cos\left(400t-44.7^\circ\right)}$$

Problem 4:

Solution:

$$v_{s(t)}=50\cos(200t)$$
$$\vec{V_s}=50\angle0^\circ$$
$$\omega=200$$
$$Z_R=10$$
$$Z_L=i\cdot200\cdot0.02=i4$$
$$Z_C=\frac{1}{i\cdot200\cdot0.005}=-i$$
$$Z_{eq}=10+i4-i1=10+i3$$
$$|Z_{eq}|=\sqrt{10^2+3^2}=10.44$$
$$\theta=\arctan\left(\frac{3}{10}\right)=16.7^\circ$$
$$\vec{I}=50\angle0\frac{\circ}{10.44\angle16.7^\circ}=4.79\angle(-16.7^\circ)$$
$$i(t)=\boxed{4.79\cos\left(200t-16.7^\circ\right)}$$

Problem 5:

Solution:

$-14i + 25i \parallel 16 = \frac{176i}{16+11i} \times \frac{16-11i}{16-11i} = $ `frac` $(176, 377)(11+16i)$

`frac` $(176, 377)(11+16i) + 4 + 20i = \frac{3444}{377} + \frac{10356i}{377} = 9.135 + 27.469i$

$\vec{I} = \frac{\vec{V}}{\vec{Z}} = \frac{12}{9.135+27.469i} = \frac{287-863i}{2194} = 0.1308 - 0.3933i$

$\theta = \arctan(\frac{-0.3933}{0.1308}) = -71.6°$

$I_o = \sqrt{0.1308^2 + 0.3933^2} = 0.41448$

$$i(t) = 0.41448\cos(10t - 71.6°)A$$

Problem 6:

Solution:

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j200 \cdot 50 \cdot 10^{-6}} = -j100$$

$$Z_{\text{par}} = \frac{50 \cdot Z_C}{50 + Z_C} = \frac{50(-j100)}{50 - j100} = 40 - j20 \quad (|Z_{\text{par}}| = 44.72, \theta = -26.57°)$$

$$Z_L = j\omega L = j200 \cdot 0.1 = j20$$

$$\vec{V}_o = 60 \cdot \frac{Z_L}{Z_{\text{par}} + Z_L} = 60 \cdot \frac{j20}{40 - j20 + j20} = 60 \cdot \frac{j20}{40} = j30$$

$$|\vec{V}_o| = 30, \quad \theta = 90°$$

$$v_o(t) = \boxed{30\cos(200t + 90°)}\text{V}$$

Problem 7:

Solution:

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j377 \cdot 50 \cdot 10^{-6}} = -j53.05 \quad (|Z_C| = 53.05, \theta = -90°)$$

$$Z_L = j\omega L = j377 \cdot 60 \cdot 10^{-3} = j22.62 \quad (|Z_L| = 22.62, \theta = 90°)$$

$$Z_{\text{par}} = \frac{Z_L \cdot 40}{Z_L + 40} = \frac{j22.62 \cdot 40}{j22.62 + 40} = 9.69 + j17.14 \quad (|Z_{\text{par}}| = 19.69, \theta = 60.51°)$$

$$Z_{\text{eq}} = 12 + Z_C + Z_{\text{par}} = 12 + (-j53.05) + (9.69 + j17.14)$$

$$Z_{\text{eq}} = (12 + 9.69) + j(-53.05 + 17.14) = 21.69 - j35.91$$

$$|Z_{\text{eq}}| = \sqrt{21.69^2 + 35.91^2} = 41.95$$

$$\theta = \arctan\left(\frac{-35.91}{21.69}\right) = -58.87°$$

$$\boxed{Z_{\text{eq}} = 41.95\angle - 58.87°}$$

Problem 8:

Solution:

$$Z_{\text{top}} = 4 - j6 \quad \left(|Z_{\text{top}}| = 7.21, \theta = -56.31^\circ\right)$$

$$Z_{\text{bot}} = 3 + j4 \quad (|Z_{\text{bot}}| = 5.0, \theta = 53.13^\circ)$$

$$Z_{\text{par}} = \frac{Z_{\text{top}} \cdot Z_{\text{bot}}}{Z_{\text{top}} + Z_{\text{bot}}} = \frac{(4 - j6)(3 + j4)}{(4 - j6) + (3 + j4)} = \frac{(12 + 16j - 18j + 24)}{7 - j2}$$

$$Z_{\text{par}} = \frac{36 - j2}{7 - j2} = 4.83 + j1.09 \quad \left(|Z_{\text{par}}| = 4.95, \theta = 12.77^\circ\right)$$

$$Z_{\text{eq}} = 2 + Z_{\text{par}} = 2 + 4.83 + j1.09 = 6.83 + j1.09 \quad \left(|Z_{\text{eq}}| = 6.92, \theta = 9.10^\circ\right)$$

$$\vec{I} = 10 + \frac{j10}{Z_{\text{eq}}} = 10 + \frac{j10}{6.83 + j1.09} = 10.23 + j1.43$$

$$|\vec{I}| = 10.33, \quad \theta = 7.94^\circ$$

$$\boxed{\vec{I} = 10.33\angle 7.94^\circ \text{A}}$$

End of Homework 1