PS12

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$$S = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

1

$$\det(S - \lambda I) = 0$$
$$(1-\lambda)(4-\lambda) - 2 \cdot 2 = 0$$
$$5\lambda + \lambda^2 = 0$$
$$\lambda(5+\lambda) = 0$$
$$\lambda_1 = 0 \, \lambda_2 = -5$$

$$\begin{bmatrix} -5 & 0 \\ 0 & 0 \end{bmatrix}$$

2

$$Null(S - 5I) = \vec{v_1}$$
$$\begin{bmatrix} -4 & 2 \\ 2 & -1 \end{bmatrix} \xrightarrow{R_2 = R_2 + \frac{1}{2}R_1} \begin{bmatrix} -4 & 2 \\ 0 & 0 \end{bmatrix} \xrightarrow{R_1 = -\frac{1}{4}R_1} \begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & 0 \end{bmatrix}$$
$$x_2 = t_2 \Rightarrow \vec{x_h} = t_2 \begin{bmatrix} -\frac{1}{2} \\ 1 \end{bmatrix} \Rightarrow \therefore \vec{v_1} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$
$$Null(S) = \vec{v_2}$$
$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \xrightarrow{R_2 = R_2 - 2R_1} \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$$
$$x_2 = t_2 \Rightarrow \vec{x_h} = t_2 \begin{bmatrix} -2 \\ 1 \end{bmatrix} \Rightarrow \therefore \vec{v_2} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$
$$v_1 \cdot v_2 = v_1^T v_2 = \begin{bmatrix} -1 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \end{bmatrix} = 0$$
$$\therefore \text{ angle between } \vec{v_1}, \vec{v_2} = 90^\circ$$