Homework 2

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Problem 1 - Section A

System 1

A

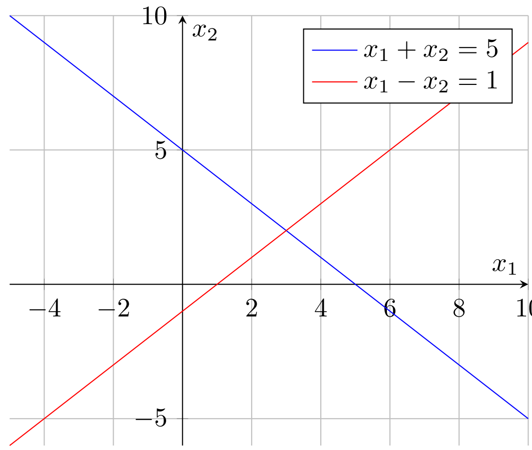

Figure 1: Plot of $x_1 + x_2 = 5$ and $x_1 - x_2 = 1$

B

A unique solution since there is one intersection point between the lines.

C

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

D

$$\begin{bmatrix} 1 & 1 & 5 \\ 1 & -1 & 1 \end{bmatrix}$$

E

$$\begin{bmatrix} 1 & 1 & 5 \\ 1 & -1 & 1 \end{bmatrix} \xrightarrow{R_2 = R_2 - R_1} \begin{bmatrix} 1 & 1 & 5 \\ 0 & -2 & -4 \end{bmatrix}$$

F

$$\begin{bmatrix} 1 & 1 & 5 \\ 0 & -2 & -4 \end{bmatrix} \xrightarrow{R_2=-\frac{1}{2}R_2} \begin{bmatrix} 1 & 1 & 5 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{R_1=R_1-R_2} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \end{bmatrix}$$

Therefore, given this RREF, it follows that

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

System 2

A

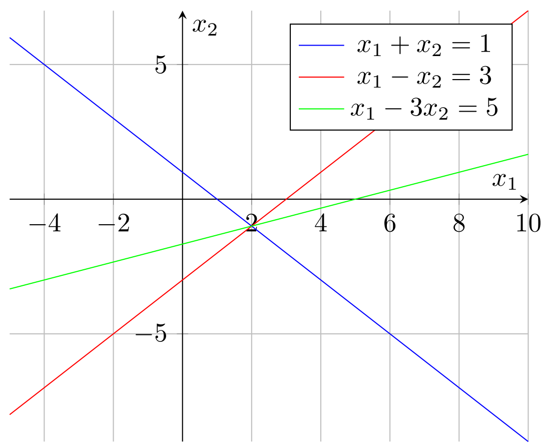

Figure 2: Plot of $x_1 + x_2 = 1$, $x_1 - x_2 = 3$, and $x_1 - 3x_2 = 5$

B

The SLE will have one solution since all lines intersect at the same point.

C

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & -3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

D

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 3 \\ 1 & -3 & 5 \end{bmatrix}$$

E

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 3 \\ 1 & -3 & 5 \end{bmatrix} \xrightarrow{R_2=R_2-R_1, R_3=R_3-R_1} \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 2 \\ 0 & -4 & 4 \end{bmatrix} \xrightarrow{R_3=R_3-2R_2} \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 2 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2=-\frac{1}{2}R_2} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1=R_1-R_2} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Therefore, given this RREF, it follows that

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

System 3

A

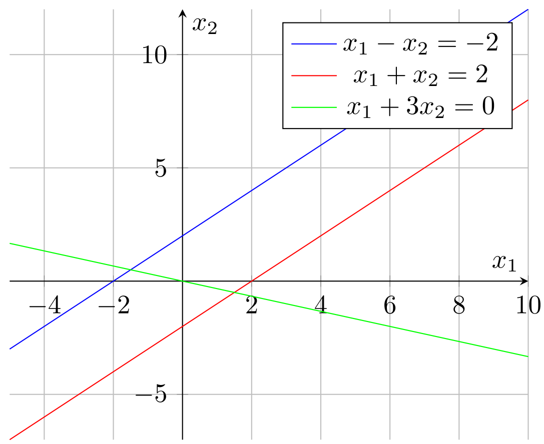

Figure 3: Plot of $x_1 - x_2 = -2$, $x_1 + x_2 = 2$, and $x_1 + 3x_2 = 0$

B

No solution to SLE since the lines do not all have common intersection point.

C

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}$$

D

$$\begin{bmatrix} 1 & -1 & -2 \\ 1 & 1 & 2 \\ 1 & 3 & 0 \end{bmatrix}$$

E

$$\begin{bmatrix} 1 & -1 & -2 \\ 1 & 1 & 2 \\ 1 & 3 & 0 \end{bmatrix} \xrightarrow{R_2 = R_2 - R_1, R_3 = R_3 - R_1} \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 4 \\ 0 & 4 & 2 \end{bmatrix} \xrightarrow{R_3 = R_3 - 2R_2} \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 4 \\ 0 & 0 & -6 \end{bmatrix}$$

F

The last column of the REF effectively shows that $x_1 \cdot 0 = -6$, which is sufficient to determine that the SLE is not consistent.

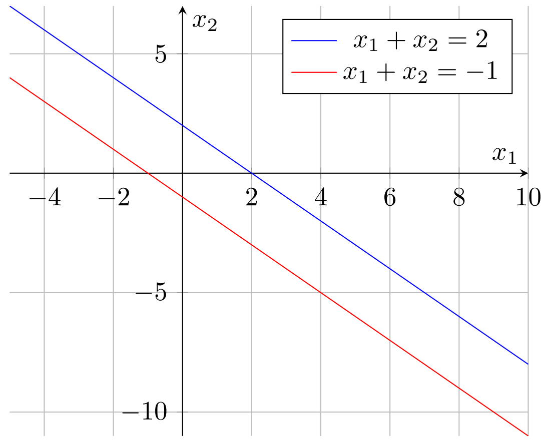

Figure 4: Plot of $x_1 + x_2 = 2$ and $x_1 + x_2 = -1$

Problem 1 - Section B

System 4

B

The SLE will have no solutions since the lines are parallel and therefore will never intersect.

C

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & -2 \end{bmatrix} \xrightarrow{R_2 = R_2 - R_1} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix} \xrightarrow{R_2 = -\frac{1}{4}R_2} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 = R_1 - 2R_2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

D

```
A = [1,1;1,1];
disp(rref(A))
```

```
1     1
0     0
```

E

```
A = [1,1,2;1,1,-2];
disp(rref(A))
```

```
1     1     0
0     0     1
```

F

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Column 1: Pivot Column 2: Free variable Column 3: Pivot

G

Since there is a pivot in the last column, from which is follows that $x_2 \cdot 0 = 1$, the SLE is in fact not consistent.

System 5

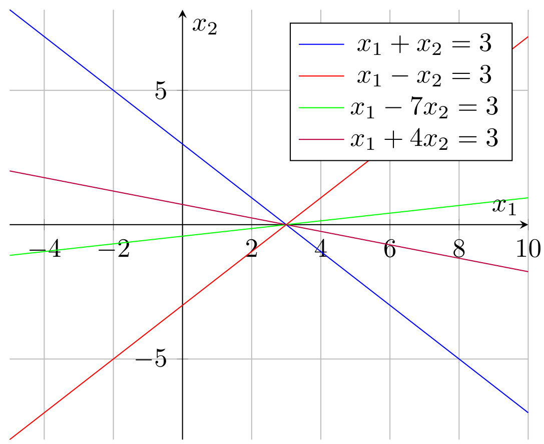

Figure 5: Plot of $x_1 + x_2 = 3$, $x_1 - x_2 = 3$, $x_1 - 7x_2 = 3$, and $x_1 + 4x_2 = 3$

B

Unique solution since there exists a common intersection point of the lines.

C

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & -1 & 3 \\ 1 & -7 & 3 \\ 1 & 4 & 3 \end{bmatrix} \xrightarrow{R_2=R_2-R_1, R_3=R_3-R_1, R_4=R_4-R_1} \begin{bmatrix} 1 & 1 & 3 \\ 0 & -2 & 0 \\ 0 & -8 & 0 \\ 0 & 3 & 0 \end{bmatrix} \xrightarrow{R_4=2R_4} \begin{bmatrix} 1 & 1 & 3 \\ 0 & -2 & 0 \\ 0 & -8 & 0 \\ 0 & 6 & 0 \end{bmatrix}$$

$$\xrightarrow{R_4=R_4+3R_3, R_3=R_3-4R_2} \begin{bmatrix} 1 & 1 & 3 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2=-frac12R_2} \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1=R_1-R_2} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

D

```
A = [1,1;1,-1;1,3;1,4];
disp(rref(A))
```

```
1    0
0    1
0    0
0    0
```

E

```
A = [1,1,3;1,-1,3;1,-7,3;1,4,3];
disp(rref(A))
```

```
1    0    3
0    1    0
0    0    0
0    0    0
```

F

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Column 1: Pivot Column 2: Pivot

G

Yes, the SLE is indeed consistent and unique as predicted. By expanding the augmented matrix back to two equations we are left with $x_1 = 3, x_2 = 0$, which is indeed the same point shown in Fig.5.

System 6

A

-

B

Since there are 3 equations in 5 unknowns, if the system is consistent it must have infinitely many solutions (with two free variables).

C

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 4 \\ -1 & 1 & -1 & 1 & 1 & 2 \\ 2 & 2 & 0 & -1 & 5 & 4 \end{bmatrix} \xrightarrow{R_2=R_2+R_1, \quad R_3=R_3-2R_1} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 4 \\ 0 & 2 & 0 & 2 & 2 & 6 \\ 0 & 0 & -2 & -3 & 3 & -4 \end{bmatrix}$$

$$\xrightarrow{R_2=\frac{1}{2}R_2, \quad R_3=-\frac{1}{2}R_3} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 4 \\ 0 & 1 & 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & \frac{3}{2} & -\frac{3}{2} & 2 \end{bmatrix}$$

$$\xrightarrow{R_1=R_1-R_2} \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & \frac{3}{2} & -\frac{3}{2} & 2 \end{bmatrix}$$

$$\xrightarrow{R_1=R_1-R_3} \begin{bmatrix} 1 & 0 & 0 & -\frac{3}{2} & \frac{3}{2} & -1 \\ 0 & 1 & 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & \frac{3}{2} & -\frac{3}{2} & 2 \end{bmatrix}$$