EK307: Circuits

Lecture notes for Circuits (EK307)

Giacomo Cappelletto

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Contents

Chapter 1: Current, Voltage, Charge and Power

Variables and Fundamental Quantities

Electric Charge — Definition 1.1

Charge is a fundamental property of matter that determines electromagnetic interaction. It comes in two types (positive and negative) and is conserved in all physical processes. Important facts:

- Unit: coulomb (C). The elementary charge carried by an electron has magnitude $e = 1.602 \times 10^{-19}$C.
- Conservation of charge: In any isolated system, the algebraic sum of charge remains constant.

Electric Current — Definition 1.2

Current measures the rate at which charge flows past a reference point in a circuit:

$$i(t) = \frac{dq(t)}{dt} \quad [1]$$

where $q(t)$ is the algebraic charge that has crossed the reference. Key points:

- Unit: ampere (A) with $1\text{A} = 1\ \text{C/s}$.
- Current direction follows the *conventional* positive-charge flow from higher to lower potential; electron flow is opposite.
- If a reference direction is chosen, a negative value of $i(t)$ indicates actual flow opposite to that reference.

Transferred Charge over an Interval — Definition 1.3

The algebraic charge transferred between t_0 and t is

$$q(t) - q(t_0) = \int_{t_0}^{t} i(\tau)d\tau \quad [2]$$

and, equivalently, $i(t) = \frac{dq(t)}{dt}$.

DC vs AC Current — Note 1.1

DC (direct current) means the current maintains one direction over time (its sign does not change). AC (alternating current) changes direction periodically.

From q(t) to i(t) — Example 1.1

Suppose the transferred charge is piecewise linear (in μC)

$$q(t) = \begin{cases} 0 & \text{if } t < 0 \\ 30t & \text{if } 0 \le t < 1 \\ 30 - 30(t-1) & \text{if } 1 \le t < 2 \\ -30 + 15(t-2) & \text{if } 2 \le t < 4 \\ 0 & \text{if } t \ge 4 \end{cases} \quad [3]$$

with t in seconds. Find $i(t)$ and comment on current direction.

Solution: Differentiate $q(t)$ on each interval (and convert to amperes by μC/s = μA):

$$i(t) = \begin{cases} 0 & \text{if } t < 0 \\ 30\mu\text{A} & \text{if } 0 \le t < 1 \\ -30\mu\text{A} & \text{if } 1 \le t < 2 \\ 15\mu\text{A} & \text{if } 2 \le t < 4 \\ 0 & \text{if } t \ge 4 \end{cases} \quad [4]$$

Intervals with negative slope give negative current, meaning actual flow opposite to the chosen reference direction during $1 \le t < 2$.

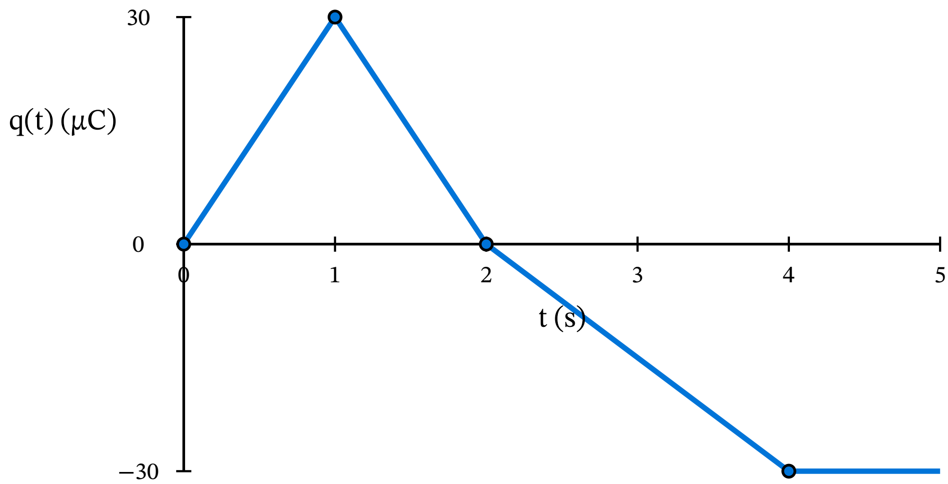

Figure 1: Piecewise linear charge function q(t)

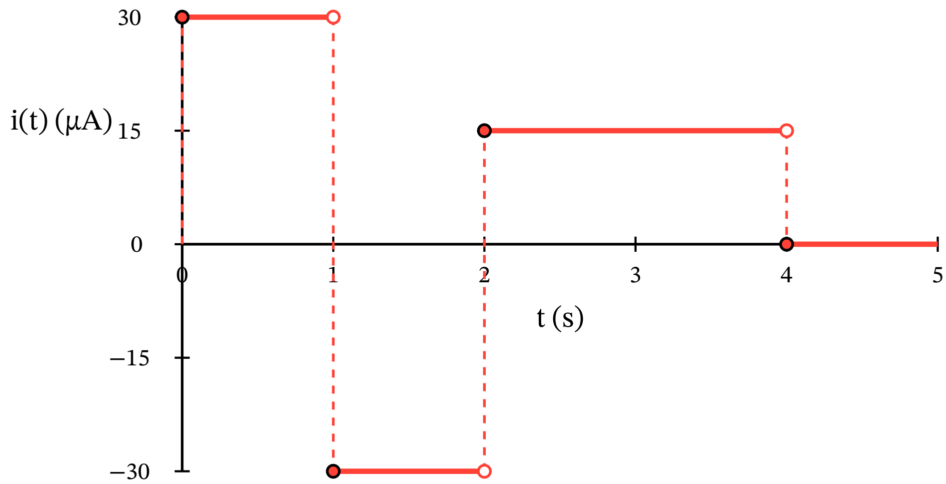

Figure 2: Piecewise constant current function i(t) = dq/dt

Voltage (Potential Difference)

Voltage — **Definition 1.4**

Voltage is the change in potential energy per unit charge between two points:

$$v(t) = \frac{dw}{dq}, \quad 1\text{V} = 1 \text{ J/C} \tag{5}$$

Properties and usage:

- Voltage is always measured *between* two points and is a relative quantity; a reference point ("ground") is often chosen to report node voltages.
- A "voltage drop" is the potential decrease across an element following a specified reference polarity.

Resistance and Conductance

Resistance and Ohm's Law — **Definition 1.5**

Resistance models opposition to the flow of charge. For an ohmic element,

$$v = iR \quad \text{or} \quad i = Gv \tag{6}$$

where R is resistance in ohms (Ω) and $G = \frac{1}{R}$ is conductance in siemens (S). In the i–v plane the slope is $\frac{di}{dv} = G$ (a straight line through the origin for an ideal resistor).

Power and Energy

Instantaneous Power Definition 1.6

Electrical power is the rate of change of energy with respect to time:

$$p(t) = \frac{dw}{dt} = \frac{dw}{dq}\frac{dq}{dt} = v(t)i(t) \quad [7]$$

For a resistor using Ohm's law,

$$\boxed{p = vi = i^2R = \frac{v^2}{R}} \quad [8]$$

Under the passive sign convention, $p > 0$ indicates the element absorbs power, while $p < 0$ indicates it delivers power.

Passive Sign Convention Definition 1.7

By conservation of energy, the power absorbed by all elements in a system is equal to the power delivered by the other elements. Therefore sum_i^n p_i = 0

Circuit Elements

Circuit elements are the building blocks of electrical circuits. They fall into two main categories:

Passive Elements Definition 1.8

Passive elements can only absorb or store energy - they cannot generate energy. Examples include:

- Resistors: Convert electrical energy to heat (always absorb power)
- Capacitors: Store energy in electric fields
- Inductors: Store energy in magnetic fields

Under the passive sign convention, passive elements have $p \geq 0$ when current enters the positive terminal.

Active Elements Definition 1.9

Active elements can supply energy to a circuit. The primary active elements are:

- Voltage Sources: Maintain a specified voltage across their terminals
- Current Sources: Maintain a specified current through them

Active elements can have $p < 0$ (delivering power) under the passive sign convention.

Independent Sources

Independent sources provide a specified voltage or current that does not depend on other circuit variables.

Voltage Sources

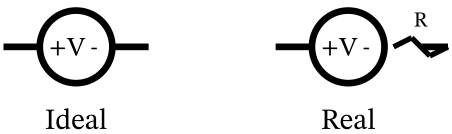

Figure 3: Ideal vs Real Voltage Sources

Ideal Voltage Source — Definition 1.10

An ideal voltage source maintains a constant voltage V_s across its terminals regardless of the current flowing through it. Key properties:

- Terminal voltage is always V_s (independent of current)
- Can supply unlimited current if needed
- Internal resistance $R_s = 0$
- Open circuit: $V = V_s$, $I = 0$
- Short circuit: $V = 0$, $I = \infty$ (not physically realizable)

Real Voltage Source — Definition 1.11

A real voltage source has internal resistance R_s in series with an ideal voltage source. Properties:

- Terminal voltage: $V = V_s - IR_s$
- Open circuit: $V = V_s$, $I = 0$
- Short circuit: $V = 0$, $I = \frac{V_s}{R_s}$
- Maximum power transfer occurs when load resistance equals R_s

Current Sources

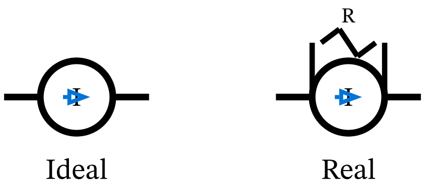

Figure 4: Ideal vs Real Current Sources

Ideal Current Source — Definition 1.12

An ideal current source maintains a constant current I_s through it regardless of the voltage across its terminals. Key properties:

- Current is always I_s (independent of voltage)
- Can develop unlimited voltage if needed
- Internal resistance $R_s = \infty$
- Open circuit: $I = 0$, $V = \infty$ (not physically realizable)
- Short circuit: $I = I_s$, $V = 0$

Real Current Source — Definition 1.13

A real current source has internal resistance R_s in parallel with an ideal current source. Properties:

- Terminal current: $I = I_s - \frac{V}{R_s}$
- Open circuit: $I = 0$, $V = I_s R_s$
- Short circuit: $I = I_s$, $V = 0$
- Norton equivalent circuit representation

Dependent Sources

Dependent (controlled) sources have outputs that depend on other voltages or currents in the circuit. They are essential for modeling active devices like transistors and operational amplifiers.

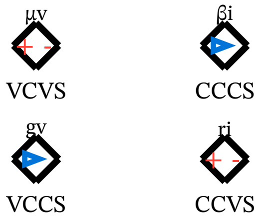

Figure 5: Four types of dependent sources (diamond symbols)

Voltage Controlled Voltage Source (VCVS) — Definition 1.14

Output voltage depends on a controlling voltage elsewhere in the circuit:

$$v_{\text{out}} = \mu v_{\text{control}} \quad [9]$$

where μ is the voltage gain (dimensionless). Used to model voltage amplifiers.

Current Controlled Current Source (CCCS) — Definition 1.15

Output current depends on a controlling current elsewhere in the circuit:

$$i_{\text{out}} = \beta i_{\text{control}} \quad [10]$$

where β is the current gain (dimensionless). Used to model current amplifiers like BJTs.

Voltage Controlled Current Source (VCCS) — Definition 1.16

Output current depends on a controlling voltage elsewhere in the circuit:

$$i_{\text{out}} = g v_{\text{control}} \quad [11]$$

where g is the transconductance (units: S = A/V). Used to model devices like FETs.

Current Controlled Voltage Source (CCVS) — Definition 1.17

Output voltage depends on a controlling current elsewhere in the circuit:

$$v_{\text{out}} = r i_{\text{control}} \quad [12]$$

where r is the transresistance (units: Ω = V/A). Less commonly used in practice.

Power Calculation with Dependent Sources — Example 1.2

Consider a circuit with a CCCS where $\beta = 0.6$ and the controlling current is $i_{\text{control}} = 3\text{A}$.

If the dependent source has 5V across its terminals:

- Output current: $i_{\text{out}} = \beta i_{\text{control}} = 0.6 \times 3\text{A} = 1.8\text{A}$
- Power delivered: $p = vi = 5\text{V} \times 1.8\text{A} = 9\text{W}$

This demonstrates that dependent sources can deliver power to a circuit, making them active elements.

Understanding the Passive Sign Convention

The passive sign convention (PSC) is crucial for determining whether a circuit element absorbs or delivers power. The key insight is that both current direction and voltage polarity are reference choices - we can choose them arbitrarily, but the power calculation depends on how we choose them relative to each other.

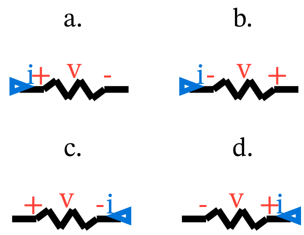

Figure 6: Four possible combinations of current direction and voltage polarity references

Power Calculations for Each Case Note 1.2

The power absorbed by each resistor depends on the relative orientation of current and voltage:

Cases a & d (PSC satisfied): Current enters the positive terminal

- Power: $p = +vi$ (positive = absorbing power)

Cases b & c (PSC not satisfied): Current enters the negative terminal

- Power: $p = -vi$ (can be positive or negative depending on actual values)

Key insight:

- If $p > 0$: Element absorbs power (acts like a load)
- If $p < 0$: Element delivers power (acts like a source)

The passive sign convention simply provides a consistent framework for determining the sign of power calculations based on our chosen reference directions.

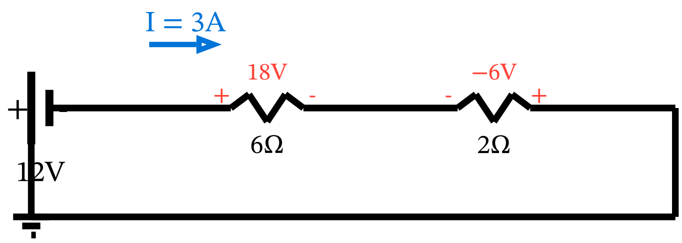

Figure 7: Series circuit with 12V battery, 6Ω and 2Ω resistors (showing voltage polarity references)

Passive Sign Convention and Power Balance Note 1.3

This circuit demonstrates the passive sign convention with I = 3A throughout. The devices have the following voltage and power characteristics:

Power calculations using passive sign convention:

- 12V Battery (delivers power): $p_{\text{battery}} = -vi = -12\text{V} \times 3\text{A} = -36\text{W}$
- 18V Device (6Ω, absorbs power): $p_1 = +vi = +18\text{V} \times 3\text{A} = +54\text{W}$
- 6V Device (2Ω, flipped polarity): $p_2 = -vi = -6\text{V} \times 3\text{A} = -18\text{W}$

Verification of power balance:

$$\sum p_i = p_{\text{battery}} + p_1 + p_2 = -36\text{W} + 54\text{W} + (-18\text{W}) = 0\text{W} \quad [13]$$

✓

Key insight: The flipped polarity on the 6V device means it has a negative power sign in this reference frame, even though it's still physically absorbing power. The algebraic sum of all powers equals zero, confirming energy conservation.

Note: In this idealized circuit, we're treating the devices as having fixed voltage drops at 3A current, demonstrating the passive sign convention rather than pure resistive behavior.

Chapter 2: Circuit Topology: Nodes, Branches, and Loops

Understanding circuit topology is essential for analyzing electrical circuits systematically. We need to identify the basic structural elements that define how components are connected.

Definitions

Node — Definition 2.1

A node is a point where two or more circuit elements connect. All points connected by ideal wires (zero resistance) are considered to be at the same node and have the same voltage.

Branch — Definition 2.2

A branch is a single circuit element or a series combination of elements between two nodes. Current through all elements in a branch is identical.

Loop — Definition 2.3

A loop is any closed path through the circuit that starts and ends at the same node without passing through any node more than once.

Circuit Topology Examples

The following three diagrams show the same circuit with different aspects highlighted:

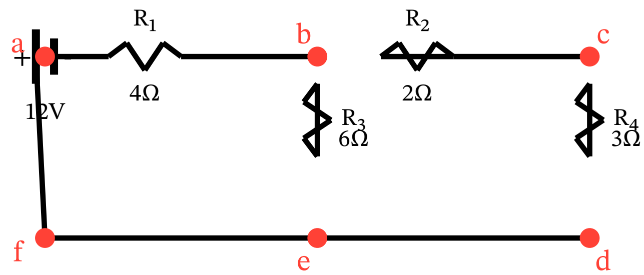

Figure 8: Circuit with nodes highlighted (red circles). This circuit has 6 nodes: a, b, c, d, e, f

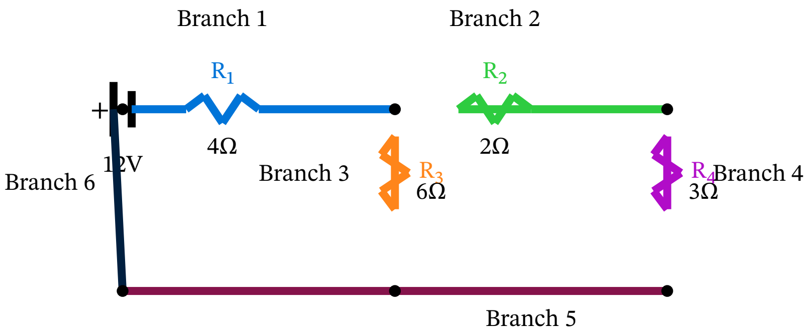

Figure 9: Circuit with branches highlighted (different colors). This circuit has 6 branches: 4 resistors, 1 voltage source, and 1 connecting wire

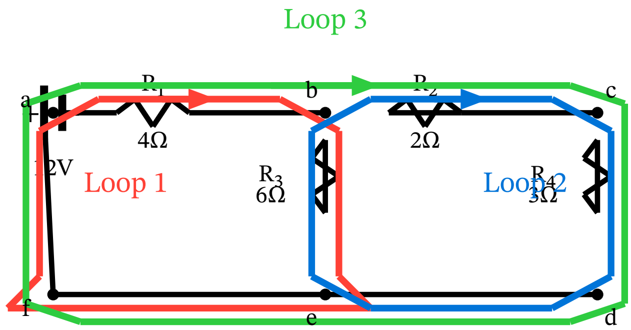

Figure 10: Circuit with loops highlighted (colored arrows). This circuit has several possible loops, with three examples shown

Counting Circuit Elements — Note 2.1

For this example circuit:

- Nodes: 6 total (a, b, c, d, e, f)
- Branches: 6 total (voltage source + 4 resistors + 1 connecting wire)
- Loops: Many possible loops exist. The three shown are:
  - Loop 1: a → R_1 → b → R_3 → e → (bottom wire) → f → (voltage source) → a
  - Loop 2: b → R_2 → c → R_4 → d → (bottom wire) → e → R_3 → b
  - Loop 3: Outer loop through all components a → R_1 → b → R_2 → c → R_4 → d → (bottom wire) → e → (bottom wire) → f → (voltage source) → a

Understanding these topological elements is essential for applying systematic circuit analysis methods like nodal analysis and mesh analysis.

Chapter 3: Resistor Combinations

Resistors can be combined in two fundamental ways: series and parallel connections. Understanding these combinations allows us to simplify complex circuits by finding equivalent resistances.

Series Connection

Series Resistors — Definition 3.1

Resistors are in series when they share the same current (i.e., when current has only one path to flow through all of them). The equivalent resistance is the sum of individual resistances:

$$\boxed{R_{\text{eq}} = \sum_{i=1}^{n} R_i = R_1 + R_2 + R_3 + \ldots} \qquad [14]$$

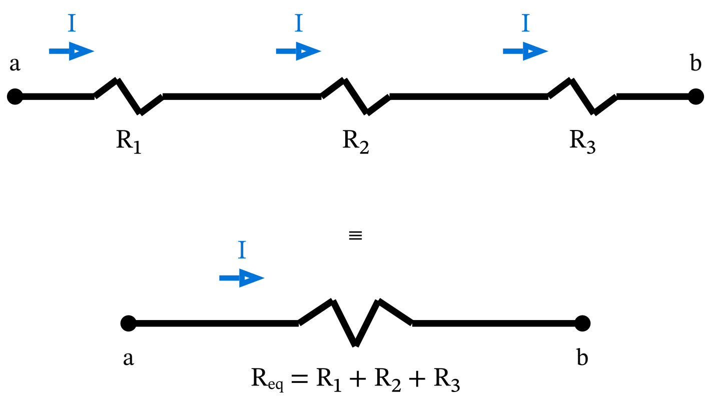

Figure 11: Series resistor combination: same current flows through each resistor

Series Resistor Properties — Note 3.1

In a series connection:
- Same current: $I_1 = I_2 = I_3 = I$
- Voltages add: $V_{\text{total}} = V_1 + V_2 + V_3$
- Individual voltages: $V_i = IR_i$
- Total resistance increases: $R_{\text{eq}} > R_{\text{largest}}$

Parallel Connection

Parallel Resistors — Definition 3.2

Resistors are in parallel when they share the same voltage (i.e., when they are connected across the same two nodes). The reciprocal of equivalent resistance equals the sum of reciprocals:

$$\boxed{\frac{1}{R_{\text{eq}}} = \sum_{i=1}^{n} \frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots} \quad [15]$$

For two resistors: $R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2}$ (product over sum)

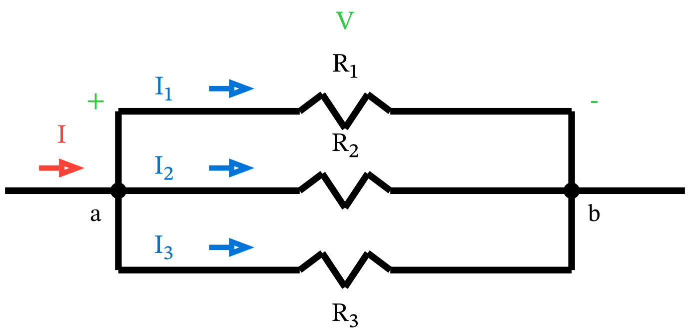

Figure 12: Parallel resistor combination: same voltage across each resistor

Parallel Resistor Properties — Note 3.2

In a parallel connection:
- Same voltage: $V_1 = V_2 = V_3 = V$
- Currents add: $I_{\text{total}} = I_1 + I_2 + I_3$
- Individual currents: $I_i = \frac{V}{R_i}$
- Total resistance decreases: $R_{\text{eq}} < R_{\text{smallest}}$
- Current divides inversely with resistance

Voltage Division

When resistors are connected in series, the total voltage divides among them proportionally to their resistance values.

Voltage Divider Rule — Definition 3.3

For resistors in series, the voltage across any resistor is:

$$\boxed{V_i = V_{\text{total}} \times \frac{R_i}{R_{\text{total}}}} \quad [16]$$

where $R_{\text{total}} = R_1 + R_2 + \ldots + R_n$ is the sum of all series resistances.

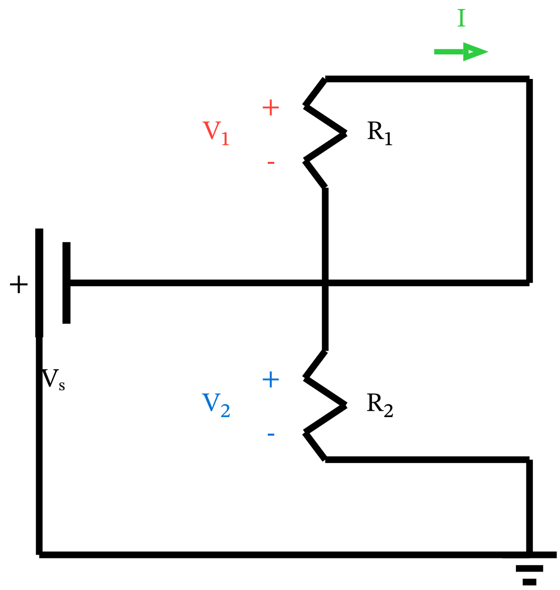

Figure 13: Voltage divider circuit showing voltage division across series resistors

Voltage Divider Calculation Example 3.1

Given: $V_s = 12\text{V}$, $R_1 = 8\text{k}\Omega$, $R_2 = 4\text{k}\Omega$

Solution:

1. Total resistance: $R_{\text{total}} = R_1 + R_2 = 8 + 4 = 12\text{k}\Omega$
2. Voltage across R_1: $V_1 = V_s \times \frac{R_1}{R_{\text{total}}} = 12\text{V} \times \frac{8\text{k}\Omega}{12\text{k}\Omega} = 8\text{V}$
3. Voltage across R_2: $V_2 = V_s \times \frac{R_2}{R_{\text{total}}} = 12\text{V} \times \frac{4\text{k}\Omega}{12\text{k}\Omega} = 4\text{V}$

Verification: $V_1 + V_2 = 8\text{V} + 4\text{V} = 12\text{V} = V_s$ ✓

Note: The larger resistance (R_1) gets the larger voltage drop.

Current Division

When resistors are connected in parallel, the total current divides among them inversely proportional to their resistance values.

Current Divider Rule Definition 3.4

For resistors in parallel, the current through any resistor is:

$$\boxed{I_i = I_{\text{total}} \times \frac{R_{\text{eq}}}{R_i} = I_{\text{total}} \times \frac{1/R_i}{\sum_{k=1}^{n} 1/R_k}} \quad [17]$$

For two resistors: $I_1 = I_{\text{total}} \times \frac{R_2}{R_1+R_2}$ and $I_2 = I_{\text{total}} \times \frac{R_1}{R_1+R_2}$

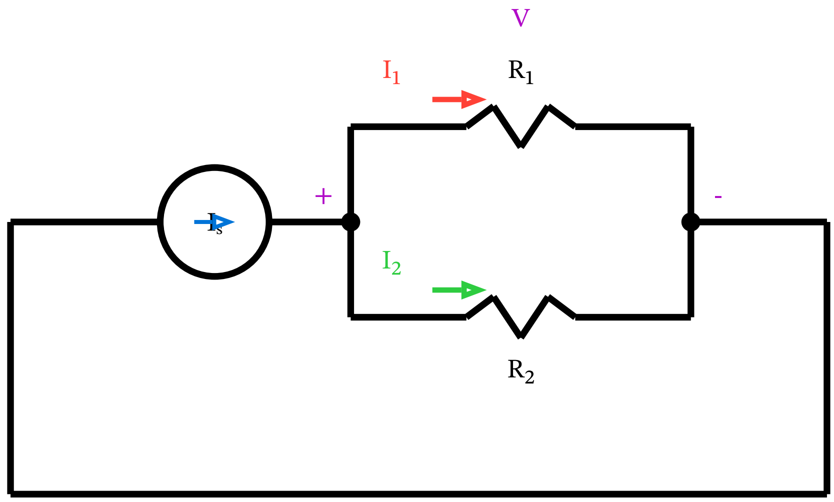

Figure 14: Current divider circuit showing current division across parallel resistors

Current Divider Calculation — Example 3.2

Given: $I_s = 6\text{A}$, $R_1 = 3\Omega$, $R_2 = 6\Omega$

Solution: Using the two-resistor current divider formula:

1. Current through R_1: $I_1 = I_s \times \frac{R_2}{R_1+R_2} = 6\text{A} \times \frac{6\Omega}{3\Omega+6\Omega} = 6\text{A} \times \frac{6}{9} = 4\text{A}$
2. Current through R_2: $I_2 = I_s \times \frac{R_1}{R_1+R_2} = 6\text{A} \times \frac{3\Omega}{3\Omega+6\Omega} = 6\text{A} \times \frac{3}{9} = 2\text{A}$

Verification: $I_1 + I_2 = 4\text{A} + 2\text{A} = 6\text{A} = I_s$ ✓

Note: The smaller resistance (R_1) gets the larger current (inverse relationship).

Key Relationships — Note 3.3

Voltage Division (Series):

- Voltage divides directly with resistance
- Larger R gets larger V
- $\frac{V_i}{V_{\text{total}}} = \frac{R_i}{R_{\text{total}}}$

Current Division (Parallel):

- Current divides inversely with resistance
- Smaller R gets larger I
- $\frac{I_i}{I_{\text{total}}} = \frac{\frac{1}{R_i}}{\sum \frac{1}{R_k}}$

These relationships are fundamental for analyzing more complex circuits.