ENG EK103: Problem Set 5 (Spring 2025) Due: March 6

1. Determine whether or not the vectors **u** and **v** are linearly independent, where

a) $\boldsymbol{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \boldsymbol{v} = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$

b) $\boldsymbol{u} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \boldsymbol{v} = \begin{bmatrix} -2 \\ 6 \end{bmatrix}$

c) $\boldsymbol{u} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}, \boldsymbol{v} = \begin{bmatrix} 4 \\ 5 \\ -6 \end{bmatrix}$

d) $\boldsymbol{u} = \begin{bmatrix} 2 \\ 4 \\ -8 \end{bmatrix}, \boldsymbol{v} = \begin{bmatrix} 3 \\ 6 \\ -12 \end{bmatrix}$

Clearly state your reason. (Hint: This problem should not require row reduction).

2. Determine whether or not the following vectors in $\mathbf{R}^3$ are linearly independent, where

a) $\boldsymbol{u} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}, \boldsymbol{v} = \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}, \boldsymbol{w} = \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix}$

b) $\boldsymbol{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \boldsymbol{v} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \boldsymbol{w} = \begin{bmatrix} 1 \\ 5 \\ 6 \end{bmatrix}$

c) $\boldsymbol{u} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}, \boldsymbol{v} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \boldsymbol{w} = \begin{bmatrix} 2 \\ 5 \\ 7 \end{bmatrix}, \boldsymbol{x} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$

Clearly state your reason for each part and show all your steps.

3. Find the inverse, if they exist, of the following 2x2 matrices (C and D) using the formula below.

$$\textbf{Inverse Formula}: A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$C = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$.

Check your answer using the **inv** command in MATLAB.

4. Show that $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{bmatrix}$ are inverses without using the row reduction algorithm.

Check your answer using the **inv** command in MATLAB.

5. Now find the inverse of A = $\begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$ using the row reduction algorithm. Show all your steps in the algorithm.