Homework 2

Giacomo Cappelletto

February 9, 2025

1 Linear Transformations

A)

I)

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

II)

$$ScalarMatrix$$

B)

I

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

II)

$$Horizontalshear$$

C)

I)

$$\begin{bmatrix} \cos(270°) & -\sin(270°) \\ \sin(270°) & \cos(270°) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

II)

$$RotationMatrix$$

1.1 E)

I)

$$\begin{bmatrix} \cos(180°) & -\sin(180°) \\ \sin(180°) & \cos(180°) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

II)

$$ReflectionMatrix$$

1.2 F)

I)

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

II)

$$Projection(matrix)$$

2 Rotations

A)

$$\cos(\theta + \phi) = \frac{v_1}{r} \Rightarrow v_1 = r \cdot \cos(\theta + \phi)$$

$$v = \begin{bmatrix} r \cdot \cos(\theta + \phi) \\ r \cdot \sin(\theta + \phi) \end{bmatrix}$$

$$\cos(\theta) = \frac{z_1}{r} \Rightarrow z_1 = r \cdot \cos(\theta)$$

$$z = \begin{bmatrix} r \cdot \cos(\phi) \\ r \cdot \sin(\phi) \end{bmatrix}$$

B)

Since

$$\cos(\theta + \phi) = \cos(\theta)\cos(\phi) - \sin(\theta)\sin(\phi)$$

Then

$$v_1 = r \cdot \cos(\theta + \phi) = r \cdot \cos(\theta)\cos(\phi) - r \cdot \sin(\theta)\sin(\phi)$$

C)

Given

$$z_1 = r \cdot \cos(\theta), z_2 = \cdot \sin(\theta)$$

Then

$$v_1 = z_1 \cdot \cos(\theta) - z_2 \cdot \sin(\theta)$$

D)

$$v_1 = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \end{bmatrix} \cdot \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = z_1 \cdot \cos(\theta) - z_2 \cdot \sin(\theta)$$

E)

Since

$$\sin(\theta + \phi) = \sin(\theta)\cos(\phi) + \cos(\theta)\sin(\phi)$$

Then

$$v_2 = r \cdot \sin(\theta + \phi) = r \cdot \sin(\theta)\cos(\phi) + r \cdot \cos(\theta)\sin(\phi)$$

F)

Given

$$z_1 = r \cdot \cos(\theta), z_2 = \cdot \sin(\theta)$$

Then

$$v_2 = z_1 \cdot \sin(\theta) + z_2 \cdot \cos(\theta)$$

G)

$$v_2 = \begin{bmatrix} \sin(\theta) & \cos(\theta) \end{bmatrix} \cdot \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = z_1 \cdot \sin(\theta) + z_2 \cdot \cos(\theta)$$

H)

$$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \cdot \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

3 Homogeneous Transformations

A)

From the given T stops it is given that

$$L_1 = 1km, L_2 = 2km, L_3 = 3km$$

and

$$\theta_1 = -45°, \theta_2 = 45°, \theta_3 = 45°$$

Therefore the Homogeneous Transformation Matrices are

$$T_0 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(-45°) & -\sin(-45°) & 0 \\ \sin(-45°) & \cos(-45°) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_1 = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & L_1 \\ \sin(\theta_2) & \cos(\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(45°) & -\sin(45°) & 1 \\ \sin(45°) & \cos(45°) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3) & L_2 \\ \sin(\theta_3) & \cos(\theta_3) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(45°) & -\sin(45°) & 2 \\ \sin(45°) & \cos(45°) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 2 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_3 = \begin{bmatrix} \cos(0) & -\sin(0) & L_3 \\ \sin(0) & \cos(0) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Working out T_{ee} gives

$$T_{ee} = T_0 \cdot T_1 \cdot T_2 \cdot T_3$$

$$= \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 2 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & \frac{\sqrt{2}}{2} \\ 0 & 1 & -\frac{\sqrt{2}}{2} \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & \frac{4+\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & \frac{4+2\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Therefore the position of Government center compared to BU central is

$$\begin{bmatrix} \frac{4+2\sqrt{2}}{2} \\ 0 \end{bmatrix} = \begin{bmatrix} 3.4142 \\ 0 \end{bmatrix}$$

and its orientation is

$$\begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} (\theta = 45°)$$

4 MATLAB

```
% Put together a list of ‘n‘ 2x2 simple geometric transformations
clear
A_lst(:,:,1) = [1 0; 0 1];                              % First matrix
A_lst(:,:,2) = [-1 0; 0 1];                             % Second matrix
A_lst(:,:,3) = [1 0; 0 -1];                             % Third matrix
A_lst(:,:,4) = [cos(pi/2) -sin(pi/2); sin(pi/2) cos(pi/2)]; % Fourth matrix
A_lst(:,:,5) = [1/2 0; 0 1];                            % Fifth matrix
A_lst(:,:,6) = [1 0; 0 1/2];                            % Sixth matrix
A_lst(:,:,7) = [1 0.5; 0 1];                              % Seventh matrix
A_lst(:,:,8) = [1 0; 0.5 1];                              % Eighth matrix

% Define the house
H = [[0;0], [0;1], [1;1.5], [1;1], [1;0], [0;0]];

% Create the first figure
figure(1)

% Plot the house
plot(H(1,:), H(2,:), '-o', 'Color', 'blue', 'MarkerFaceColor', 'red')
title('Original image')

xlim([-2 2])
axis equal

% Get the number of 2x2 matrices
numMatrices = size(A_lst,3);

% Create a second figure
figure(2)
set(gcf, 'Position', [100, 100, 800, 1200]);

transf = {"Scaling by factor of 1", "Reflection about y-axis", "Reflection about x-axis", "Rotation by 

for i = 1:numMatrices
    H_transformed = A_lst(:,:,i) * H;
    subplot(ceil(sqrt(numMatrices)), ceil(sqrt(numMatrices)), i); % Create subplots
    plot(H_transformed(1,:), H_transformed(2,:), '-o', 'Color', 'blue', 'MarkerFaceColor', 'red');
    title(['Transformation ' transf(i)], "FontSize", 10);
    xlim([-2 2]);
```

```
    ylim([-2 2]);
    axis equal;
end
```