# ENG EK 103: Computational Linear Algebra: Problem Set 1

Only use MATLAB to check your answers, or where asked. **No Explanation = No Credit**. For each problem, all derivation processes should be written down, your steps should be shown, and described in a clear fashion. For accurate grading, be sure to write down your name, BU ID, and homework number (PS1 ) on all pages you submit.

For those in different sections of EK 103, **remember** that both an over arrow ($\overrightarrow{v}$) or alternatively bold typeface ($\mathbf{v}$) indicate "this variable is a vector."

Each instructor will assign a point value to each question according to their own grading scheme (example, if the homeworks are out of 100 points or out of 40 points or...)

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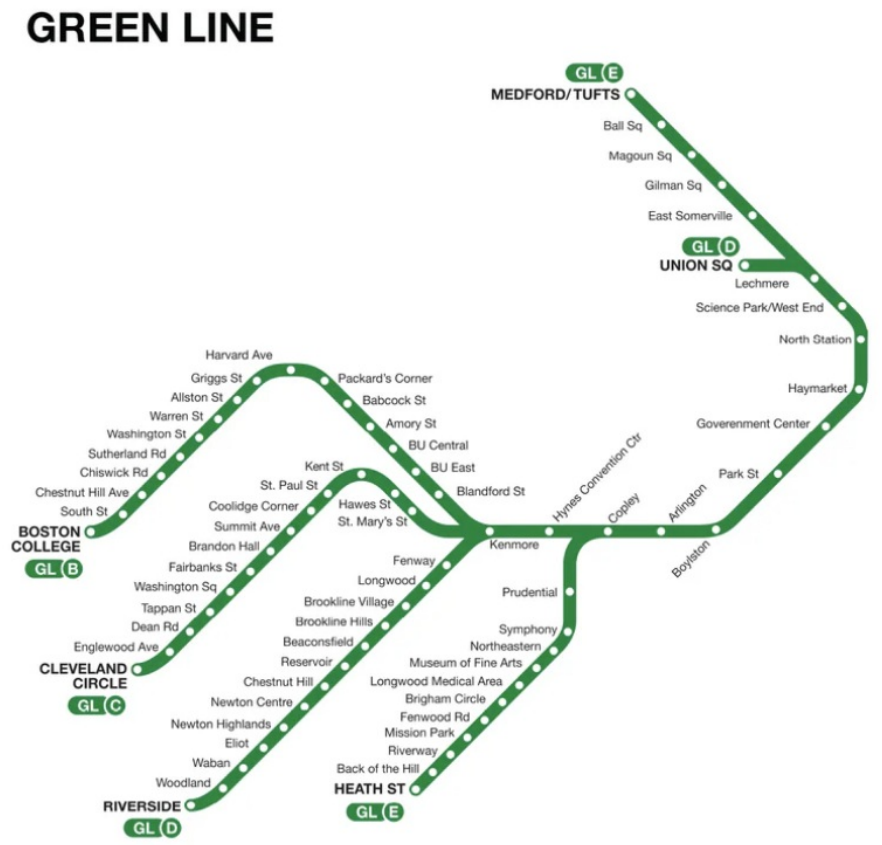
1. Plotting vectors, adding vectors, lengths of vectors

   You've been hired by the MBTA. They need you to re-draw a map of the Green Line specifically for students at BU.

   As a reminder, here is the current map of the Green Line:



The surveyors for the T have made the following measurements (not accurate, just easier for your calculations!).

- From Copley to BU: 12 miles west, 5 miles north
- From Harvard Ave to BU: 4 miles east, 3 miles south
- From Copley to Lechmere: 3 miles east, 8 miles north
- From Government Center to Lechmere: 0 miles east, 4 miles north
- From Harvard Ave to Medford/Tufts: 15 miles east, 3 miles north

Let's put BU at the origin of the map (0,0). The point for each stop should be labelled with its first letter ($H, B, C, G, L, M$). Assume that east is positive in the $x_1$ direction, and that north is positive in the $x_2$ direction.

Throughout this problem, you should write a vector representing a translation from stop "A" to stop "B" as $\mathbf{v}_{AB}$ for example:

$$\mathbf{v}_{LC} = \begin{bmatrix} -3 \\ -8 \end{bmatrix}$$

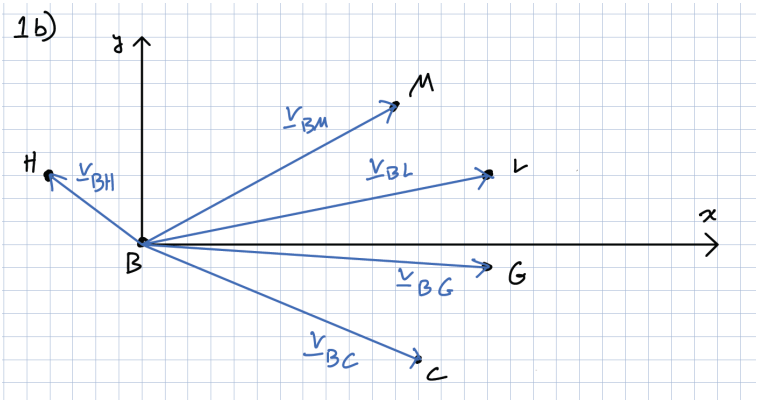
Notice that $\mathbf{v}_{AB} = -\mathbf{v}_{BA}$.

(a) Calculate the position vector of each stop of the five stops in relation to BU. To do so, write a vector equation that uses the surveyor's measurements. For example, your answer for Lechmere is named $\mathbf{v}_{BL}$ and is:

$$\mathbf{v}_{BL} = \mathbf{v}_{BC} + \mathbf{v}_{CL} = \begin{bmatrix} 12 \\ -5 \end{bmatrix} + \begin{bmatrix} 3 \\ 8 \end{bmatrix} = \begin{bmatrix} 15 \\ 3 \end{bmatrix}$$

(b) On a piece of graph paper, or your tablet with a grid background, draw all of these vectors. Label all stops with their letter and their $(x, y)$ coordinates.

You can compare against my result here (which needs to have the coordinates filled in):




(c) Calculate the vectors representing the T track from one stop to the next, following the Green Line's ordering of stops from left to right (starting in the west, then going east/north).

(d) On a piece of graph paper, or your tablet with a grid background, draw a map of this section of the green line. To do this, draw and label all the vectors you calculated in part (c).

(e) How long is this section of the green line, from west to east? Calculate this total length as the sum of lengths of each track section, $\ell_{HB} + \ell_{BC} + \ldots$

Use the "norm" symbol, $||\mathbf{v}_{AB}||$, which you might remember is defined per the Pythagorean Theorem as:

$$\ell = ||\mathbf{v}|| = \sqrt{\mathbf{v} \cdot \mathbf{v}} = \sqrt{\mathbf{v}^\top \mathbf{v}}$$

Chapter 6.1 may help if this is surprising.

We suggest writing the "length" symbol like we do, as "squiggly letter ell," to distinguish it versus the number "one."

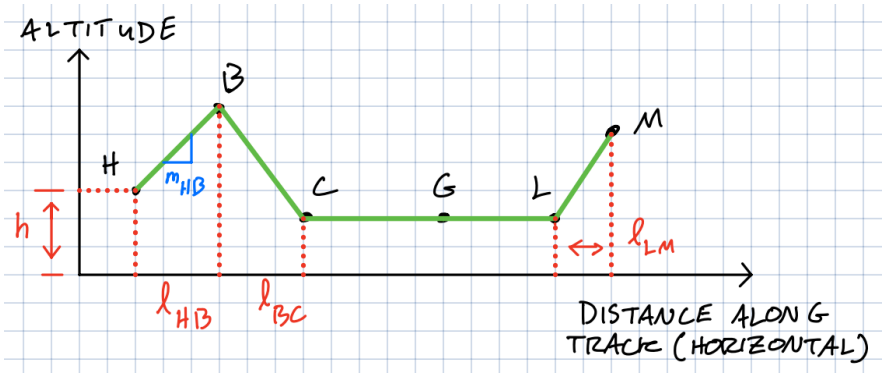
2. Interpreting a vector as a list of numbers

Sometimes, a vector is just a list of numbers, not meant to be plotted in 2D or 3D.

For this problem, each section of this Green Line track has a constant slope (change in altitude above sea level, per mile). The MBTA surveyors have measured these slopes and given them to you as a vector:

$$\mathbf{a} = \begin{bmatrix} m_{HB} \\ m_{BC} \\ m_{CG} \\ m_{GL} \\ m_{LM} \end{bmatrix} = \begin{bmatrix} 0.2 \\ -0.1 \\ 0 \\ 0 \\ 0.1 \end{bmatrix}$$

Here's a visualization, not to scale:




(a) Write the lengths you calculated of each section on the map of the Green Line as a vector:

$$\mathbf{b} = \begin{bmatrix} \ell_{HB} \\ \ell_{BC} \\ \ell_{CG} \\ \ell_{GL} \\ \ell_{LM} \end{bmatrix} =$$

(b) What is the total altitude change between Harvard Ave to Medford? Calculate using a vector operation. *Hint:* remember that slope is rise over run. What calculation would you do that multiplies these track distances by their corresponding slopes?

3. Matrix-vector multiplication

(a) Here is a matrix and a vector:

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}, \qquad \mathbf{v}_1 = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

Multiply to get the vector $\mathbf{v}_2 = A\mathbf{v}_1$.

Can you multiply $\mathbf{v}_3 = \mathbf{v}_1 A$? Why or why not? Answer using *rows and columns*. If possible, calculate $\mathbf{v}_3$.

(b) The MBTA is trying to protect the Green Line against climate change. Each station needs to be a certain height above sea level, but not too high since water still has to drain properly along the tracks when it rains.

Government Center is too low of an altitude. We have a few options for adjusting the slope of the track on the sections between $H, B, C$, and $G$. Here are four alternatives from Q2, for the sections of the track with slopes $m_{HB}$, $m_{BC}$, and $m_{CG}$.

$$\mathbf{a}_1 = \begin{bmatrix} 0.1 \\ -0.1 \\ 0.1 \end{bmatrix}, \qquad \mathbf{a}_2 = \begin{bmatrix} 0 \\ -0.01 \\ 0 \end{bmatrix}, \qquad \mathbf{a}_3 = \begin{bmatrix} 0.5 \\ -0.2 \\ 0.1 \end{bmatrix}, \qquad \mathbf{a}_4 = \begin{bmatrix} -0.01 \\ 0 \\ -0.01 \end{bmatrix}$$

Using the track lengths as in Q2, write a matrix $A$ and a vector $\mathbf{x}$ so that multiplying $A\mathbf{x}$ will calculate the altitude of Government Center for each option.
*Hint,* the vector $\mathbf{x}$ should be distances along the track (map distances) of the style in Q2.

$$A = ? \qquad \mathbf{x} = ?$$

Finally, calculate $A\mathbf{x}$.

(c) Assume that Harvard Ave is at $h = 0.15$ mile above sea level.

Which of these options put Government Center above sea level (greater than zero) but no higher than Harvard Ave?

4. Matrix-Matrix multiplication

(a) Here are two matrices:

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 4 & -3 \\ 5 & 6 \end{bmatrix}$$

Mutiply to get the matrix $C_1 = AB$.

Can you multiply $C_2 = BA$? Why or why not? Answer using *rows and columns*. If it is possible, calculate $C_2$.

(b) Now, the MBTA is considering changing the length of the tracks in Q3, in addition to changing their slopes.
Here are three possible sets of track lengths between Harvard Ave and Government Center:

$$\mathbf{b}_1 = \begin{bmatrix} 5 \\ 10 \\ 8 \end{bmatrix}, \qquad \mathbf{b}_2 = \begin{bmatrix} 6 \\ 11 \\ 6 \end{bmatrix}, \qquad \mathbf{b}_3 = \begin{bmatrix} 8 \\ 6 \\ 13 \end{bmatrix}$$

Write two matrices, $A$ and $B$, so that calculating $AB$ will give you the altitude change between Harvard Ave and Government Center for each combination of track configurations (both slope changes and length changes). Use the same slope options as Q3.

$$A = ? \qquad\qquad B = ?$$

(c) Open the MATLAB file `ps1_q4.m`.

Type in your matrices $A$ and $B$ and calculate the result $C = AB$.
Publish your code using the guide we posted on Slack, and attach the PDF to your submission of this homework. **BE SURE THAT YOUR CALCULATION RESULT PRINTS OUT**.

Then, knowing the absolute elevation of Harvard Ave from the previous question, calculate the elevation above sea level of Government Center in each case.

*Hint:* MATLAB requires careful attention to matrix and vector dimensions, but sometimes lets you take shortcuts. For example, `A*B+h` is a valid calculation in MATLAB when `h` is a scalar.

Which of these options put Government Center above sea level (greater than zero) but no higher than Harvard Ave?

5. Bonus Points Question

Use a generative AI of your choice (ChatGPT, Claude, etc.) to check your answers to this homework. Compare and contrast your solution versus the AI's answer. What did it interpret correctly, and what did it not interpret correctly?