P.S. 5

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1)

A)

$$\begin{bmatrix} 1 & 3 \\ 2 & -5 \end{bmatrix} \xrightarrow{R_2=R_2-2R_1} \begin{bmatrix} 1 & 3 \\ 0 & -11 \end{bmatrix}$$

Linearly Independent, as number of columns is equal to number of pivots

B)

$$\begin{bmatrix} 1 & -2 \\ -3 & 6 \end{bmatrix} \xrightarrow{R_2=R_2+3R_1} \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}$$

Linearly Dependent because $\vec{v} = \vec{u} \cdot -2$

C)

$$\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ -3 & -6 \end{bmatrix} \xrightarrow{\substack{R_2 = R_2 - 2R_1 \\ R_3 = R_3 + 3R_1}} \begin{bmatrix} 1 & 4 \\ 0 & -3 \\ 0 & 6 \end{bmatrix}$$

Linearly Independent because number of columns is equal to number of pivots (and therefore only solution to $A\vec{x} = \vec{0}$ is the trivial one)

D)

$$\begin{bmatrix} 2 & 3 \\ 4 & 6 \\ -8 & -12 \end{bmatrix} \xrightarrow{\substack{R_2 = R_2 - 2R_1 \\ R_3 = R_3 + 4R_1}} \begin{bmatrix} 2 & 3 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Linearly Dependent because $\vec{v} = \vec{u} \cdot \frac{3}{2}$

2)

A)

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 5 & 5 \\ 5 & 1 & 2 \end{bmatrix} \xrightarrow{\substack{R_2 = R_2 - 2R_1 \\ R_3 = R_3 - 5R_1}} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 0 & -9 & -3 \end{bmatrix} \xrightarrow{R_3=R_3+9R_2} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 24 \end{bmatrix}$$

Linearly Independent because number of columns is equal to number of pivots (and therefore only solution to $A\vec{x} = \vec{0}$ is the trivial one)

B)

$$\begin{bmatrix}1&0&1\\2&0&5\\3&0&6\end{bmatrix} \xrightarrow[R_3=R_3-3R_1]{R_2=R_2-2R_1} \begin{bmatrix}1&0&1\\0&0&3\\0&0&3\end{bmatrix} \xrightarrow{R_3=R_3-R_2} \begin{bmatrix}1&0&1\\0&0&3\\0&0&0\end{bmatrix}$$

Linearly Dependent due to the presence of the column $\vec{0}$, which immediately makes the set dependent, since now the number of pivots cannot exceed 2, 1 less than number of columns

C)

$$\begin{bmatrix}1&1&2&3\\2&3&5&1\\5&1&7&4\end{bmatrix} \xrightarrow[R_3=R_3-5R_1]{R_2=R_2-2R_1} \begin{bmatrix}1&1&2&3\\0&1&1&-5\\0&-4&-3&-11\end{bmatrix} \xrightarrow{R_3=R_3+4R_2} \begin{bmatrix}1&1&2&3\\0&1&1&-5\\0&0&1&-31\end{bmatrix}$$

Linearly Dependent because of free variable in column 4

3)

$$C^{-1} = \frac{1}{-2}\begin{bmatrix}5&-3\\-4&2\end{bmatrix} = \begin{bmatrix}-\frac{5}{2}&\frac{3}{2}\\2&-1\end{bmatrix}$$

$$\det(D) = 0 \implies D^{-1}\, DNE$$

4)

$$A \times B = \begin{bmatrix}1&0&2\\2&-1&3\\4&1&8\end{bmatrix} \times \begin{bmatrix}-11&2&2\\-4&0&1\\6&-1&-1\end{bmatrix} = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

5)

$$\left[\begin{array}{cccccc}1&0&2&1&0&0\\2&-1&3&0&1&0\\4&1&8&0&0&1\end{array}\right] \xrightarrow[R_3=R_3-4R_1]{R_2=R_2-2R_1} \left[\begin{array}{cccccc}1&0&2&1&0&0\\0&-1&-1&-2&1&0\\0&1&0&-4&0&1\end{array}\right] \xrightarrow{R_3=R_3+R_2} \left[\begin{array}{cccccc}1&0&2&1&0&0\\0&-1&-1&-2&1&0\\0&0&-1&-6&1&1\end{array}\right]$$

$$\xrightarrow[R_3=-R_3-5R_1]{R_2=-R_2} \left[\begin{array}{cccccc}1&0&2&1&0&0\\0&1&1&2&-1&0\\0&0&1&6&-1&-1\end{array}\right] \xrightarrow[R_2=R_2-R_3]{R_1=R_1-2R_3} \left[\begin{array}{cccccc}1&0&0&-11&2&2\\0&1&0&-4&0&1\\0&0&1&6&-1&-1\end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix}-11&2&2\\-4&0&1\\6&-1&-1\end{bmatrix}$$