

Posted JAN 23

[1] How many ways are there to form a three letter sequence using the letters a,b,c,d,e,f

(a) without repetition of letters ?

(b) with repetition of letters allowed ?

(c) without repetition and containing the letter "e" once ?

Solution;

(a) $6 \cdot 5 \cdot 4 = 120$

(b) 6^3

(c) $3 \cdot 5 \cdot 4 = 60$

[2] how many non empty different collections can be formed from five identical apples and eight identical oranges ?

Solution:

$5+1 = 6$ (there are 6 possible values for apples and 9 possible values for oranges. Subtract the one collection which has neither apples nor oranges since problem asked for non empty collection)

[3] How many different sequences of Heads and Tails are possible if a coin is flipped 100 times ??

Solution ; 2^{100}

- [4] How many ways are there to pick 2 different cards from a standard deck of 52 cards such that ;
- (a) the first card is an Ace and second card is NOT a Queen ?
 - (b) the first card is a spade and second card is NOT a Queen ? (beware the Queen of Spades)

Solution;

- (a) $4 \cdot 47 = 188$
- (b) $(1 \cdot 48) + (12 \cdot 47) = 612$ (break up into two mutually exclusive events. First card picked is a Queen of spade or Not)

[5]

- (a) how many 4 letter words (sequence of letters with repetition) are there in which the first and last letters are vowels (ie: a,e,i,o,u) ?
- (b) what if vowels appear only (if at all) in the first and last letter ?

Solution;

- (a) $5^2 \cdot 26^2$
- (b) $26^2 \cdot 21^2$ (the middle two letters must not be vowels so 21^2 while first and last letters can be anything)

[6] how many different 8 digit binary sequences are there with six 1's and two 0's ??

Solution

$$\binom{8}{6}$$

[7] A coin is flipped 15 times. How many different sequences of 15 Heads and Tails are possible in which the third Tail appears on the twelfth flip ?

solution;

$\binom{11}{2} 2^3 = 440$ choose 2 Tails in the first 11 flips followed by Tail on 12th flip. Then remainder can be anything which is 2^3

[8] there are 10 white balls and 5 red balls in a box. How many ways can you select 3 white and 2 red balls ?
What is the probability that this will occur ?

Solution;

$\binom{10}{3} \binom{5}{2}$, probability is $\frac{\binom{10}{3} \binom{5}{2}}{\binom{15}{5}}$

[9] How many ways are there to arrange the letters REPLETE with no consecutive E's ? (ie; cannot have any E's together) what is the probability that this will occur ?

Solution;

$\binom{5}{3} 4!$ consider the letters - R - P - L - T - wherever there is a dash you can insert letter E. This will

Result in "word" that does NOT have any consecutive E's. Once you have inserted the E's, the other letters RPLT can be arranged in 4! Ways. Probability is just this divided by total number of ways to arrange 7 letters

of which there are 3 of a kind. So denominator is $\frac{7!}{3!}$ so probability is $\frac{\binom{5}{3} 4!}{\frac{7!}{3!}}$