

$$\text{scal}_{\mathbf{w}} \mathbf{u} = |\mathbf{u}| \cdot \cos \theta = \frac{\mathbf{w} \cdot \mathbf{u}}{|\mathbf{w}|}$$

$$\text{proj}_{\mathbf{w}} \mathbf{u} = \left(\frac{\mathbf{w} \cdot \mathbf{u}}{\mathbf{w} \cdot \mathbf{w}} \right) \mathbf{w}$$

$$V_{\text{parallelepiped}} = \vec{u} \cdot (\vec{v} \times \vec{w})$$

$$S(t) = \int_a^t |\vec{r}'(s)| ds$$

$$\kappa(t) = \frac{|\vec{T}'(t)|}{|\vec{r}'(t)|}$$

$$\kappa(t) = \frac{|\vec{a} \times \vec{v}|}{|\vec{v}|^3}, \quad \kappa(t) = \frac{1}{|\vec{v}|} \left| \frac{d\vec{T}}{dt} \right|, \quad \kappa(t) = \left| \frac{d\vec{T}}{ds} \right|$$

$$|\vec{a}_{\perp}(t)| = \frac{|\vec{v}(t) \times \vec{a}(t)|}{|\vec{v}(t)|} = |\vec{v}(t)|^2 \cdot \kappa(t)$$

$$|\vec{a}_{\parallel}(t)| = \frac{d}{dt} |\vec{v}(t)| = \frac{d^2}{dt^2} \int_0^t |\vec{v}(p)| dp = \frac{d^2}{dt^2} s(t)$$

$$\vec{a}(t) = |\vec{a}_{\perp}(t)| \cdot \frac{\vec{T}'(t)}{|\vec{T}'(t)|} + |\vec{a}_{\parallel}(t)| \cdot \frac{\vec{r}'(t)}{|\vec{r}'(t)|}$$

$$\vec{B} = \vec{T} \times \vec{N} = \frac{\vec{v} \times \vec{a}}{|\vec{v} \times \vec{a}|}$$

$$\vec{T} = \frac{\vec{r}'(t)}{|\vec{r}'(t)|}$$

$$\vec{N} = \frac{\vec{T}'(t)}{|\vec{T}'(t)|}$$

$$\frac{d\vec{B}}{dt} = -\tau \vec{N}$$

$$\tau = \frac{\vec{B}'(t) \cdot \vec{N}(t)}{|\vec{r}'(t)|}$$

$$\tau = \frac{(\vec{v} \times \vec{a}) \cdot \vec{a}'}{|\vec{v} \times \vec{a}|^2}$$

$$H(x, y) = \begin{bmatrix} f_{xx}(x, y) & f_{xy}(x, y) \\ f_{yx}(x, y) & f_{yy}(x, y) \end{bmatrix}$$

1. If $\det(H) > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a *local minimum*.
2. If $\det(H) > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a *local maximum*.
3. If $\det(H) < 0$, then (a, b) is a *saddle point*.
4. If $\det(H) = 0$, the test is *inconclusive*.

Curve/Surface	Equation
$y = f(x)$	$y = f'(a)(x - a) + f(a)$
$z = f(x, y)$	$z = c + \nabla f(a, b) \cdot \langle x - a, y - b \rangle$
$F(x, y, z) = D$	$\nabla F(a, b, c) \cdot \langle x - a, y - b, z - c \rangle = 0$

$$D_{\mathbf{u}} f(a, b) = \langle f_x(a, b), f_y(a, b) \rangle \cdot \mathbf{u}$$

$$\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{dy}{dx} = \frac{f'(\theta) \sin \theta + f(\theta) \cos \theta}{f'(\theta) \cos \theta - f(\theta) \sin \theta}$$

$$A = \int_{\alpha}^{\beta} \frac{1}{2} f(\theta)^2 d\theta$$

$$S_r = \int_{\theta_1}^{\theta_2} \sqrt{(f'(\theta))^2 + (f(\theta))^2} d\theta$$

$$\det(J) = \begin{vmatrix} \frac{\partial g}{\partial u} & \frac{\partial g}{\partial v} \\ \frac{\partial h}{\partial u} & \frac{\partial h}{\partial v} \end{vmatrix} = \frac{\partial g}{\partial u} \cdot \frac{\partial h}{\partial v} - \frac{\partial g}{\partial v} \cdot \frac{\partial h}{\partial u}$$

$$\int_C f(x, y) ds = \int_a^b f(\mathbf{r}(t)) |\mathbf{r}'(t)| dt$$

$$\begin{aligned} \int_C \mathbf{F} \cdot d\mathbf{r} &= \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) dt = \\ \int_a^b \left[P(x(t), y(t)) \frac{dx}{dt} + Q(x(t), y(t)) \frac{dy}{dt} \right] dt &= \\ \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA \end{aligned}$$

To determine whether a vector field $\mathbf{F} = (P(x, y), Q(x, y))$ is a gradient field: $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$.

$$\phi(x, y) = \int P(x, y) dx + h(y)$$

$$\text{curl}(\mathbf{F}) = \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$$

$$\text{Area}(R) = \int_C x dy = - \int_C y dx = \frac{1}{2} \int_C (x dy - y dx)$$

$$\int_C \mathbf{F} \cdot \mathbf{n} ds = \int_a^b [f(x(t), y(t))y'(t) - g(x(t), y(t))x'(t)] dt =$$

$$\int_C f dy - g dx$$

$$\int_C \mathbf{F} \cdot \mathbf{n} ds = \iint_R \text{div}(\mathbf{F}) dA$$

$$\text{div}(\mathbf{F}) = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y}$$