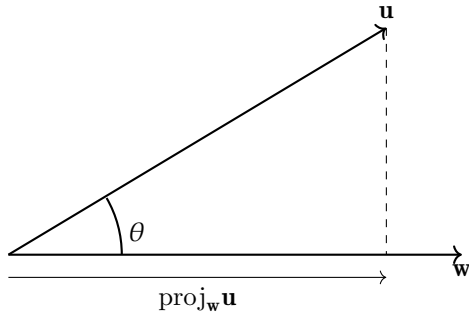


CASMA225 CHEATSHEET

Dot Product Value	Interpretation	Relationship Between Vectors
$\vec{v} \cdot \vec{w} = 0$	$\cos \theta = 0$	Vectors are perpendicular (orthogonal), $\theta = 90^\circ$
$\vec{v} \cdot \vec{w} > 0$	$0 < \theta < 90^\circ$	Vectors form an acute angle , pointing in the same general direction
$\vec{v} \cdot \vec{w} < 0$	$90^\circ < \theta < 180^\circ$	Vectors form an obtuse angle , pointing in opposite general directions
$\vec{v} \cdot \vec{w} = \vec{v} \vec{w} $	$\cos \theta = 1$	Vectors are parallel and point in the same direction , $\theta = 0^\circ$
$\vec{v} \cdot \vec{w} = - \vec{v} \vec{w} $	$\cos \theta = -1$	Vectors are parallel but point in opposite directions , $\theta = 180^\circ$

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = (a_2b_3 - a_3b_2)\hat{i} - (a_1b_3 - a_3b_1)\hat{j} + (a_1b_2 - a_2b_1)\hat{k}$$



$$\text{scal}_{\mathbf{w}} \mathbf{u} = |\mathbf{u}| \cdot \cos \theta = \frac{\mathbf{w} \cdot \mathbf{u}}{|\mathbf{w}|}$$

$$\text{proj}_{\mathbf{w}} \mathbf{u} = |\mathbf{u}| \cos \theta \left(\frac{\mathbf{w}}{|\mathbf{w}|} \right)$$

$$\text{proj}_{\mathbf{w}} \mathbf{u} = \left(\frac{\mathbf{w} \cdot \mathbf{u}}{\mathbf{w} \cdot \mathbf{w}} \right) \mathbf{w}$$

$$S(t) = \int_a^t |\vec{r}'(s)| \, ds.$$

$$\frac{d}{dt} \vec{r}(S^{-1}(t)) = \frac{\vec{r}'(S^{-1}(t))}{|\vec{r}'(S^{-1}(t))|}.$$

$$\kappa(t) = \frac{|\vec{T}'(t)|}{|\vec{r}'(t)|}$$

$$\kappa(t) = \frac{|\vec{a} \times \vec{v}|}{|\vec{v}|^3}, \quad \kappa(t) = \frac{1}{|\vec{v}|} \left| \frac{d\vec{T}}{dt} \right|, \quad \kappa(t) = \left| \frac{d\vec{T}}{ds} \right|$$

$$|\vec{a}_{\perp}(t)| = \frac{|\vec{v}(t) \times \vec{a}(t)|}{|\vec{v}(t)|} = |\vec{v}(t)|^2 \cdot \kappa(t)$$

$$|\vec{a}_{\parallel}(t)| = \frac{d}{dt} |\vec{v}(t)| = \frac{d^2}{dt^2} \int_0^t |\vec{v}(p)| dp = \frac{d^2}{dt^2} s(t)$$

$$\vec{a}(t) = |\vec{a}_{\perp}(t)| \cdot \frac{\vec{T}'(t)}{|\vec{T}'(t)|} + |\vec{a}_{\parallel}(t)| \cdot \frac{\vec{r}'(t)}{|\vec{r}'(t)|}$$

$$\vec{B} = \vec{T} \times \vec{N} = \frac{\vec{v} \times \vec{a}}{|\vec{v} \times \vec{a}|}$$

$$\vec{T} = \frac{\vec{r}'(t)}{|\vec{r}'(t)|}$$

$$\vec{N} = \frac{\vec{T}'(t)}{|\vec{T}'(t)|}$$

$$\frac{d\vec{B}}{dt} = -\tau \vec{N}$$

$$\tau = \frac{\vec{B}'(t) \cdot \vec{N}(t)}{|\vec{r}'(t)|}$$

$$\vec{B} = \frac{\vec{v} \times \vec{a}}{|\vec{v} \times \vec{a}|}$$

$$\tau = \frac{(\vec{v} \times \vec{a}) \cdot \vec{a}'}{|\vec{v} \times \vec{a}|^2}$$

$$\det(M) = \det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

$$H(x, y) = \begin{bmatrix} f_{xx}(x, y) & f_{xy}(x, y) \\ f_{yx}(x, y) & f_{yy}(x, y) \end{bmatrix}$$

1. If $\det(H) > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a *local minimum*.
2. If $\det(H) > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a *local maximum*.
3. If $\det(H) < 0$, then (a, b) is a *saddle point*.
4. If $\det(H) = 0$, the test is *inconclusive*.

Curve/Surface	Equation
$y = f(x)$	$y = f'(a)(x - a) + f(a)$
$z = f(x, y)$	$z = c + \nabla f(a, b) \cdot \langle x - a, y - b \rangle$
$F(x, y, z) = D$	$\nabla F(a, b, c) \cdot \langle x - a, y - b, z - c \rangle = 0$