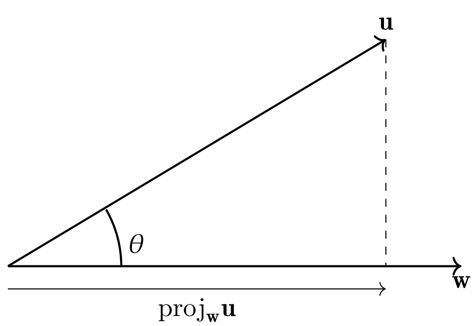
CASMA225 CHEATSHEET

Dot Product Value	Interpretation	Relationship Between Vectors
$\vec{v} \cdot \vec{w} = 0$	$\cos\theta = 0$	Vectors are **perpendicular** (orthogonal), $\theta = 90^\circ$
$\vec{v} \cdot \vec{w} > 0$	$0 < \theta < 90^\circ$	Vectors form an **acute angle**, pointing in the same general direction
$\vec{v} \cdot \vec{w} < 0$	$90^\circ < \theta < 180^\circ$	Vectors form an **obtuse angle**, pointing in opposite general directions
$\vec{v} \cdot \vec{w} = \|\vec{v}\|\|\vec{w}\|$	$\cos\theta = 1$	Vectors are **parallel** and point in the **same direction**, $\theta = 0^\circ$
$\vec{v} \cdot \vec{w} = -\|\vec{v}\|\|\vec{w}\|$	$\cos\theta = -1$	Vectors are **parallel** but point in **opposite directions**, $\theta = 180^\circ$

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = (a_2b_3 - a_3b_2)\hat{\imath} - (a_1b_3 - a_3b_1)\hat{\jmath} + (a_1b_2 - a_2b_1)\hat{k}$$



$$\text{scal}_{\mathbf{w}}\mathbf{u} = |\mathbf{u}| \cdot \cos\theta = \frac{\mathbf{w} \cdot \mathbf{u}}{|\mathbf{w}|}$$

$$\text{proj}_{\mathbf{w}}\mathbf{u} = |\mathbf{u}| \cos\theta \left(\frac{\mathbf{w}}{|\mathbf{w}|}\right)$$

$$\text{proj}_{\mathbf{w}}\mathbf{u} = \left(\frac{\mathbf{w} \cdot \mathbf{u}}{\mathbf{w} \cdot \mathbf{w}}\right)\mathbf{w}$$

$$S(t) = \int_a^t \left|\vec{r}'(s)\right| ds.$$

$$\frac{d}{dt}\vec{r}(S^{-1}(t)) = \frac{\vec{r}'(S^{-1}(t))}{\left|\vec{r}'(S^{-1}(t))\right|}.$$

$$\kappa(t) = \frac{|\vec{T}'(t)|}{|\vec{r}'(t)|}$$

$$\kappa(t) = \frac{|\vec{a} \times \vec{v}|}{|\vec{v}|^3}, \quad \kappa(t) = \frac{1}{|\vec{v}|}\left|\frac{d\vec{T}}{dt}\right|, \quad \kappa(t) = |\frac{d\vec{T}}{ds}|$$

$$|\vec{a}_\perp(t)| = \frac{|\vec{v}(t) \times \vec{a}(t)|}{|\vec{v}(t)|} = |\vec{v}(t)|^2 \cdot \kappa(t)$$

$$|\vec{a}_\parallel(t)| = \frac{d}{dt}|\vec{v}(t)| = \frac{d^2}{dt^2}\int_0^t |\vec{v}(p)|dp = \frac{d^2}{dt^2}s(t)$$

$$\vec{a}(t) = |\vec{a}_\perp(t)| \cdot \frac{\vec{T}'(t)}{|\vec{T}'(t)|} + |\vec{a}_\parallel(t)| \cdot \frac{\vec{r}'(t)}{|\vec{r}'(t)|}$$

$$\vec{B} = \vec{T} \times \vec{N} = \frac{\vec{v} \times \vec{a}}{|\vec{v} \times \vec{a}|}$$

$$\vec{T} = \frac{\vec{r}'(t)}{|\vec{r}'(t)|}$$

$$\vec{N} = \frac{\vec{T}'(t)}{|\vec{T}'(t)|}$$

$$\frac{d\vec{B}}{dt} = -\tau\vec{N}$$

$$\tau = \frac{\vec{B}'(t) \cdot \vec{N}(t)}{|\vec{r}'(t)|}$$

$$\vec{B} = \frac{\vec{v} \times \vec{a}}{|\vec{v} \times \vec{a}|}$$

$$\tau = \frac{(\vec{v} \times \vec{a}) \cdot \vec{a}'}{|\vec{v} \times \vec{a}|^2}$$

$$\det(M) = \det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

$$H(x,y) = \begin{bmatrix} f_{xx}(x,y) & f_{xy}(x,y) \\ f_{yx}(x,y) & f_{yy}(x,y) \end{bmatrix}$$

1. If $\det(H) > 0$ and $f_{xx}(a,b) > 0$, then $f(a,b)$ is a *local minimum.*
2. If $\det(H) > 0$ and $f_{xx}(a,b) < 0$, then $f(a,b)$ is a *local maximum.*
3. If $\det(H) < 0$, then (a,b) is a *saddle point.*
4. If $\det(H) = 0$, the test is *inconclusive.*

Curve/Surface	**Equation**
$y = f(x)$	$y = f'(a)(x-a) + f(a)$
$z = f(x,y)$	$z = c + \nabla f(a,b) \cdot \langle x-a, y-b \rangle$
$F(x,y,z) = D$	$\nabla F(a,b,c) \cdot \langle x-a, y-b, z-c \rangle = 0$

Suppose $D = \{(x,y,z) : a \leqslant x \leqslant b,\ g(x) \leqslant y \leqslant h(x),\ G(x,y) \leqslant z \leqslant H(x,y)\}$, where the bounds of y depend on x and the bounds of z depend on x and y.

Then

$$V = \iiint_D f(x,y,z)\, dV = \int_a^b \int_{g(x)}^{h(x)} \int_{G(x,y)}^{H(x,y)} f(x,y,z)\, dz\, dy\, dx$$