Temporary Doc
Calc 3

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Contents

Chapter 1

Vector Valued Functions $f : \mathbb{R} \longrightarrow \mathbb{R}^n$

Example 1.0.1 (Problem with Multiple Surfaces)

The solid common to the cylinders bounded by $z = \sin x$ and $z = \sin y$ over the region

$$R = \{(x, y) : 0 \leqslant x \leqslant \pi,\ 0 \leqslant y \leqslant \pi\}.$$

We define the domain D as

$$D = \{(x, y, z) : (x, y) \in R,\ 0 \leqslant z \leqslant \min(\sin x, \sin y)\}.$$

To solve this problem, we will:

1. Consider one-fourth of the volume under the intersection of the two cylinders, taking advantage of symmetry.
2. Identify the bounding conditions for z in each quadrant.
3. Set up and compute the triple integral over D to find the volume.

Given the symmetry, we restrict our analysis to the region

$$0 \leqslant x \leqslant \frac{\pi}{2}, \quad 0 \leqslant y \leqslant \frac{\pi}{2}.$$

In this quadrant, z is bounded by $\min(\sin x, \sin y)$, which means $z \leqslant \sin y$ in the region R_1, where $\sin y \leqslant \sin x$.
For region R_2 (where $\sin x \leqslant \sin y$), we have $z \leqslant \sin x$. By observing symmetry, the volume in each quadrant contributes equally, so we can calculate the volume in this restricted region and multiply by 4. The bounds for x and y are:

$$0 \leqslant y \leqslant \pi, \quad y \leqslant x \leqslant \pi - y.$$

Thus, we set up the volume integral as follows:

$$V = 4 \int_0^{\pi/2} \int_0^{\sin y} \int_y^{\pi - y} dx\, dy\, dz.$$

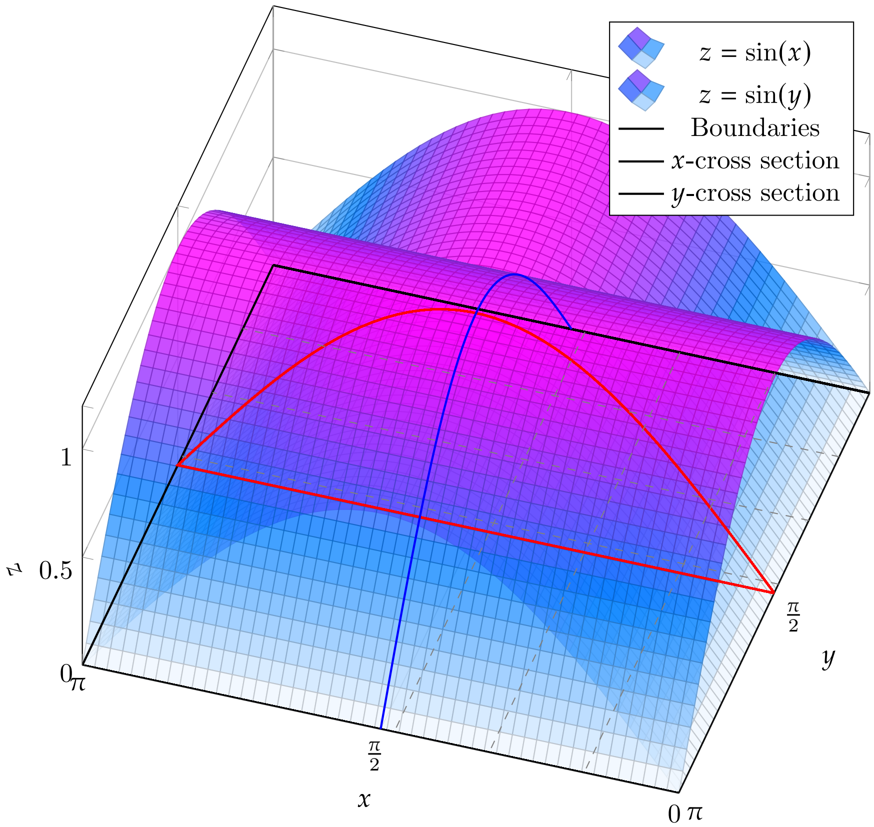
Evaluating the integral:

$$V = 4 \int_0^{\pi/2} \int_0^{\sin y} \left(\int_y^{\pi - y} dx \right) dy\, dz.$$

Upon computation, this integral yields:

$$V = \pi - 2.$$

The key idea was to leverage the symmetry of the intersection region, focusing on one-fourth of the area and then scaling up by a factor of 4. By analyzing the geometry, we found that z was bounded by $\sin y$ in region R_1. From there, the triple integral was computed over x, y, and z to yield the final volume of the region.



$z = \sin(x)$
$z = \sin(y)$
Boundaries
x-cross section
y-cross section
z
1
0.5
0
π
$\frac{\pi}{2}$
x
0
π
$\frac{\pi}{2}$
y
0