MA193
Discrete Mathematics

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Contents

Chapter 1

Fundamental Principles of Counting

1.1 Counting with Repetitions

Note:-

These notes cover the basic counting principles (often called the "Rule of Sum" and the "Rule of Product"), along with a brief discussion of permutations and combinations, including the formula for permutations of multiset objects (i.e., repeated elements).

Definition 1.1.1: Rule of Sum ("OR")

If a certain task can be done in n ways and another independent task can be done in m ways, and these tasks are mutually exclusive, then there are $n + m$ ways to do *one* of the two tasks.

Definition 1.1.2: Rule of Product ("AND")

If a procedure can be broken into two consecutive steps such that the first step can be done in n ways and the second step can be done in m ways (independently of how the first step is done), then there are $n \times m$ ways to do the entire procedure.

Note:-

In many counting problems, we break a larger procedure into a series of smaller steps and then apply either the Rule of Sum or the Rule of Product (or both) as needed.

Definition 1.1.3: Arrangements of n Distinct Objects

If you want to arrange n distinct objects in a row (i.e., an ordered list), there are $n!$ ways to do so. Order matters here, and this number is referred to as the number of permutations of n distinct items.

Definition 1.1.4: Permutations of Multisets

Suppose we have n total objects, but they are not all distinct. Instead, let there be n_1 objects of type 1, n_2 objects of type 2, …, and n_k objects of type k. Clearly

$$n_1 + n_2 + \cdots + n_k = n.$$

Then the number of distinct ways to arrange all n objects is

$$\frac{n!}{n_1!\, n_2! \ldots n_k!}.$$

Example 1.1.1 (Examples of Counting with Repetitions)

1. **ABCD:** All letters are distinct, so the number of ways to arrange A, B, C, D is
$$4! = 24.$$
2. **AABC:** Here we have 4 total letters, with A repeated twice. The number of distinct arrangements is
$$\frac{4!}{2!} = \frac{24}{2} = 12.$$
3. **AABB:** Now we have 2 A's and 2 B's (4 letters total). The number of arrangements is
$$\frac{4!}{2!\,2!} = \frac{24}{2 \times 2} = 6.$$
4. **SUCCESS:** The word "SUCCESS" has 7 letters total: 3 S's, 2 C's, 1 U, and 1 E. The number of distinct permutations is
$$\frac{7!}{3!\,2!\,1!\,1!} = \frac{5040}{(6)(2)(1)(1)} = 420.$$