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1/26/23

COEN 379 HW 2

1. For a graph with exactly n vertices, according to the proof we went over in class, the probability that the randomized min-cut algorithm succeeds is with probability 2/[n(n-1)], which will return one particular random min-cut possibility. Because this is a probability, and because the total possible amount of min-cuts are all independent and different from each other, we know that each of these probabilities will add up to a maximum value of 1. Thus, dividing this whole fraction from the max total probability of 1 (i.e. 1/(2/[n(n-1)]) = [n(n-1)]/2) gives us the max total number of different min-cutsets given n number of vertices.

2. For each comparison, there is ½ chance that it is a non-pivot swap. Thus:

   E[N] = 1/2E[C]

   A[1] and A[2] are always in the left partition when compared against a pivot A[j] when j > 3. If A[2] is picked as pivot before A[1], then A[1] will be the only element in the left partition and never is the pivot since rqs stops when A is of size 1. Thus, the probability that A[i] becomes the pivot and gets swapped is ½.

   The same logic above can be applied to the right partition, thus A[n] is involved in a pivot iff it's chosen as pivot before A[n-1], making the probability also ½.

   Lastly, A[i-1], A[i], and A[i+1] would also go into the same partition, but the last one selected as pivot will be in A of size 1 and thus will never be swapped as a pivot, making the probability ⅔.

   Let's define Pi as equal to 1 if A[i] gets swapped as a pivot, and 0 otherwise.

   E[P] = E[Sum(Pi)] = ½ + ⅔+ … + ⅔ + ½ = (2n-1)/2

   E[R] = E[P] because for each partition called, there is also a random swap.

   E[S] = E[N] +E[P] + E[R]
   = 1/2E[C] + 2 * (2n-1)/2
   = 1/2E[C] + 2n - 1