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COEN 239 HW 7

1. During a 1-hour observation interval, the name server of a distributed system received 5400 requests. The mean service time of these requests was observed to be half a second. If the queueing system behaves as a M/M/1 queue, calculate the utilization of this queue, the number of the jobs waiting in the queue, average number of jobs in the system, average waiting time in the queue, and the average response time of a job.
   ($\rho = 0.75,\ L_q = 2.25, W_q = 1.5, L = 3$, and $W = 2$)

   Customer Arrival Rate = 5400/4600 = 3/2 Customers/sec
   Mean Service Time = 1/u = 0.5 sec
   Utliization = 3/2(0.5) = 0.75
   Number of jobs waiting in queue = (0.75)^2/(1-0.75) = 0.5625/0.25 = 2.25
   Average # of jobs in system = 0.75/0.25 = 3
   Average waiting time in queue = 3 * 0.5 = 1.5 sec
   Average response time of job = 0.5/0.25 = 2 sec

2. Consider an M/M/4/4 queue. In this queue $\rho = 4$, and the average service time is 200 seconds. Compute the blocking probability of a job. Also compute the average number of busy servers, and the average response time of the customers who entered the system.
   ($P_B = 0.31068, L = 2.76, W = 200$)

   M/M/4/4; *l* = 4; Ts = 200 sec
   PB = B(4, *l*) = $\frac{l4/4!}{\sum_{n=0}^{4} \frac{ln}{n!}}$ = 0.31068
   Average response time of customers who entered system = 200 sec
   Average number of busy servers = 4(1-0.31068) = 4*0.69832 = 2.75728 Jobs

3. Consider an M/M/∞ queue. This queue has infinite number of servers. For this server, the arrival rate is 60 jobs per second, and the average service time is 50 milliseconds.

   In parts b) and c) write expression, do not perform numerical calculations.

   a) Compute the traffic intensity and utilization of the system. ($\rho = 3, U = (1 - e^{-\rho})$)

   b) Probability that there are exactly 5 jobs in the system. ($e^{-\rho}\rho^5/5!$)

   c) Probability that there are at most 5 jobs in the system.

   d) Mean number of jobs in the system.

   e) Variance of the number of jobs in the system.

   f) Mean response time

a) Traffic intensity = 60 jobs/(1/0.05 sec) = 3
   Utilization = 1 - e^(-3) = 0.95
b) Probability that there are exactly 5 jobs in the system = e^(-3) * 3^5/5!
c) Probability that there are <= 5 jobs = $\sum_{n=0}^{5}$ Pi; Pi = e^(-3) * 3^i/5!; i = 0,1,2…
d) Mean number of jobs in the system = 3
e) Variance of number of Jobs in the System = 3
f) Mean response time = 0.05 seconds

4. A storage system consists of a single disk drive. The average time to service an I/O request is 80 milliseconds. The I/O requests arrive to the disk system at a rate of 10 requests per second. Using an M/M/1 model for this system, determine the following:

   a) Average disk drive utilization.

   b) Probability of the system being idle.

   c) Probability of queueing

   d) Average number of jobs in the system.

   e) average number of jobs waiting in the buffer (queue).

   f) Mean response time

   g) Average waiting time in the buffer

a) U = 10/0.08 = 0.8
b) PI = 1 - 0.8 = 0.2
c) PQ = 0.8
d) L = 0.8/(1-0.8) = 0.8/0.2 = 4
e) Lq = 0.8^2/(1-0.8) = 3.2 sec
f) W = 0.08/0.2 = 0.4 sec
g) Wq = 4*0.08 = 0.32 sec

5. page 545, (Jain), Problem 31.3, parts a, b, c, d, e.

a) U = (60-10)/60 = ⅚
b) 3 = Ts/1-(⅚) = 1/2sec
c) Arrival rate of queries = ⅚ * 2 = 5/3 queries/sec
   Observation interval = 60 sec
   # queries completed = 60*(5/3) = 100
d) L = ⅚ / ⅙ = 5
e) $\sum_{n=11}^{infity}$ Pm = (⅚)^11