Homework 9: Decidability

CSCI 161

Due 9:00pm, Saturday, November 21, 2020

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Overview. This week's homework primarily concerns the decidability of languages. You will give (partial) classifications for six languages. The following gives additional per-problem guidance:

- The bonus question is an ungraded practice question. I'll present a solution in office hours next week upon request as an example of a decider algorithm involving creative and mechanical problem solving. Although the language is analogous, the solution does not directly influence the solution to Question 6.
- As with most decidability arguments, showing decidability of each of the languages in Questions 1-4 will involve giving an algorithm and arguing that it correctly accepts or rejects any possible input in finite time. Your algorithms can use procedures discussed in class – and in several cases this will probably be helpful! – but you should describe at a high level whatever algorithm you are calling on. You can probably rely on the correctness of your algorithms to speak for themselves, but a sentence or two describing why they work as designed may help with the clarity of your answer.
- Turing recognizability can be established by giving an algorithm that is guaranteed to accept all inputs in the language in finite time, and co-Turing-recognizability (Question 5) can be established by giving an algorithm that is guaranteed to reject all inputs not in the language in finite time.
- Undecidability is generally established by using an assumed decider for the language to construct a contradiction. The hint for Question 6 suggests what contradiction you may wish to construct.

As usual, edit this tex document and submit it with your compiled pdf for 5% extra credit.

Question 0. List all collaboration on this assignment.

Bonus Q. Show that $REV_{\text{DFA}} = \left\{\langle D\rangle \mid D \text{ is a DFA and } L(D) = \left\{w^{\mathcal{R}} \mid w \in L(D)\right\}\right\}$ is decidable.

Question 1. Consider the problem of determining whether a DFA and a regular expression are equivalent. Express this problem as a language EQ_{DR}, and show that it is decidable.

Question 2. Let $SUBSET_{\text{REX}} = \{\langle R, S\rangle \mid R, S \text{ are regular expressions and } L(R) \subseteq L(S)\}$. Show that $SUBSET_{\text{REX}}$ is decidable.

Question 3. Any regular language intersected with a context-free language is context-free. Use this fact to show that $IMB_{\text{DFA}} = \{\langle D\rangle \mid D \text{ is a DFA and } w \in L(D) \text{ for some } w \text{ with more 1's than 0's}\}$ is decidable.

Question 4. Let $A\epsilon_{\text{CFG}} = \{\langle G\rangle \mid G \text{ is a CFG and } \epsilon \in L(G)\}$. Show that $A\epsilon_{\text{CFG}}$ is decidable.

Question 5. Recall that $EQ_{\text{CFG}} = \{\langle G, H\rangle \mid G, H \text{ are CFGs and } L(G) = L(H)\}$. Show that EQ_{CFG} is co-Turing-recognizable.

Question 6. Show that $REV_{\text{TM}} = \left\{\langle M\rangle \mid M \text{ is a TM and } L(M) = \left\{w^{\mathcal{R}} \mid w \in L(M)\right\}\right\}$ is undecidable.

Hint: Show $A_{\text{TM}} \to REV_{\text{TM}}$.