Homework 10: The Asymmetry of Recognizability

CSCI 161
Due 9:00pm, December 3, 2020
Krehbiel

Overview. This last homework focuses on the differences between decidability and recognizability including questions about mapping reductions. Submit your edited copy of this text file with its compiled pdf for 5% extra credit. You may submit late only in accordance with the syllabus policy of 10% deduction per hour, but only up until 2:15 on the last day of classes (no extensions!) so we can discuss solutions for this and any other homework on our last review day.

Question 0. Who did you collaborate with on this homework? Optionally if you are willing to have me share your advice for students who take this course in the future, please let me know what was most and least helpful about studying with other students for this class this quarter.

Question 1. (6pts) Recall that an LBA is a Turing machine whose tape head never leaves the portion of the tape initially occupied by the input, and let $EQ_{\text{LBA}} = \{\langle M_1, M_2\rangle \mid M_1, M_2 \text{ are LBAs and } L(M_1) = L(M_2)\}$. Prove whether EQ_{LBA} is decidable, whether it is Turing-recognizable, and whether it is co-Turing-recognizable.

Question 2. (4pts) Recall that A_{TM} reduces to $REGULAR_{\text{TM}} = \{\langle M\rangle \mid M \text{ is a TM and } L(M) \text{ is regular}\}$. Prove whether $REGULAR_{\text{TM}}$ is Turing-recognizable and whether it is co-Turing-recognizable.

Question 3. (4pts) Let $J = \left\{w \mid w = 0x \text{ for some } x \in A_{\text{TM}} \text{ or } w = 1y \text{ for some } y \in \overline{A_{\text{TM}}}\right\}$. Prove that J is neither Turing-recognizable nor co-Turing-recognizable. For 2pts extra credit, also show $J \leq_m \overline{J}$.

Question 4. (4pts) Prove that a language A is Turing-recognizable if and only if $A \leq_m A_{\text{TM}}$.

Structuring your proof: Remember that a good strategy to prove an if and only if statement is to give two arguments for the two implications. Assuming A is recognizable, show there must be a mapping reduction. Assuming there is a mapping reduction, show that A must be recognizable. Use the definitions of recognizability and mapping reductions!

Brain teaser. Is there a nonregular language that can be map-reduced to a regular language? Can every nonregular language be map-reduced to a regular language?