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HW 9

0) I collaborated with Carlo Bilbao on this assignment.

1) The language EQ_{DR} = {<D,R> | D is a DFA and R is a regex w/ L(D) = L(R)} is decidable. According to Lemma 1.6, the language of any DFA is described by some regex. Thus, we can convert the regex into a DFA and then apply Theorem 4.5, which states that EQ_{DFA} is decidable, thus proving EQ_{DR} is also decidable.

2) The language $SUBSET_{REX}$ = {<R, S> | R, S are regular expressions and L(R) ⊆ L(S)} is decidable. We can convert both R and S into DFAs using Lemma 1.6. Because DFAs are closed under complement, we can then obtain the DFA S' from S. Then, we can compare the intersection of R and S' and create a DFA T from that. If any string lies within this intersection, then R is not a subset of S. Using Theorem 4.4, which states that E_{DFA} is decidable, we can plug in T, which will either be accepted or rejected for every input, thus proving $SUBSET_{REX}$ is also decidable.

3) The language IMB_{DFA} = {<D> | D is a DFA and w ∈ L(D) for some w with more 1's than 0's} is decidable. Assume there is a CFG that accepts w if it has more 1's than 0's. Then, using the fact that any regular language intersected with a CFL is context-free, by intersecting D and this CFG, IMB_{DFA} must also be context free. By proving that IMB_{DFA} is context-free, we can then apply Theorem 4.7, which states that A_{CFG} is decidable. Hence, IMB_{DFA} is also decidable.

4) The language $A_{\varepsilon CFG}$ = {<G> | G is a CFG and ε ∈ L(G)} is decidable. Convert G into CNF. Then, if any rule R where some variable V → ε exists, then accept the language. If not, then reject. Thus, $A_{\varepsilon CFG}$ is decidable.

5) The language EQ_{CFG} = {<G, H> | G, H are CFGs and L(G) = L(H)} is co-Turing-recognizable. Convert both G and H into CNF. Then, run CYK on both of them for any possible string, and for any if one of them accepts and the other rejects, then the languages are proven to be nonequivalent, proving EQ_{CFG} to be co-Turing-recognizable.

6) The language REV_{TM} = {<M> | M is a TM and L(M) = w^R | w $\in$ L(M)} is undecidable. Assume for contradiction that a TM exists that decides REV_{TM}. Thus, there is an acceptor for strings that are in this language, and a rejector for strings that are not in this language. This decides A_{TM}, but according to theorem 4.11, A_{TM} is undecidable, contradicting the decidability of REV_{TM}, proving it is not decidable by reduction.