Justin Li
Dr. Krehbiel
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HW 10

0) I collaborated with Carlo Bilbao on this assignment. I would recommend for students in the future when studying with others to make sure to rewatch lectures and review the notes and important concepts discussed in class, and to ask questions whenever there is confusion on particular concepts, as I found it very helpful to talk with my partner whenever we didn't get particular parts, and were mostly able to come out with a better understanding of the material.

1) Assume for contradiction that EQ_{LBA} is decidable. First, we design the following TM D with input <M, w> where M is an LBA. Next, design a TM E where L(M) is empty, in which case E doesn't accept anything. Then, run a TM H, which decides the EQ_{LBA} problem on input <D,E> such that E rejects all inputs. If H accepts, then D accepts. Else, if H rejects, then D rejects, meaning that one is not empty. By doing this, we created a decider for EQ_{LBA}. However, since we know that EQ_{LBA} is undecidable, this is a contradiction. Hence, EQ_{LBA} is undecidable, and rather is only co-recognizable.

2) $REGULAR_{TM}$ is not recognizable. To show this, we begin by creating a turing machine D as follows:
   D with input <M, w> where M is a TM, and w is an input, where L(M) is regular
   We can create a computable function f(M,w) such that we output TM D(x) where if ($x = 0^n1^n$) then we simulate M(w), and otherwise reject
   If (M,w) passes A_{TM}, then f(M,w) = D accepts $\{0^n1^n\}$, and if it doesn't pass A_{TM}, then f(M,w) = D accepts nothing. Thus, (M,w) doesn't pass A_{TM} if and only if f(M,w) passes $REGULAR_{TM}$.
   Hence, by proof by reduction, $REGULAR_{TM}$ is not recognizable, but is co-recognizable.

3) J is neither recognizable nor co-recognizable. First, we design the following TM D. On input (M,w), create a tape with 0 followed by <M,w>. Then, <M,w> passes A_{TM} if and only if the output of D is in J. By doing this, we create a mapping reduction of A_{TM} to J. Next, we design the following TM E. On input (M,w), create a tape with 1 followed by <M,w>. Then, <M,w> doesn't pass A_{TM} if and only if the output of E is in J, or <M,w> passes A_{TM} if and only if the output of E is in ~J. By

doing this, we create a mapping reduction of A_{TM} to ~J. Since A_{TM} mapping reduces to J, ~A_{TM} mapping reduces to ~J. Hence, J is non-co-recognizable because A_{TM} is recognizable. Additionally, since A_{TM} mapping reduces to ~J, ~A_{TM} mapping reduces to J. Hence, J is non-recognizable.

4) A language A is Turing-recognizable if and only if A <= $_m$ATM. First, if A <= $_m$ATM, then A is Turing-recognizable because A_{TM} is Turing recognizable. On the other side, if A is Turing recognizable, then there exists some TM D that recognizes A. This means that D receives some input w and accepts if w passes in D. Then, we design a TM where on input w, we create a tape with <D,w>. Then, <D,w> is in A_{TM} if and only if w is in A. Thus A mapping reduces to $A_{TM.}$ Hence, we proved that the language A is Turing-recognizable if and only if A <= $_m$ATM.