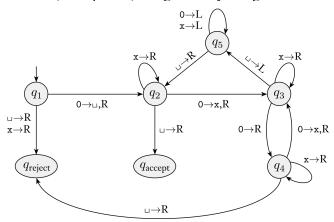
Due 9pm, Saturday, November 14, 2019

**Overview.** This week's homework concerns the mechanics of Turing machines discussed in Chapter 3. Submit a file containing all your solutions including a screenshot of your JFLAP Turing machine for Q2, but also submit a JFLAP file containing your Q2 Turing machine. Q2 will be auto-graded for correctness based on several test inputs, so you should confirm that it is formatted correctly and produces the correct output for strings in and out of the language. As usual, 5% extra credit for editing this file to generate your solutions file.

Question 1. Give pseudocode for a Turing machine that decides the language  $L=\{a^n\#b^n\#c^n\mid n\geq 0\}$  .

Question 2. Give a JFLAP Turing machine that corresponds to your algorithm for Q1.

Question 3. The machine below (Example 3.7) recognizes any string of 0s whose length is a power of two.



- a) Give the configuration of the machine on input  $0^{(2^3)}$  immediately after its second  $q_2$  to  $q_3$  transition.
- b) Give the reject configuration of the machine on input  $0^7$ .
- c) Give the reject configuration of the machine on input  $0^6$ .

Question 4. At the end of Unit 2, we remarked that nondeterministic PDAs are more powerful than deterministic ones without going into detail why. A deterministic PDA can recognize the language  $\{w\#w^{\mathcal{R}}\mid w\in\{0,1\}^*\}$  but only nondeterministic PDAs can recognize  $\{ww^{\mathcal{R}}\mid w\in\{0,1\}^*\}$ . (Think about why!) Of course even nondeterministic PDAs cannot recognize  $\{ww\mid w\in\{0,1\}^*\}$  since we saw that this language fails the conditions of the context-free pumping lemma, but enhanced memory power allows a Turing machine to recognize this language. Although nondeterminism can be simulated (often inefficiently!) on a Turing machine (Section 3.3), Turing machines themselves are deterministic. For this question, you will explore the idea of determinism on Turing machines in the context of two context-sensitive languages:

$$L_1 = \{ w \# w \mid w \in \{0, 1\}^* \} \text{ and } L_2 = \{ ww \mid w \in \{0, 1\}^* \}.$$

Document your exploration by answering the following questions. All of your answers can be high-level:

- a) Give high-level pseudocode for a Turing machine that recognizes  $L_1$  in quadratic  $O(|w|^2)$  time.
- b) Describe how you could modify this algorithm *nondeterministically* to recognize  $L_2$  in quadratic time. How long would a deterministic simulation of this nondeterministic algorithm take? (A deterministic simulation of a nondeterministic process considers all possible computation paths in sequence.)
- c) Describe a more clever Turing machine that deterministically recognizes  $L_2$  in quadratic time.