2 “Practical” Optimisation Examples

Games and Medication

League of Legends

- Given

  Health Score : *H*

  Armour quantity : *A*

  Starting number of tokens : *T*

If extra armour costs *18 tokens* and improved health *2.5 tokens*, how should an allocation of *T* be used to maximize *Effective Health*, *E*, defined to be

$$E = \frac{H(100+A)}{100} \text{ ?}$$

League of Legends – Analysis

If *x* tokens are spent on *Improved Health* then

$$\frac{T - 2.5x}{18}$$

are available for *Armour*.

Hence wish to find value for *x* (Improved Health spending) to maximize

$$E(x) = \frac{x(100 + (T - 2.5x)/18)}{100} = \left(1 + \frac{T}{1800}\right)x - \frac{2.5x^2}{1800}$$

League of Legends – Analysis

Critical Points

$$E'(x) = \left(1 + \frac{T}{1800}\right) - \frac{5x}{1800} = \frac{1800 + T - 5x}{1800}$$

Critical point is:

$$x = \frac{1800 + T}{5} = 360 + \frac{T}{5}$$

This ***is*** a *maximum*

$$E''(x) = \frac{-5}{1800} < 0$$

Intravenous Injection – Dosage Effectiveness

- Given

Dosage : *D*

Absorption rate : α

Elimination rate : β

What is the optimal length time, *t*, that will maximise the dosage concentration?

$$c(t) = \left(\frac{D}{1-\beta/\alpha}\right)\left(\frac{1}{\exp(\alpha t)} - \frac{1}{\exp(\beta t)}\right)$$

D, α, and β are *constants*

Dosage Effectiveness – Analysis

Noting that *D*, α and β are constants.

$$c'(t) = \left(\frac{D}{1-\beta/\alpha}\right)\left(\frac{-\alpha}{\exp(\alpha t)} + \frac{\beta}{\exp(\beta t)}\right)$$

Critical point will be the value, t, at which $\beta \exp(\alpha t) - \alpha \exp(\beta t) = 0$

That is for which

$$\log \beta + \alpha t = \log \alpha + \beta t$$

Hence,

$$t = \frac{\log \beta - \log \alpha}{\beta - \alpha}$$