VECTORS AND VECTOR OPERATIONS

A Sense of Order

Well how was I to know it was going to be so complicated! I thought it was just going to be a straightforward bash on the nut up an alley.

The Missing Page (from BBC TV Series, Hancock's Half Hour, 1960)

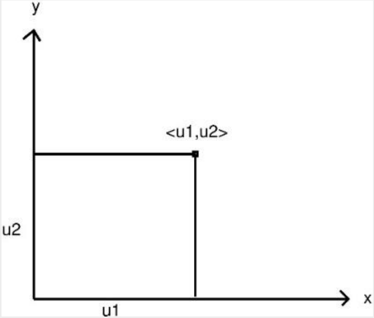
Ray Galton & Alan Simpson

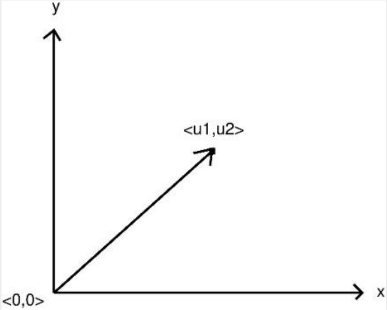
Why do we need order?

- On COMP109 and COMP107 you will have seen notions such as “*relation*” and “*tuple*”
- These allow data to be structured in ways that support complicated processes: querying databases, ordering symbol sequences, etc.
- The concept of vector builds on “tuples of *numbers* (chosen within an underlying set)”.

Where have we seen vectors?

- Cartesian co-ordinates ($\langle x, y\rangle$; $\langle x, y, z\rangle$, etc) not only describe points in 2 (3)-dimensions but also "*lines from the origin*": called *2-vectors* and *3-vectors*.





n-vectors : definitions

- A set of numbers, H, eg H is one of N, W, Z, Q, or R
- A Natural number, $n \in N$
- An n-*vector* over H, has n *components*

  $$\underline{\mathbf{x}} = \langle x_1, x_2, \dots, x_n \rangle$$

- Each $x_k \in$ H.
- This is *not a set*: we can have $x_s = x_t$ and $s \neq t$
- The *order* of components matters: $\langle 1,2,1 \rangle \neq \langle 1,1,2 \rangle$

n-vectors : notation & operations

- Arbitrary vectors: $\underline{\mathbf{x}}$, $\underline{\mathbf{y}}$, $\underline{\mathbf{u}}$, $\underline{\mathbf{v}}$, $\underline{\mathbf{w}}$ etc
- Writing a vector in full: $\langle u_1, u_2 \ , \ldots , u_n \rangle$
- For points we use $(u_1, u_2 \ , \ldots , u_n)$
- Operations of interest:

Addition (+)

Scalar Multiple (*)

Size or *Length* ($\|u\|$ or $\|u\|_2$)

"*Product*" (·,×) (can use: $u \cdot v = \ \|u\|\|v\| \cos \theta$)

n-vector operations

- Given n-vectors: **u** , **v** , **w** etc and a constant $\alpha \in H$.
- *Addition* (+): **w** = **u** + **v** : $w_k = u_k + v_k \; \forall k \; 1 \le k \le n$
- *Scalar Multiple* (*): **w** = α**u** : $w_k = \alpha u_k \; \forall k \; 1 \le k \le n$
- *Size* ($\|u\|$ or $\|u\|_2$):

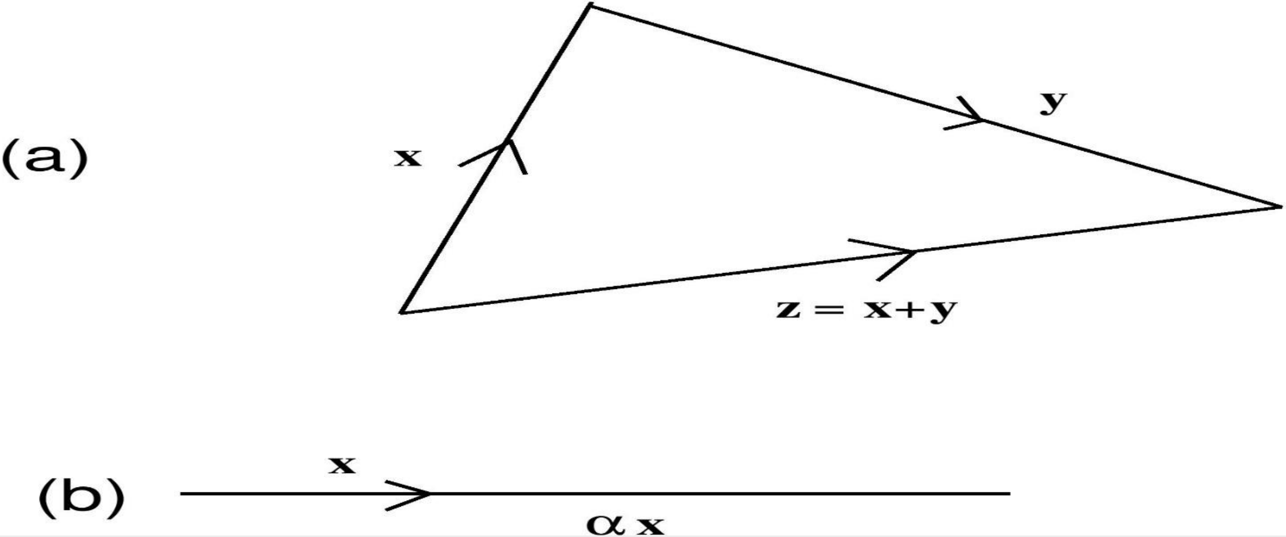
$$\|u\| = \sqrt{\sum_{k=1}^{n} |u_k|^2}$$

- *Product* ($\cdot, \times$) – see pages 69-75 course text.

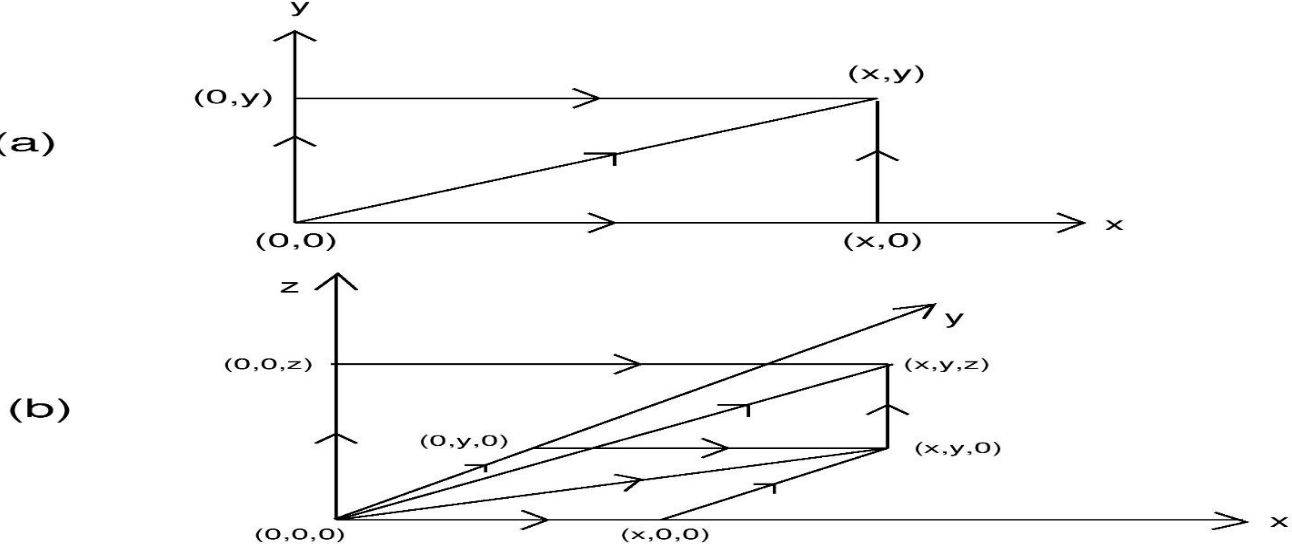
n-vector attributes

- A vector, <u>**u**</u>, has a *size* ($\|u\|$) and a *direction*.
- A vector does *not* have a "*position*".
- The operations can be interpreted geometrically.
- The length of a vector resulting from adding two vectors may be written in terms of basic trigonometric relations (page 67)

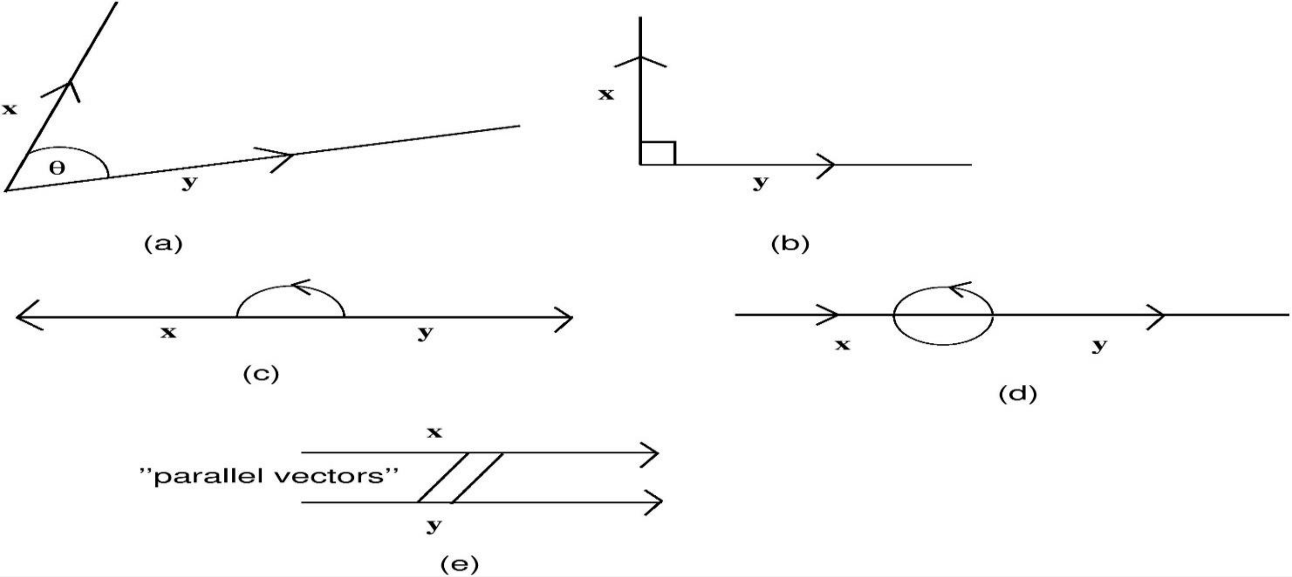
Addition and Scalar Multiple



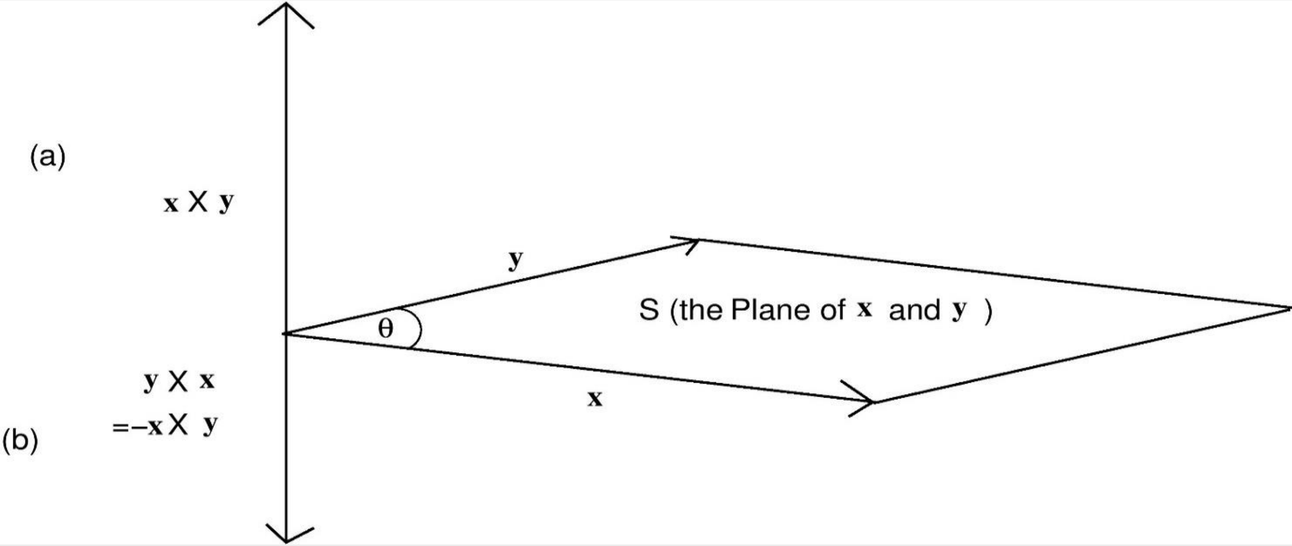
Length



Vector “dot” product



Vector “cross” product



“Special” n-vectors

- The “*zero vector*”: **<u>0</u>** (all components are 0).
- The “*basis vector*”: e_k - all components 0, except the kth which is equal to 1. There are n basis vectors (one for each k).
- *Unit vectors*: those vectors whose *size* is 1.
- *Orthogonal vectors*: **<u>u</u>** and **<u>v</u>** with **<u>u</u>** · **<u>v</u>** = 0.

Summary

- The manipulation of objects described as vectors underpins many areas of CS:

  Basic computer graphics effects

  Web page analytics (eg search outcomes)

  Multivariable optimization problems

  Data science

  Computational Geometry

- We next look at *Vector Spaces* and *Linear Transformations* which play an important role in developing these ideas.