Foundations of Computer Science (COMP109)

Tutorial VIII, Week 18.11.2024 – 22.11.2024

A reasonable attempt at answering Question (VIII.2.) should be submitted on Canvas by ***14:00 on Tuesday 19.11.2024*** *as a text entry, a text file (txt), a pdf file, or a photo of the hand-written answer. This assignment makes up 1% of your final mark. We want to encourage you to discuss the questions with your fellow students in person or on the Canvas discussion board but do not copy your answer from anybody else.*

VIII.1. Let $X = \{1, 2, 3, 4\}$ and $Y = \{a, b, c\}$. Let $R \subseteq X \times X$ be a relation on X given by xRy if, and only if, $x < y$. Let $S \subseteq X \times Y$ be a relation between X and Y given by the matrix:

$$M_S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

(a) Represent R as a set of ordered pairs;

(b) Represent S as a set of ordered pairs;

(c) Represent S in graphical form;

(d) Find the matrix M_R representing the relation R

(e) Use Boolean matrix multiplication to find the matrix $M_{S \circ R}$ representing the composition $S \circ R$.

VIII.2. Let $A = \{1, 2, 3, 4\}$ and let R, S, T and U be the following relations:

$R = \{(1, 3), (3, 2), (2, 1), (4, 4)\}$,

$S = \{(2, 1), (3, 3), (4, 2)\}$,

$T = \{(4, 1), (4, 2), (3, 1), (3, 2), (1, 2)\}$,

$U = \{(x, y) \mid x > y\}$.

(a) For each of R, S, T and U determine whether they are functional, reflexive, symmetric, anti-symmetric or transitive.

Explain your answer in each case, showing why your answer is correct.

(b) What is the transitive closure of R?

(c) Explain why R^*, the transitive closure of R, is an equivalence relation. Describe the equivalence classes E_x into which the relation partitions the set A.

VIII.3. Prove or give a counterexample to the following statement: for any relation R both R and $R \circ R$ always have the same transitive closure.

VIII.4. Is there a mistake in the following proof that any transitive and symmetric relation R is reflexive? If so, what is it?

> *Let aRb. By symmetry, bRa. By transitivity, if aRb and bRa, then aRa. This proves reflexivity.*