Foundations of Computer Science (COMP109)

Tutorial VI, Week 06.11.2023 – 10.11.2023

*A reasonable attempt at answering Question (VI.6.) should be submitted on Canvas by **23:59 on Tuesday 07.11.2023** either as a text entry, a text file (txt), a pdf file, or a photo of the handwritten answer. This assignment makes up 1% of your final mark. We would like to encourage you to discuss the questions with your fellow students in person or on the Canvas discussion board, but do not copy your answer from anybody else.*

VI.1. What is the cardinality of the set $\{x \in \mathbb{Z} \mid x = y^2, \text{ for some } y \in \mathbb{Z}, -10 \leq y \leq 10\}$

VI.2. Determine the power set $Pow(A)$ for

- $A = \{1, 2, L\}$;
- $A = \emptyset$.

VI.3. List all distinct functions from the set $A = \{1, 2\}$ to the set $B = \{a, b\}$. How many such distinct functions exist?

VI.4. Consider $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^2$. What are the domain, codomain and range of f?

VI.5. Which of the following functions are injective? Which are surjective?

(a) $f : \mathbb{Z} \to \mathbb{Z}$ given by $f(x) = x^2 + 1$.

(b) $g : \mathbb{N} \to \mathbb{N}$ given by $g(x) = 2^x$.

(c) $h : \mathbb{R} \to \mathbb{R}$ given by $h(x) = 5x - 1$.

VI.6. Use the pigeonhole principle to give solutions to the following problems:

(a) How many times must a single die be rolled to guarantee that some number is obtained at least twice?

(b) How many times must two dice be rolled to guarantee that the same total score is obtained at least twice?

(c) How many times must two dice be rolled to guarantee that the same total score is obtained at least three times?