COMP111: Artificial Intelligence

Section 6. Adversarial search (Game playing)

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Outline

We will look at how search can be applied to playing games

- Types of Games
- Perfect play:
  - minimax algorithm
  - α–β pruning
- Playing with limited resources (heuristics)

Games vs. search problems

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- A method is needed for selecting good moves that stand a good chance of achieving a winning state whatever the opponent does!
- Because of combinatorial explosion, in practice we must approximate using heuristics.

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- Some games are deterministic: chess, go.
- Others have an element of chance: backgammon, monopoly, bridge, poker

The games we consider

We consider special kinds of games

- Deterministic
- Two-player
- Zero-sum:
  - the utility values at the end are equal and opposite
  - example: one wins $(+1)$ the other loses (-1).
- Perfect information

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- Terminal test
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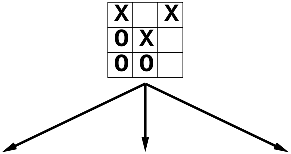
- Utility function
  - Numeric value for terminal states
  - E.g. Chess +1, -1, 0
  - E.g. Backgammon +192 to -192

Possible Development



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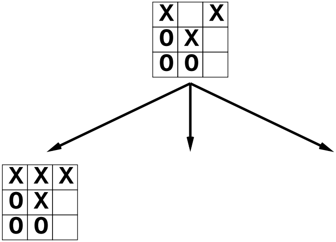

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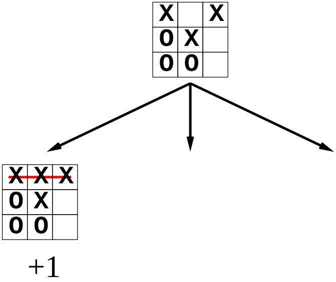

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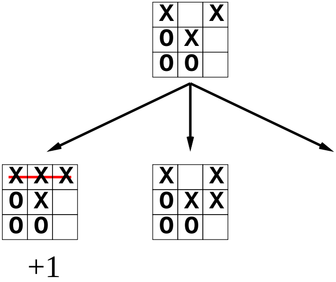

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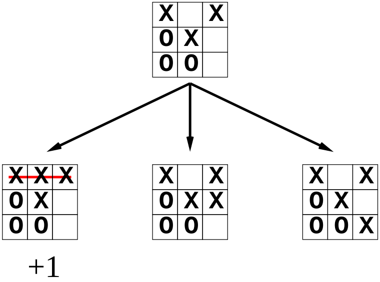

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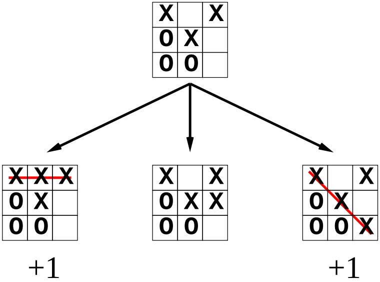

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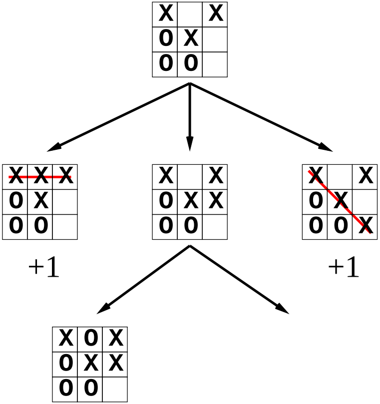

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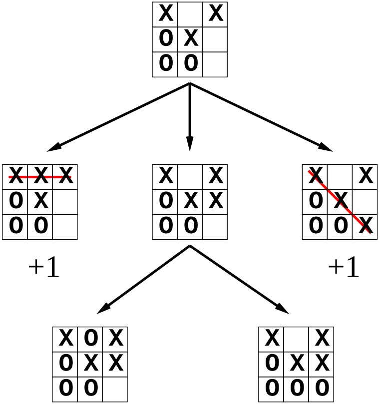

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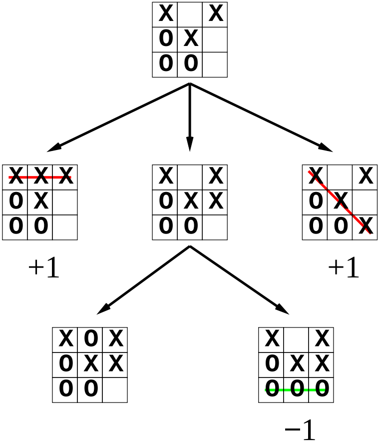

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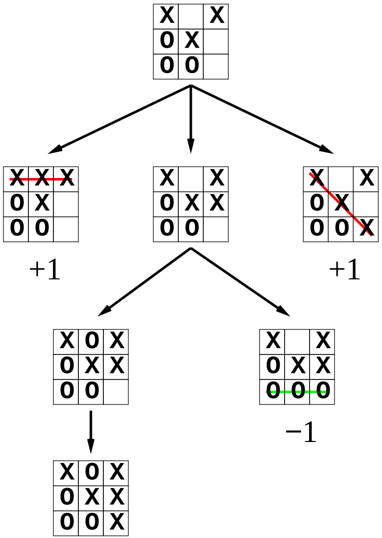

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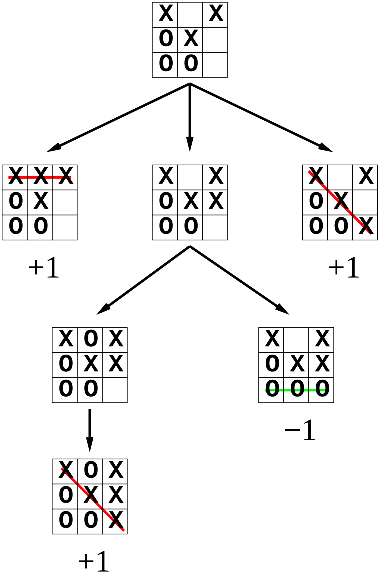

Game Tree

- Each level labelled with player to move
- Each level represents a ply
  - Half a turn
- Represents what happens with competing agents

Introducing MIN and MAX

MIN and MAX are two players:

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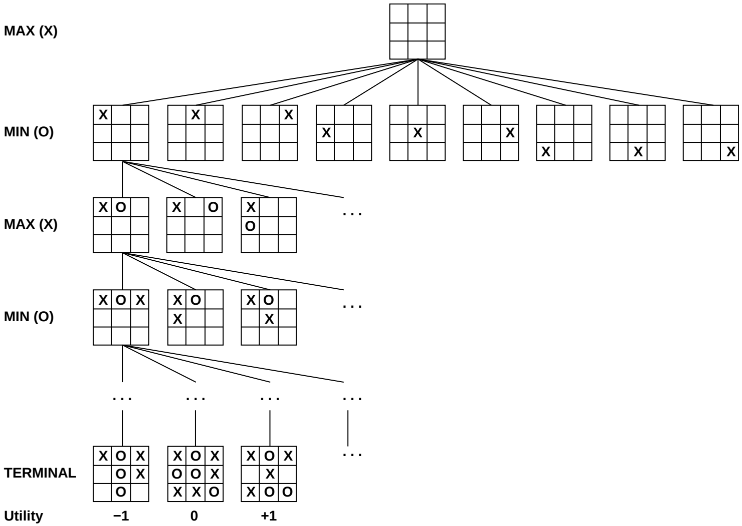
Both players will play to the best of their ability

- MAX wants a strategy for maximising utility assuming MIN will do best to minimise MAX's utility
- Consider minimax value of each state: the utility of a state given that both players play optimally.

Example Game Tree



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Formally, let $\mathsf{Succ}(s)$ denote the set of successors states of state s. Define the function $\mathsf{MinimaxV}(s)$ recursively as follows:

$$\mathrm{MinimaxV}(s) = \begin{cases} \mathrm{Utility}(s) & s \text{ is Terminal} \\ \max_{n \in \mathrm{Succ}(s)} \mathrm{MinimaxV}(n) & \text{MAX moves in } s \\ \min_{n \in \mathrm{Succ}(s)} \mathrm{MinimaxV}(n) & \text{MIN moves in } s \end{cases}$$

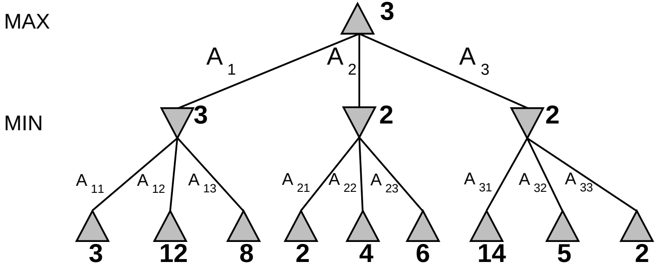
Minimax algorithm

- Calculate minimax value of each state using the equation above starting from the terminal states.
- Game tree as minimax tree:

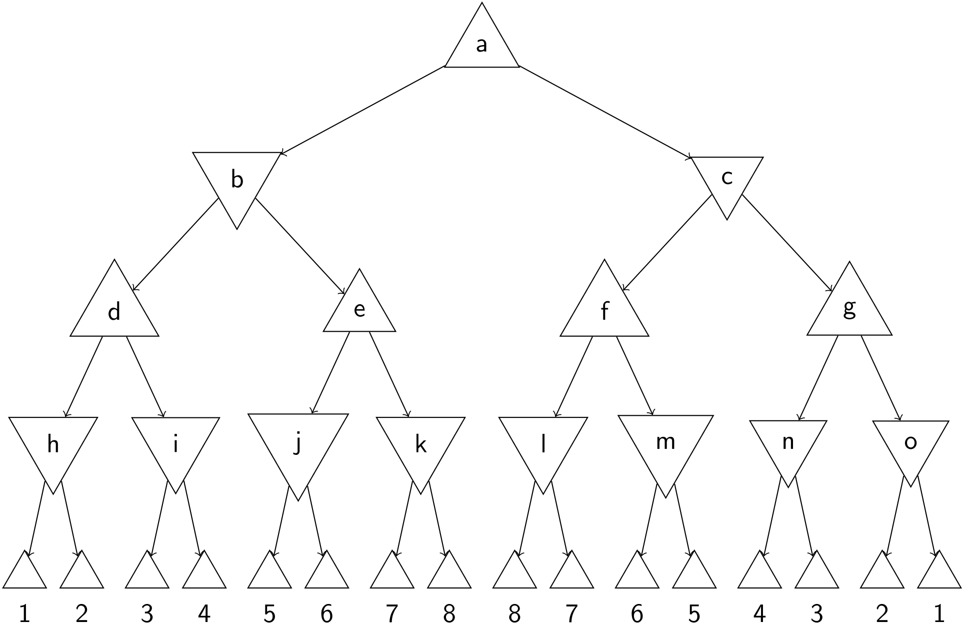
Minimax Tree

- MIN takes the minimal value from its successors.
- MAX takes the maximal value from its successors.

Consider



What is the minimax value of *a*?



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Assume MAX always move to the state with the maximal minimax value.

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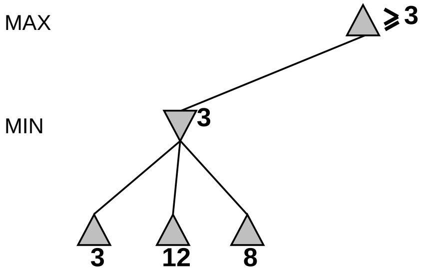
For chess, $b \approx 35$, $m \approx 100$ for “reasonable” games

- 10^{154} paths to explore
- infeasible

But do we need to explore every path?

Removing redundant information: α-β-Pruning

If you know half-way through a calculation that it will succeed or fail, then there is no point in doing the rest of it!



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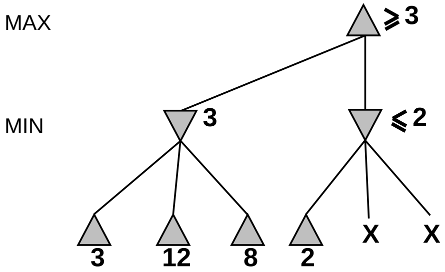

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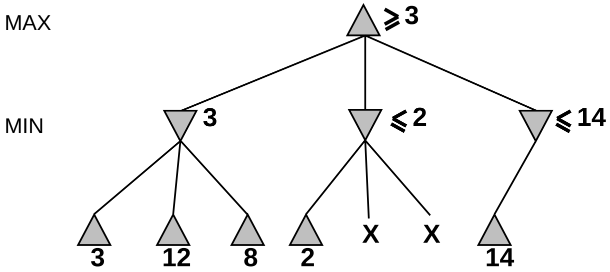

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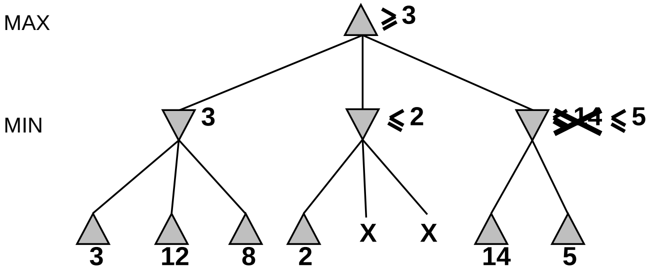

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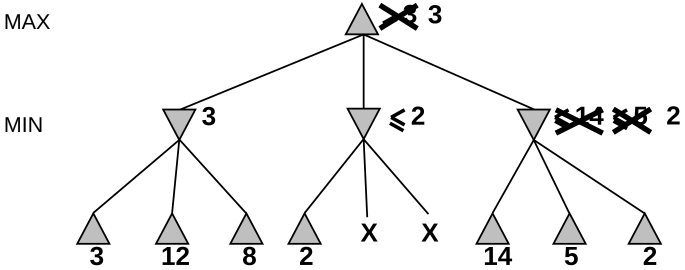
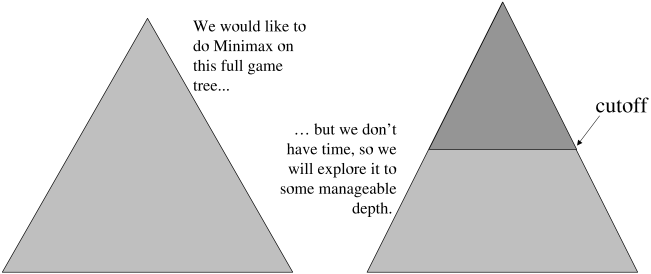


Cutoffs and Heuristics

- Cutoff search according to some cutoff test.
- Problem: utitilies are defined only at terminal states.
- Solution: Evaluate the pre-terminal leaf states using heuristic evaluation function rather than using the actual utility function.



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Assume that we can compute a function Evaluation(s) which gives us a utility value for any state s which we do not want explore (every cutoff state).
Then define CutOffV(s) recursively:

$$\mathrm{CutoffV}(s) = \begin{cases} \mathrm{Utility}(s) & s \text{ is Terminal} \\ \mathrm{Evaluation}(s) & s \text{ is Cutoff} \\ \max_{n \in \mathrm{Succ}(s)} \mathrm{CutoffV}(n) & s \text{ is MAX} \\ \min_{n \in \mathrm{Succ}(s)} \mathrm{CutoffV}(n) & s \text{ is MIN} \end{cases}$$

Example: Chess (I)

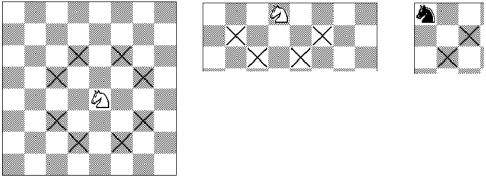
- Assume MAX is white
- Assume each piece has the following material value:
  - pawn $= 1$;
  - knight $= 3$;
  - bishop $= 3$;
  - rook $= 5$;
  - queen $= 9$;
- let w = sum of the value of white pieces
- let b = sum of the value of black pieces

$$\mathrm{Evaluation}(s) = \frac{w - b}{w + b}$$

Example: Chess (II)

- The previous evaluation function naively gave the same weight to a piece regardless of its position on the board...
- Let X_i be the number of squares the ith piece attacks

$$\mathrm{Evaluation}(s) = piece_1 value * X_1 + piece_2 value * X_2 + \ldots$$



(a) (b) (c)

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- Not efficient enough for games such as chess, go, etc.
- Evaluation functions are needed to replace terminal states by cutoff states.
- Various approaches to define evaluation function.
- Most successful approach: machine learning. Evaluate positions using experience from previous games.