Chaos, Fractals, Composition

Complex Numbers in AI

Complex Number Sequences

- The behaviour of certain types of Complex Number sequences has inspired their use in two particular areas of the specialized AI field of *Computer Creativity*.

  Building Intricate Graphical Forms

  Musical Composition

- We look at the main ideas supporting these in this lecture.
- The key concept is that of an *Iterated Function sequence*.
- These define how to construct a *sequence* of Complex Numbers starting from an *initial Complex Value*.

Iterated Function Sequences

- Let $f : C \rightarrow C$ be some Complex Valued Function.
- The *n'th iterate of f* denoted $f_{(n)}$ when applied to $z \in C$ is the Complex Number, denoted z_n, that results by applying f repeatedly (to the initial value z_0). Thus

$$f_{(n)}(z) = \begin{cases} z & \text{if } n = 0 \\ f\left(f_{(n-1)}(z)\right) & \text{if } n > 0 \end{cases}$$

Bounded Modulus, Escape Radius and Orbits

- In a number of cases $f : C \rightarrow C$ behaves as follows.
- The Complex Plane can be *divided* into *two disjoint* parts relative to some parameter, r, called the *Escape Radius* of f.

$$in(f) = \{z \in C : \forall\, n \left|f_{(n)}(z)\right| \leq r\}$$
$$out(f) = \{z \in C : \exists\, k \left|f_{(k)}(z)\right| > r\}$$

- An *orbit* (with length k) of such a function is a sequence

$$\left(z_0, z_1, \ldots, z_j, z_{j+1}, \ldots, z_k\right)$$

for which $f(z_k) = z_0$; $f\left(z_j\right) = z_{j+1} \forall\, 0 \leq j < k$

- The escape radius can often be set as $r = 2$.

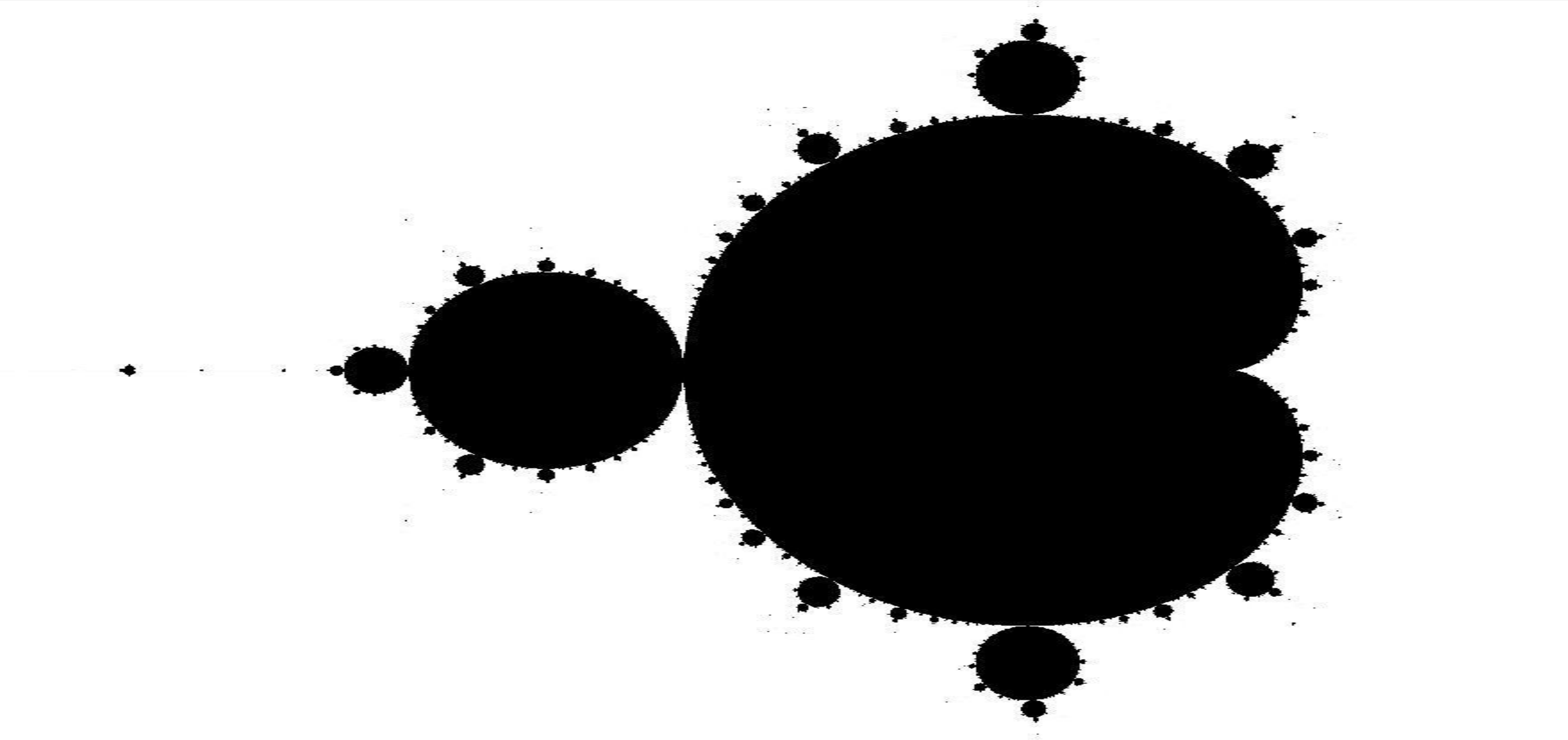
Quadratic Complex Valued Cases

- The behaviour of *quadratic functions* has been found to give rise to interesting patters of behaviour, particularly those of the form

$$f(z) = z^2 + c \text{ for } c \in C$$

- The well-known *Mandelbrot Set* classifies those c for which $\left|f_{(n)}(0)\right| \leq 2 \;\; \forall n \geq 0$.
- If c is fixed the subset (using radius 2) $in(z^2 + c)$ is called a *Julia set* while the set $out(z^2 + c)$ is called the *Fatou set*.

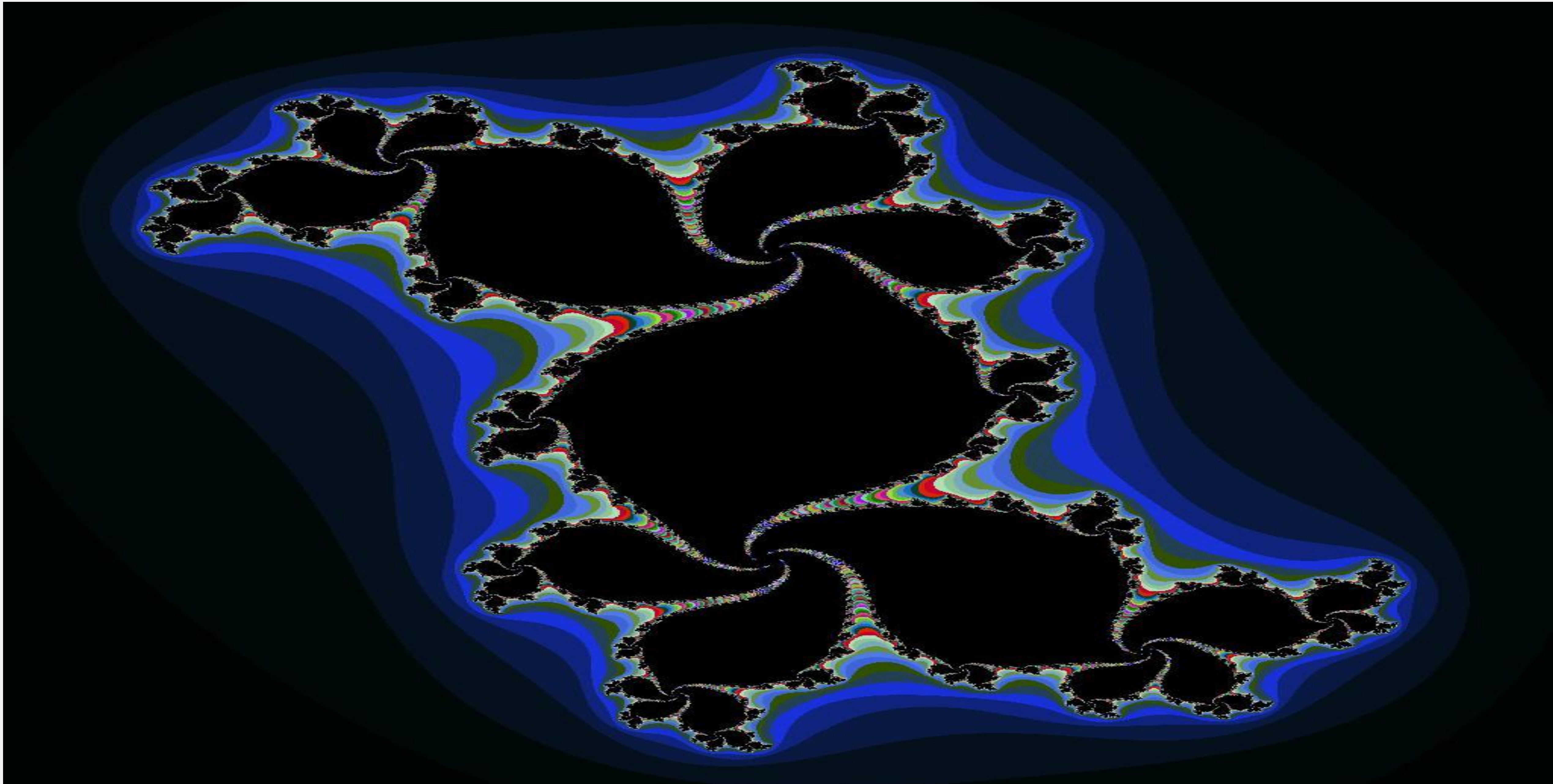
The Mandelbrot Set in the Complex Plane



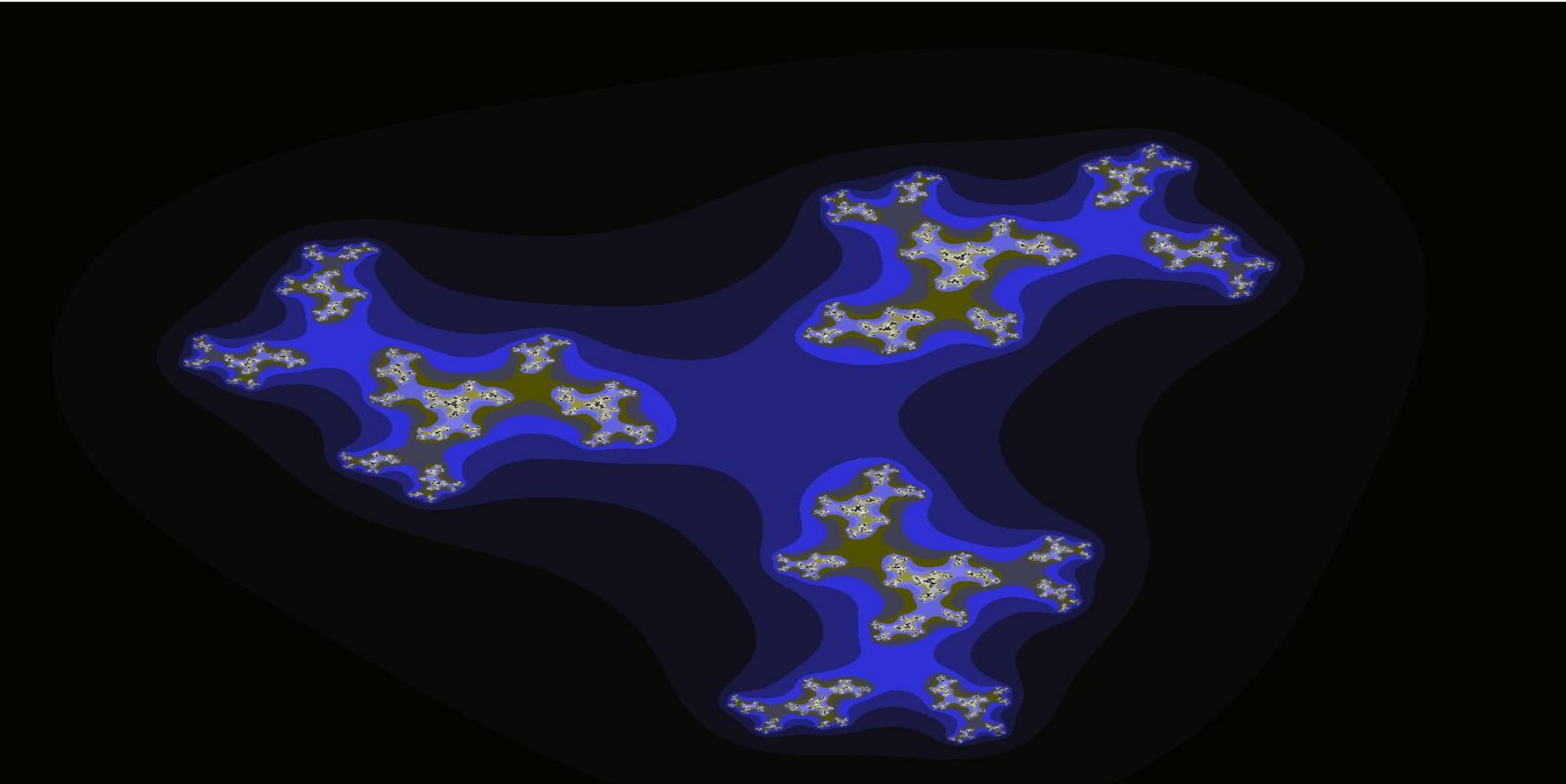
Computing and Displaying Julia Sets

- It is, of course, not possible to validate $\left|f_{(n)}(z)\right| \leq 2\ \forall n \geq 0$.
- In practice, however, iterating some *fixed number* of times (eg ~ 1000) gives a "*good enough approximation*".
- In displaying Julia Sets a "*colour mapping*" convention is often used to highlight the structure.
- This may be based on the *number of iterations* needed for a point to exceed the escape radius.
- We can, in fact, generate these forms for any Complex Rational function $\frac{f(z)}{g(z)}$ with $f(z), g(z)$ both *polynomials*.

$$f(z) = z^2 + 0.27 + 0.5i$$



$$f(z) = z^3 + 0.65 - 0.68i$$



Orbits in Julia Sets and AI Music

- In her 2001 NYU Master's thesis the US composer *Elaine Walker* explored the use of *orbits in Julia Sets* as a *compositional device*.
- A single Julia Set may contain *many orbits* of *varying length*.
- These provide a *unifying collection* of *building blocks* to experiment with compositional themes.
- The elements in an orbit are Complex Numbers.
- By mapping the Real and Imaginary parts (which are Reals) to "*standard*" *sound conventions*, eg MIDI, intricate musical scores can be produced.
- Improvised scores (so-called *Chaos Melody*) are produced by different (not necessarily orbital) start points.

${}^{1}/_{f}$–noise and “fractal dimension”

- The assertion that the structures developed from Julia Sets are “aesthetically satisfying” originates, in part, from the concept of “*fractal dimension*”.
- Instead of classical Euclidean dimension based on *Whole* numbers, fractal sets have “*dimension*” which is in Q – the *Rationals*.
- The US Physicists Voss and Clark conducted experiments to examine the hypothesis that a fractal dimension of ~ 1 in Music was found to be the “most appealing”.
- Voss developed a *random* $\frac{1}{f}$ *- music algorithm* based on “*dice throwing*”. This allows a mapping to “more traditional” scores.

Some Example Output from Voss' Algorithm I

Some Example Output from Voss’ Algorithm II

Example

Summary – Complex Numbers in AI

- The exploration of ideas arising from the characteristics of iterating Complex Functions has provided a rich source of ideas for the AI sub-field of *Computer Creativity*.
- Many of these take as their starting point features arising from the study of "*fractal sets*".
- In addition to "*abstract images*" from Julia sets and variations these also offer a "more realistic" method for *CGI* creation of "*artificial landscapes*" as used in *PC Games* and "*science fiction/fantasy*" *film*.
- The use in *musical composition* has been considered by many modern composers (eg Stockhausen, Boulez) and groups, eg IRCAM