Foundations of Computer Science Comp109

University of Liverpool

Boris Konev

konev@liverpool.ac.uk

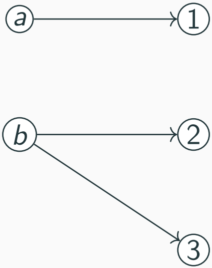
Olga Anosova

O.Anosova@liverpool.ac.uk

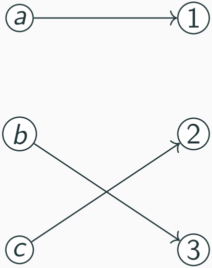
Recap: functions, function classification and pigeonhole principle.

- A **function** $f : A \to B$ is an assignment such that
- A is called the , B is the , $f(A)$ is the .
- **Injective** function:
- **Surjective** function:
- **Bijective** function:
- **Inverse function** $f^{-1} : Y \to X$ such that $f(a) = b \iff f^{-1}(b) = a$.

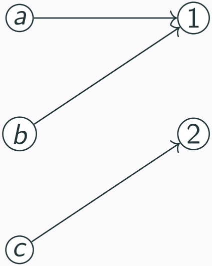
Recap quiz: Classify $f : \{a, b, c\} \to \{1, 2, 3\}$ given by



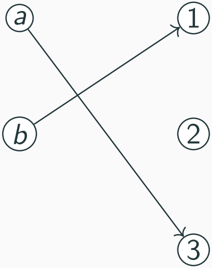
Recap quiz: Classify $g : \{a, b, c\} \to \{1, 2, 3\}$ given by



Recap quiz: Classify $h : \{a, b, c\} \to \{1, 2\}$ given by



Recap quiz: Classify $h_1 : \{a, b, c\} \to \{1, 2, 3\}$ given by



Recap: The pigeonhole principle (Dirichlet's principle).

Let $f : A \to B$ be a function where A and B are finite sets.

The *pigeonhole principle* states that

Recap Example solution: birthday problem.

Problem. There are 15 people on a bus. Show that at least two of them have a birthday in the same month of the year.

DIY Example solution: dark socks puzzle

Imagine you have 10 pairs of socks in a drawer. How many socks would you need to pull out blindly in a completely dark room to ensure you have at least one matching pair?

Example: car number plates

How many different standard GB car number plates should be considered to guarantee that at least two of them begin with the same letter of the alphabet and end with the same letter of the alphabet?

Assume for that a standard number plate consists of 2 letters+2 numbers+3 letters: BD51 SMR

and assume for simplicity that all letters are possible.

Example: ensure the total

What is the smallest number that should be selected from the numbers $1, 2, 3, 4, 5, 6, 7, 8$ to guarantee that there will always be two of the numbers that sum to 9?

Extended pigeonhole principle

Consider a function $f : A \to B$ where A and B are finite sets and $|A| > k|B|$ for some natural number k. Then there is a value of f which occurs at least $k+1$ times.

Example: extended car plate coverage

In the assumptions made earlier, how many different standard GB car number plates should be considered to guarantee that at least five of them begin with the same letter of the alphabet and end with the same letter of the alphabet?

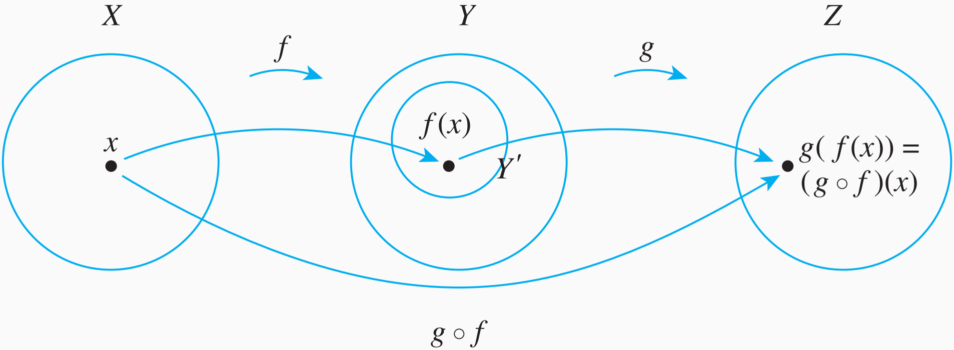
Example: Theorem of Friends and Strangers (Ramsay's theorem)

Show that in any group of six people there are either three who all know each other or three complete strangers.

Composition of functions

If $f : X \to Y$ and $g : Y \to Z$ are functions, then their *composition* $g \circ f$ is a function from X to Z given by

$$(g \circ f)(x) = g(f(x)).$$



Example

Consider $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^2$ and $g : \mathbb{R} \to \mathbb{R}$ given by $g(x) = 4x + 3$.

- $g \circ f(x) =$

- $f \circ g(x) =$

- $f \circ f(x) =$

- $g \circ g(x) =$

DIY: clasification of compositions

What can you state about the composition of two

- injective functions,
- surjective functions,
- bijective functions?

Summary

- General properties of functions could guarantee some general properties of function outputs.
- The **extended pigeonhole principle**: if $|A| > k|B|$ for some $k \in \mathbb{N}$ and a function $f : A \to B$, then there is a value of f which occurs at least $k+1$ times.
- **Composition** $g \circ f : X \to Z$ of two functions $f : X \to Y$ and $g : Y \to Z$ is defined by $(g \circ f)(x) = g(f(x))$.