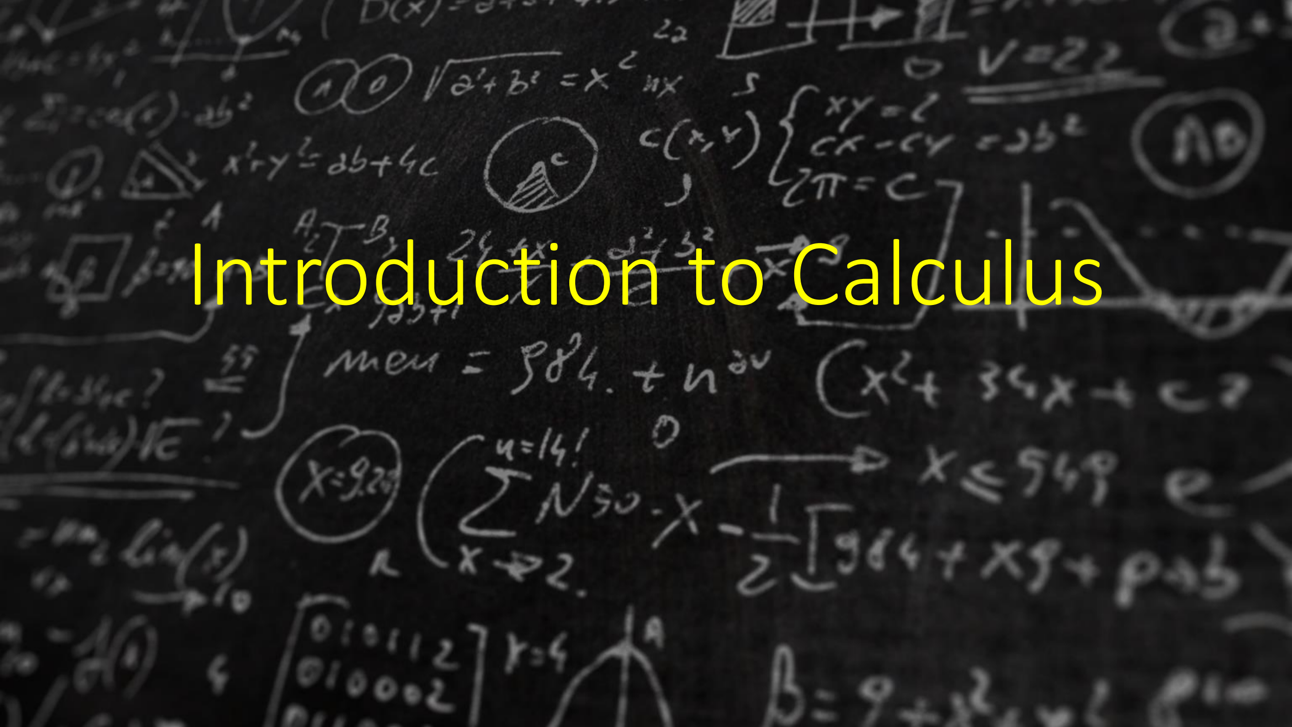

Introduction to Calculus

Another obscure quote

"*And one and all they had a longing to get away from this painfulness, this ceremony which had reminded them of things they could not bear to think about – to get away quickly and go about their business and forget.*"

John Galsworthy

The Man of Property (from *The Forsyte Saga*)

Static vs. Dynamic

- The models we have looked at (Numbers, Polynomials, Vectors, and Matrix-vector product) are rather "*fixed*" and "*unchanging*".
- We understand how to *evaluate* a polynomial at a *given point*, **but** …
- we have no tools to analyze *how* that polynomial *changes* as the *values change*.
- *Differential Calculus* has its origins in trying to find a rigorous mechanism by which to study *change*.

Functions

- A *function* is just a formal description of how a given *input value* should be changed to produce a particular *output*.
- Input values are restricted to those within some set (the *domain* of the function)
- Output values are required to be within some set (the *range* of the function).
- A function, f, with domain D and range R is written as

$$f : D \to R$$

- We will consider only (at this point) functions whose domain and range are the Reals.

Some examples

- Polynomial Functions
- Trigonometric Functions: $\sin\theta$, $\cos\theta$, $\tan\theta$
- Logarithm and Exponential Functions: $\log x$, $\ln x$, $\exp x$
- Rational functions: ${f(x)}/{g(x)}$
- "*Radicals*" : $\sqrt[k]{x}, x^{1/k}$
- These have *very different* behaviours:

  Some are "well-behaved" for *all* Real values

  Some are "undefined" for *particular* Real values.

  Some may result in *any* Real number

  Some may only produce Reals in a given "*interval*"

More about the examples

- Polynomial Functions: well-behaved for any $x \in R$
- Trigonometric Functions:
  $\sin\theta, \cos\theta : -1 \leq f(x) \leq 1$
  $\tan\theta : -\infty \leq \tan\theta \leq \infty \ \forall\theta$
- Logarithm and Exponential: $\log x, \exp x$:
  $\exp x > 0 \ \forall x$
  $\log x$: "ill-defined" when $x \leq 0$.
- Rational functions: $f(x)/_{g(x)}$: "ill-defined" for $\alpha : g(\alpha) = 0$
- "*Radicals*" : $\sqrt[k]{x}, x^{1/k}$: "ill-defined" for $k \geq 2, \ x < 0$

Linear functions

- These are functions that have the form
$$f(x) = mx + C$$
- This is often written instead as
$$y = mx + C$$
- Each $x \in R$ defines a *unique* $y \in R$: an (x, y) *co-ordinate*.
- The parameter, *m*, is called the *gradient* (of the line).
- The parameter, *C*, is the *offset* (of the line).
- At a very informal level "*differential calculus*" concerns the behaviour of (the gradient of) *lines* related to *functions*.

Properties of Linear functions

- Given any two *distinct* points - $(x_1, y_1), (x_2, y_2)$:

  there is a *unique* line function connecting these.

- Given any single point (x, y) and gradient m :

  there is a *unique* line function whose gradient is m and which contains the point (x, y).

- Finding gradients underpins the basis of calculus.

Computing Gradients I

- Given any two *distinct* points - $(x_1, y_1), (x_2, y_2)$:
- The gradient measures how "*height*" changes with "*distance*".
- If $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ are all on the same line, then (assuming these are ordered by increasing x-coordinate) then the gradient of the line between $(x_1, y_1), (x_2, y_2)$ is *exactly the same* as that between $(x_2, y_2), (x_3, y_3)$
- In other words *gradient* describes "how much the y-value ***should*** change relative to how much the x-value ***has*** changed."

Computing Gradients II

- This gives the formula for the gradient, m, of the line joining $(x_1, y_1), (x_2, y_2)$ as:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- To compute the offset, C, we just need to find the y value corresponding to $x = 0$. In other words, the value C, with

$$m = \frac{y_1 - C}{x_1}$$

(assuming, of course, that $x_1 \neq 0$)

Gradients in Differential Calculus

- The gradient of importance when analysing a function $f(x)$ are those of "*the line touching (the curve)* $(x, f(x))$".
- Such gradients are described by the function called the *first derivative* of $f(x)$ which is denoted either as $f'(x)$ or ${}^{dy}/_{dx}$.
- We shall ***avoid*** using terminology such as "gradient of the ***tangent*** … "
- In the next part we look at how such derivatives are found.