UNIVERSITY OF
LIVERPOOL

FIRST SEMESTER EXAMINATIONS 2022/23

FOUNDATIONS OF COMPUTER SCIENCE

TIME ALLOWED : TWO Hours

INSTRUCTIONS TO CANDIDATES

NAME OF CANDIDATE .... SEAT NO ....

USUAL SIGNATURE ....

READ THE FOLLOWING CAREFULLY:

1. Each of the following questions comprise 5 statements, for which you should select the one most appropriate answer. Each question is worth the same amount of points.

2. The exam mark is based on the overall number of correctly answered questions. The more questions you answer correctly the higher your mark, incorrectly answered questions do not count against you.

3. Enter your name and examination number IN PENCIL on the computer answer sheet according to the instructions on that sheet.

4. When you have completed this exam paper, read the instructions on the computer answer sheet carefully and transfer your answers from the exam paper. Use a HB pencil to mark the computer answer sheet and if you change your mind be sure to erase the mark you have made. You may then mark the alternative answer.

5. At the end of the examination, be absolutely sure to hand in BOTH this exam paper AND the computer answer sheet.

6. Calculators are NOT permitted.

THIS PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

This mock paper only has 25 questions; the real exam paper is 40 question.

For Questions 1 and 2, suppose that $S = \langle 5, 6, 7, 1, 2, 3, 4 \rangle$ is the sequence of all elements under consideration, $A = \{x \in \mathbb{N} \mid 2 < x < 7\}$, $B = \{3, 7\}$, and $C = \{3, 4, 5, 6\}$.

1. The set $B \cap (A \cup C)$ is

☐ **A.** $\emptyset$

☐ **B.** $\{7\}$

■ **C.** $\{3\}$

☐ **D.** $\{3, 3, 4, 4, 7\}$

☐ **E.** $\{3, 4, 5, 6, 7\}$

2. What is the characteristic vector of $(A \triangle B) \cap C$?

☐ **A.** $(0, 0, 0, 0, 0, 0, 0)$

☐ **B.** $(0, 0, 1, 1, 1, 0, 0)$

■ **C.** $(1, 1, 0, 0, 0, 0, 1)$

☐ **D.** $(1, 1, 1, 1, 1, 1, 1)$

☐ **E.** $(0, 1, 1, 1, 0, 0, 0)$

3. Which of the following are true?

   (1) $\{a, b, c\} \subseteq \{b, c, a\} - \{b\}$
   (2) $\{a\} \subseteq \{a, b\}$
   (3) $\{b, c\} \subseteq \{c, b\}$
   (4) $\{a, b, c\} \triangle \{b\} = \{a, c\}$

☐ **A.** (2) only

☐ **B.** (1) and (3) only

☐ **C.** (2) and (3) only

■ **D.** (2) and (3) and (4) only

☐ **E.** (1), (2), (3) and (4)

4. Which of the following statements are true?

(1) $x \in \{x\}$

(2) $\{x\} \subseteq \{x\}$

(3) $\{x\} \in \{x\}$

(4) $\{x\} \in \{\{x\}\}$

(5) $\emptyset \subseteq \{x\}$

(6) $\emptyset \in \{x\}$

☐ **A.** (1), (2), (3), (4) and (5) only

☐ **B.** (3) and (6) only

☐ **C.** (2), (3), (4), (5) and (6) only

■ **D.** (1), (2), (4) and (5) only

☐ **E.** (1) and (2) only

5. Which of the following are true for every negative integer x and every negative integer y?

(1) $x - y$ is a positive integer.

(2) $x \times y$ is a natural number.

(3) $x + y$ is a negative integer.

(4) x/y is a rational.

(5) y/x is a rational.

☐ **A.** (1), (2) and (4) only

☐ **B.** (1), (2) and (5) only

☐ **C.** (1), (3) and (4) only

☐ **D.** (2), (3) and (4) only

■ **E.** (2), (3), (4) and (5) only

6. How many prime numbers are even?

☐ **A.** There are no even prime numbers.

■ **B.** There is exactly one even prime number.

☐ **C.** There are exactly two even prime numbers.

☐ **D.** There are exactly four even prime numbers.

☐ **E.** There are infinitely many even prime numbers.

7. Let $A = \{1, 3, 5\}$ and $B = \{x, y, z\}$. Define $f : A \to B$ by specifying that

$$f(1) = z, f(3) = y, f(5) = y$$

Then f is

☐ **A.** injective, surjective and bijective

☐ **B.** injective and not surjective

☐ **C.** surjective and not injective

■ **D.** not injective, not surjective and not bijective

☐ **E.** injective and surjective and not invertible

8. Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by $f(x) = 2 \times x + 1$. Then f is

■ **A.** injective, surjective and bijective

☐ **B.** injective and not surjective

☐ **C.** surjective and not injective

☐ **D.** not injective, not surjective and not bijective

☐ **E.** injective and surjective and not invertible

For Questions 9–11, let $X = \{1, 2, 3\}$, $Y = \{a, b, c, d, e\}$ and $Z = \{x, y, z\}$. Define functions $f : X \to Y$ and $g : Y \to Z$ by specifying that

$$f(1) = b, f(2) = a, f(3) = c$$

and

$$g(a) = x, g(b) = y, g(c) = z, g(d) = x, g(e) = y.$$

9. Which of the following holds?

☐ **A.** $g \circ f(1) = x, g \circ f(2) = y, g \circ f(3) = z$

☐ **B.** $g \circ f(1) = x, g \circ f(2) = x, g \circ f(3) = y$

■ **C.** $g \circ f(1) = y, g \circ f(2) = x, g \circ f(3) = z$

☐ **D.** $g \circ f(1) = y, g \circ f(2) = x, g \circ f(3) = y$

☐ **E.** $g \circ f(1) = z, g \circ f(2) = y, g \circ f(3) = x$

10. The range of $g \circ f$ is

☐ **A.** $\emptyset$

☐ **B.** $\{x, y\}$

☐ **C.** $\{x, z\}$

☐ **D.** $\{y, z\}$

■ **E.** $\{x, y, z\}$

11. The function $g \circ f$ is

■ **A.** injective, surjective and bijective

☐ **B.** injective and not surjective

☐ **C.** surjective and not injective

☐ **D.** not injective, not surjective and not bijective

☐ **E.** injective and surjective and not invertible

12. Let $X = \{1, 2\}$, $Y = \{a, b, c\}$ and $Z = \{x, y, z\}$. Let $f : X \to Y$ and $g : Y \to Z$ be such that

$$g(a) = x, g(b) = y, g(c) = z,$$

and

$$g \circ f(1) = x, g \circ f(2) = x.$$

Then

■ **A.** The only possibility for f is $f(1) = a, f(2) = a$

☐ **B.** The only possibility for f is $f(1) = b, f(2) = c$

☐ **C.** The only possibility for f is $f(1) = c, f(2) = a$

☐ **D.** The only possibility for f is $f(1) = a, f(2) = c$

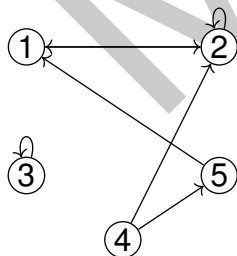☐ **E.** There is more that one possibility for f

13. Let $A = \{2, 3, 4\}$ and $B = \{a, b\}$. Which of the following is $A \times B$?

☐ **A.** $\{2, 3, 4, a, b\}$

☐ **B.** $\{(a, 2), (a, 3), (a, 4), (b, 2), (c, 3), (c, 4)\}$

■ **C.** $\{(2, a), (2, b), (3, a), (3, b), (4, a), (4, b)\}$

☐ **D.** $\{(2, 3), (3, 4), (3, 4)\}$

☐ **E.** $B \times A$

14. Let $|A| = m$ and $|B| = n$. Then the cardinality of the set $A \times B$ is

☐ **A.** 0

☐ **B.** m^n

■ **C.** mn

☐ **D.** $m + n$

☐ **E.** $\mathbb{N}$

15. Which relation is represented by the following digraph?

☐ **A.** $\{(2, 1), (4, 2), (4, 5), (5, 1)\}$

☐ **B.** $\{(2, 1), (4, 2), (4, 5), (5, 1), (5, 4)\}$

☐ **C.** $\{(1, 2), (2, 2), (3, 3), (2, 4), (5, 4), (1, 5), (4, 5)\}$

■ **D.** $\{(1, 2), (2, 1), (2, 2), (3, 3), (4, 2), (4, 5), (5, 1)\}$

☐ **E.** $\{(2, 1), (2, 2), (3, 3), (4, 2), (4, 5), (5, 1), (5, 4)\}$

In Questions 16–18 sets A, B and C are defined as $A = \{a, b, c\}$, $B = \{1, 2, 3, 4\}$ and $C = \{3, 4, 5\}$.

16. Which of the following sets of ordered pairs represents relation R given by the following matrix?

$$M = \begin{bmatrix} F & T & T & T \\ F & F & T & T \\ F & F & T & T \end{bmatrix}$$

■ **A.** $\{(a, 2), (a, 3), (a, 4), (b, 3), (b, 4), (c, 3), (c, 4)\}$

☐ **B.** $\{(a, 1), (a, 2), (a, 3), (a, 4), (b, 3), (c, 4), (c, 3), (c, 4)\}$

☐ **C.** $\emptyset$

☐ **D.** $A \times B$

☐ **E.** $\{(2, a), (3, a), (4, a), (3, b), (4, b), (3, c), (4, c)\}$

17. Let $S \subseteq B \times C$ be given by

$$S = \{(x, y) \mid x < y\}.$$

Which of the following sets of ordered pairs represent the inverse relation S^{-1} of S.

☐ **A.** $\{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5), (3, 4), (3, 5), (4, 5)\}$

■ **B.** $\{(3, 1), (4, 1), (5, 1), (3, 2), (4, 2), (5, 2), (4, 3), (5, 3), (5, 4)\}$

☐ **C.** $\emptyset$

☐ **D.** $A \times B$

☐ **E.** $\{(2, 1), (3, 1), (4, 1), (3, 2), (4, 2), (3, 1), (4, 2)\}$

18. What is the *second line* in the matrix representation of $S \circ R$.

☐ **A.** $T\ T\ T$

■ **B.** $F\ T\ T$

☐ **C.** $T\ F\ T$

☐ **D.** $T\ T\ F$

☐ **E.** $F\ T\ T\ T$

19. Let $A = \{1, 2, 3, 4\}$ and $R \subseteq A \times A$ be given by

$$R = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 3), (4, 3)\}.$$

☐ **A.** R is reflexive, symmetric, antisymmetric, and not transitive

☐ **B.** R is reflexive, not symmetric, antisymmetric, and not transitive

☐ **C.** R is reflexive, not symmetric, not antisymmetric, and not transitive

☐ **D.** R is not reflexive, symmetric, antisymmetric, and transitive

■ **E.** R is not reflexive, not symmetric, not antisymmetric, and not transitive

20. Let $T = \{(2, 3), (1, 2), (3, 1)\}$. Which of the following five sets represents the transitive closure of T?

■ **A.** $\{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$

☐ **B.** $\{(1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 2)\}$

☐ **C.** $\{(1, 1), (1, 2), (2, 2), (2, 3), (3, 1), (3, 3)\}$

☐ **D.** $\{(1, 2), (2, 3), (3, 1)\}$

☐ **E.** $\emptyset$

21. Which of the following sentences are propositions?

   (1) London is the capital of Paris

   (2) A banana is larger than its skin

   (3) $2 + 3 = 5$

   (4) $5 + 7 = 10$

   (5) Answer this question

☐ **A.** (1), (2) and (5) only

☐ **B.** (3) only

☐ **C.** (3) and (4) only

☐ **D.** all of them

■ **E.** (1), (2), (3) and (4) only

22. Solve the following logical puzzle. You are on an island that has two kinds of inhabitants, knights, who always tell the truth, and their opposites, knaves, who always lie. You encounter two people *A* and *B*. What are *A* and *B* if *A* says "*B* is a knave" and *B* says "The two of us are knights."

☐ **A.** *A* is a knight and *B* is a knight

■ **B.** *A* is a knight and *B* is a knave

☐ **C.** *A* is a knave and *B* is a knight

☐ **D.** *A* is a knave and *B* is a knave

☐ **E.** It is not possible to give an answer to this puzzle

23. Which of the following is true?

☐ **A.** $(p_0 \vee p_1) \equiv (p_0 \wedge p_1)$

■ **B.** $(p_0 \wedge p_1) \equiv (p_0 \wedge (p_0 \rightarrow p_1))$

☐ **C.** $(p_0 \wedge (p_0 \rightarrow p_1)) \equiv p_1$

☐ **D.** $(p_1 \wedge (p_0 \rightarrow p_1)) \equiv \neg p_0$

☐ **E.** $(p_0 \rightarrow p_1) \equiv (\neg p_1 \vee p_0)$

24. Which of the following is the binary of 196?

■ **A.** 11000100

☐ **B.** 11100100

☐ **C.** 10010100

☐ **D.** 11000010

☐ **E.** 11010010

25. In how many ways can a 2-person subcommittee be chosen from a 6-person committee?

☐ **A.** 2×6

■ **B.** $\binom{6}{2}$

☐ **C.** 6^2

☐ **D.** $6 + 5$

☐ **E.** 6×5