Artificial Intelligence (COMP111)

Exercise 4

Your answers to Questions 2 and 3 should be submitted on canvas for assignment *Exercise 4* either as a text entry, a text file (txt), a pdf file, or a photo of the handwritten solution. The deadline is Monday, 21st of October, at 8pm. You should also attempt to answer the other questions before your tutorial (but not submit them).

You obtain 1 point (1 percent of the final mark) if you make a reasonable attempt to answer Questions 2 and 3. You should also actively participate in your tutorial in the week starting Monday 21st of October.

We would like to encourage you to discuss the questions with your fellow students, but do not copy your answer from anybody else.

1. The BBC maintains a variety of knowledge bases (ontologies). In its sport ontology it introduces class names

   – Competition, DevisionalCompetition,
   – MultiRoundCompetition, KnockoutCompetition,
   – LeagueCompetion, UnitCompetition,
   – Match, MedalCompetition

   and states the following set K_r of rules about their relationships:

   – $\mathsf{DevisionalCompetition}(x) \rightarrow \mathsf{Competition}(x)$
   – $\mathsf{MultiRoundCompetition}(x) \rightarrow \mathsf{Competition}(x)$
   – $\mathsf{MedalCompetition}(x) \rightarrow \mathsf{Competition}(x)$
   – $\mathsf{UnitCompetition}(x) \rightarrow \mathsf{Competition}(x)$
   – $\mathsf{KnockoutCompetition}(x) \rightarrow \mathsf{MultiRoundCompetition}(x)$
   – $\mathsf{LeagueCompetition}(x) \rightarrow \mathsf{MultiRoundCompetition}(x)$
   – $\mathsf{Match}(x) \rightarrow \mathsf{UnitCompetition}(x)$

   Now consider the following set K_a atomic assertions:

   – LeagueCompetition(PremierLeague).
   – KnockoutCompetition(FACup).

Using the algorithm given in the lecture notes, compute the set *DerivedAssertions* for the sets K_r and K_a above.

Let K be a knowledge base containing the rules K_r:

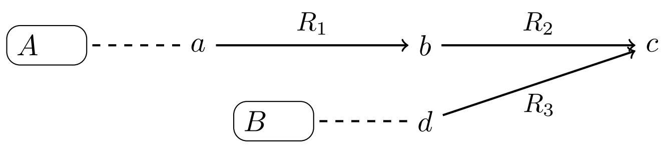
- $A_1(x) \wedge A_2(x) \rightarrow A(x)$
- $A_2(x) \wedge A_3(x) \rightarrow B(x)$
- $A(x) \rightarrow C(x)$

and the atomic assertions K_a:

- $A_1(a)$, $A_2(a)$, and $A_1(b)$.

2. Compute the set *DerivedAssertions* for the knowledge base K.
3. Using your answer to Question 2, decide whether $K \models B(a)$ and whether $K \models C(a)$.
4. Which of the following statements are correct? Explain you answers.
   - $\{A(a), B(b), A(x) \rightarrow B(x)\} \models B(a)$?
   - $\{A(a), B(b), A(x) \rightarrow B(x)\} \models A(b)$?
5. Let K be a knowledge base containing the rules K_r:
   - $A(x) \wedge R_3(x, y) \rightarrow C(x)$
   - $R_1(x, y) \wedge R_2(y, z) \rightarrow R_3(x, z)$

   and the following knowledge graph



   Which of the following statements are correct?
   - $K \models C(a)$?
   - $K \models C(d)$?
6. Let K be a knowledge base containing the rules:
   - $R(x, y) \rightarrow R^*(x, y)$
   - $R^*(x, y) \wedge R^*(y, z) \rightarrow R^*(x, z)$
   - $R^*(x, y) \wedge C(y) \wedge R^*(y, z) \rightarrow S(x, z)$

Assume that K_a contains $R(a_1, a_2)$, $R(a_2, a_3)$, $R(a_3, a_4)$, $R(a_4, a_5)$, $C(a_3)$.

For which (a_i, a_j) does it hold that $K \models S(a_i, a_j)$?

7. We reduce the problem of finding a path from the start state to a goal state in a search graph to a logical deduction problem.

   Let S be the set of states of a search graph G, s_{start} its start state and S_{goal} its goal states. We can regard G as a knowledge graph K_a as follows: regard the states in S as individual names and take a class name S_{goal} (note that we use S_{goal} both as a name for the set of goal states and as a class name) and a binary relation symbol $\text{action}(x, y)$. Then the knowledge graph K_a representing G contains

   – $S_{\text{goal}}(s)$ for every goal state s of G;
   – $\text{action}(s_1, s_2)$ if s_1, s_2 are states in S and $s_1 \to s_2$ holds in G.

   Take one more class name, Reach, and assume that the knowledge base K contains K_a and the following rules:

   – $S_{\text{goal}}(x) \to \text{Reach}(x)$;
   – $\text{action}(x, y) \wedge \text{Reach}(y) \to \text{Reach}(x)$.

   Now argue that there is a path in the search graph from s_{start} to a goal state if, and only if, $K \models \text{Reach}(s_{\text{start}})$.

   To answer the question, you might first want to consider an example. Take, for instance, the search graph from Exercise 1 with states $\{a, b, c, d, e, f\}$, start state $s_{\text{start}} = a$, and with $S_{\text{goal}} = \{f\}$:

$$\begin{tikzcd}
 & b \arrow[dr] & & \\
a \arrow[ur] \arrow[r] \arrow[dr] & c \arrow[r] & e \arrow[r] & f \\
 & d \arrow[ur] & &
\end{tikzcd}$$

   Then K_a contains $\text{action}(a, b)$, $\text{action}(a, c)$, $\text{action}(a, d)$, $\text{action}(b, e)$, $\text{action}(c, e)$, $\text{action}(d, e)$, $\text{action}(e, f)$, and $S_{\text{goal}}(f)$.

   Now check how you derive $K \models \text{Reach}(a)$ in this case.