COMP111: Artificial Intelligence

Introduction to Tree Search Problems

Frank Wolter

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- Show how the search tree is generated from the search graph;
- Introduce main blind (also called uninformed) tree search algorithms:
  - breadth first-search (BFS);
  - depth first-search (DFS).
- Introduce main performance measures for tree search algorithms and analyse BFS and DFS.

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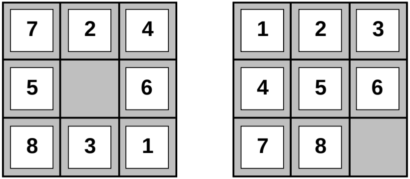
- Once the problem has been formulated, looking for a sequence of actions that lead to a goal state and is optimal is called search.

Four Examples

- 8-Puzzle
- Holiday in Romania
- Vacuum cleaner world
- The 8 Queens Problem

Example 1: 8-Puzzle

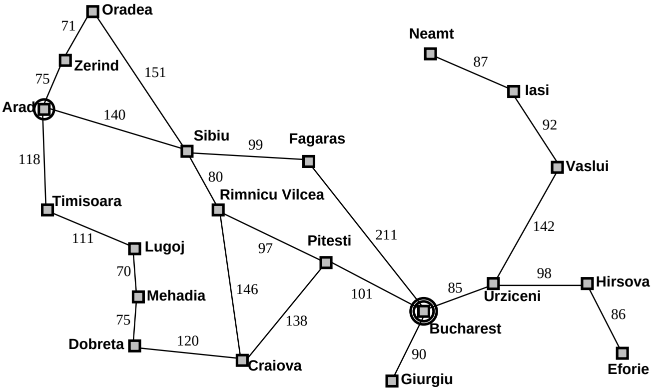


Start State   Goal State

The 8-puzzle is a sliding puzzle that consists of a frame of numbered square tiles in random order with one tile missing. The object of the puzzle is to place the tiles in order by making sliding moves that use the empty space.

Example 2: Holiday in Romania

On holiday in Romania; currently in Arad.
Drive as quickly as possible to Bucharest.



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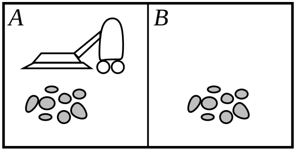
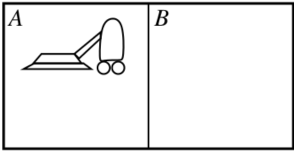


Possible Start

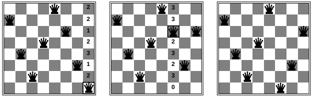


Possible Goal

The vacuum cleaner world is composed of

- two rooms
- each room might contain dirt
- a vacuum cleaner located in one of the rooms
  - vacuum actions: move left/right, suck up the dirt, do nothing
- the goal is to clean all rooms

Example 4: The 8 Queens Problem



- This is a problem from chess.
- Place 8 queens on chess board such that no queen attacks any other.

(A queen attacks any piece in the same row, column or diagonal.)

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We determine the search graph for our four problems.

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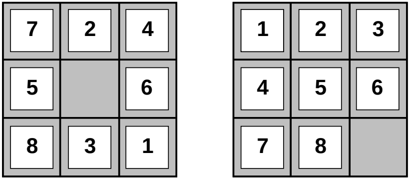

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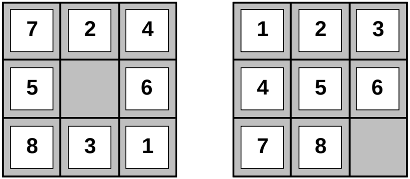

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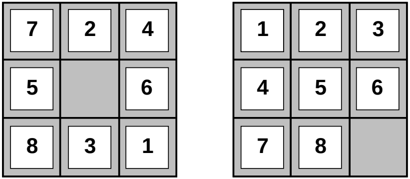

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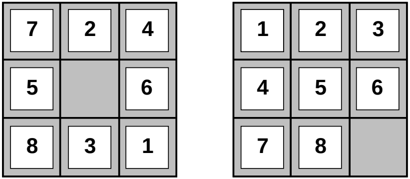

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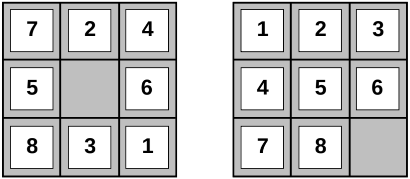

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- Four actions: move the tile to left of empty square to right; etc..
- Cost function: all actions have the same cost. (We set cost$=1$ for each action.) We want a shortest sequence of actions that leads from s_{start} to the goal.

Exercise

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- When drawing a search graph, we often abstract from the actions. Instead, we just draw arrows from any state s to the states that can be reached by a single action. They are called the successor states of s.
- Draw a partial picture of the search graph of the 8-puzzle by listing the successor states of the start state from the previous slide and the predecessor states of the goal state. (We call a state s' is a predecessor of a state s if s is a sucessor of s').

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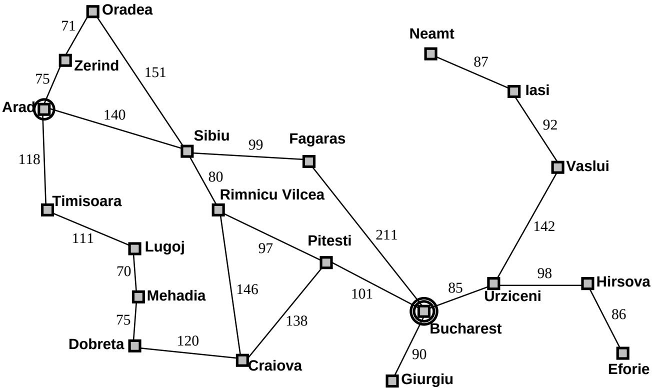

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In this case, the map from the previous slide is already a representation of the search graph!

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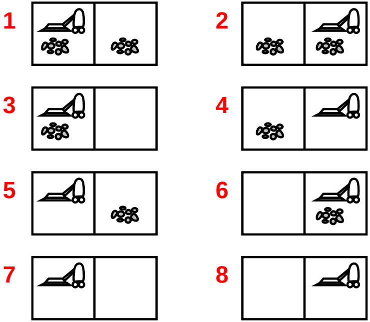

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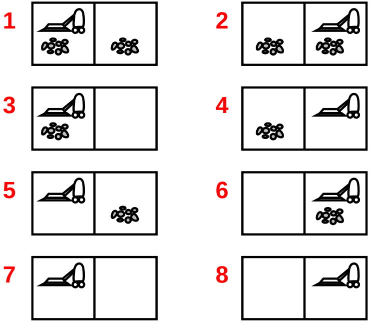

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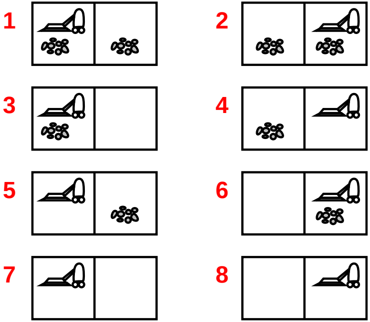

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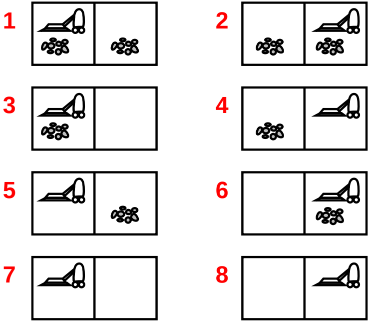

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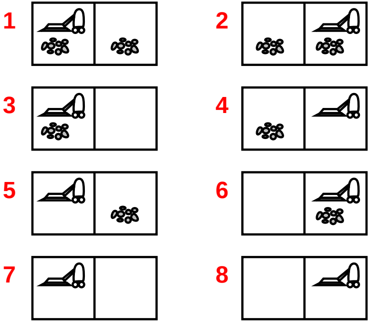

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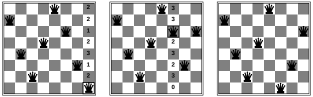
- Actions: move left/right, suck up the dirt, do nothing;
- Cost function: assume all actions equally expensive (cost=1).

Example 4: 8 Queens Problem



- Place 8 queens on chess board so that no queen can be taken by another.

(A queen attacks any piece in the same row, column or diagonal.)

Search graph for 8 Queens Problem

- Set S of states: any arrangement of 0 to 8 queens on the board;
- Start state s_{start}: empty chess board;
- Set S_{goal} of goal states: 8 queens on chess board such that no queen attacks another;
- Actions: add a queen to an empty square;
- Cost function: none needed.

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- For guaranteed realisability, *any* real state "in Arad" must get to real state "in Zerind"
- Abstract solution = set of real paths that are solutions in the real world

Abstraction should be "easier" than the original problem!

Size of the search graph

In the toy examples given above the search space is still rather small. In real search problems the state space is huge:

Problem	States	Brute-Force Search (10 million states/sec)
8 Puzzle	10^6	0.01 seconds
15 Puzzle	10^{13}	6 days
Rubik's Cube	10^{18}	68, 000 years
24 Puzzle	10^{26}	12 billion years
Checkers	10^{40}	
Chess	10^{120}	

In what follows we assume we can't store the whole search graph explicitly. It is imaginary.

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Note that the whole resulting search tree is imaginary. Typically it cannot be stored.

Search Tree for holiday in Romania

The paths starting in Arad are:

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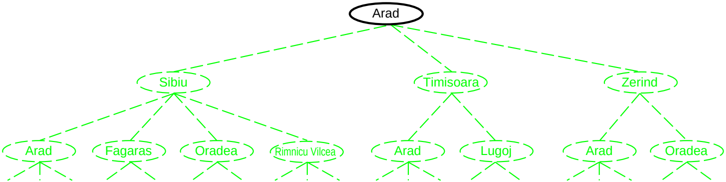
The set of all paths can be depicted as a tree with root Arad:



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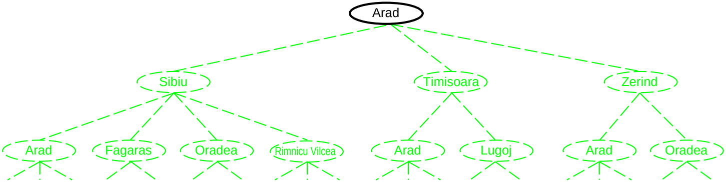
- The root of the tree is the start state s_{start}
- Applying actions, we generate longer paths by adding successor states to a path (also called expanding a path/state)



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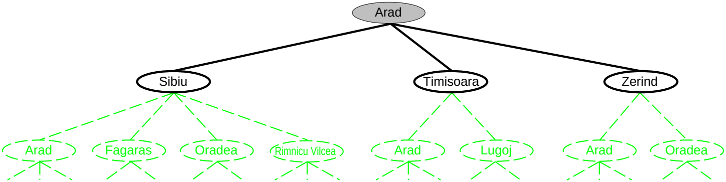

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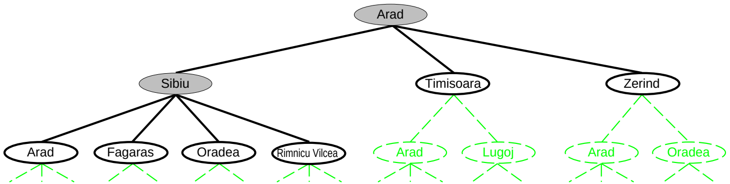
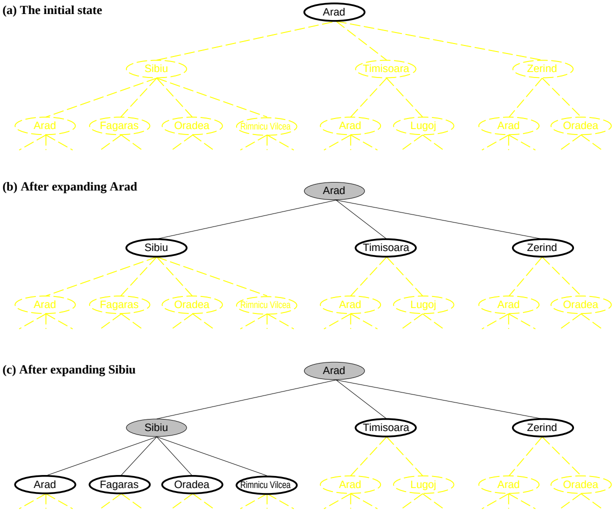


The Search Tree



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In this module we give algorithms (such as seach algorithms) in pseudocode, informal high-level descriptions of algorithms that are intended for human reading rather than machine reading.

Algorithms in pseudocode often abstract from implementation details and leave certain steps (or subroutines) unspecified.

We now give a generic algorithm for search in pseudocode that, in fact, leaves a very important step unspecified.

Generic Algorithm for Search

Input: a start state s_0
for each state s the successors of s
a test goal(s) checking whether s is a goal state

Set frontier := $\{s_0\}$
while frontier is not empty **do**
select and remove from frontier a path $s_0....s_k$
if goal(s_k) **then**
return $s_0 \dots s_k$ (and terminate)
else for every successor s of s_k add $s_0 \dots s_k s$ to frontier
end if
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- It terminates and returns the path if its final state is a goal state.
- Otherwise it adds to frontier all paths obtained by expanding (adding a successor to) the removed path.
- It then repeats the same steps by selecting and removing a path from the updated frontier.

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- Two obvious strategies:
  - breadth first search, BFS, (always select the path first added to frontier);
  - depth first search, DFS, (always select the path last added to frontier).

Breadth First Search (BFS)

- Start by selecting (expanding) start state — gives tree of depth 1.

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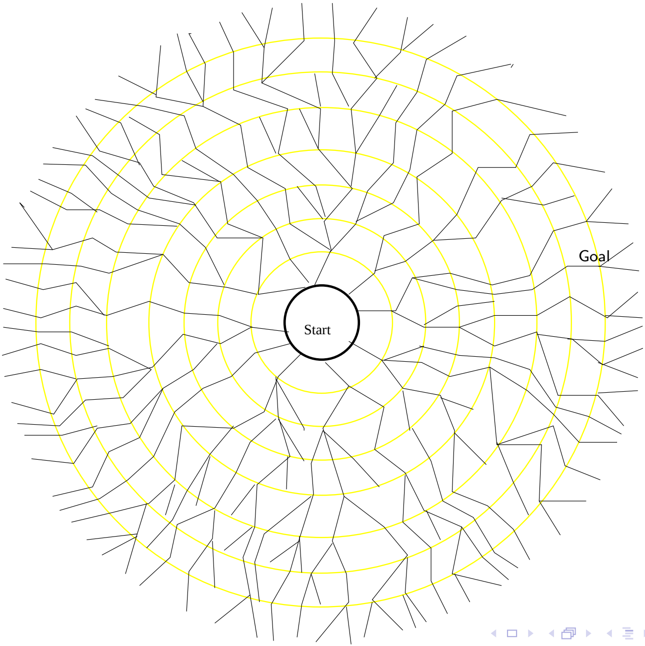
- and so on.
- In general: select (expand) all paths of depth n before depth $n+1$.

Example: BFS in a Maze



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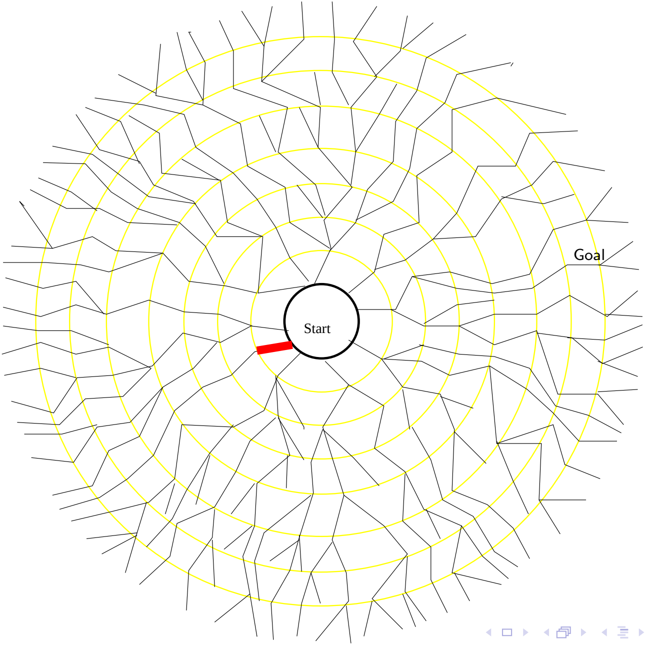

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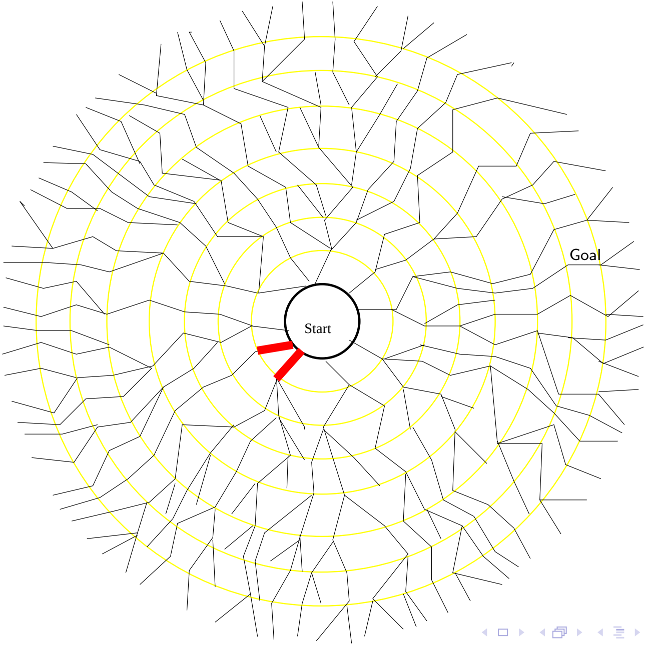

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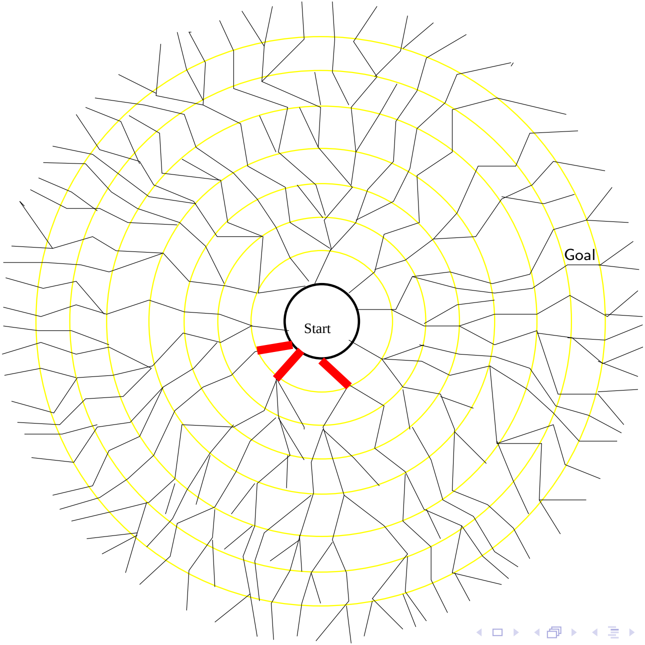

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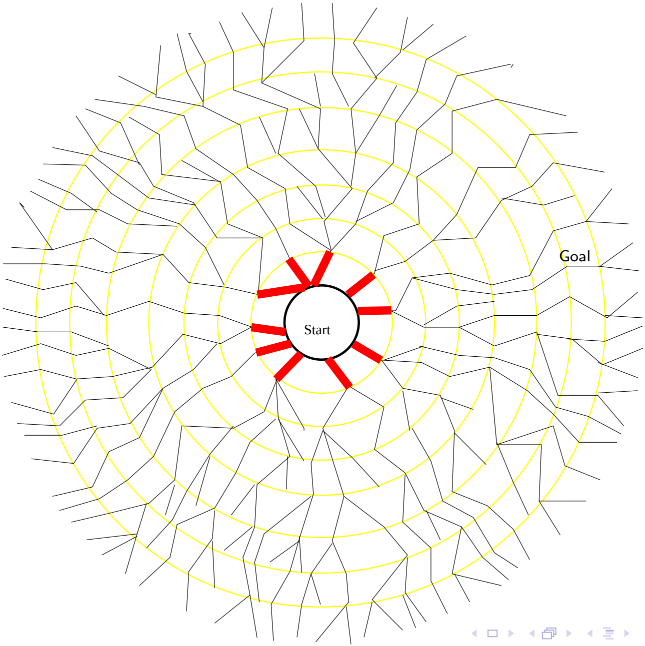

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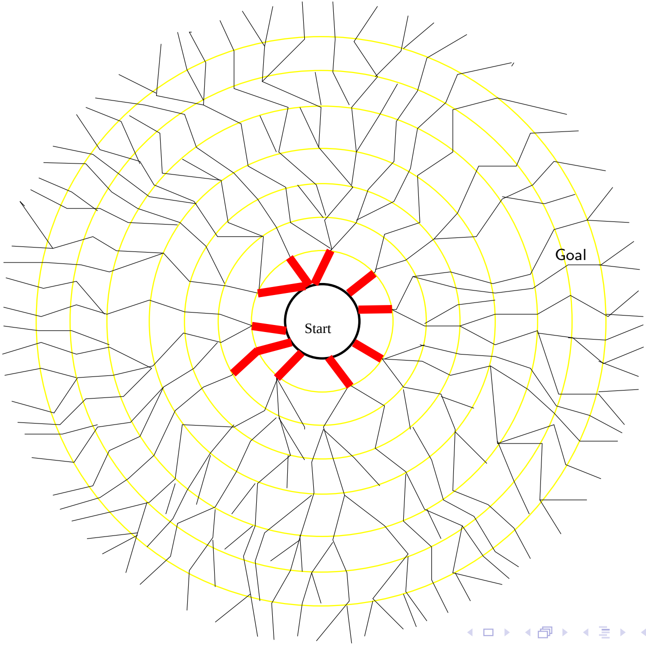

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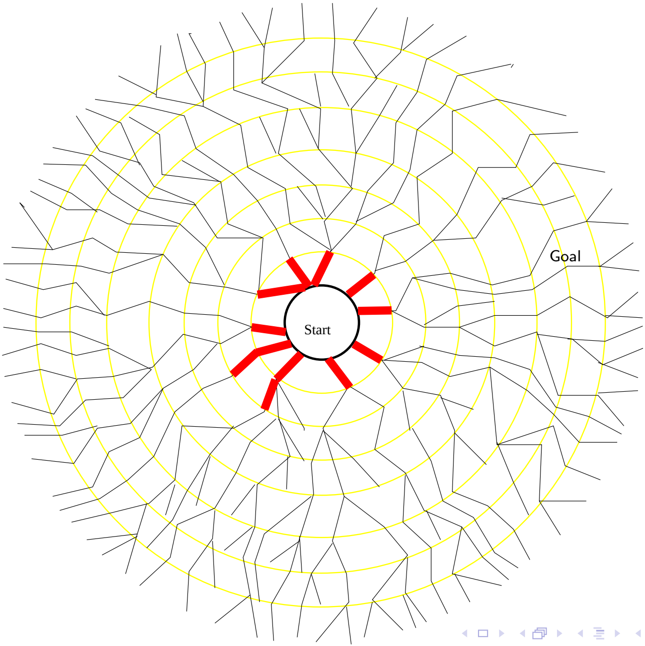

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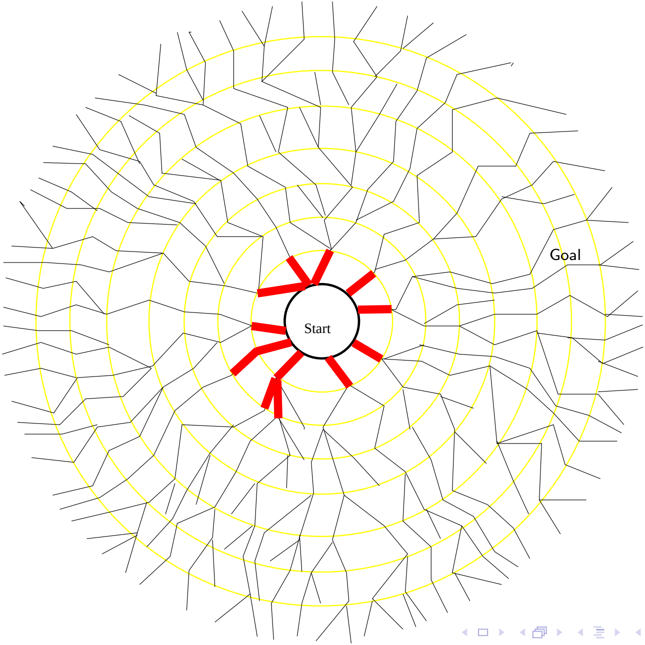

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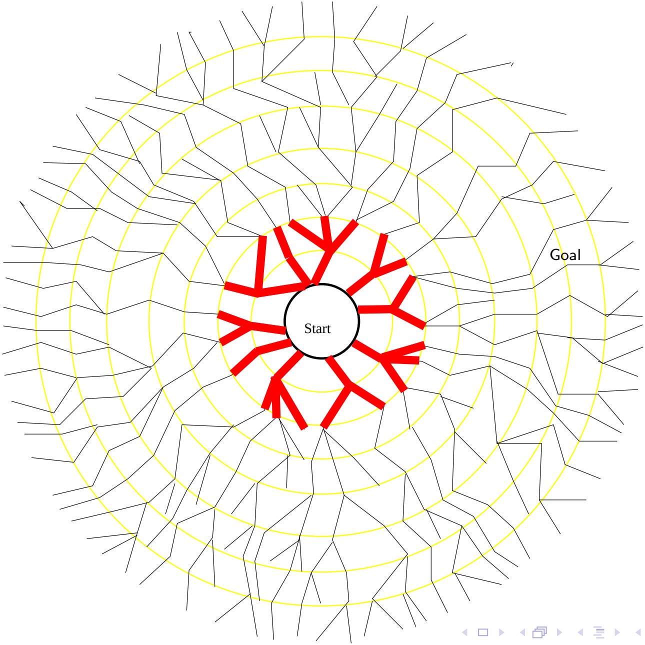

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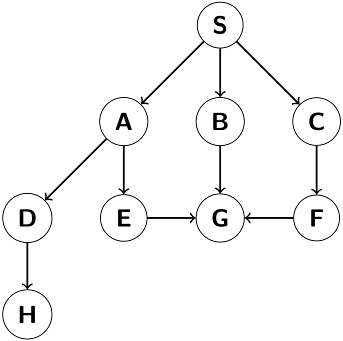
Breadth-First Example

Reaching G from S

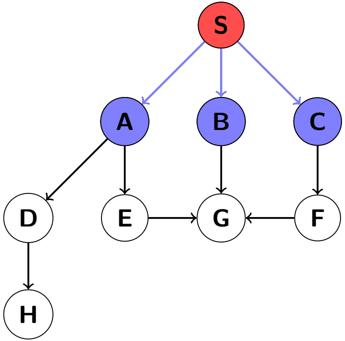
Exp. paths	Frontier
	$\{S\}$



Breadth-First Example

Reaching G from S

Exp. paths	Frontier
	$\{S\}$
S not goal	$\{SA, SB, SC\}$

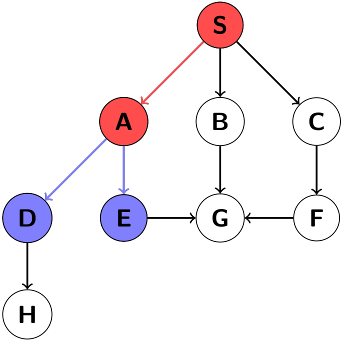


Selected path: S
Is the last state in S_{goal}? No
Expand S: add $SA, SB,$ and SC to the frontier

Breadth-First Example

Reaching G from S

Exp. paths	Frontier
	$\{S\}$
S not goal	$\{SA, SB, SC\}$
SA not goal	$\{SB, SC, SAD, SAE\}$



Selected path: SA

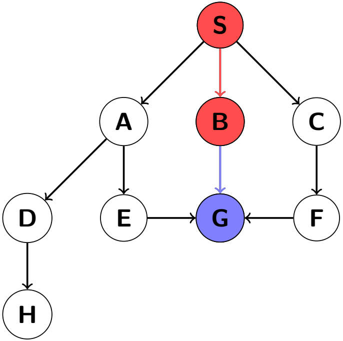
Is the last state in S_{goal}? No

Expand SA: add SAD and SAE to the frontier

Breadth-First Example

Reaching G from S

Exp. paths	Frontier
	$\{S\}$
S not goal	$\{SA, SB, SC\}$
SA not goal	$\{SB, SC, SAD, SAE\}$
SB not goal	$\{SC, SAD, SAE\ SBG\}$

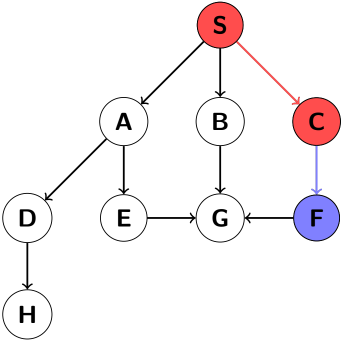


Selected path: SB
Is the last state in S_{goal}? No
Expand SB: add SBG to the frontier

Breadth-First Example

Reaching G from S

Exp. paths	Frontier
	$\{S\}$
S not goal	$\{SA, SB, SC\}$
SA not goal	$\{SB, SC, SAD, SAE\}$
SB not goal	$\{SC, SAD, SAE\ SBG\}$
SC not goal	$\{SAD, SAE, SBG, SCF\}$



Selected path: SC

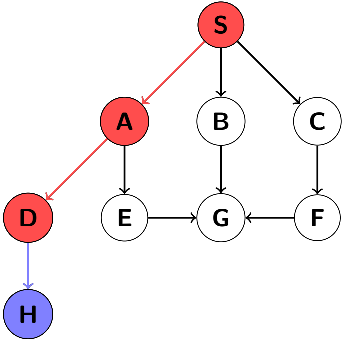
Is the last state in S_{goal}? No

Expand SC: add SCF to the frontier

Breadth-First Example

Reaching G from S

Exp. paths	Frontier
	$\{S\}$
S not goal	$\{SA, SB, SC\}$
SA not goal	$\{SB, SC, SAD, SAE\}$
SB not goal	$\{SC, SAD, SAE\ SBG\}$
SC not goal	$\{SAD, SAE, SBG, SCF\}$
SAD not goal	$\{SAE, SBG, SCF, SADH\}$



Selected path: SAD
Is the last state in S_{goal}? No
Expand SAD: add $SADH$ to the frontier

Breadth-First Example

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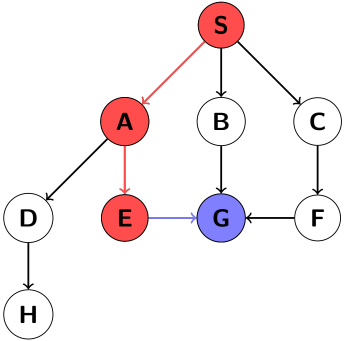

Selected path: SAE
Is the last state in S_{goal}? No
Expand SAE: add $SAEG$ to the frontier

Breadth-First Example

Reaching G from S

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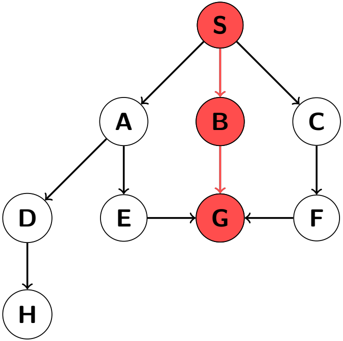
SAE not goal	$\{SBG, SCF, SADH, SAEG\}$
SBG goal!	$\{SCF, SADH, SAEG\}$



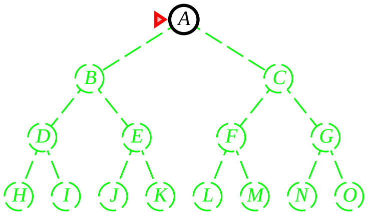
Selected path: SBG

Is the last state in S_{goal}? Yes!

Path found: SBG

Depth First Search (DFS)

- Start by selecting start state.
- Select one of the paths resulting from 1st step.
- Select one of the paths resulting from 2nd step.
- Select one of the paths resulting from 3nd step. And so on.
- Always select longest (also called deepest) path.
- Follow one “branch” of search tree.



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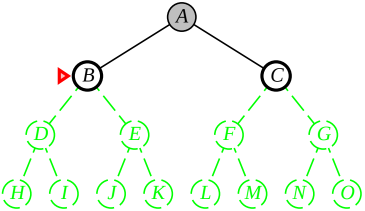

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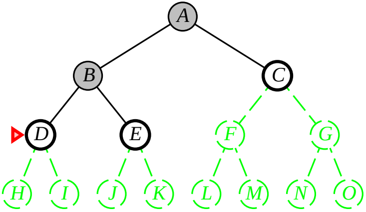

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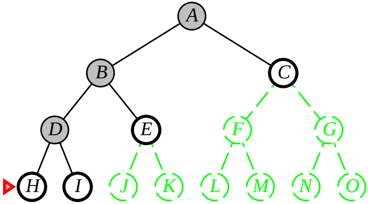

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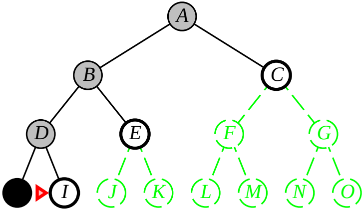

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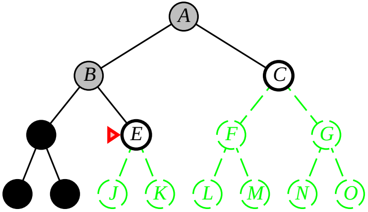

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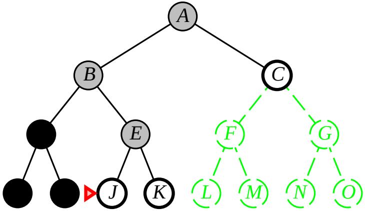

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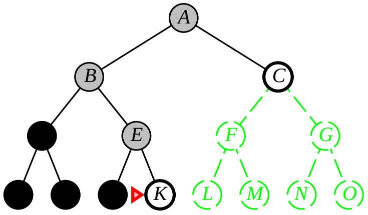

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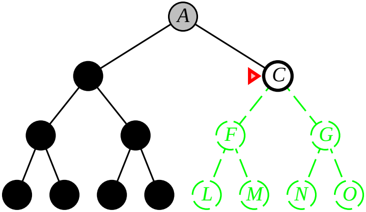

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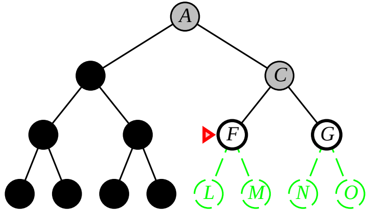

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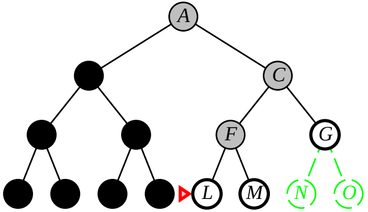

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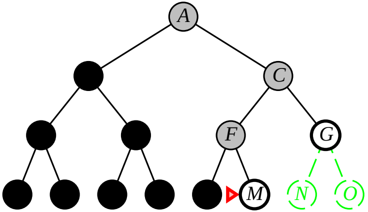

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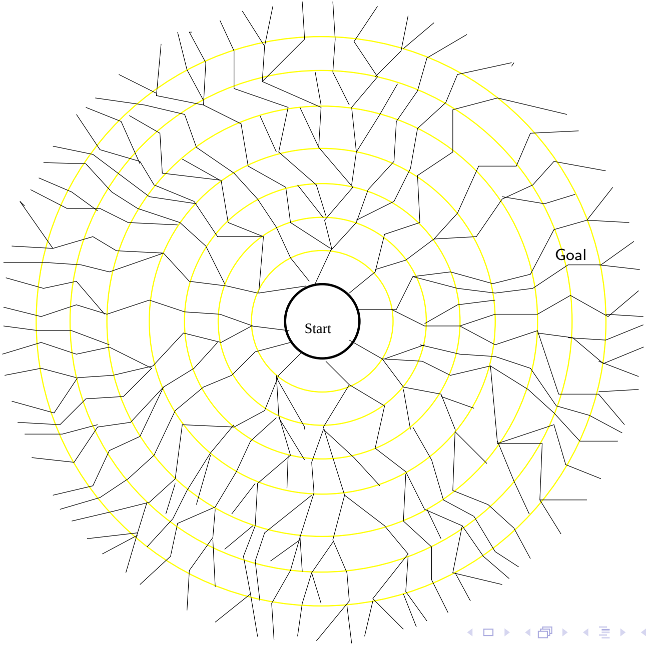

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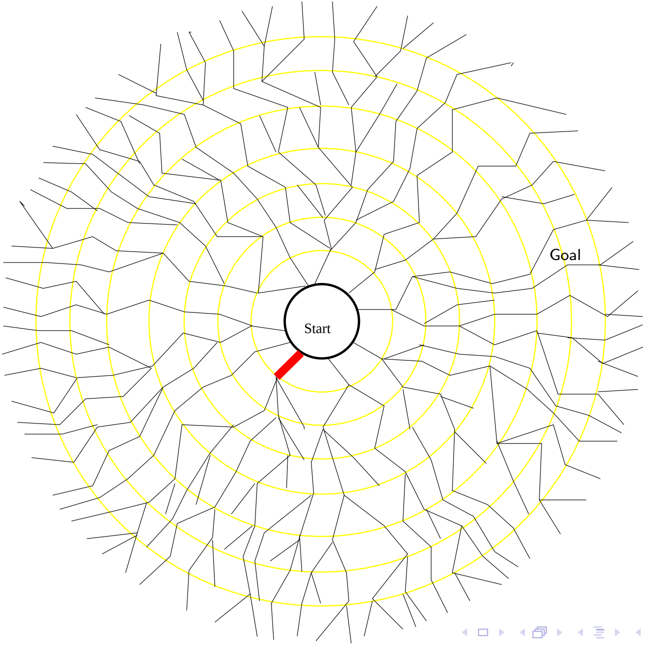

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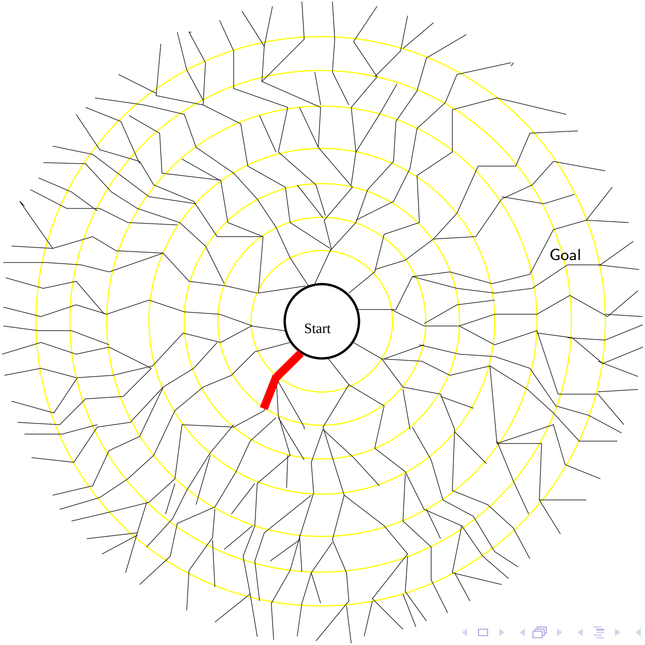

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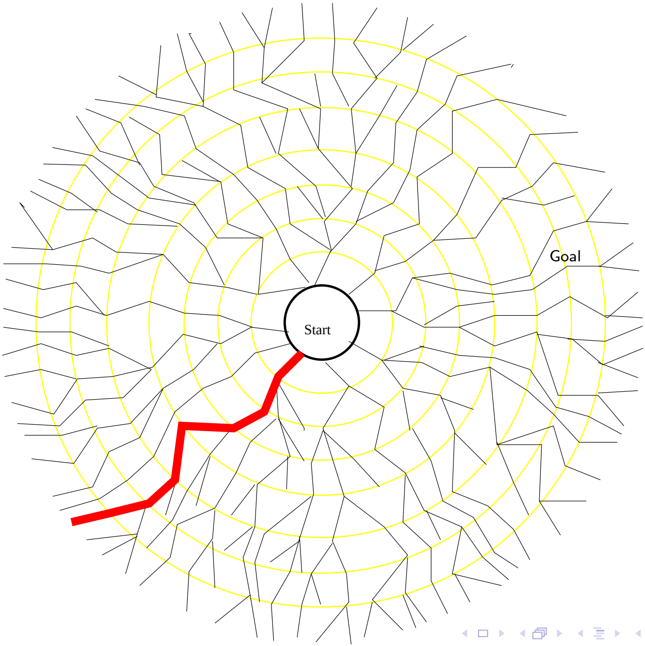

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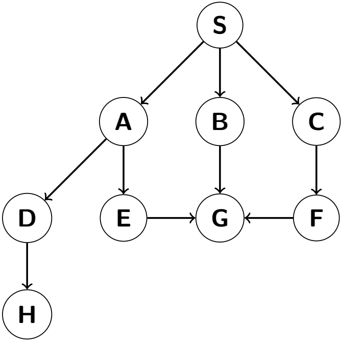
Depth-First Example

Reaching G from S

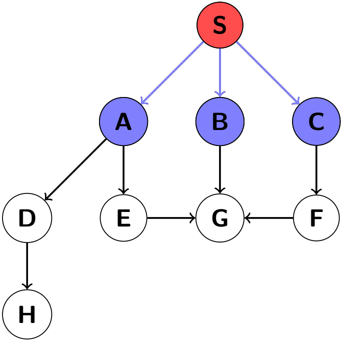
Exp. paths	Frontier
	$\{S\}$



Depth-First Example

Reaching G from S

Exp. paths	Frontier
	$\{S\}$
S not goal	$\{SA, SB, SC\}$



Selected path: S

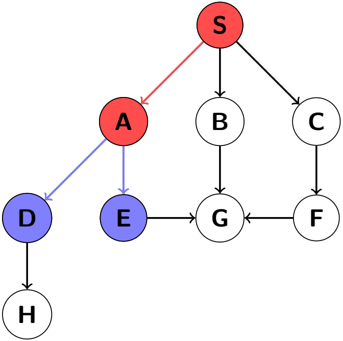
Is the last state in S_{goal}? No

Expand S: add $SA, SB,$ and SC to the frontier

Depth-First Example

Reaching G from S

Exp. paths	Frontier
	$\{S\}$
S not goal	$\{SA, SB, SC\}$
SA not goal	$\{SB, SC, SAD, SAE\}$



Selected path: SA

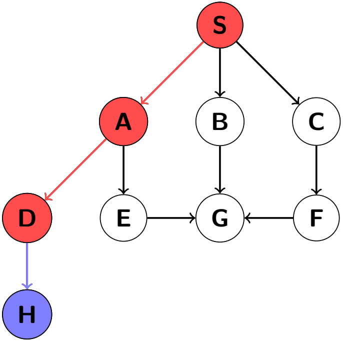
Is the last state in S_{goal}? No

Expand SA: add SAD and SAE to the frontier

Depth-First Example

Reaching G from S

Exp. paths	Frontier
	$\{S\}$
S not goal	$\{SA, SB, SC\}$
SA not goal	$\{SB, SC, SAD, SAE\}$
SAD not goal	$\{SB, SC, SAE, SADH\}$



Selected path: SAD

Is the last state in S_{goal}? No

Expand SAD: add $SADH$ to the frontier

Depth-First Example

Reaching G from S

Exp. paths	Frontier
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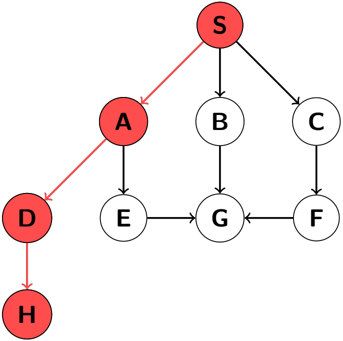

Selected path: $SADH$
Is the last state in S_{goal}? No
Expand $SADH$: nothing to add

Depth-First Example

Reaching G from S

Exp. paths	Frontier
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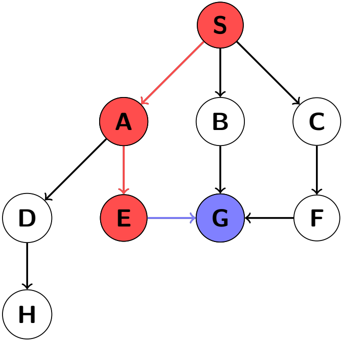
S not goal	$\{SA, SB, SC\}$
SA not goal	$\{SB, SC, SAD, SAE\}$
SAD not goal	$\{SB, SC, SAE, SADH\}$
$SADH$ not goal	$\{SB, SC, SAE\}$
SAE not goal	$\{SB, SC, SAEG\}$



Selected path: SAE

Is the last state in S_{goal}? No

Expand SAE: add $SAEG$ to the frontier

Depth-First Example

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SA not goal	$\{SB, SC, SAD, SAE\}$
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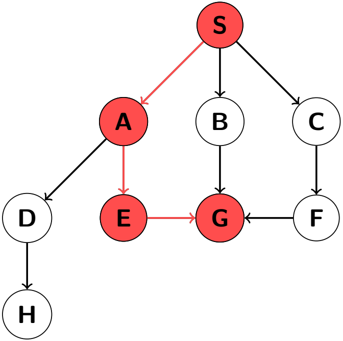
$SADH$ not goal	$\{SB, SC, SAE\}$
SAE not goal	$\{SB, SC, SAEG\}$
$SAEG$ goal	$\{SB, SC\}$



Selected path: $SAEG$
Is the last state in S_{goal}? Yes!
Path found: $SAEG$

Properties of Search Strategies

- Completeness: does the algorithm always find a solution if one exists?

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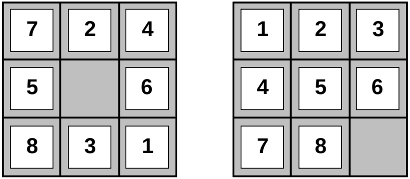
  - d—depth of the optimal solution: this is the length of the shortest path from the start state to a goal state.
  - m—maximum depth of the state space: this is the length of the longest path in the state space (may be ∞)

Example 1: 8-Puzzle



Start State Goal State

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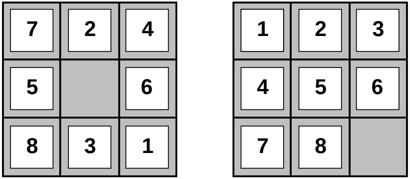

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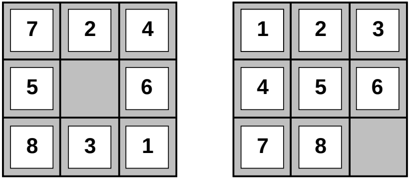

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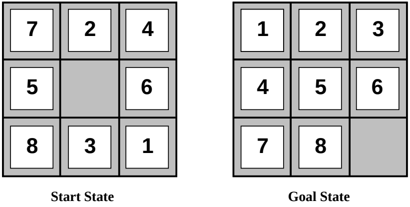

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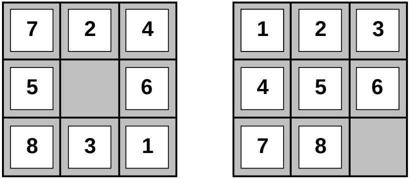

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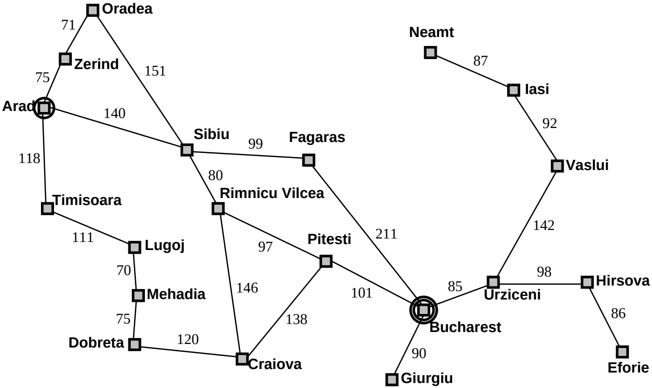
In fact, for the following two start states the minimal solution paths have length 31.

8	7	6			8		6
	4	1			5	4	7
2	5	3			2	3	1

Example 2: Holiday in Romania

Drive as quickly as possible from Arad to Bucharest.



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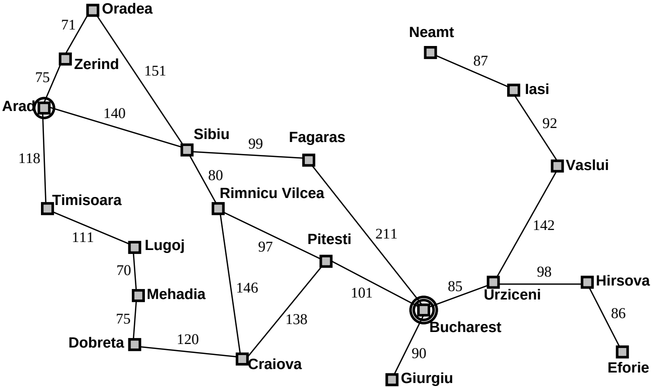

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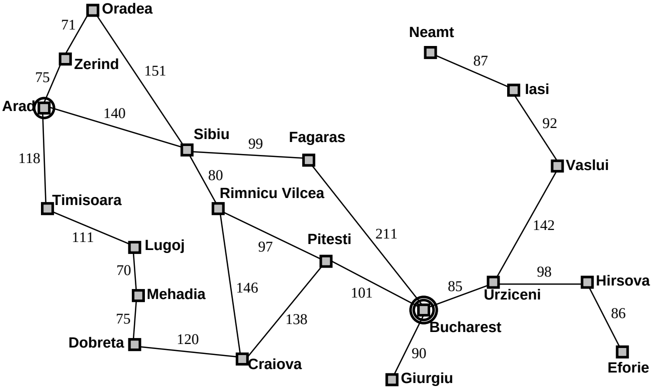

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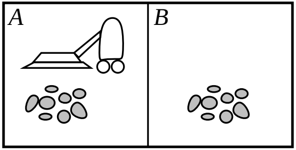
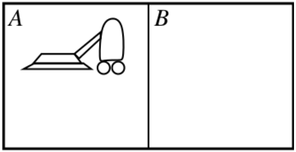


Possible Start



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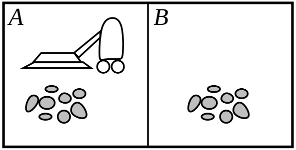

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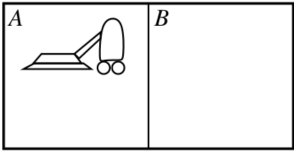

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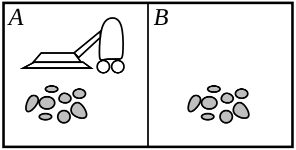

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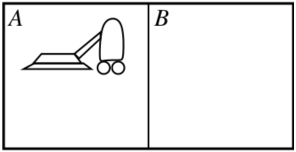

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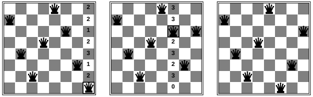
Example 4: The 8 Queens Problem



- This is a problem from chess.
- Place 8 queens on chess board such that no queen attacks any other.

(A queen attacks any piece in the same row, column or diagonal.)

- the maximum branching factor is $b = 8 \times 8$.

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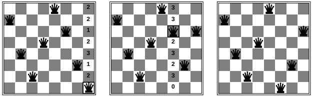

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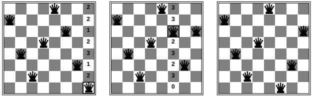

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- the length of all paths from the start state is at most 8. Thus $m = 8$.

Properties of Breadth First Search

Facts. (1) The number of paths of length d in a search tree with maximum branching factor b is at most b^d.
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- Advantage of BFS: it is optimal (always finds the shortest solution).

Complexity

Depth	paths	Time
0	1	1 msec
1	11	.01 sec
2	111	.1 sec
4	11,111	11 secs
6	10^6	18 mins
8	10^8	31 hours
10	10^{10}	128 days
12	10^{12}	35 years

Combinatorial explosion!

Time for breadth first search, assuming a branching factor of 10 and approximately 1000 states are expanded per second.

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- Space complexity: If the length of the longest path starting at a goal state is m, then in the worst case the frontier can contain

$$b \times m$$

  paths. If m is infinite, then the space requirement is infinite in the worst case. But if DFS finds a path to the goal state, then memory requirement is much less than for BFS.

Basic Search Strategies

- BFS is complete but expensive.
- DFS is cheap in space complexity but incomplete
- Can't we do better than this?

Next Lectures

Search strategies:

- *Blind* search strategies
  no additional information about states beyond that provided in the problem definition
  - Breadth First Search
  - Depth First Search
  - ...
- *Heuristic* search strategies
  Know whether one non-goal state is "more promising" than another
  - greedy search
  - A^* search