Artificial Intelligence (COMP111)
Exercise 5

Your answers to Questions 2 and 3(c) should be submitted on canvas for assignment *Exercise 5* either as a text entry, a text file (txt), a pdf file, or a photo of the handwritten solution. The deadline is Monday, 28th of October, at 8pm. You should also attempt to answer the other questions before your tutorial (but not submit them).

You obtain 1 point (1 percent of the final mark) if you make a reasonable attempt to answer Questions 2 and 3(c). You should also actively participate in your tutorial in the week starting Monday 28th of October.

We would like to encourage you to discuss the questions with your fellow students, but do not copy your answer from anybody else.

1. Assume that $P = \neg(p_1 \vee \neg p_2)$. Write down the truth table for P. Give the list of interpretations I under which P is true. How many different interpretations are there that make P true?

2. Assume that $P = (((p_1 \wedge \neg p_2) \wedge p_3) \Rightarrow p_2)$. Write down the truth table for P. How many different interpretations make P true. Give the list of interpretations I under which P false.

3. A formula P is called *satisfiable* if there exists an interpretation I such that $I(P) = 1$. Which of the following formulas are satisfiable? Check using truth tables.

   (a) $(\neg p_1 \wedge p_2)$.

   (b) $(p_1 \Rightarrow \neg p_1)$.

   (c) $(p_1 \Leftrightarrow \neg p_1)$.

   (d) $(p_1 \wedge (\neg p_2 \vee \neg p_1))$.

4. Consider a formula P with n atomic propositions. How many rows does the truth table for P have?

5. Consider $P = (((((\neg p_1 \wedge p_2) \vee p_3) \wedge p_4) \vee p_5) \Rightarrow p_1)$. Show without using truth tables that P is satisfiable.

6. Consider $P = (((((\neg p_1 \wedge p_2) \vee p_3) \wedge p_4) \vee p_5) \wedge (p_6 \wedge \neg p_6))$. Show without using truth tables that P is not satisfiable.

7. Which of the following are true? Check using truth tables or defining relevant interpretations directly.

   (a) $\{p_1, (p_1 \Rightarrow p_2), (p_2 \Rightarrow p_3)\} \models p_3$.

   (b) $\{(p_1 \Rightarrow p_2)\} \models (p_2 \Rightarrow p_1)$.

8. Three boxes are presented to you. One contains gold, the other two are empty. Each box has imprinted on it a clue. The clues are:

   – Box 1: "The gold is not here".

   – Box 2: "The gold is not here".

   – Box 3: "The gold is in Box 2".

   Only one clue is true; the other two are false. Which box contains the gold? Formalize the puzzle in propositional logic and find the solution using a truth table.