$y = \log_a x \qquad x = a^y$

$\log_3(9) = 2 \qquad 3^2 = 9$

$x = a^y \qquad \frac{d}{dx} \stackrel{?}{=} c\frac{d}{dy}$

$y = \log_a x$

$\frac{d}{dy} x = e^y \frac{d}{dy} \rightarrow \frac{dx}{dy} = \frac{1}{y} e^y$

$\frac{d}{dx} y = \ln x \frac{d}{dx} \rightarrow \frac{dy}{dx} =$

$$\underbrace{\frac{d}{dx}\ln(x)}_{\nabla} = \lim_{h\to 0}\left(\ln(x+h) - \ln(x)\right)/h$$

$$\nabla = \frac{\ln(x+h) - \ln(x)}{h} \text{, where } h \to 0$$

$$h\nabla = \ln(x+h) - \ln(x)$$

$$e^{h\nabla} = e^{\ln(x+h) - \ln(x)}$$

$$= \frac{e^{\ln(x+h)}}{e^{\ln x}}$$

$$= \frac{x+h}{x}$$

$$= 1 + \frac{h}{x}$$

$(e^{h\nabla})^{\frac{1}{h}} = (1+\frac{h}{x})^{\frac{1}{h}}$

$h\nabla = \ln(1+\frac{h}{x})$

$\nabla = \frac{\ln(1+\frac{h}{x})}{h}$

$\nabla = \frac{\ln(1+h/x)}{e^{\ln}}$

$\ln(x)/y \overset{?}{=} \ln(x-y)$

$e^{\ln(x)/y} = x-y$

$e^{\ln(x)} = (x-y)^y$

$x = (x-y)^y$

$e^{h\nabla} = 1+\frac{h}{x}$

$= e^0 + \frac{h}{x}$

$e^{h\nabla}/e^0 = (\frac{h}{x})/e^0$

$e^{h\nabla - 0} = \frac{h+1}{x}$

$e^x - e^y \overset{?}{=} e^{\ln(x-y)}$

$x-y = \ln(x-y)$

×

$y\ln(x) \overset{?}{=} \ln(x^y)$

$e^{y\ln x} = x^y$

$(e^{\ln x})^y = (x^y)$

$e^{\ln x} = (x^y)^{\frac{1}{y}}$

$x = x$

$h\nabla = \ln(\frac{x+h}{x})$

$\nabla = \ln(\frac{x+h}{x})\frac{1}{h}$

$\nabla = \ln((\frac{x+h}{x})^{-h})$

$\nabla = \ln(1^0) = 0$ ×

$\rightarrow \frac{\ln(x+h)-\ln(x)}{h}$

$\downarrow$

$\cancel{\ln((x+h)^{\frac{1}{h}}) - \ln(x^{1/h})}$

$\ln(\frac{x+h}{x})/h$

$h(1+\frac{h}{x})/h$

$\ln(1)\cdot\ln(\frac{h}{x})/h$

$\ln(1+\frac{h}{x})/h$

$\ln(1+t)/xt$

$\ln((1+t)^{1/t})/x \rightarrow \frac{\ln(1)^{1/t}\cdot\ln(t^{1/t})}{x}$

$\searrow \frac{1}{x}\ln((1+t)^{1/t})$