

$$\begin{aligned}
 & \frac{a^{x+h} - a^x}{h} \\
 & \lim_{h \rightarrow 0} \frac{a^{x+h} - a^x}{h} \\
 & = \log_a \left(\frac{a^{x+h} - a^x}{h} \right) \\
 & = \frac{a^x a^h - a^x}{h} \\
 & = a^x \left(\frac{a^h - 1}{h} \right)
 \end{aligned}$$

so, all that's left is to find a st. $\lim_{h \rightarrow 0} \left[\frac{a^h - 1}{h} \right] = 1$

$$so \quad a^h - 1 = h$$

$$a^h = h + 1$$

$$a = (h + 1)^{1/h} \quad \lim_{h \rightarrow 0} \dots$$

✓