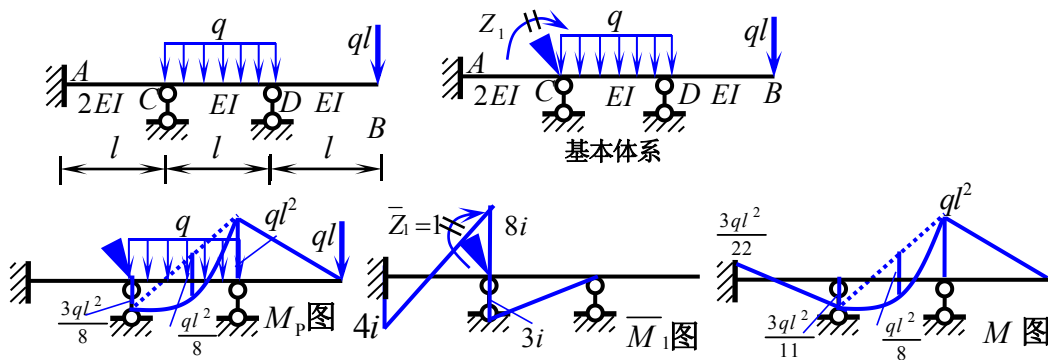


第五章 超静定结构位移法 答案

【题1】 用位移法计算图示结构，并作弯矩图。

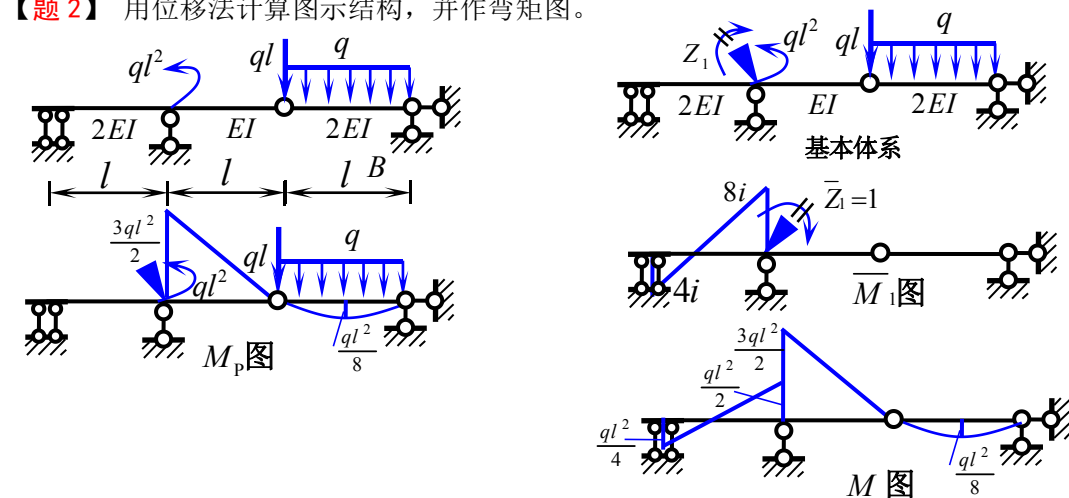


令线刚度: $i = EI/l$

$$k_{11}Z_1 + F_{1P} = 0 \quad k_{11} = 11i \quad F_{1P} = \frac{3ql^2}{8} \quad Z_1 = -\frac{3ql^2}{88i}$$

由 $M = \bar{M}_1 Z_1 + M_p$ 作最终弯矩图。

【题2】 用位移法计算图示结构，并作弯矩图。



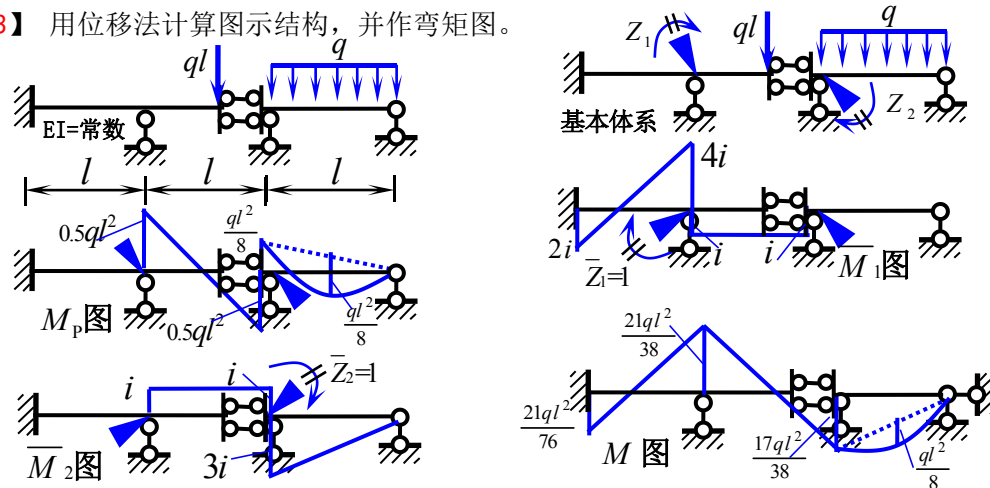
令线刚度: $i = EI/l$

$$k_{11}Z_1 + F_{1P} = 0$$

$$k_{11} = 8i \quad F_{1P} = -\frac{ql^2}{2} \quad Z_1 = \frac{ql^2}{16i}$$

由 $M = \bar{M}_1 Z_1 + M_p$ 作最终弯矩图。

【题3】 用位移法计算图示结构，并作弯矩图。



令线刚度: $i = EI/l$

$$\begin{aligned} k_{11}Z_1 + k_{12}Z_2 + F_{1P} &= 0 \\ k_{21}Z_1 + k_{22}Z_2 + F_{2P} &= 0 \end{aligned}$$

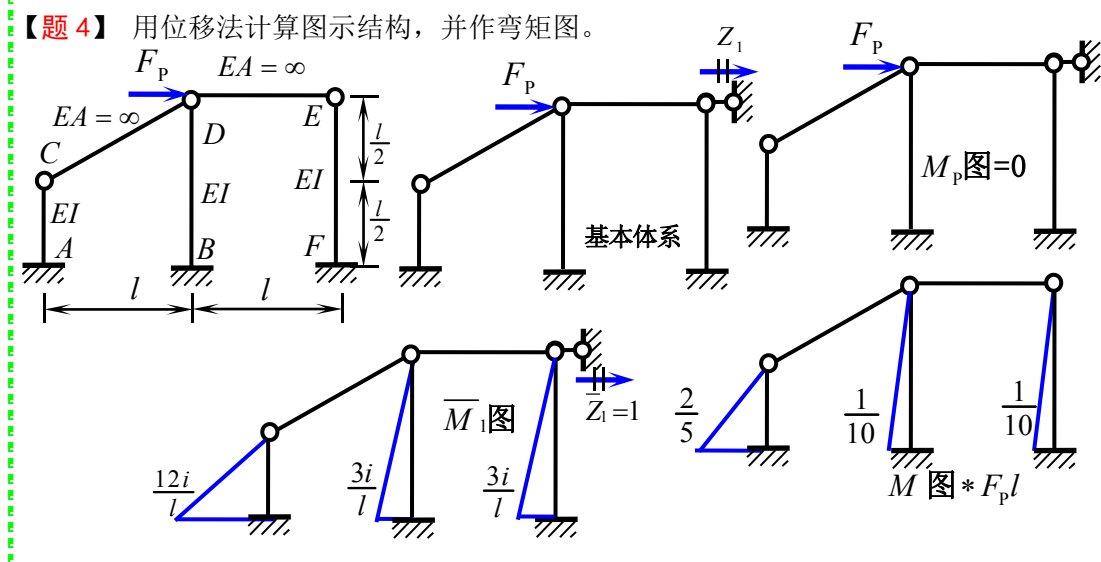
$$k_{11} = 5i \quad k_{22} = 4i \quad k_{21} = k_{12} = -i \quad F_{1P} = -ql^2/2 \quad F_{2P} = -5ql^2/8$$

$$\begin{aligned} 5iZ_1 - iZ_2 - ql^2/2 &= 0 \\ -iZ_1 + 4iZ_2 - 5ql^2/8 &= 0 \end{aligned}$$

$$Z_1 = 21ql^2/152i \quad Z_2 = 29ql^2/152i$$

由 $M = \overline{M}_1Z_1 + \overline{M}_2Z_2 + M_P$ 作最终弯矩图。

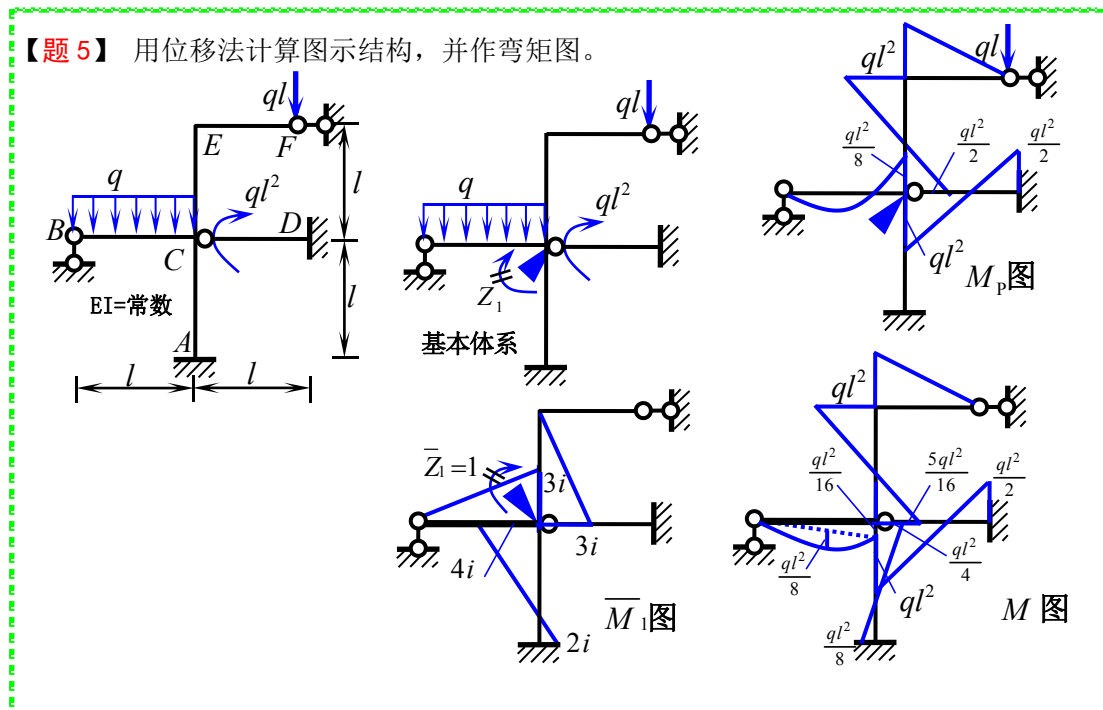
【题4】 用位移法计算图示结构，并作弯矩图。



令线刚度: $i = EI/l$

$$k_{11}Z_1 + F_{1P} = 0 \quad k_{11} = 30i/l^2 \quad F_{1P} = -F_P \quad Z_1 = F_P l^2 / 30i$$

由 $M = \bar{M}_1 Z_1 + M_P$ 作最终弯矩图。

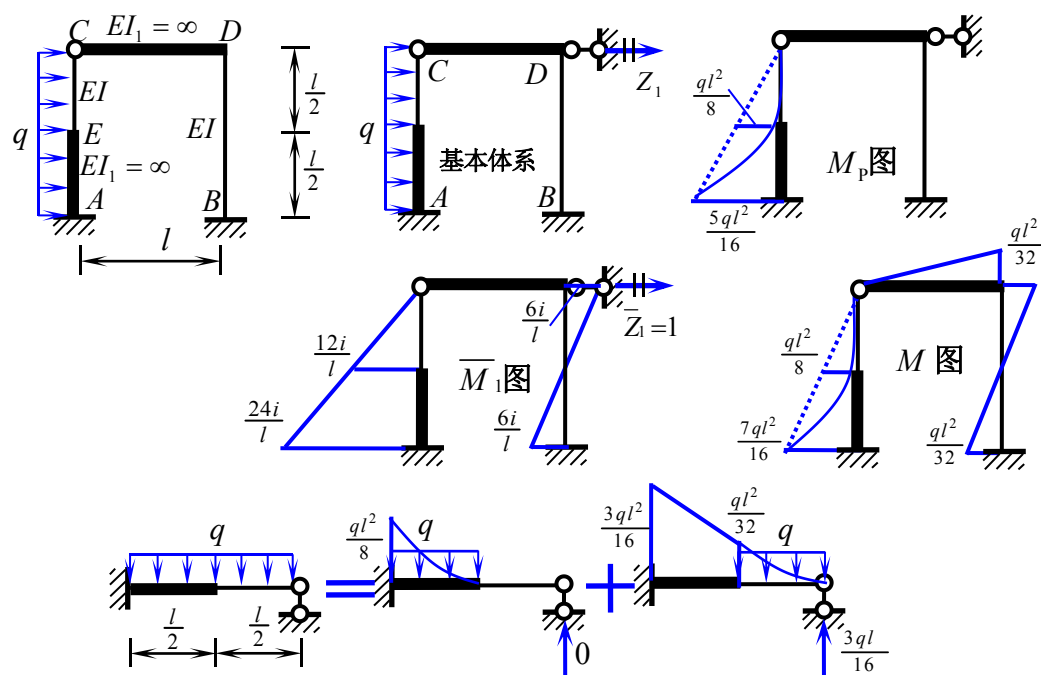


令线刚度: $i = EI/l$

$$k_{11}Z_1 + F_{1P} = 0 \quad k_{11} = 10i \quad F_{1P} = 5ql^2/8 \quad Z_1 = -ql^2/16i$$

由 $M = \bar{M}_1 Z_1 + M_P$ 作最终弯矩图。

【题6】 用位移法计算图示结构，并作弯矩图。

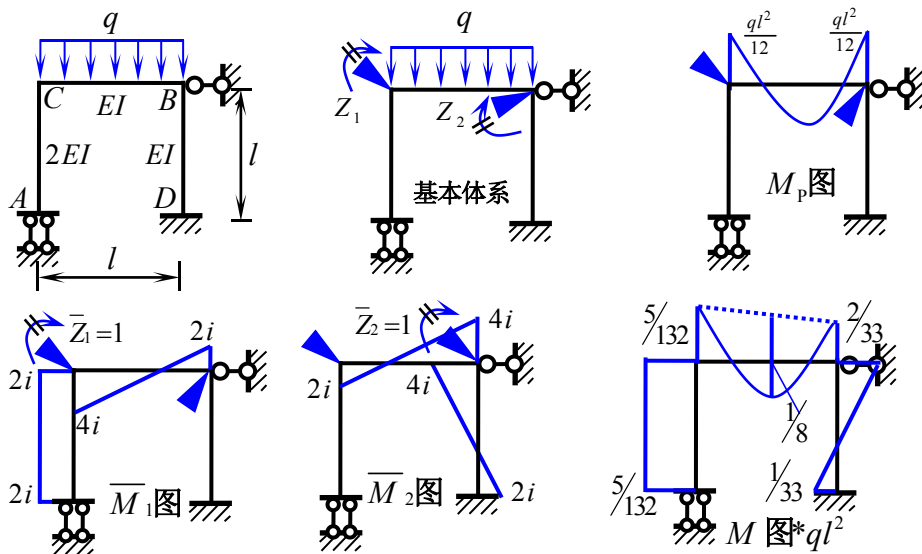


令线刚度： $i = EI/l$

$$k_{11}Z_1 + F_{1P} = 0 \quad k_{11} = 36i/l^2 \quad F_{1P} = -3ql/16 \quad Z_1 = ql^3/192i$$

由 $M = \bar{M}_1 Z_1 + M_P$ 作最终弯矩图。

【题7】 用位移法计算图示结构，并作弯矩图。



令线刚度： $i = EI/l$

$$k_{11}Z_1 + k_{12}Z_2 + F_{1P} = 0$$

$$k_{21}Z_1 + k_{22}Z_2 + F_{2P} = 0$$

$$k_{11} = 6i \quad k_{22} = 8i \quad k_{21} = k_{12} = 2i \quad F_{1P} = -ql^2/12 \quad F_{2P} = ql^2/12$$

$$\begin{aligned} 6iZ_1 + 2iZ_2 - ql^2/12 &= 0 \\ 2iZ_1 + 8iZ_2 + ql^2/12 &= 0 \end{aligned} \quad Z_1 = 5ql^2/264i \quad Z_2 = -ql^2/66i$$

由 $M = \bar{M}_1 Z_1 + \bar{M}_2 Z_2 + M_P$ 作最终弯矩图。

令线刚度: $i = EI/l$

$$k_{11}Z_1 + k_{12}Z_2 + F_{1P} = 0$$

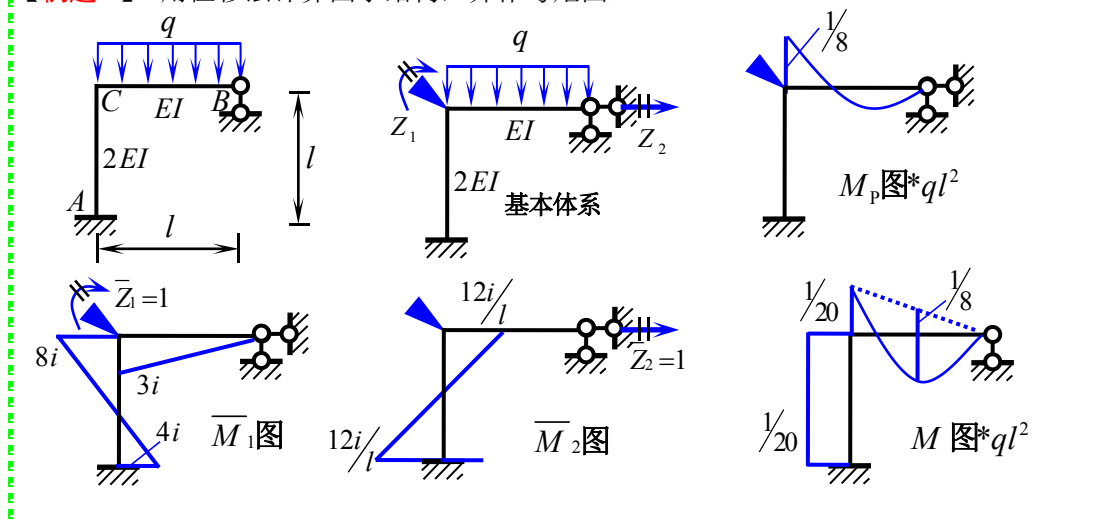
$$k_{21}Z_1 + k_{22}Z_2 + F_{2P} = 0$$

$$k_{11} = 12i \quad k_{22} = 12i/l^2 \quad k_{21} = k_{12} = -6i/l \quad F_{1P} = -ql^2/12 \quad F_{2P} = 0$$

$$\begin{aligned} 12iZ_1 - 6i/l Z_2 - ql^2/12 &= 0 \\ -6i/l Z_1 + 12i/l^2 Z_2 &= 0 \end{aligned} \quad Z_1 = ql^2/108i \quad Z_2 = ql^3/216i$$

由 $M = \bar{M}_1 Z_1 + \bar{M}_2 Z_2 + M_P$ 作最终弯矩图。

【例题 2】用位移法计算图示结构，并作弯矩图。



令线刚度: $i = EI/l$

$$k_{11}Z_1 + k_{12}Z_2 + F_{1P} = 0$$

$$k_{21}Z_1 + k_{22}Z_2 + F_{2P} = 0$$

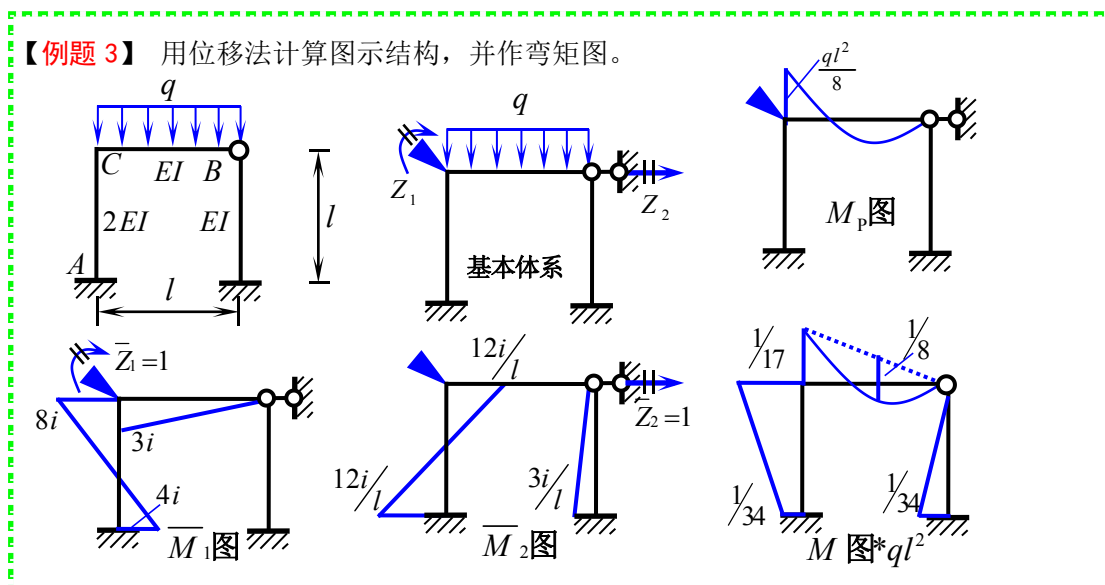
$$k_{11} = 11i \quad k_{22} = 24i/l^2 \quad k_{21} = k_{12} = -12i/l \quad F_{1P} = -ql^2/8 \quad F_{2P} = 0$$

$$11iZ_1 - 12i/l Z_2 - ql^2/8 = 0$$

$$-12i/l Z_1 + 24i/l^2 Z_2 = 0$$

$$Z_1 = ql^2/40i \quad Z_2 = ql^3/80i$$

由 $M = \overline{M}_1 Z_1 + \overline{M}_2 Z_2 + M_P$ 作最终弯矩图。



令线刚度： $i = EI/l$

$$k_{11}Z_1 + k_{12}Z_2 + F_{1P} = 0$$

$$k_{21}Z_1 + k_{22}Z_2 + F_{2P} = 0$$

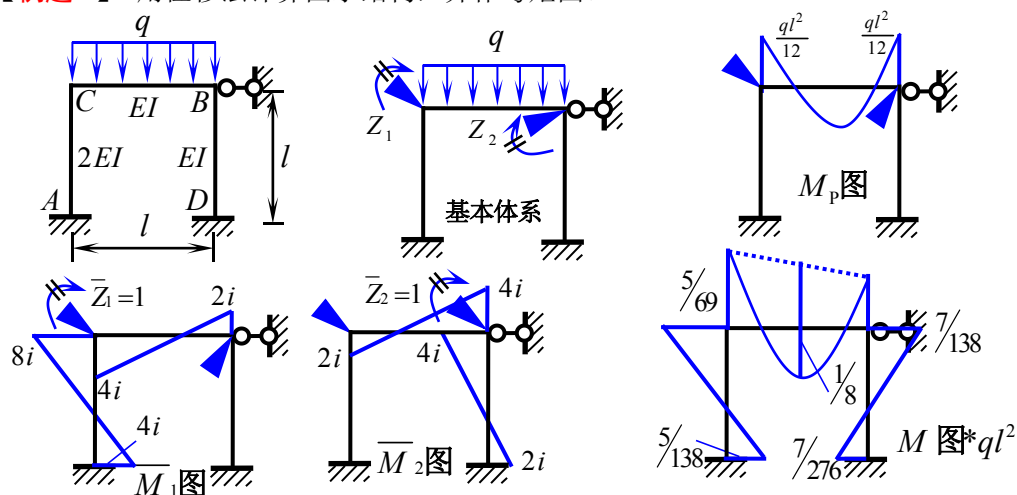
$$k_{11} = 11i \quad k_{22} = 27i/l^2 \quad k_{21} = k_{12} = -12i/l \quad F_{1P} = -ql^2/8 \quad F_{2P} = 0$$

$$11iZ_1 - 12i/l Z_2 - ql^2/8 = 0$$

$$-12i/l Z_1 + 27i/l^2 Z_2 = 0 \quad Z_1 = 3ql^2/136i \quad Z_2 = ql^3/102i$$

由 $M = \overline{M}_1 Z_1 + \overline{M}_2 Z_2 + M_P$ 作最终弯矩图。

【例题 4】 用位移法计算图示结构，并作弯矩图。



令线刚度： $i = EI/l$

$$k_{11}Z_1 + k_{12}Z_2 + F_{1P} = 0$$

$$k_{21}Z_1 + k_{22}Z_2 + F_{2P} = 0$$

$$k_{11} = 12i \quad k_{22} = 8i \quad k_{21} = k_{12} = 2i \quad F_{1P} = -ql^2/12 \quad F_{2P} = ql^2/12$$

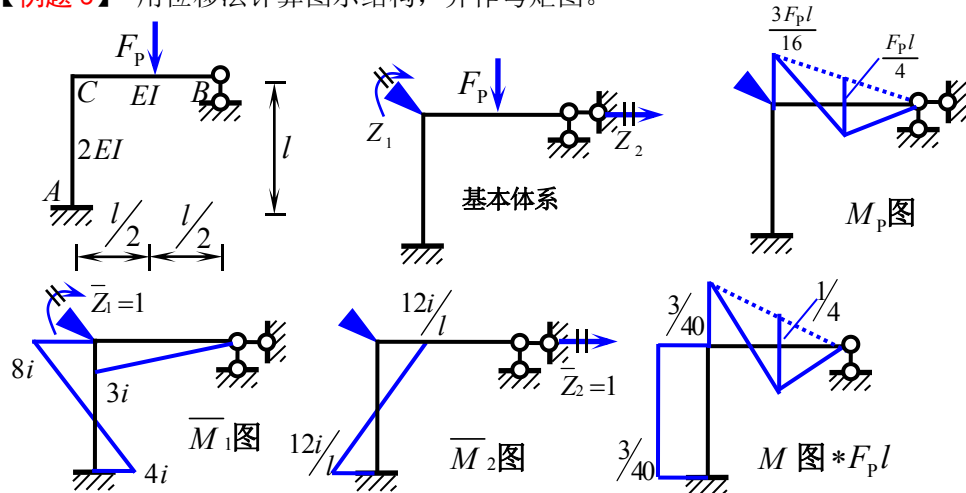
$$12iZ_1 + 2iZ_2 - ql^2/12 = 0$$

$$2iZ_1 + 8iZ_2 + ql^2/12 = 0$$

$$Z_1 = 5ql^2/552i \quad Z_2 = -7ql^2/552i$$

由 $M = \bar{M}_1Z_1 + \bar{M}_2Z_2 + M_p$ 作最终弯矩图。

【例题 5】 用位移法计算图示结构，并作弯矩图。



令线刚度： $i = EI/l$

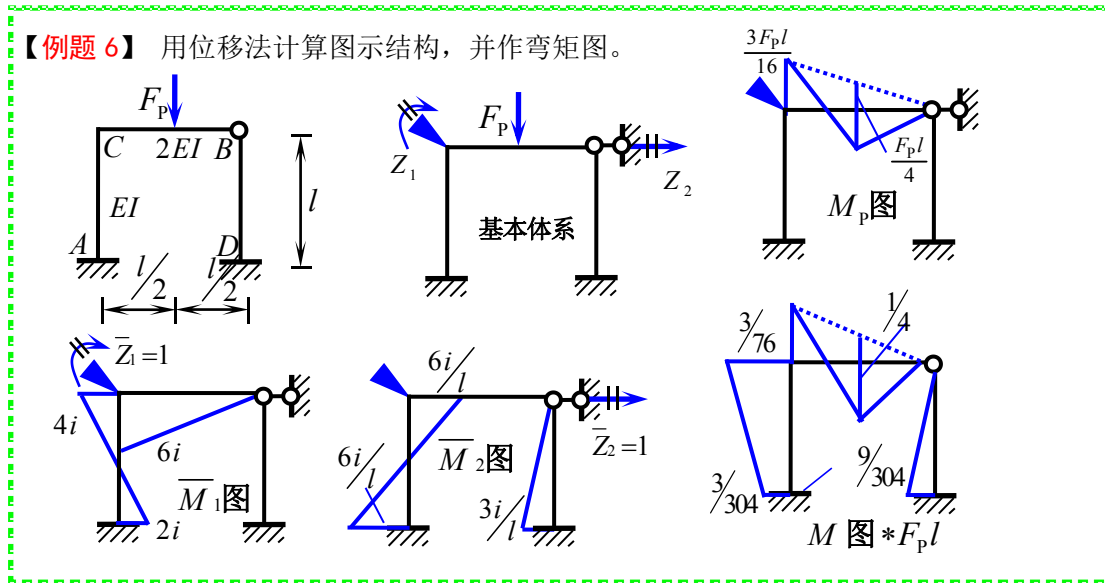
$$k_{11}Z_1 + k_{12}Z_2 + F_{1P} = 0$$

$$k_{21}Z_1 + k_{22}Z_2 + F_{2P} = 0$$

$$k_{11} = 11i \quad k_{22} = 24i/l^2 \quad k_{21} = k_{12} = -12i/l \quad F_{1P} = -3F_P l/16 \quad F_{2P} = 0$$

$$\begin{aligned} 11iZ_1 - 12i/l Z_2 - 3F_P l/16 &= 0 \\ -12i/l Z_1 + 24i/l^2 Z_2 &= 0 \end{aligned} \quad Z_1 = 3F_P l/80i \quad Z_2 = 3F_P l^2/160i$$

由 $M = \bar{M}_1 Z_1 + \bar{M}_2 Z_2 + M_P$ 作最终弯矩图。



令线刚度： $i = EI/l$

$$k_{11}Z_1 + k_{12}Z_2 + F_{1P} = 0$$

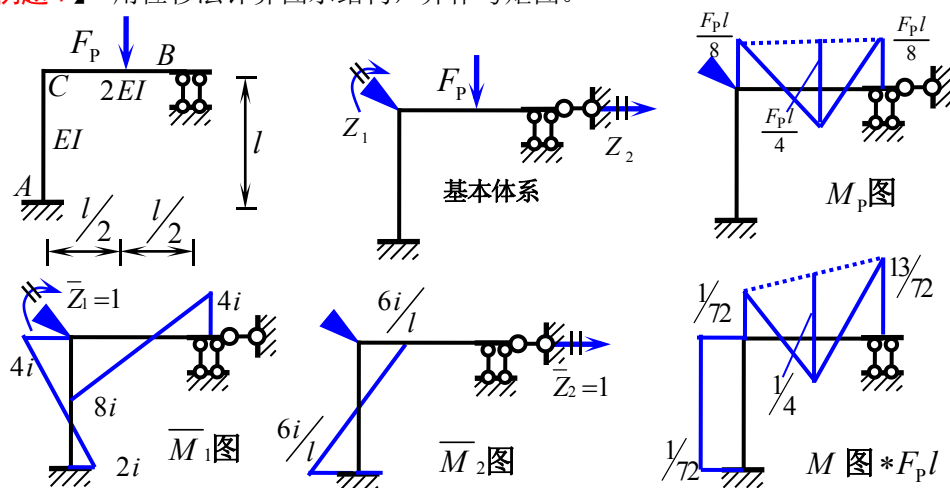
$$k_{21}Z_1 + k_{22}Z_2 + F_{2P} = 0$$

$$k_{11} = 10i \quad k_{22} = 15i/l^2 \quad k_{21} = k_{12} = -6i/l \quad F_{1P} = -3F_P l/16 \quad F_{2P} = 0$$

$$\begin{aligned} 10iZ_1 - 6i/l Z_2 - 3F_P l/16 &= 0 \\ -6i/l Z_1 + 15i/l^2 Z_2 &= 0 \end{aligned} \quad Z_1 = 15F_P l/608i \quad Z_2 = 3F_P l^2/304i$$

由 $M = \bar{M}_1 Z_1 + \bar{M}_2 Z_2 + M_P$ 作最终弯矩图。

【例题 7】 用位移法计算图示结构，并作弯矩图。



令线刚度: $i = EI/l$

$$k_{11}Z_1 + k_{12}Z_2 + F_{1P} = 0$$

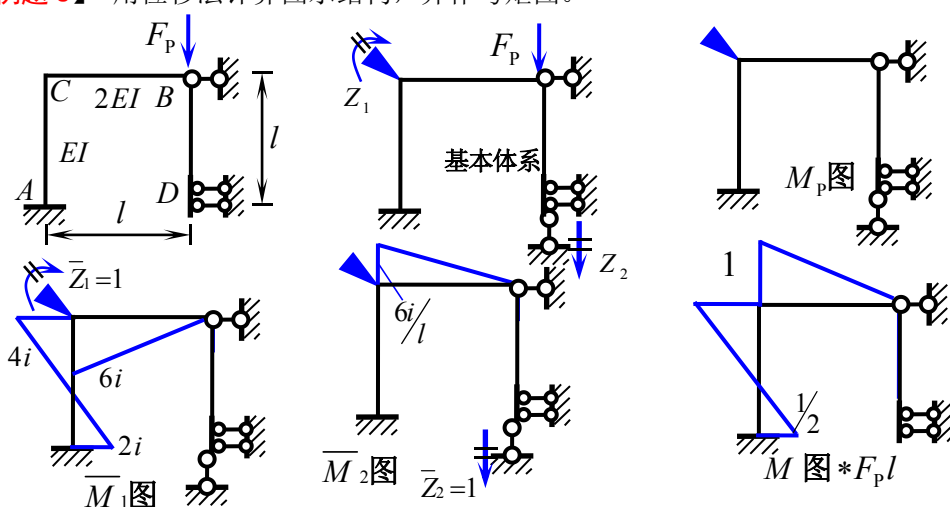
$$k_{21}Z_1 + k_{22}Z_2 + F_{2P} = 0$$

$$k_{11} = 12i \quad k_{22} = 12i/l^2 \quad k_{21} = k_{12} = -6i/l \quad F_{1P} = -F_P l/8 \quad F_{2P} = 0$$

$$\begin{aligned} 12iZ_1 - 6i/l Z_2 - F_P l/8 &= 0 \\ -6i/l Z_1 + 12i/l^2 Z_2 &= 0 \end{aligned} \quad Z_1 = F_P l/72i \quad Z_2 = F_P l^2/144i$$

由 $M = \bar{M}_1 Z_1 + \bar{M}_2 Z_2 + \bar{M}_p$ 作最终弯矩图。

【例题 8】 用位移法计算图示结构，并作弯矩图。



令线刚度: $i = EI/l$

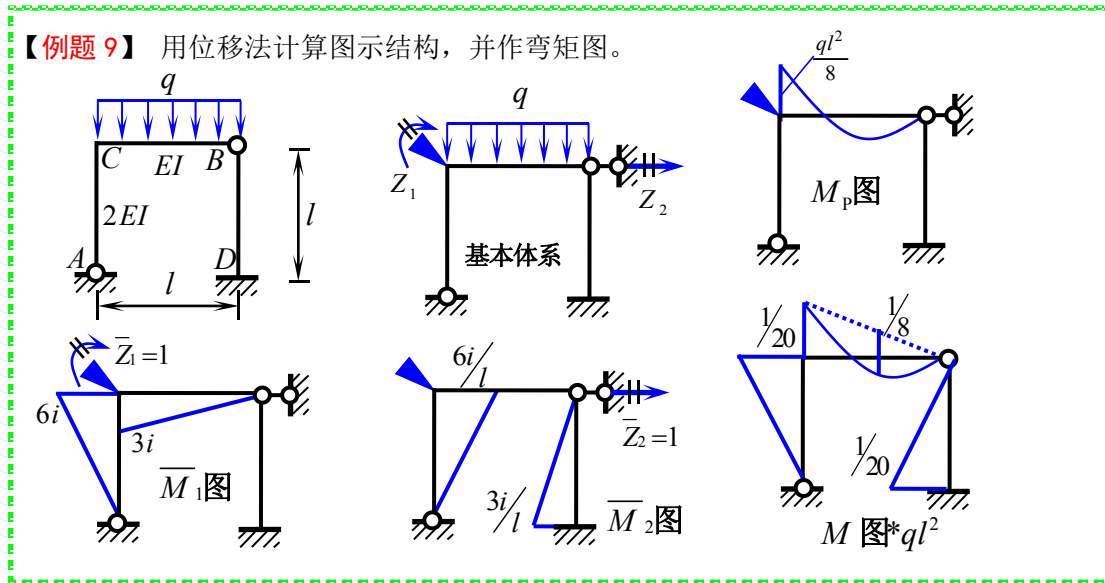
$$k_{11}Z_1 + k_{12}Z_2 + F_{1P} = 0$$

$$k_{21}Z_1 + k_{22}Z_2 + F_{2P} = 0$$

$$k_{11} = 10i \quad k_{22} = \frac{6i}{l^2} \quad k_{21} = k_{12} = -\frac{6i}{l} \quad F_{1P} = 0 \quad F_{2P} = -F_P$$

$$\begin{aligned} 10iZ_1 - \frac{6i}{l}Z_2 &= 0 \\ -\frac{6i}{l}Z_1 + \frac{6i}{l^2}Z_2 - F_P &= 0 \end{aligned} \quad Z_1 = \frac{F_P l}{4i} \quad Z_2 = \frac{5F_P l^2}{12i}$$

由 $M = \overline{M}_1 Z_1 + \overline{M}_2 Z_2 + M_P$ 作最终弯矩图。



令线刚度: $i = EI/l$

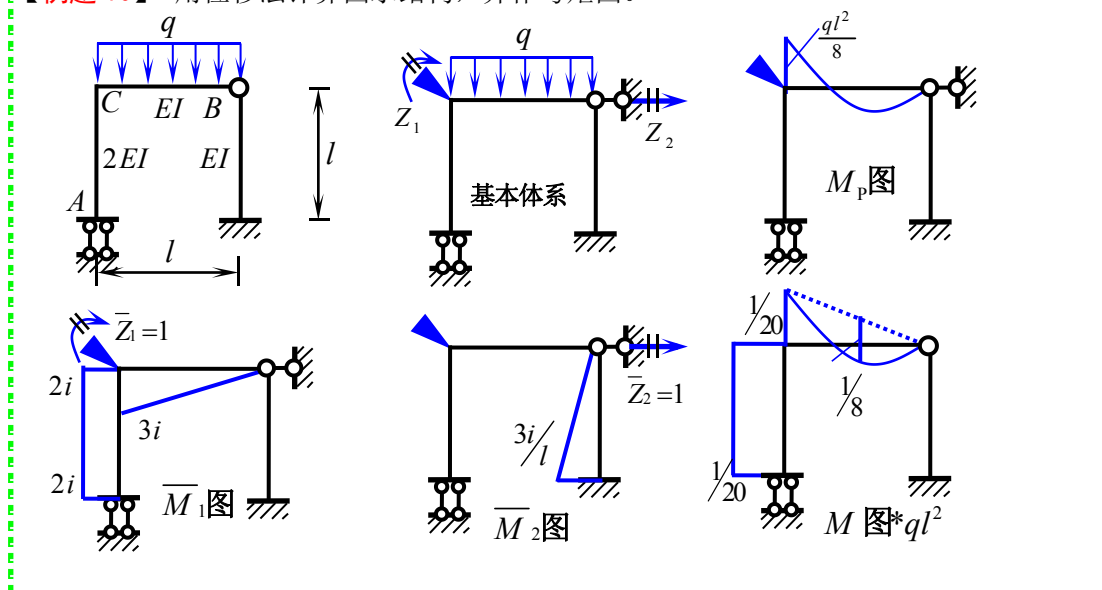
$$\begin{aligned} k_{11}Z_1 + k_{12}Z_2 + F_{1P} &= 0 \\ k_{21}Z_1 + k_{22}Z_2 + F_{2P} &= 0 \end{aligned}$$

$$k_{11} = 9i \quad k_{22} = \frac{9i}{l^2} \quad k_{21} = k_{12} = -\frac{6i}{l} \quad F_{1P} = -\frac{ql^2}{8} \quad F_{2P} = 0$$

$$\begin{aligned} 9iZ_1 - \frac{6i}{l}Z_2 - \frac{ql^2}{8} &= 0 \\ -\frac{6i}{l}Z_1 + \frac{9i}{l^2}Z_2 &= 0 \end{aligned} \quad Z_1 = \frac{ql^2}{40i} \quad Z_2 = \frac{ql^3}{60i}$$

由 $M = \overline{M}_1 Z_1 + \overline{M}_2 Z_2 + M_P$ 作最终弯矩图。

【例题 10】 用位移法计算图示结构，并作弯矩图。



令线刚度: $i = EI/l$

$$k_{11}Z_1 + k_{12}Z_2 + F_{1P} = 0$$

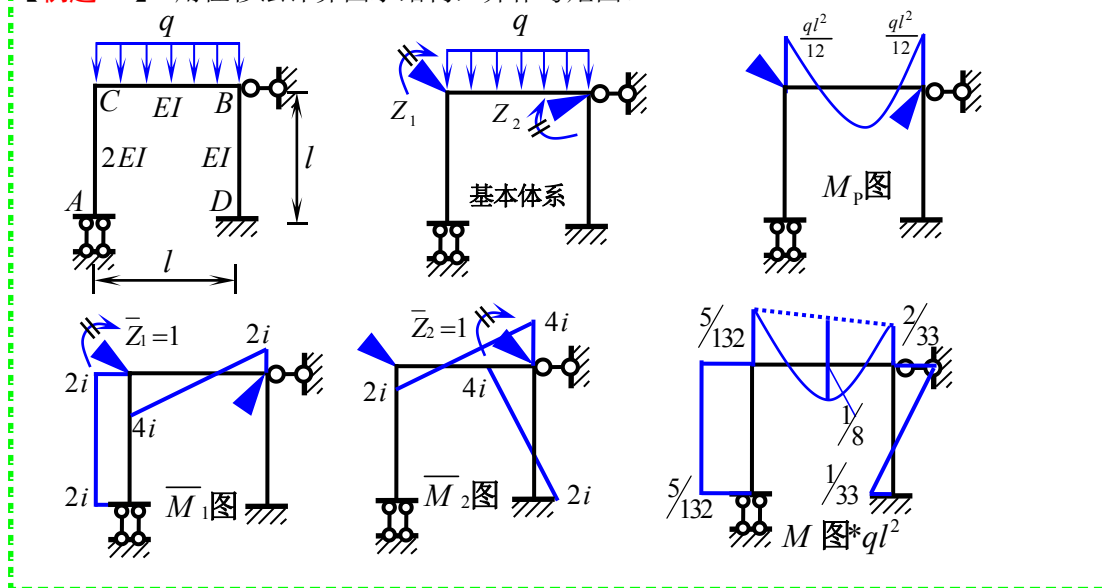
$$k_{21}Z_1 + k_{22}Z_2 + F_{2P} = 0$$

$$k_{11} = 5i \quad k_{22} = \frac{3i}{l^2} \quad k_{21} = k_{12} = 0 \quad F_{1P} = -\frac{ql^2}{8} \quad F_{2P} = 0$$

$$\begin{aligned} 5iZ_1 - \frac{ql^2}{8} &= 0 \\ \frac{3i}{l^2}Z_2 &= 0 \end{aligned} \quad Z_1 = \frac{ql^2}{40i} \quad Z_2 = 0$$

由 $M = \bar{M}_1 Z_1 + \bar{M}_2 Z_2 + M_p$ 作最终弯矩图。

【例题 11】 用位移法计算图示结构，并作弯矩图。



令线刚度： $i = EI/l$

$$k_{11}Z_1 + k_{12}Z_2 + F_{1P} = 0$$

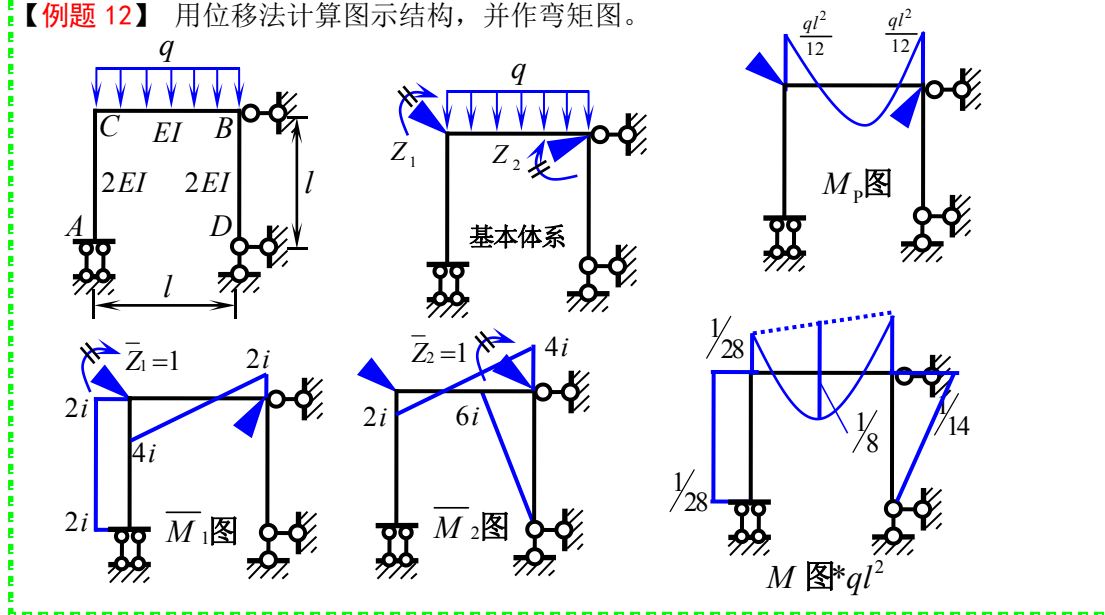
$$k_{21}Z_1 + k_{22}Z_2 + F_{2P} = 0$$

$$k_{11} = 6i \quad k_{22} = 8i \quad k_{21} = k_{12} = 2i \quad F_{1P} = -ql^2/12 \quad F_{2P} = ql^2/12$$

$$\begin{aligned} 6iZ_1 + 2iZ_2 - ql^2/12 &= 0 \\ 2iZ_1 + 8iZ_2 + ql^2/12 &= 0 \end{aligned} \quad \begin{aligned} Z_1 &= 5ql^2/264i \\ Z_2 &= -ql^2/66i \end{aligned}$$

由 $M = \overline{M}_1Z_1 + \overline{M}_2Z_2 + M_P$ 作最终弯矩图。

【例题 12】 用位移法计算图示结构，并作弯矩图。



令线刚度: $i = EI/l$

$$k_{11}Z_1 + k_{12}Z_2 + F_{1P} = 0$$

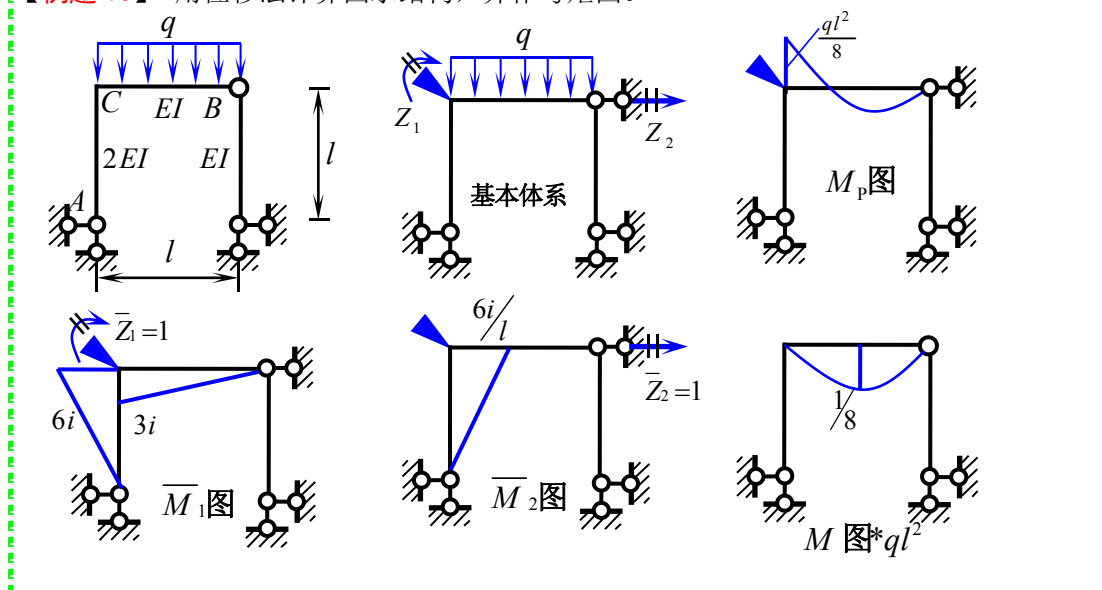
$$k_{21}Z_1 + k_{22}Z_2 + F_{2P} = 0$$

$$k_{11} = 6i \quad k_{22} = 10i \quad k_{21} = k_{12} = 2i \quad F_{1P} = -ql^2/12 \quad F_{2P} = ql^2/12$$

$$\begin{aligned} 6iZ_1 + 2iZ_2 - ql^2/12 &= 0 \\ 2iZ_1 + 10iZ_2 + ql^2/12 &= 0 \end{aligned} \quad \begin{aligned} Z_1 &= ql^2/56i \\ Z_2 &= -ql^2/84i \end{aligned}$$

由 $M = \bar{M}_1Z_1 + \bar{M}_2Z_2 + M_P$ 作最终弯矩图。

【例题 13】 用位移法计算图示结构，并作弯矩图。



本题为静定结构，也可以用位移法求解，有两个未知数。

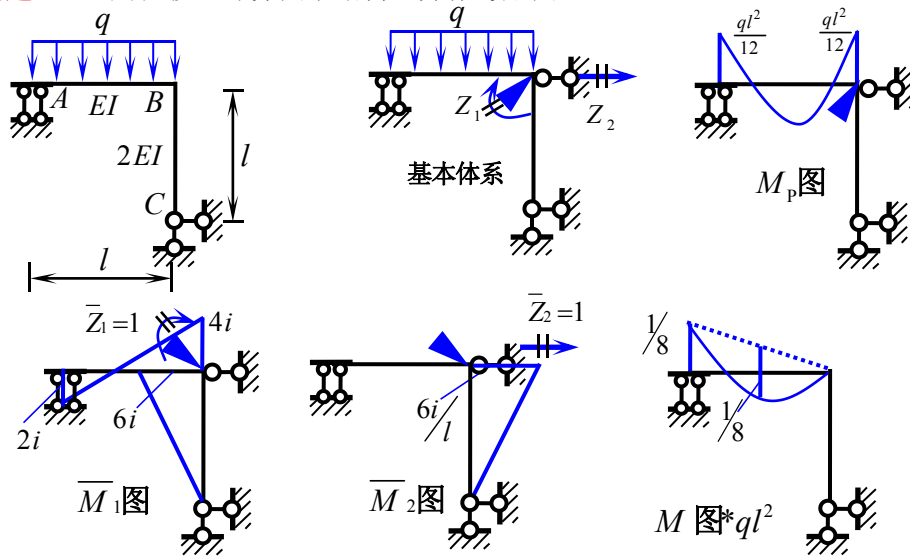
$$\text{令线刚度: } i = EI/l \quad \begin{aligned} k_{11}Z_1 + k_{12}Z_2 + F_{1P} &= 0 \\ k_{21}Z_1 + k_{22}Z_2 + F_{2P} &= 0 \end{aligned}$$

$$k_{11} = 9i \quad k_{22} = \frac{6i}{l^2} \quad k_{21} = k_{12} = -\frac{6i}{l} \quad F_{1P} = -\frac{ql^2}{8} \quad F_{2P} = 0$$

$$\begin{aligned} 9iZ_1 - \frac{6i}{l}Z_2 - \frac{ql^2}{8} &= 0 \\ -\frac{6i}{l}Z_1 + \frac{6i}{l^2}Z_2 &= 0 \end{aligned} \quad \begin{aligned} Z_1 &= \frac{ql^2}{24i} \\ Z_2 &= \frac{ql^3}{24i} \end{aligned}$$

由 $M = \overline{M}_1Z_1 + \overline{M}_2Z_2 + M_p$ 作最终弯矩图。

【例题 14】 用位移法计算图示结构，并作弯矩图。



令线刚度: $i = EI/l$

$$\begin{aligned} k_{11}Z_1 + k_{12}Z_2 + F_{1P} &= 0 \\ k_{21}Z_1 + k_{22}Z_2 + F_{2P} &= 0 \end{aligned}$$

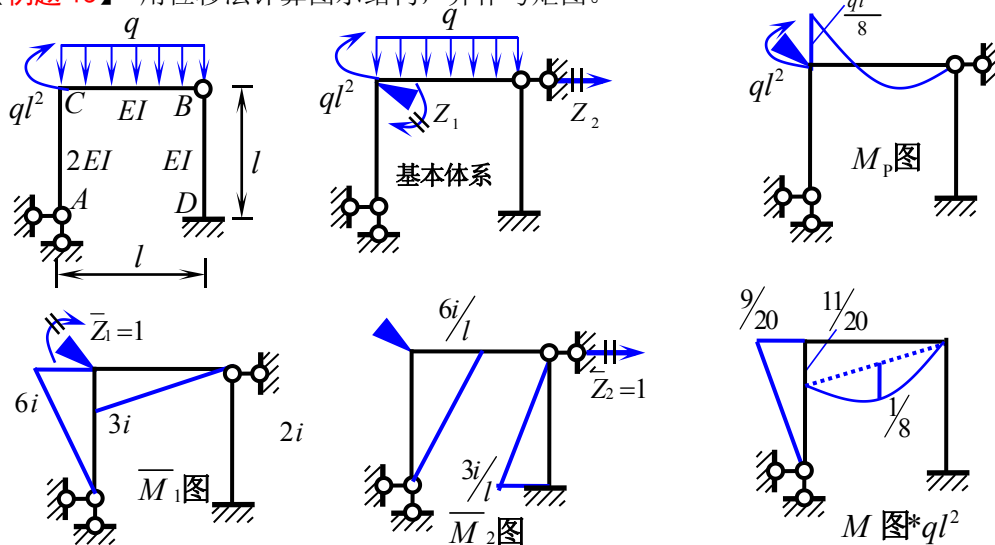
$$k_{11} = 10i \quad k_{22} = \frac{6i}{l^2} \quad k_{21} = k_{12} = -\frac{6i}{l} \quad F_{1P} = \frac{ql^2}{12} \quad F_{2P} = 0$$

$$\begin{aligned} 10iZ_1 - \frac{6i}{l}Z_2 + \frac{ql^2}{12} &= 0 \\ -\frac{6i}{l}Z_1 + \frac{6i}{l^2}Z_2 &= 0 \end{aligned}$$

$$Z_1 = -\frac{ql^2}{48i} \quad Z_2 = -\frac{ql^3}{48i}$$

由 $M = \overline{M}_1Z_1 + \overline{M}_2Z_2 + M_p$ 作最终弯矩图。

【例题 15】 用位移法计算图示结构，并作弯矩图。

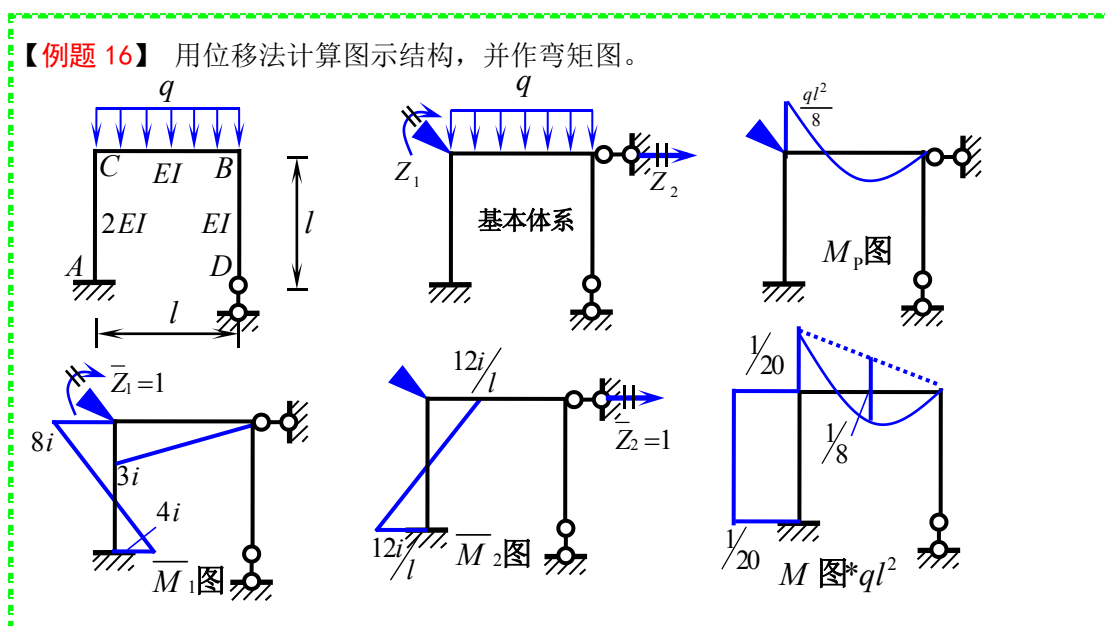


$$\text{令线刚度: } i = EI/l \quad \begin{aligned} k_{11}Z_1 + k_{12}Z_2 + F_{1P} &= 0 \\ k_{21}Z_1 + k_{22}Z_2 + F_{2P} &= 0 \end{aligned}$$

$$k_{11} = 9i \quad k_{22} = \frac{9i}{l^2} \quad k_{21} = k_{12} = -\frac{6i}{l} \quad F_{1P} = -\frac{9ql^2}{8} \quad F_{2P} = 0$$

$$\begin{aligned} 9iZ_1 - \frac{6i}{l}Z_2 - \frac{9ql^2}{8} &= 0 \\ -\frac{6i}{l}Z_1 + \frac{9i}{l^2}Z_2 &= 0 \end{aligned} \quad \begin{aligned} Z_1 &= \frac{9ql^2}{40i} \\ Z_2 &= \frac{3ql^3}{20i} \end{aligned}$$

由 $M = \overline{M}_1Z_1 + \overline{M}_2Z_2 + M_P$ 作最终弯矩图。



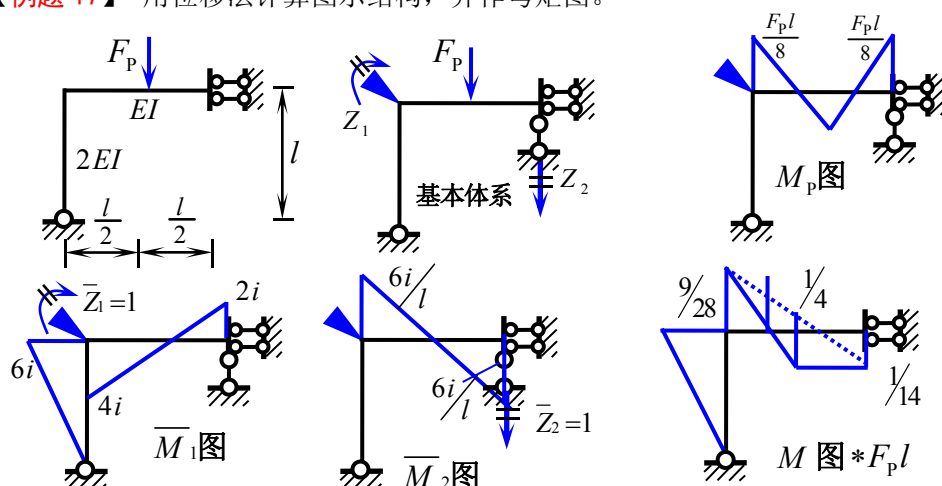
$$\text{令线刚度: } i = EI/l \quad \begin{aligned} k_{11}Z_1 + k_{12}Z_2 + F_{1P} &= 0 \\ k_{21}Z_1 + k_{22}Z_2 + F_{2P} &= 0 \end{aligned}$$

$$k_{11} = 11i \quad k_{22} = \frac{24i}{l^2} \quad k_{21} = k_{12} = -\frac{12i}{l} \quad F_{1P} = -\frac{ql^2}{8} \quad F_{2P} = 0$$

$$\begin{aligned} 11iZ_1 - \frac{12i}{l}Z_2 - \frac{ql^2}{8} &= 0 \\ -\frac{12i}{l}Z_1 + \frac{24i}{l^2}Z_2 &= 0 \end{aligned} \quad \begin{aligned} Z_1 &= \frac{ql^2}{40i} \\ Z_2 &= \frac{ql^3}{80i} \end{aligned}$$

由 $M = \overline{M}_1Z_1 + \overline{M}_2Z_2 + M_P$ 作最终弯矩图。

【例题 17】 用位移法计算图示结构，并作弯矩图。



令线刚度: $i = EI/l$

$$k_{11}Z_1 + k_{12}Z_2 + F_{1P} = 0$$

$$k_{21}Z_1 + k_{22}Z_2 + F_{2P} = 0$$

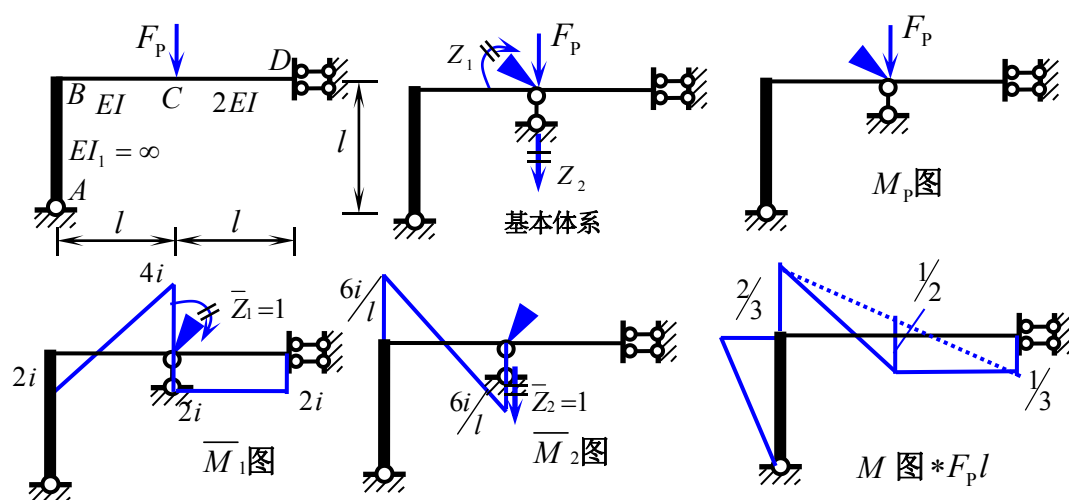
$$k_{11} = 10i \quad k_{22} = \frac{12i}{l^2} \quad k_{21} = k_{12} = -\frac{6i}{l} \quad F_{1P} = -\frac{F_P l}{8} \quad F_{2P} = -\frac{F_P}{2}$$

$$10iZ_1 - \frac{6i}{l}Z_2 - \frac{F_P l}{8} = 0 \quad Z_1 = \frac{3F_P l}{56i} \quad Z_2 = \frac{23F_P l^2}{336i}$$

$$-\frac{6i}{l}Z_1 + \frac{12i}{l^2}Z_2 - \frac{F_P}{2} = 0$$

由 $M = \bar{M}_1 Z_1 + \bar{M}_2 Z_2 + M_P$ 作最终弯矩图。

【例题 18】 用位移法计算图示结构，并作弯矩图。



令线刚度: $i = EI/l$

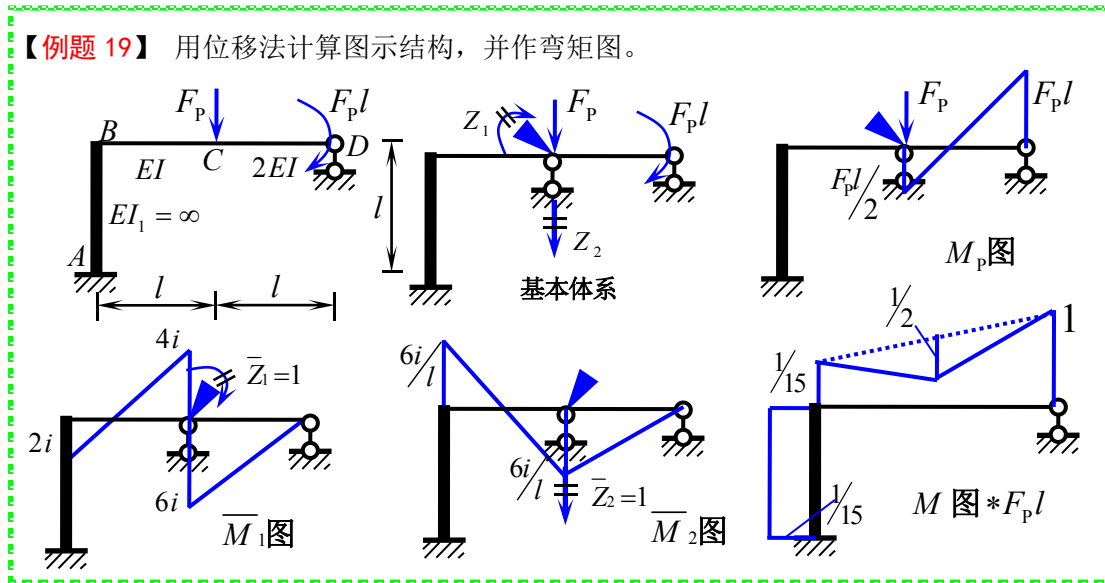
$$k_{11}Z_1 + k_{12}Z_2 + F_{1P} = 0$$

$$k_{21}Z_1 + k_{22}Z_2 + F_{2P} = 0$$

$$k_{11} = 6i \quad k_{22} = 12i/l^2 \quad k_{21} = k_{12} = -6i/l \quad F_{1P} = 0 \quad F_{2P} = -F_P$$

$$\begin{aligned} 6iZ_1 - 6i/l Z_2 &= 0 \\ -6i/l Z_1 + 12i/l^2 Z_2 - F_P &= 0 \end{aligned} \quad \begin{aligned} Z_1 &= F_P l / 6i \\ Z_2 &= F_P l^2 / 6i \end{aligned}$$

由 $M = \bar{M}_1 Z_1 + \bar{M}_2 Z_2 + M_P$ 作最终弯矩图。



令线刚度: $i = EI/l$

$$k_{11}Z_1 + k_{12}Z_2 + F_{1P} = 0$$

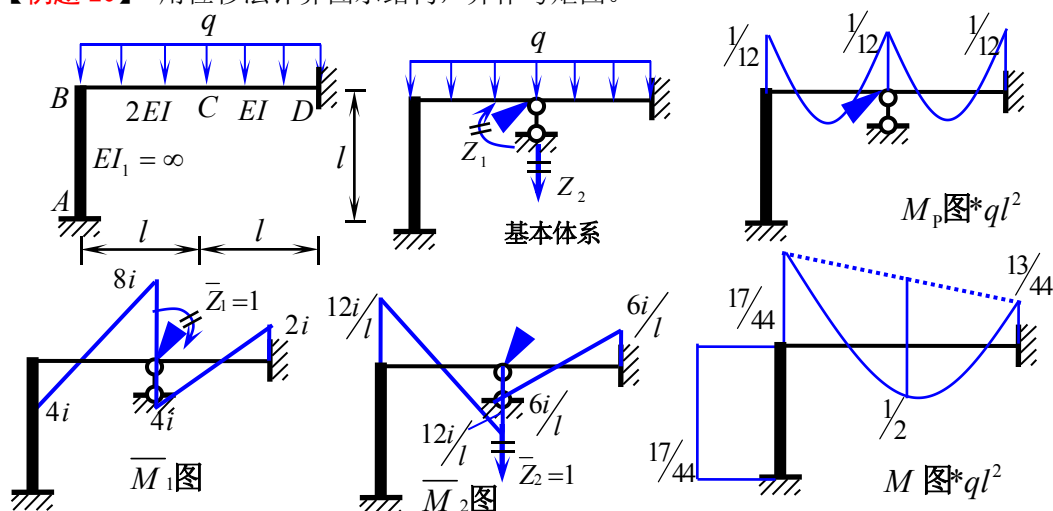
$$k_{21}Z_1 + k_{22}Z_2 + F_{2P} = 0$$

$$k_{11} = 10i \quad k_{22} = 18i/l^2 \quad k_{21} = k_{12} = 0 \quad F_{1P} = F_P l / 2 \quad F_{2P} = -F_P / 2$$

$$\begin{aligned} 10iZ_1 - F_P l / 2 &= 0 \\ 18i/l^2 Z_2 - F_P / 2 &= 0 \end{aligned} \quad \begin{aligned} Z_1 &= F_P l / 20i \\ Z_2 &= F_P l^2 / 36i \end{aligned}$$

由 $M = \bar{M}_1 Z_1 + \bar{M}_2 Z_2 + M_P$ 作最终弯矩图。

【例题 20】 用位移法计算图示结构，并作弯矩图。



令线刚度: $i = EI/l$

$$k_{11}Z_1 + k_{12}Z_2 + F_{1P} = 0$$

$$k_{21}Z_1 + k_{22}Z_2 + F_{2P} = 0$$

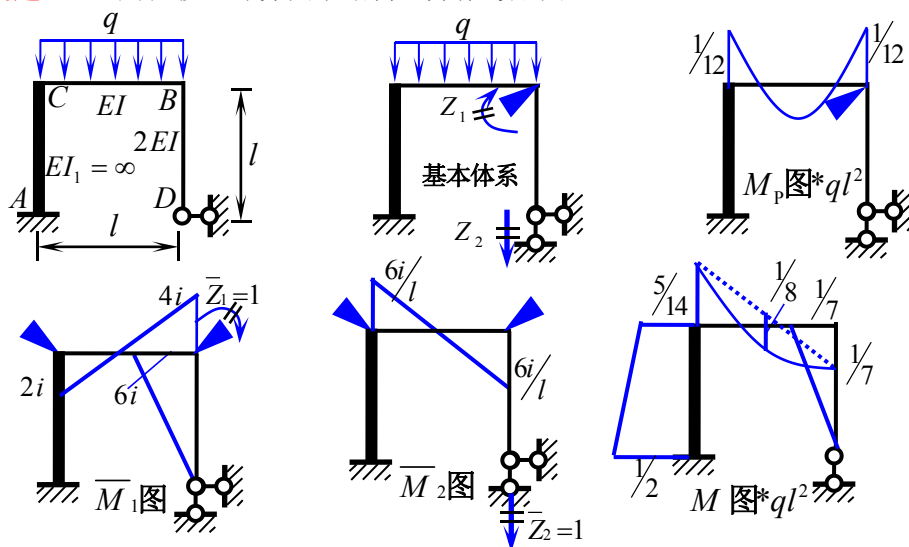
$$k_{11} = 12i \quad k_{22} = \frac{36i}{l^2} \quad k_{21} = k_{12} = -\frac{6i}{l} \quad F_{1P} = 0 \quad F_{2P} = -ql$$

$$12iZ_1 - \frac{6i}{l}Z_2 = 0$$

$$-\frac{6i}{l}Z_1 + \frac{36i}{l^2}Z_2 - ql = 0 \quad Z_1 = \frac{ql^2}{66i} \quad Z_2 = \frac{ql^3}{33i}$$

由 $M = \bar{M}_1Z_1 + \bar{M}_2Z_2 + M_P$ 作最终弯矩图。

【例题 21】 用位移法计算图示结构，并作弯矩图。

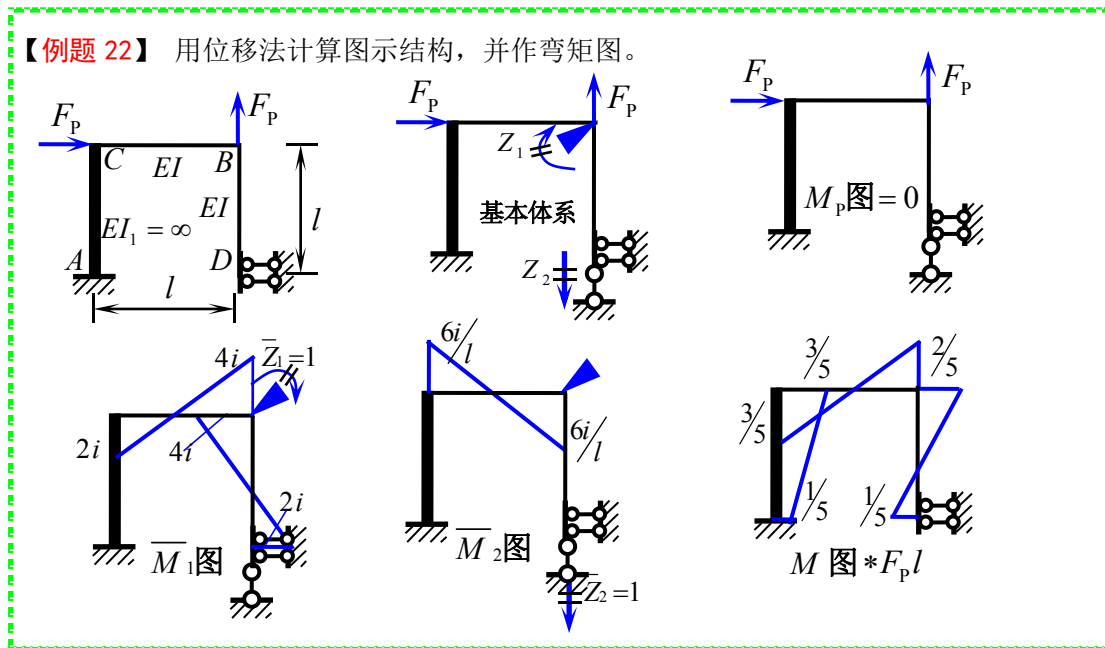


$$\text{令线刚度: } i = EI/l \quad \begin{aligned} k_{11}Z_1 + k_{12}Z_2 + F_{1P} &= 0 \\ k_{21}Z_1 + k_{22}Z_2 + F_{2P} &= 0 \end{aligned}$$

$$k_{11} = 10i \quad k_{22} = 12i/l^2 \quad k_{21} = k_{12} = -6i/l \quad F_{1P} = ql^2/12 \quad F_{2P} = -ql/2$$

$$\begin{aligned} 10iZ_1 - 6i/l Z_2 + ql^2/12 &= 0 \\ -6i/l Z_1 + 12i/l^2 Z_2 - ql/2 &= 0 \end{aligned} \quad \begin{aligned} Z_1 &= ql^2/42i \\ Z_2 &= 3ql^3/56i \end{aligned}$$

由 $M = \bar{M}_1 Z_1 + \bar{M}_2 Z_2 + M_P$ 作最终弯矩图。



$$\text{令线刚度: } i = EI/l \quad \begin{aligned} k_{11}Z_1 + k_{12}Z_2 + F_{1P} &= 0 \\ k_{21}Z_1 + k_{22}Z_2 + F_{2P} &= 0 \end{aligned}$$

$$k_{11} = 8i \quad k_{22} = 12i/l^2 \quad k_{21} = k_{12} = -6i/l \quad F_{1P} = 0 \quad F_{2P} = F_P$$

$$\begin{aligned} 8iZ_1 - 6i/l Z_2 &= 0 \\ -6i/l Z_1 + 12i/l^2 Z_2 + F_P &= 0 \end{aligned} \quad \begin{aligned} Z_1 &= -F_P l/10i \\ Z_2 &= -2F_P l^2/15i \end{aligned}$$

【例题 23】 用位移法计算图示结构，并作弯矩图。

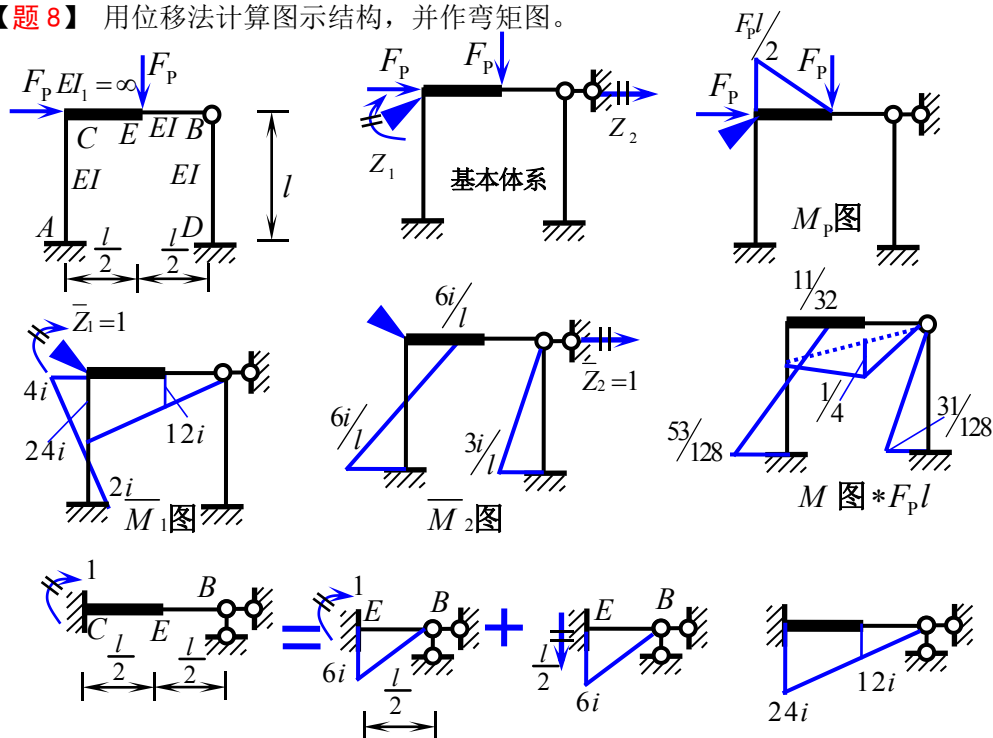
$$\text{令线刚度: } i = EI/l \quad \begin{aligned} k_{11}Z_1 + k_{12}Z_2 + F_{1P} &= 0 \\ k_{21}Z_1 + k_{22}Z_2 + F_{2P} &= 0 \end{aligned}$$

$$k_{11} = 6i \quad k_{22} = 27i/l^2 \quad k_{21} = k_{12} = -3i/l \quad F_{1p} = -ql^2/8 \quad F_{2p} = 0$$

$$\begin{aligned} 6iZ_1 - 3i/l Z_2 - ql^2/8 &= 0 \\ -3i/l Z_1 + 27i/l^2 Z_2 &= 0 \end{aligned} \quad Z_1 = 3ql^2/136i \quad Z_2 = ql^3/408i$$

由 $M = \overline{M}_1 Z_1 + \overline{M}_2 Z_2 + M_p$ 作最终弯矩图。

【题 8】 用位移法计算图示结构，并作弯矩图。



令线刚度: $i = EI/l$

$$k_{11}Z_1 + k_{12}Z_2 + F_{1P} = 0$$

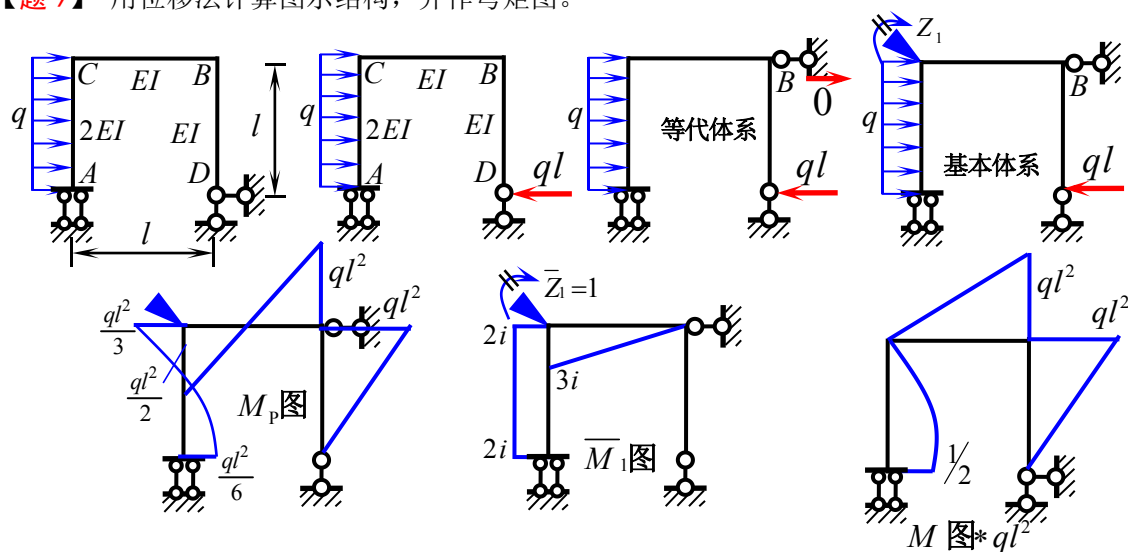
$$k_{21}Z_1 + k_{22}Z_2 + F_{2P} = 0$$

$$k_{11} = 28i \quad k_{22} = 15i/l^2 \quad k_{21} = k_{12} = -6i/l \quad F_{1P} = -F_P l/2 \quad F_{2P} = -F_P$$

$$28iZ_1 - 6i/l Z_2 - F_P l/2 = 0$$

$$-6i/l Z_1 + 15i/l^2 Z_2 - F_P = 0$$

【题 9】 用位移法计算图示结构，并作弯矩图。

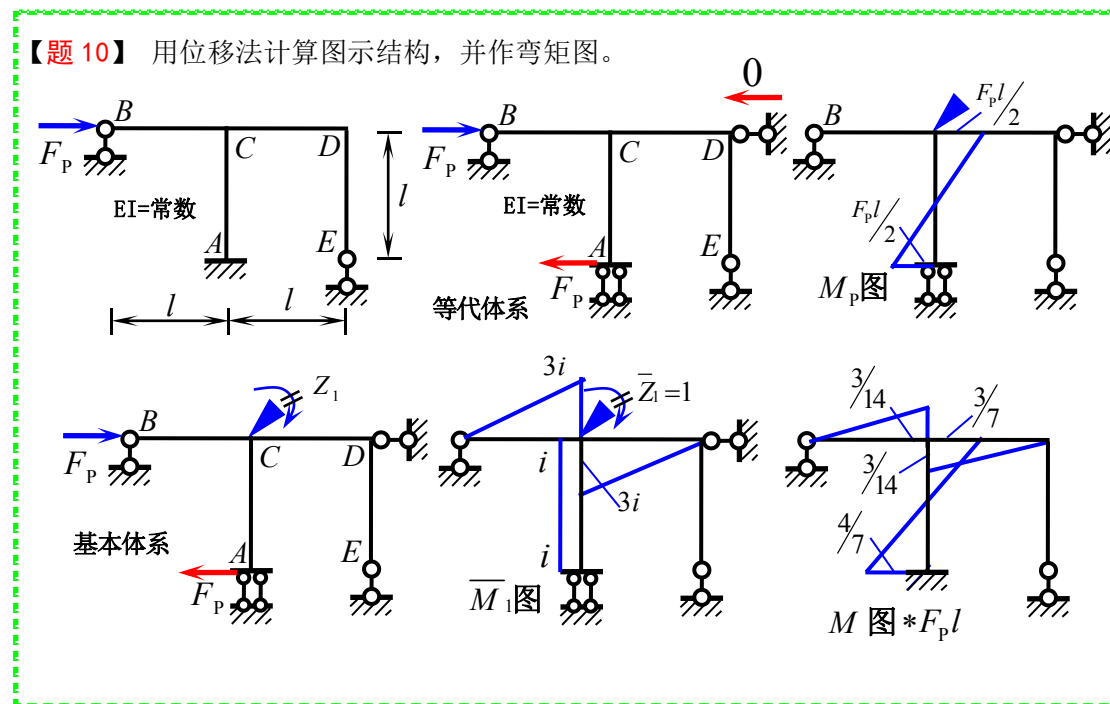


本题有两个转角位移和一个线位移共计 3 个未知数。D 处水平反力为零，可以去掉水平链杆支座，在 B 处施加一个反力为零的水平链杆支座，位移法基本未知量变成只有 1 个转角。B 处弯矩已知不用施加刚臂支座。

令线刚度： $i = EI/l$ $k_{11}Z_1 + F_{1P} = 0$

$$k_{11} = 5i \quad F_{1P} = \frac{5ql^2}{6} \quad Z_1 = -\frac{ql^2}{6i}$$

由 $M = \overline{M}_1 Z_1 + M_P$ 作最终弯矩图。



令线刚度： $i = EI/l$

$$k_{11}Z_1 + F_{1P} = 0 \quad k_{11} = 7i \quad F_{1P} = -\frac{F_P l}{2} \quad Z_1 = \frac{F_P l}{14i}$$

由 $M = \overline{M}_1 Z_1 + M_P$ 作最终弯矩图。

