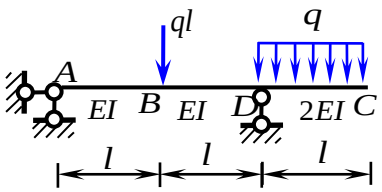
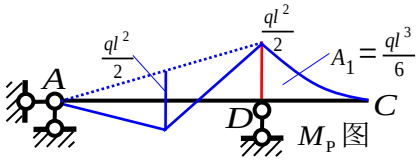
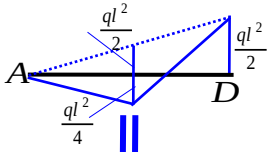
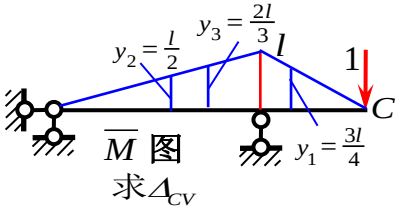
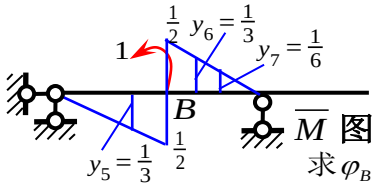
【例题1】（梁1-1）求Δ_{CV}，φ_B

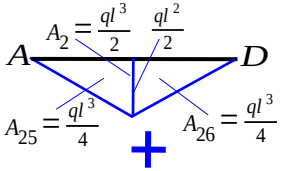


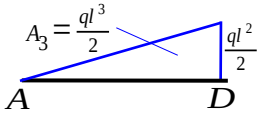


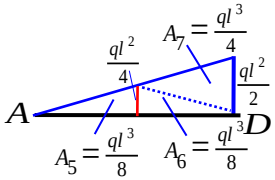












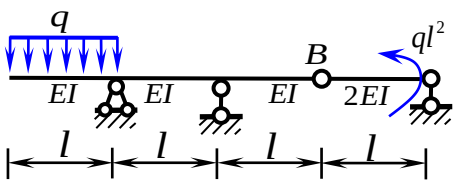
$$\Delta_{CV}=\sum\int\frac{\overline{M}\ M_P}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}=\frac{A_1y_1}{2EI}-\frac{A_2y_2}{EI}+\frac{A_3y_3}{EI}$$

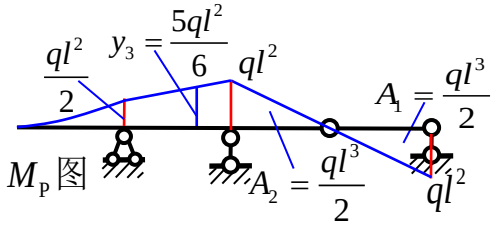
$$=\frac{1}{2EI}\times\frac{ql^3}{6}\times\frac{3l}{4}-\frac{1}{EI}\times\frac{ql^3}{2}\times\frac{l}{2}+\frac{1}{EI}\times\frac{ql^3}{2}\times\frac{2l}{3}=\frac{7ql^4}{48EI}(\downarrow)$$

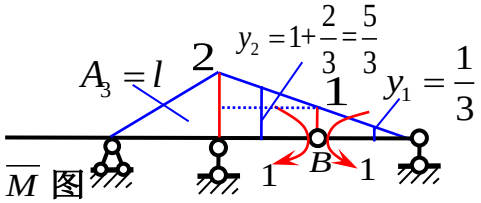
$$\varphi_B=\sum\int\frac{\overline{M}\ M_P}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}=\frac{A_{25}y_5}{EI}+\frac{-A_{26}y_6}{EI}+\frac{-A_5y_5}{EI}+\frac{A_6y_6}{EI}+\frac{A_7y_7}{EI}$$

$$=\frac{1}{EI}\times\frac{ql^3}{4}\times\frac{1}{3}+\frac{-1}{EI}\times\frac{ql^3}{4}\times\frac{1}{3}+\frac{-1}{EI}\times\frac{ql^3}{8}\times\frac{1}{3}+\frac{1}{EI}\times\frac{ql^3}{8}\times\frac{1}{3}+\frac{1}{EI}\times\frac{ql^3}{4}\times\frac{1}{6}=\frac{ql^3}{24EI}\text{（逆时针）}$$

【例题2】 求铰B 两侧截面的相对转角。 （梁1-14）





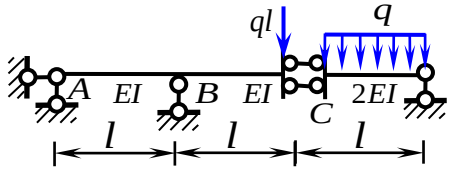


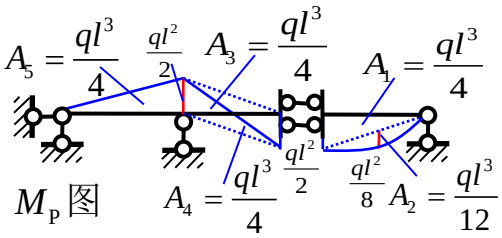
$$\Delta\varphi_B=\sum\int\frac{\overline{M}\,M_P}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}=\frac{-A_1y_1}{2EI}+\frac{A_2y_2}{EI}+\frac{A_3y_3}{EI}$$

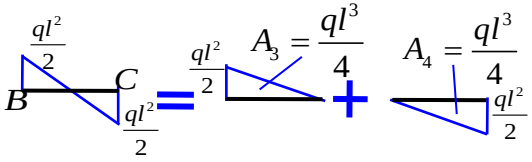
$$=\frac{-1}{2EI}\times\frac{ql^3}{2}\times\frac{1}{3}+\frac{1}{EI}\times\frac{ql^3}{2}\times\frac{5}{3}+\frac{1}{EI}\times l\times\frac{5ql^2}{6}=\frac{19ql^3}{12EI}$$

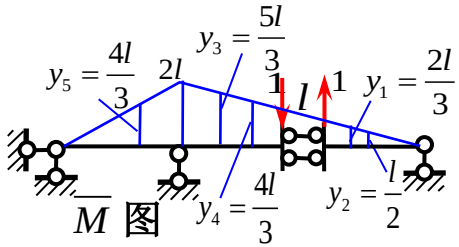
（位移方向与单位力方向相同）

【例题3】 求C 两侧截面的相对竖向位移。 （梁1-16）







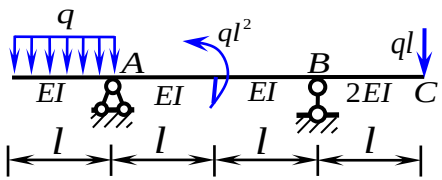


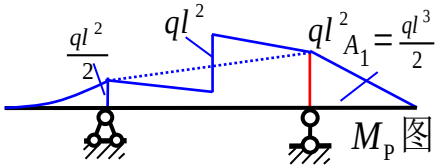
$$\Delta C_y=\sum\int\frac{\overline{M}\,M_P}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}=\frac{-A_1y_1}{2EI}-\frac{A_2y_2}{2EI}+\frac{A_3y_3}{EI}-\frac{A_4y_4}{EI}+\frac{A_5y_5}{EI}$$

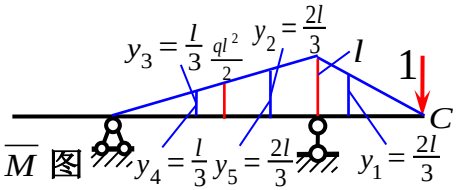
$$=\frac{-1}{2EI}\times\frac{ql^3}{4}\times\frac{2l}{3}-\frac{1}{2EI}\times\frac{ql^3}{12}\times\frac{l}{2}+\frac{1}{EI}\times\frac{ql^3}{4}\times\frac{5l}{3}-\frac{1}{EI}\times\frac{ql^3}{4}\times\frac{4l}{3}+\frac{1}{EI}\times\frac{ql^3}{4}\times\frac{4l}{3}=\frac{5ql^4}{16EI}(\downarrow\uparrow)$$

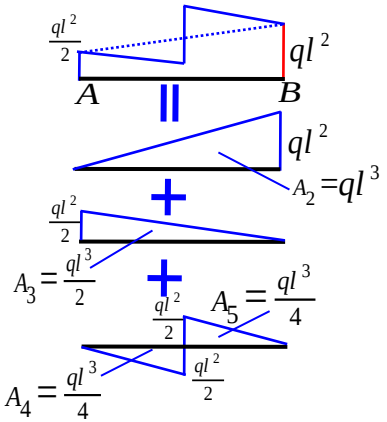
（位移方向与单位力方向相反）

【例题4】（梁1-19）求Δ_{CV}





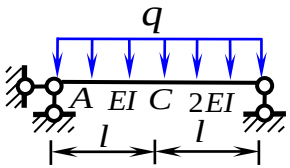


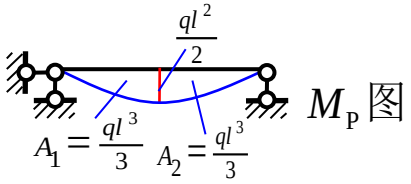


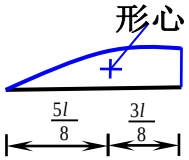
$$\Delta_{CV}=\sum\int\frac{\overline{M}\,M_P}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}=\frac{A_1y_1}{2EI}+\frac{A_2y_2}{EI}+\frac{A_3y_3}{EI}-\frac{A_4y_4}{EI}+\frac{A_5y_5}{EI}$$

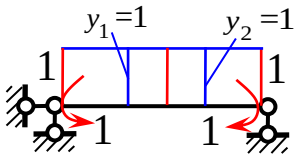
$$=\frac{1}{2EI}\times\frac{ql^3}{2}\times\frac{2l}{3}+\frac{1}{EI}\times ql^3\times\frac{2l}{3}+\frac{1}{EI}\times\frac{ql^3}{2}\times\frac{l}{3}-\frac{1}{EI}\times\frac{ql^3}{4}\times\frac{l}{3}+\frac{1}{EI}\times\frac{ql^3}{4}\times\frac{2l}{3}=\frac{13ql^4}{12EI}(\downarrow)$$

【例题5】求铰*A*、*B*两截面的相对转角。（梁1-8）





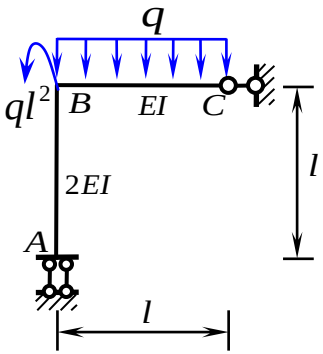


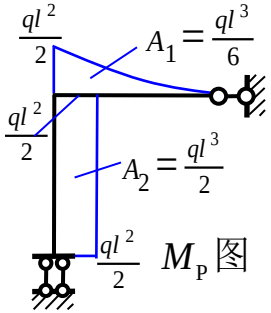


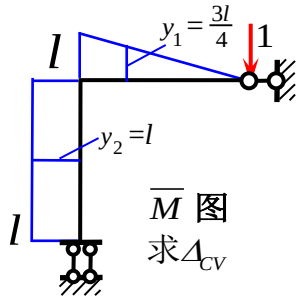
$$\Delta\varphi_{AB}=\sum\int\frac{\overline{M}\,M_P}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}=\frac{-A_1y_1}{EI}+\frac{-A_2y_2}{2EI}$$

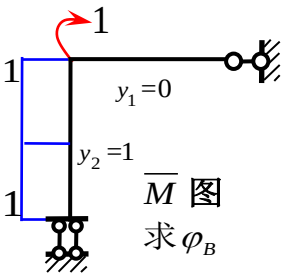
$$=\frac{-1}{EI}\times\frac{ql^3}{3}\times1+\frac{-1}{2EI}\times\frac{ql^3}{3}\times1=\frac{-ql^3}{2EI}$$

【例题6】（两刚片1-3）求Δ_{CV}，φ_B









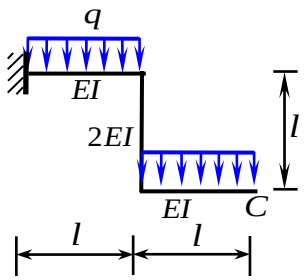
$$\Delta_{CV}=\sum\int\frac{\overline{M}\ M_P}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}=\frac{A_1y_1}{EI}-\frac{A_2y_2}{2EI}$$

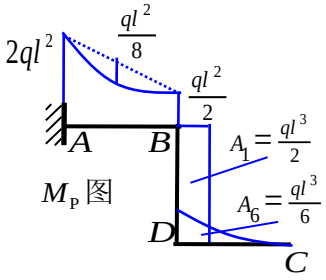
$$=\frac{1}{EI}\times\frac{ql^3}{6}\times\frac{3l}{4}-\frac{1}{2EI}\times\frac{ql^3}{2}\times l=\frac{-ql^4}{8EI}(\uparrow)$$

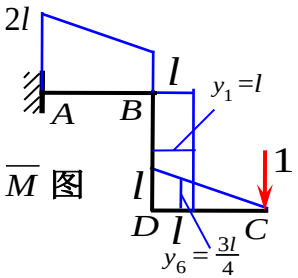
$$\varphi_B=\sum\int\frac{\overline{M}\ M_P}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}=\frac{A_1y_1}{EI}-\frac{A_2y_2}{2EI}$$

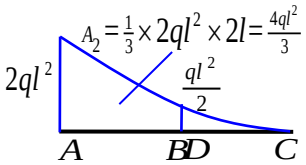
$$=\frac{1}{EI}\times\frac{ql^3}{6}\times0-\frac{1}{2EI}\times\frac{ql^3}{2}\times1=\frac{-ql^3}{4EI}(\text{逆时针})$$

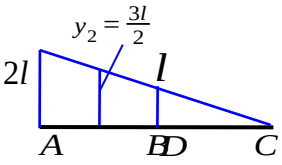
【例题7】（两刚片1-24）求Δ_{CV}

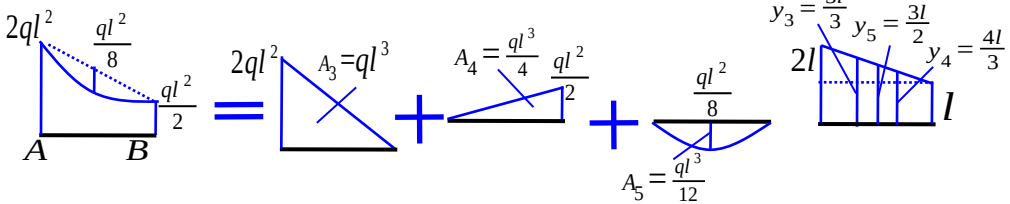












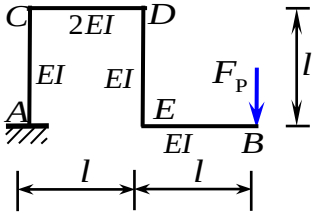
$$\Delta_{CV}=\sum\int\frac{\overline{M}\ M_P}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}=\frac{A_1y_1}{2EI}+\frac{A_2y_2}{EI}$$

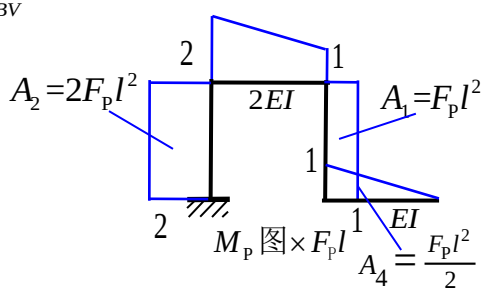
$$=\frac{1}{2EI}\times\frac{ql^3}{2}\times l+\frac{1}{EI}\times\frac{4ql^3}{3}\times\frac{3l}{2}=\frac{9ql^4}{4EI}(\downarrow)$$

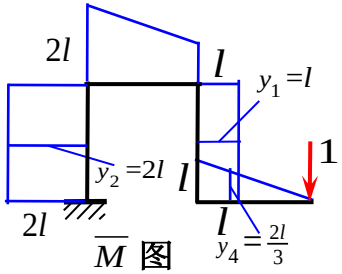
$$\Delta_{CV}=\sum\int\frac{\overline{M}\ M_P}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}=\frac{A_1y_1}{2EI}+\frac{A_6y_6}{EI}+\frac{A_3y_3}{EI}+\frac{A_4y_4}{EI}-\frac{A_5y_5}{EI}$$

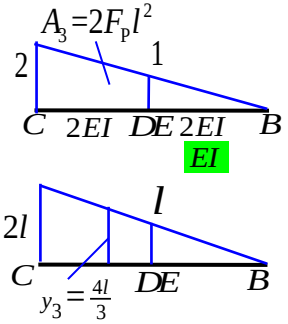
$$=\frac{1}{2EI}\times\frac{ql^3}{2}\times l+\frac{1}{EI}\times\frac{ql^3}{6}\times\frac{3l}{4}+\frac{1}{EI}\times ql^3\times\frac{5l}{3}+\frac{1}{EI}\times\frac{ql^3}{4}\times\frac{4l}{3}-\frac{1}{EI}\times\frac{ql^3}{12}\times\frac{3l}{2}=\frac{9ql^4}{4EI}(\downarrow)$$

【例题8】（两刚片1-26）求Δ_{BV}





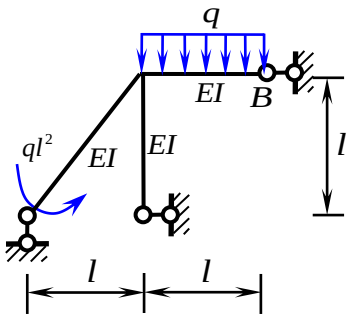


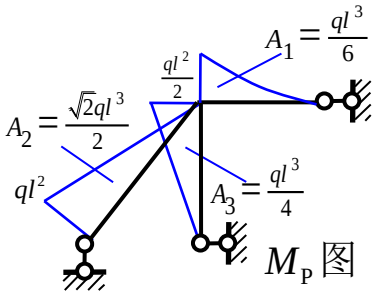


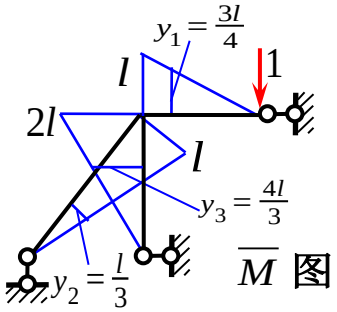
$$\Delta_{BV}=\sum\int\frac{\overline{M}\ M_P}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}=\frac{A_1y_1}{EI}+\frac{A_2y_2}{EI}+\frac{A_3y_3}{2EI}-\frac{A_4y_4}{2EI}+\frac{A_4y_4}{EI}$$

$$=\frac{1}{EI}\times F_Pl^2\times l+\frac{1}{EI}\times 2F_Pl^2\times 2l+\frac{1}{2EI}\times 2F_Pl^2\times\frac{4l}{3}-\frac{1}{2EI}\times\frac{F_Pl^2}{2}\times\frac{2l}{3}+\frac{1}{EI}\times\frac{F_Pl^2}{2}\times\frac{2l}{3}=\frac{13F_Pl^3}{2EI}(\downarrow)$$

【例题9】（两刚片1-31）求Δ_{BV}



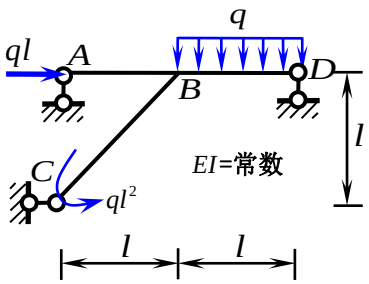


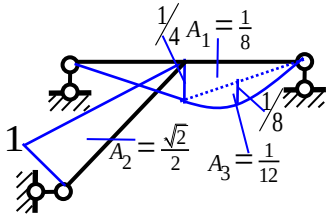


$$\Delta_{BV}=\sum\int\frac{\overline{M}\ M_P}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}=\frac{A_1y_1}{EI}-\frac{A_2y_2}{EI}+\frac{A_3y_3}{EI}$$

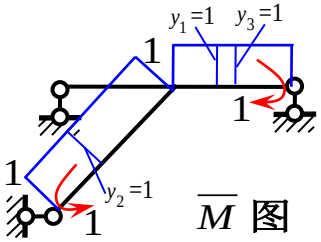
$$=\frac{1}{EI}\times\frac{ql^3}{6}\times\frac{3l}{4}-\frac{1}{EI}\times\frac{\sqrt{2}ql^3}{2}\times\frac{l}{3}+\frac{1}{EI}\times\frac{ql^3}{4}\times\frac{4l}{3}=\frac{(11-4\sqrt{2})ql^4}{24EI}(\downarrow)$$

【例题10】（两刚片1-35）求C、D两点相对转角$\Delta\varphi_{CD}$





M_P图$\times ql^2$　$A_i \times ql^3$

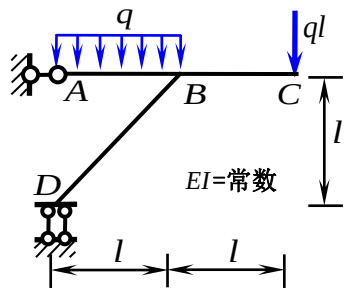


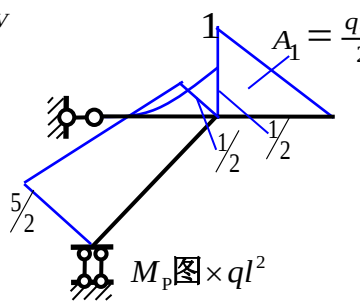
$\overline{M}$ 图

$$\Delta\varphi_{CD}=\sum\int\frac{\overline{M}\ M_P}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}=\frac{-A_1y_1}{EI}+\frac{A_2y_2}{EI}-\frac{A_3y_3}{EI}$$

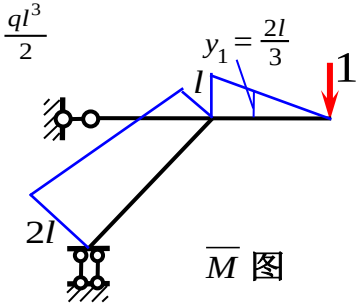
$$=\left(\frac{-1}{8}\times1+\frac{\sqrt{2}}{2}\times1-\frac{1}{12}\times1\right)\times\frac{ql^3}{EI}=\frac{\left(12\sqrt{2}-5\right)ql^3}{24EI}\text{（相对转角方向与单位力相同）}$$

【例题11】（两刚片1-39）求Δ_{CV}

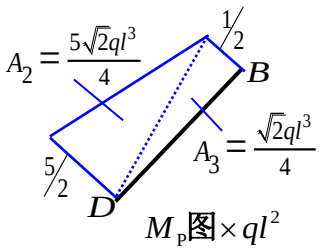




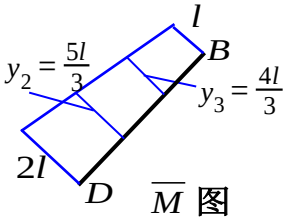
M_P图$\times ql^2$



$\overline{M}$ 图



M_P图$\times ql^2$

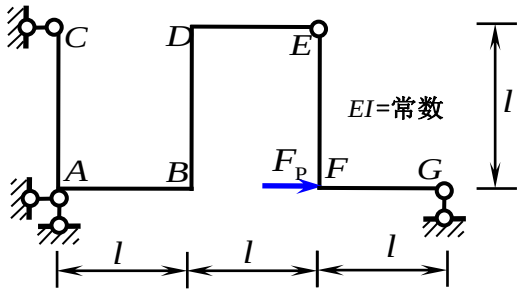


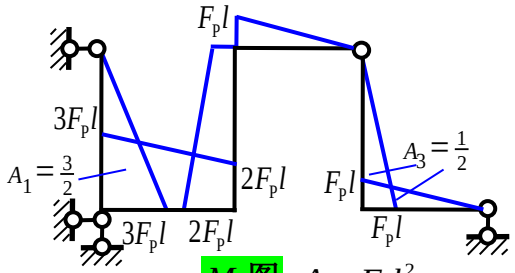
$\overline{M}$ 图

$$\Delta_{CV}=\sum\int\frac{\overline{M}\ M_P}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}=\frac{A_1y_1}{EI}+\frac{A_2y_2}{EI}+\frac{A_3y_3}{EI}$$

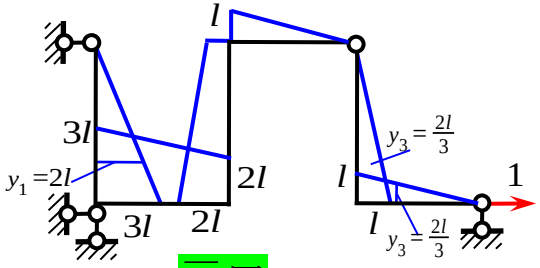
$$=\left(\frac{1}{2}\times\frac{2l}{3}+\frac{5\sqrt{2}}{4}\times\frac{5l}{3}+\frac{\sqrt{2}}{4}\times\frac{4l}{3}\right)\times\frac{ql^3}{EI}=\frac{\left(4+33\sqrt{2}\right)ql^4}{12EI}(\downarrow)$$

【例题12】（基附1-1）求Δ_{GH}

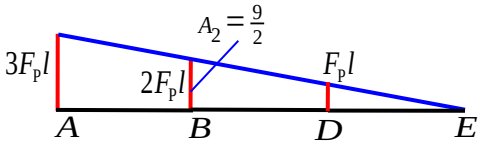


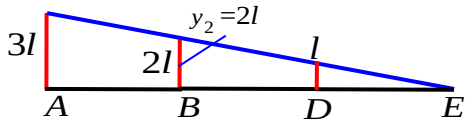


M_P图　$A_i \times F_P l^2$



$\overline{M}$ 图



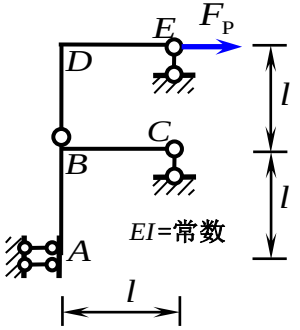


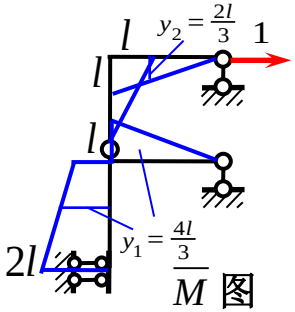
将*AB*、*BD*、*DE*三段放在一起进行图乘

$$\Delta_{GH}=\sum\int\frac{\overline{M}\ M_P}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}=\frac{A_1y_1}{EI}+\frac{A_2y_2}{EI}+\frac{2A_3y_3}{EI}$$

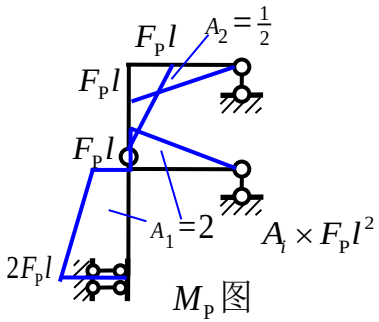
$$=\left(\frac{3}{2}\times 2l+\frac{9}{2}\times 2l+2\times\frac{1}{2}\times\frac{2}{3}\right)\times\frac{F_Pl^2}{EI}=\frac{38F_Pl^3}{3EI}(\rightarrow)$$

【例题13】（基附1-2）求Δ_{EH}





$\overline{M}$ 图



$A_i \times F_P l^2$

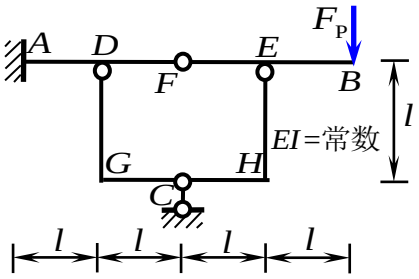
M_P图

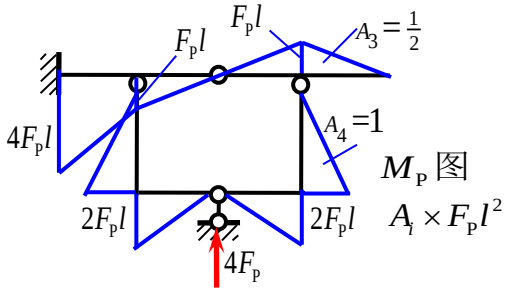
将*AB*、*BC*两段放在一起进行图乘

$$\Delta_{EH}=\sum\int\frac{\overline{M}\ M_P}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}=\frac{A_1y_1}{EI}+\frac{2A_2y_2}{EI}$$

$$=\left(2\times\frac{4l}{3}+2\times\frac{1}{2}\times\frac{2l}{3}\right)\times\frac{F_Pl^2}{EI}=\frac{10F_Pl^3}{3EI}(\rightarrow)$$

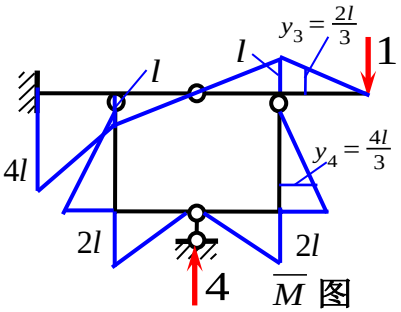
【例题14】（基附1-3）求Δ_{BV}



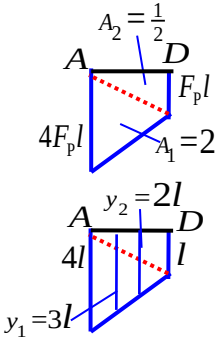


M_P 图

$A_i \times F_P l^2$



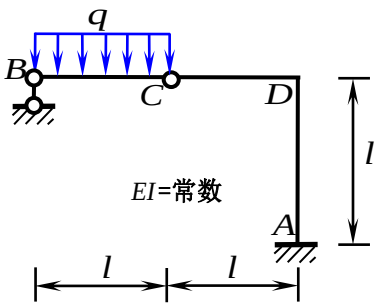
$\overline{M}$ 图

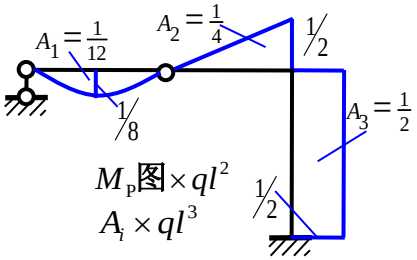


$$\Delta_{BV}=\sum\int\frac{\overline{M}\ M_P}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}=\frac{A_1y_1}{EI}+\frac{A_2y_2}{EI}+\frac{3A_3y_3}{EI}+\frac{4A_4y_4}{EI}$$

$$=\left(2\times3l+\frac{1}{2}\times2l+3\times\frac{1}{2}\times\frac{2l}{3}+4\times1\times\frac{4l}{3}\right)\times\frac{F_Pl^2}{EI}=\frac{40F_Pl^3}{3EI}(\downarrow)$$

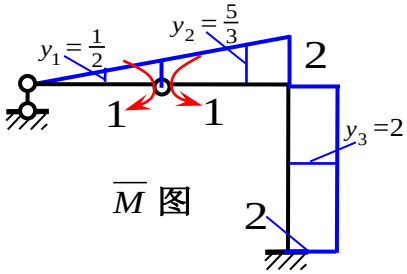
【例题15】（基附1-5）求铰C两侧截面的相对转角 $\Delta\varphi_C$





M_P图$\times ql^2$

$A_i \times ql^3$



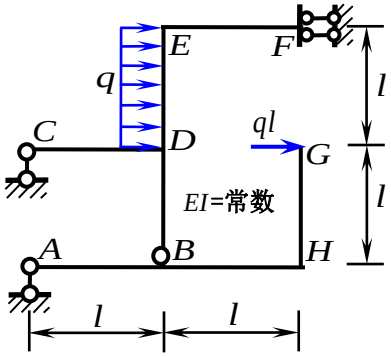
$\overline{M}$ 图

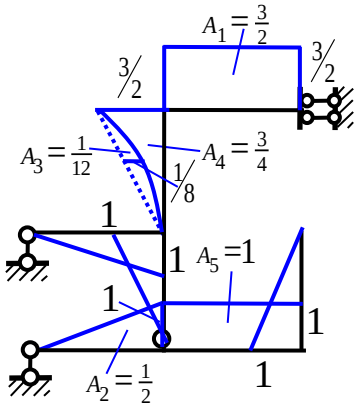
$$\Delta\varphi_C=\sum\int\frac{\overline{M}\ M_P}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}=\frac{-A_1y_1}{EI}+\frac{A_2y_2}{EI}+\frac{A_3y_3}{EI}$$

$$=\left(\frac{-1}{12}\times\frac{1}{2}+\frac{1}{4}\times\frac{5}{3}+\frac{1}{2}\times2\right)\times\frac{ql^3}{EI}=\frac{11ql^3}{8EI}$$

相对转角位移与单位力方向一致

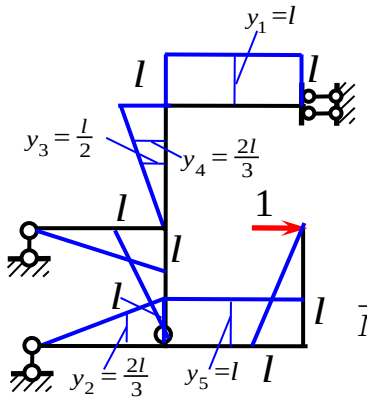
【例题16】（基附1-38）求Δ_{GH}





M_P图$\times ql^2$

$A_i\times ql^3$

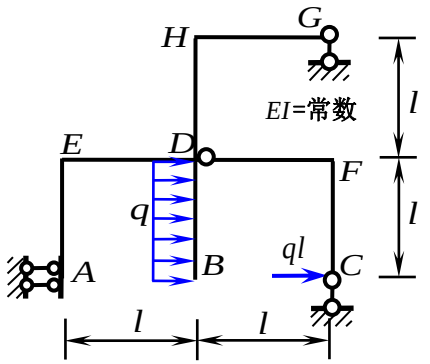


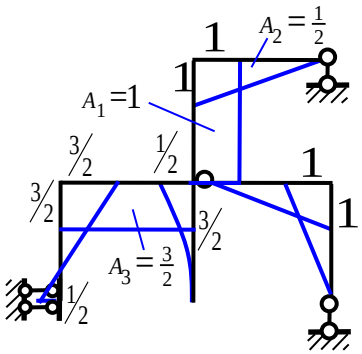
$\overline{M}$ 图

$$\Delta_{GH}=\sum\int\frac{\overline{M}\ M_P}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}=\frac{A_1y_1}{EI}+\frac{4A_2y_2}{EI}-\frac{A_3y_3}{EI}+\frac{A_4y_4}{EI}+\frac{A_5y_5}{EI}$$

$$=\left(\frac{3}{2}\times l+4\times\frac{1}{2}\times\frac{2l}{3}-\frac{1}{12}\times\frac{l}{2}+\frac{3}{4}\times\frac{2l}{3}+1\times l\right)\times\frac{ql^3}{EI}=\frac{103ql^4}{24EI}(\rightarrow)$$

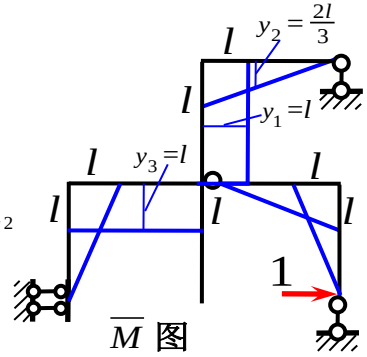
【例题17】（基附1-40） 求Δ_{CH}



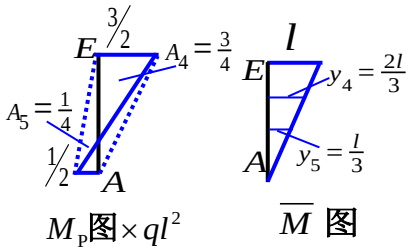


M_P图$\times ql^2$

$A_i\times ql^3$



$\overline{M}$ 图



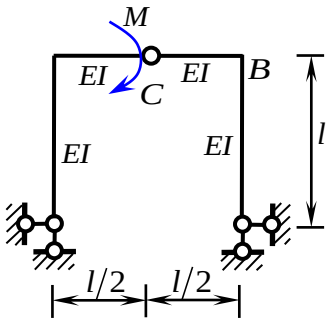
M_P图$\times ql^2$

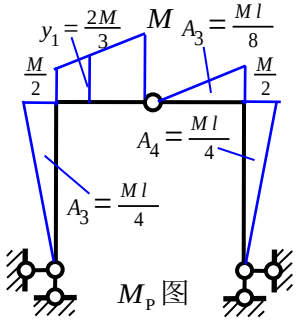
$\overline{M}$ 图

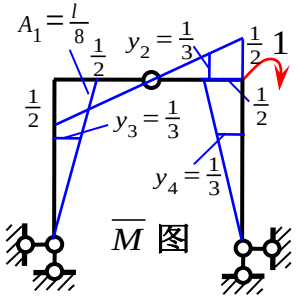
$$\Delta_{CH}=\sum\int\frac{\overline{M}\ M_P}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}=\frac{A_1y_1}{EI}+\frac{3A_2y_2}{EI}+\frac{A_3y_3}{EI}+\frac{A_4y_4}{EI}-\frac{A_5y_5}{EI}$$

$$=\left(1\times l+3\times\frac{1}{2}\times\frac{2l}{3}+\frac{3}{2}\times l+\frac{3}{4}\times\frac{2l}{3}-\frac{1}{4}\times\frac{l}{3}\right)\times\frac{ql^3}{EI}=\frac{47ql^4}{12EI}(\rightarrow)$$

【例题18】（三刚片1-4）求φ_B



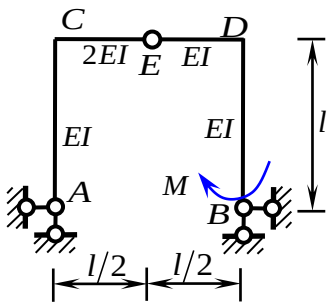


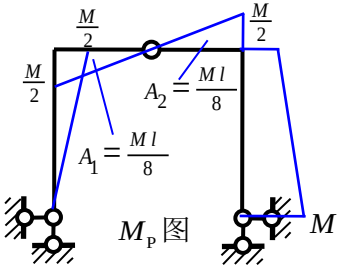


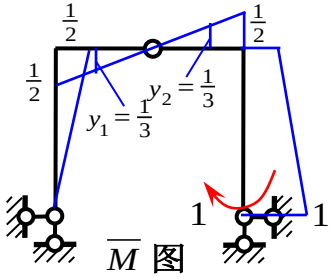
$$\varphi_B=\sum\int\frac{\overline{M}\ M_P}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}=\frac{-A_1y_1}{EI}+\frac{A_2y_2}{EI}-\frac{A_3y_3}{EI}-\frac{A_4y_4}{EI}$$

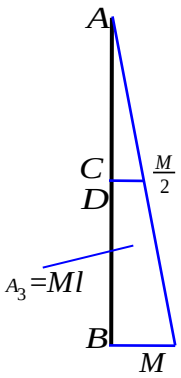
$$=\frac{-1}{EI}\times\frac{l}{8}\times\frac{2M}{3}+\frac{1}{EI}\times\frac{Ml}{8}\times\frac{1}{3}-\frac{1}{EI}\times\frac{Ml}{4}\times\frac{1}{3}-\frac{1}{EI}\times\frac{Ml}{4}\times\frac{1}{3}=\frac{-5Ml}{24EI}\text{(逆时针)}$$

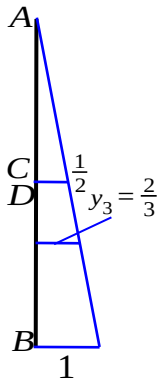
【例题19】（三刚片1-6）求φ_B







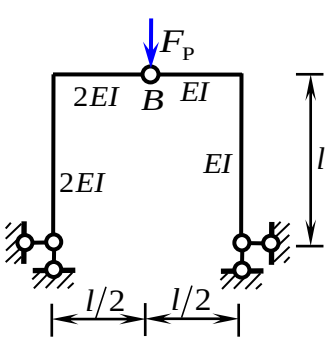


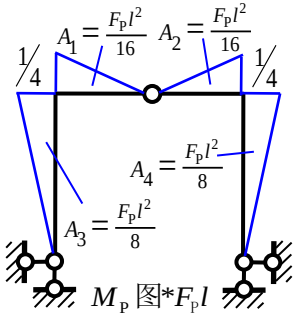


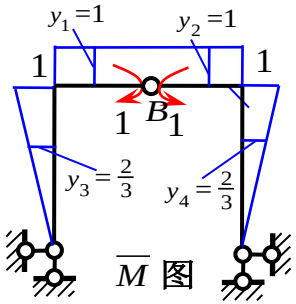
$$\varphi_B=\sum\int\frac{\overline{M}\ M_P}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}=\frac{A_1y_1}{2EI}+\frac{A_2y_2}{EI}+\frac{A_3y_3}{EI}$$

$$=\frac{1}{2EI}\times\frac{Ml}{8}\times\frac{1}{3}+\frac{1}{EI}\times\frac{Ml}{8}\times\frac{1}{3}+\frac{1}{EI}\times Ml\times\frac{2}{3}=\frac{35Ml}{48EI}\text{(逆时针)}$$

【例题20】（三刚片1-7）求*B*铰两侧相对转角。





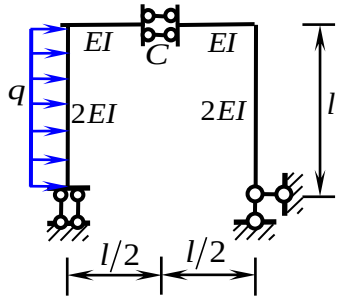


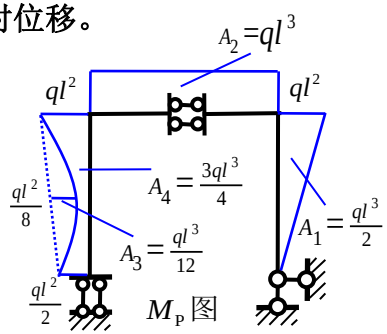
$$\Delta\varphi_B=\sum\int\frac{\overline{M}\ M_P}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}=\frac{A_1y_1}{2EI}+\frac{A_2y_2}{EI}+\frac{A_3y_3}{2EI}+\frac{A_4y_4}{EI}$$

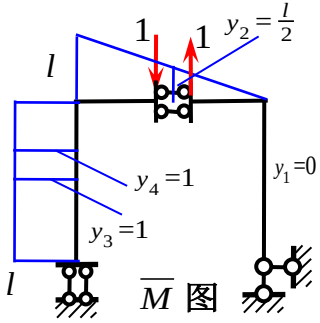
$$=\frac{1}{2EI}\times\frac{F_P l^2}{16}\times1+\frac{1}{EI}\times\frac{F_P l^2}{16}\times1+\frac{1}{2EI}\times\frac{F_P l^2}{8}\times\frac{2}{3}+\frac{1}{EI}\times\frac{F_P l^2}{8}\times\frac{2}{3}=\frac{7F_P l^2}{32EI}$$

（相对位移方向与单位力方向相同）

【例题21】（三刚片1-11）求*C*两侧相对位移。



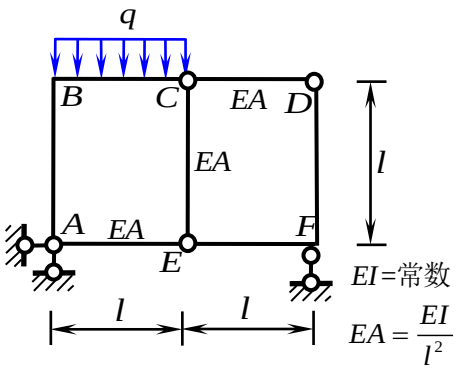


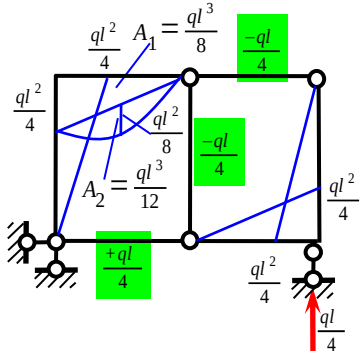


$$\Delta_{CV左右}=\sum\int\frac{\overline{M}\ M_P}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}=\frac{A_1y_1}{2EI}+\frac{A_2y_2}{EI}-\frac{A_3y_3}{2EI}+\frac{A_4y_4}{2EI}$$

$$=\frac{1}{2EI}\times\frac{ql^3}{2}\times0+\frac{1}{EI}\times ql^3\times\frac{l}{2}-\frac{1}{EI}\times\frac{ql^3}{12}\times l+\frac{1}{2EI}\times\frac{3ql^3}{4}\times l=\frac{19ql^4}{24EI}(\downarrow\uparrow)$$

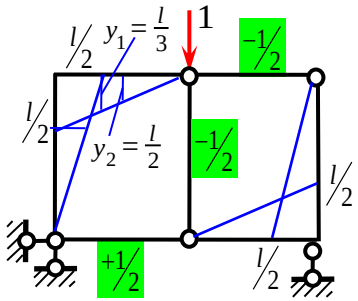
【例题22】（组合结构1-2）求Δ_{CV}





M_P 图

F_{NP} 图



$\overline{M}$ 图

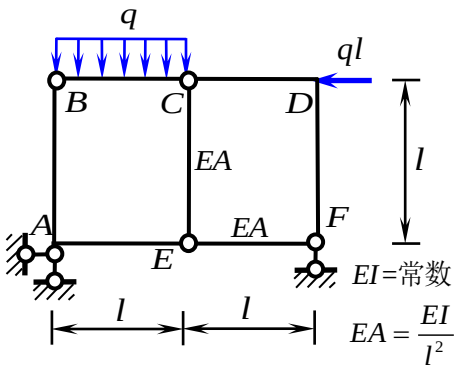
$\overline{F}_N$ 图

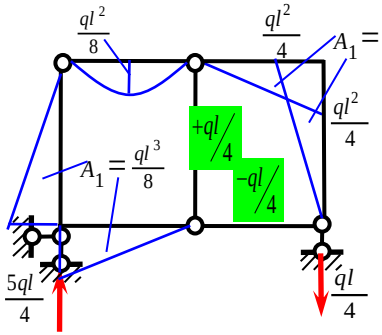
$$\Delta_{CV}=\sum\int\frac{\overline{M}\,M_P}{EI}ds+\sum\int\frac{\overline{F}_N F_{NP}}{EA}ds=\sum\frac{(\pm)Ay_0}{EI}+\sum\frac{\overline{F}_N F_{NP}l}{EA}$$

$$=4\times\frac{A_1y_1}{EI}+\frac{A_2y_2}{EI}+\sum\frac{\overline{F}_N F_{NP}l}{EA}=4\times\frac{1}{EI}\times\frac{ql^3}{8}\times\frac{l}{3}+\frac{1}{EI}\times\frac{ql^3}{12}\times\frac{l}{2}$$

$$+\frac{1}{EA}\times\frac{ql}{4}\times\frac{1}{2}\times l+\frac{1}{EA}\times\frac{-ql}{4}\times\frac{-1}{2}\times l=\frac{11ql^4}{24EI}(\downarrow)$$

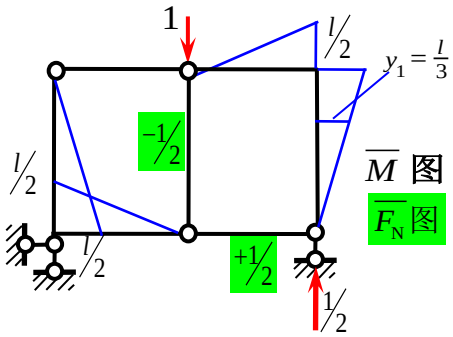
【例题23】（组合结构1-4） 求Δ_{CV}





M_P 图

F_{NP} 图



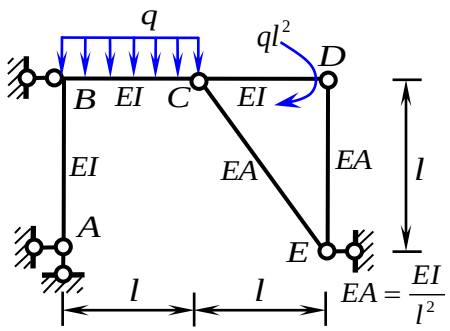
$\overline{M}$ 图

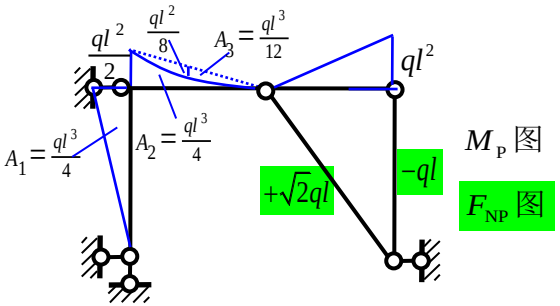
$\overline{F}_N$ 图

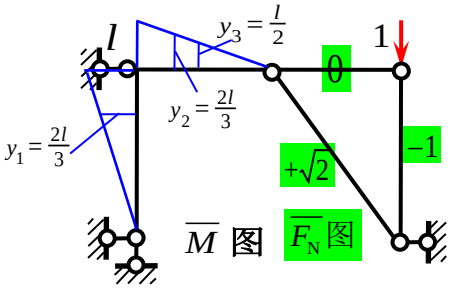
$$\Delta_{CV}=\sum\int\frac{\overline{M}\,M_P}{EI}ds+\sum\int\frac{\overline{F}_N F_{NP}}{EA}ds=\sum\frac{(\pm)Ay_0}{EI}+\sum\frac{\overline{F}_N F_{NP}l}{EA}$$

$$=4\times\frac{A_1y_1}{EI}+\sum\frac{\overline{F}_N F_{NP}l}{EA}=\frac{1}{EA}\times\frac{ql}{4}\times\frac{-1}{2}\times l+\frac{1}{EA}\times\frac{-ql}{4}\times\frac{1}{2}\times l+4\times\frac{1}{EI}\times\frac{ql^3}{8}\times\frac{2l}{3}=\frac{-ql^4}{6EI}(\uparrow)$$

【例题24】（组合结构1-9）求Δ_{DV}





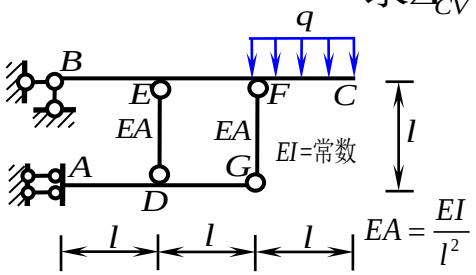


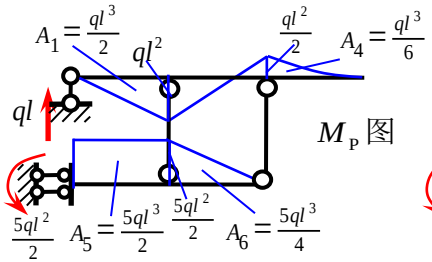
$$\Delta_{DV}=\sum\int\frac{\overline{M}\,M_P}{EI}ds+\sum\int\frac{\overline{F}_N F_{NP}}{EA}ds=\sum\frac{(\pm)Ay_0}{EI}+\sum\frac{\overline{F}_N F_{NP}l}{EA}$$

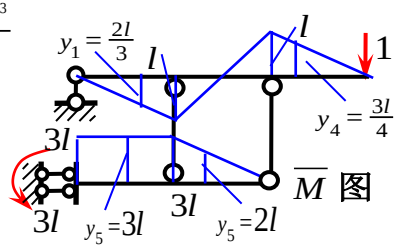
$$=\frac{A_1y_1}{EI}+\frac{A_2y_2}{EI}-\frac{A_3y_3}{EI}+\sum\frac{\overline{F}_N F_{NP}l}{EA}=\frac{1}{EA}\times\sqrt{2}ql\times\sqrt{2}\times\sqrt{2}l+\frac{1}{EA}\times ql\times 1\times l+$$

$$\frac{1}{EI}\times\frac{ql^3}{4}\times\frac{2l}{3}+\frac{1}{EI}\times\frac{ql^3}{4}\times\frac{2l}{3}-\frac{1}{EI}\times\frac{ql^3}{12}\times\frac{l}{2}=\frac{\left(31+2\sqrt{2}\right)ql^4}{24EI}(\downarrow)$$

【例题25】（组合结构1-26）求Δ_{CV}

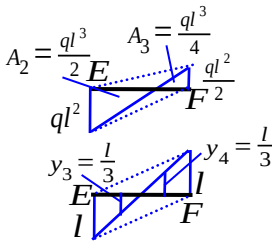






F_{NP} 图 $F_{NDE}=+\frac{5ql}{2}$ $F_{NFG}=-\frac{5ql}{2}$

$\overline{F}_N$ 图 $\overline{F}_{NDF}=+3$ $\overline{F}_{NFG}=-3$

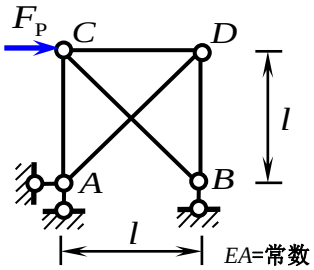


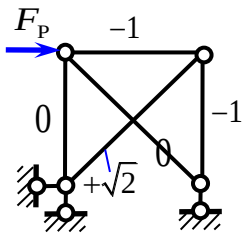
$$\Delta_{CV}=\sum\int\frac{\overline{M}\,M_P}{EI}ds+\sum\int\frac{\overline{F}_N F_{NP}}{EA}ds=\sum\frac{(\pm)Ay_0}{EI}+\sum\frac{\overline{F}_N F_{NP}l}{EA}$$

$$=\frac{A_1y_1}{EI}+\frac{A_2y_2}{EI}+\frac{A_3y_3}{EI}+\frac{A_4y_4}{EI}+\frac{A_5y_5}{EI}+\frac{A_6y_6}{EI}+\sum\frac{\overline{F}_N F_{NP}l}{EA}=\frac{1}{EA}\times\frac{5ql}{2}\times 3\times l+\frac{-1}{EA}\times\frac{5ql}{2}\times(-3)\times l+$$

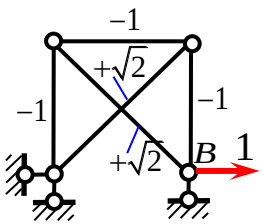
$$\frac{1}{EI}\left(\frac{ql^3}{2}\times\frac{2l}{3}+\frac{ql^3}{2}\times\frac{l}{3}+\frac{ql^3}{4}\times\frac{l}{3}+\frac{ql^3}{6}\times\frac{3l}{4}+\frac{5ql^3}{2}\times 3l+\frac{5ql^3}{4}\times 2l\right)=\frac{617ql^4}{24EI}(\downarrow)$$

【例题26】（桁架1-1）求Δ_{BH}





F_{NP}图×F_P

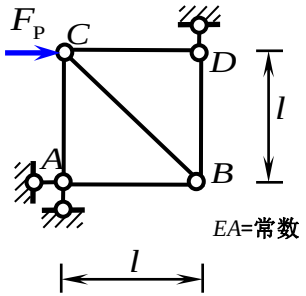


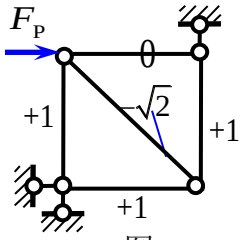
$\overline{F}_N$图

$$\Delta_{BH}=\sum\frac{\overline{F}_{N}F_{NP}l}{EA}=\frac{F_P}{EA}\{-1\times0\times l+(-1)\times(-1)\times l+(-1)\times(-1)\times l\}$$

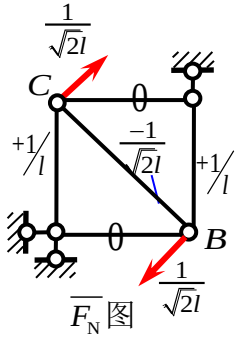
$$+\frac{F_P}{EA}\left(\sqrt{2}\times\sqrt{2}\times\sqrt{2}l+\sqrt{2}\times0\times\sqrt{2}l\right)=\frac{\left(2+2\sqrt{2}\right)F_Pl}{EA}(\rightarrow)$$

【例题27】（桁架1-3）求CB杆转角 φ_{CB}





F_{NP}图×F_P



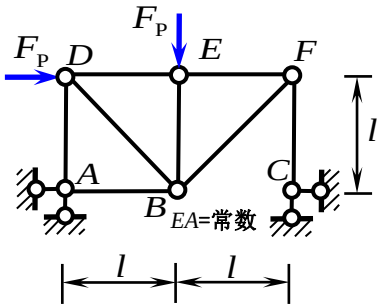
$\overline{F}_N$图

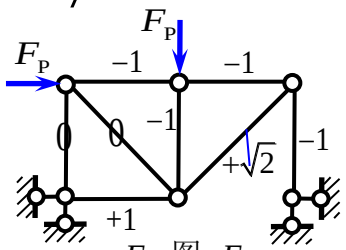
$$\varphi_{CB}=\sum\frac{\overline{F}_{N}F_{NP}l}{EA}$$

$$=\frac{F_P}{EA}\left\{\frac{1}{l}\times1\times l+\frac{1}{l}\times1\times l+0\times0\times l+0\times1\times l+\frac{-1}{\sqrt{2}l}\times\left(-\sqrt{2}\right)\times\sqrt{2}l\right\}$$

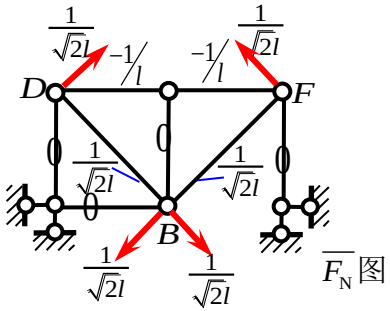
$$=\frac{\left(2+\sqrt{2}\right)F_P}{EA}(\text{顺时针})$$

【例题28】（桁架1-6）求*DB*、*BF*杆相对转角 $\Delta\varphi$





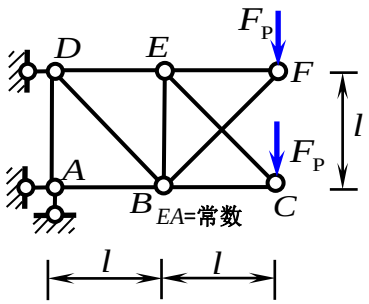
F_{NP}图×F_P

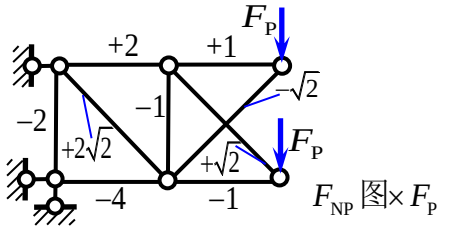


$\overline{F}_N$图

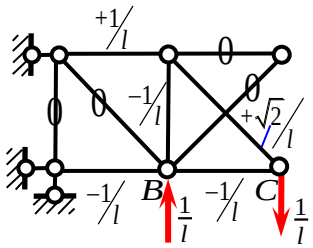
$$\Delta\varphi=\sum\frac{\overline{F}_{N}F_{NP}l}{EA}=\frac{F_P}{EA}\left\{2\times\left(\frac{-1}{l}\right)\times(-1)\times l+\frac{1}{\sqrt{2}l}\times\sqrt{2}\times\sqrt{2}l+\frac{1}{\sqrt{2}l}\times0\times\sqrt{2}l\right\}=\frac{\left(2+\sqrt{2}\right)F_P}{EA}(\text{夹角变小})$$

【例题29】（桁架1-8）求CB杆转角 φ_{CB}





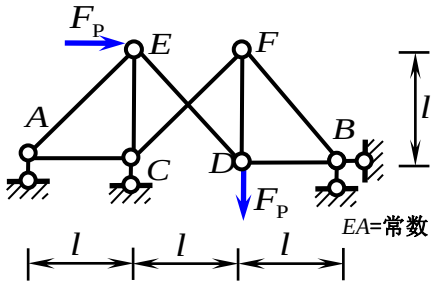
F_{NP}图×F_P

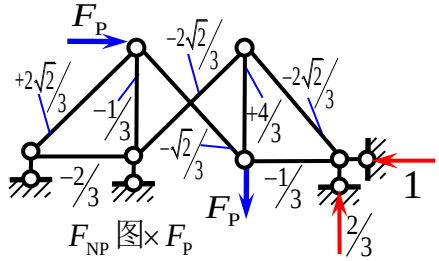


$\overline{F}_N$图

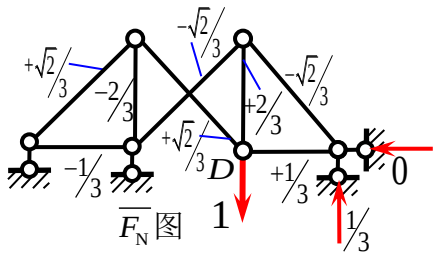
$$\varphi_{BC}=\sum\frac{\overline{F}_N F_{NP} l}{EA}=\frac{F_P}{EA}\left\{2\times\left(\frac{-1}{l}\right)\times(-1)\times l+\left(\frac{-1}{l}\right)\times(-4)\times l+\frac{1}{l}\times 2\times l+\frac{\sqrt{2}}{l}\times\left(-\sqrt{2}\right)\times\sqrt{2}l\right\}=\frac{\left(8-2\sqrt{2}\right)F_P}{EA}(\text{顺时针})$$

【例题30】（桁架1-36）求Δ_{DV}





F_{NP}图×F_P

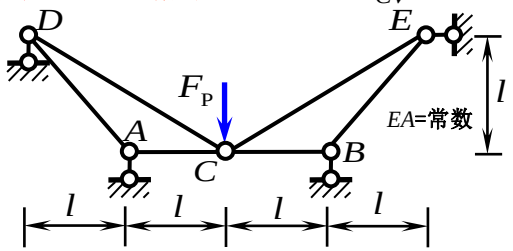


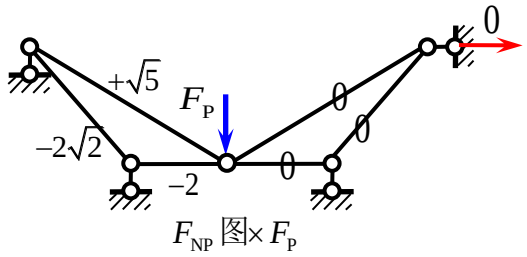
$\overline{F}_N$图

$$\Delta_{DV}=\sum\frac{\overline{F}_N F_{NP} l}{EA}=\frac{F_P}{EA}\left\{\frac{1}{3}\times\frac{-1}{3}\times l+\frac{2}{3}\times\frac{4}{3}\times l+\frac{-1}{3}\times\frac{-2}{3}\times l+\frac{-2}{3}\times\frac{-1}{3}\times l\right\}$$

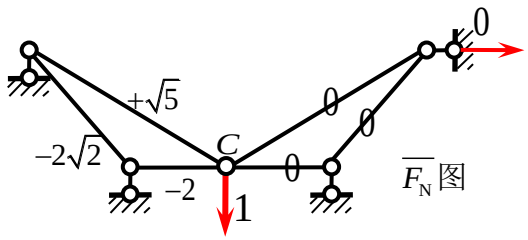
$$+\frac{F_P}{EA}\left(2\times\frac{-\sqrt{2}}{3}\times\frac{-2\sqrt{2}}{3}\times\sqrt{2}l+\frac{\sqrt{2}}{3}\times\frac{-\sqrt{2}}{3}\times\sqrt{2}l+\frac{\sqrt{2}}{3}\times\frac{2\sqrt{2}}{3}\times\sqrt{2}l\right)=\frac{\left(11+10\sqrt{2}\right)F_P l}{9EA}(\downarrow)$$

【例题31】（桁架1-36）求Δ_{CV}





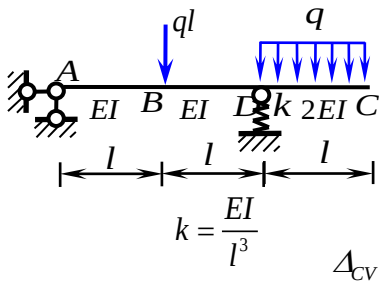
F_{NP}图×F_P

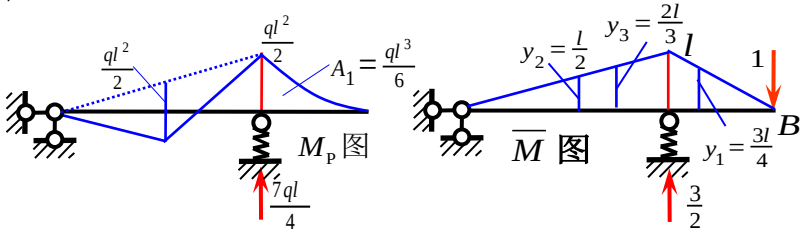


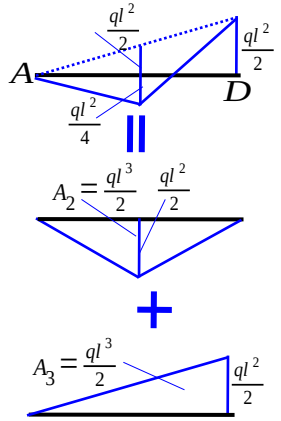
$\overline{F}_N$图

$$\Delta_{CV}=\sum\frac{\overline{F}_N F_{NP} l}{EA}=\frac{F_P}{EA}\left\{-2\times(-2)\times l+-2\sqrt{2}\times\left(-2\sqrt{2}\right)\times\sqrt{2}l\right\}+\frac{F_P}{EA}\left(\sqrt{5}\times\sqrt{5}\times\sqrt{5}l\right)=\frac{\left(4+8\sqrt{2}+5\sqrt{5}\right)F_P l}{EA}(\downarrow)$$

【例题32】（梁弹簧1-2）求Δ_{CV}



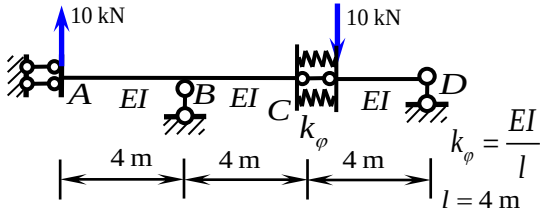


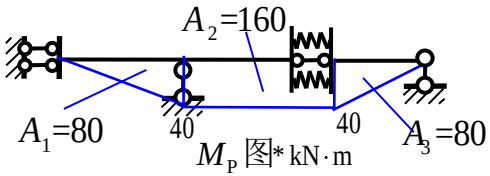


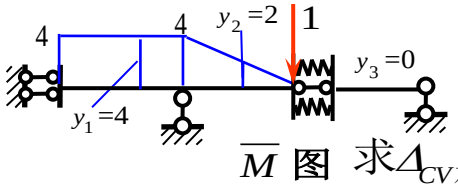
$$\Delta_{CV}=\sum\int\frac{\overline{M}\ M_P}{EI}ds+\sum\frac{\overline{F}_k F_{kP}}{k}=\sum\frac{(\pm)Ay_0}{EI}+\sum\frac{\overline{F}_k F_{kP}}{k}=\frac{A_1y_1}{2EI}-\frac{A_2y_2}{EI}+\frac{A_3y_3}{EI}+\frac{\overline{F}_k F_{kP}}{k}$$

$$=\frac{1}{2EI}\times\frac{ql^3}{6}\times\frac{2l}{3}-\frac{1}{EI}\times\frac{ql^3}{2}\times\frac{l}{2}+\frac{1}{EI}\times\frac{ql^3}{2}\times\frac{2l}{3}+\frac{1}{k}\times\frac{7ql}{4}\times\frac{3}{2}=\frac{133ql^4}{48EI}(\downarrow)$$

【例题33】（梁弹簧1-2）求Δ_{CV}

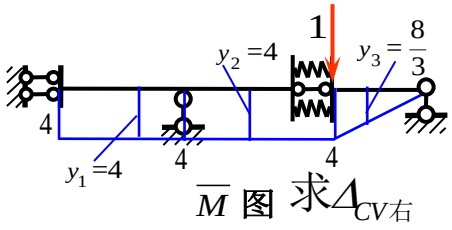






$$\Delta_{CV左}=\sum\int\frac{\overline{M}\ M_P}{EI}ds+\sum\frac{\overline{F}_k F_{kP}}{k}=\sum\frac{(\pm)Ay_0}{EI}+\sum\frac{\overline{F}_k F_{kP}}{k}=\frac{-A_1y_1}{EI}+\frac{-A_2y_2}{EI}-\frac{A_3y_3}{EI}+\frac{\overline{F}_k F_{kP}}{k_\varphi}$$

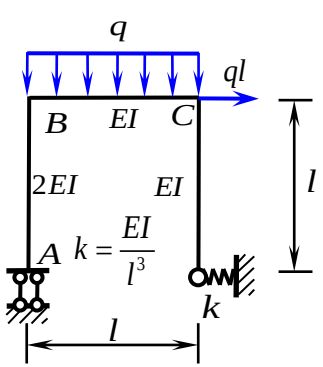
$$=\frac{-1}{EI}\times80\times4+\frac{-1}{EI}\times160\times2-\frac{1}{EI}\times80\times0+\frac{1}{k_\varphi}\times40\times0=\frac{-640\text{kN}\cdot\text{m}^3}{EI}(\uparrow)$$

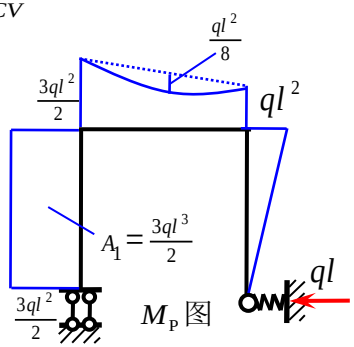


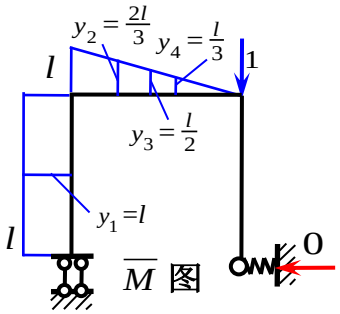
$$\Delta_{CV右}=\sum\int\frac{\overline{M}\ M_P}{EI}ds+\sum\frac{\overline{F}_k F_{kP}}{k_\varphi}=\sum\frac{(\pm)Ay_0}{EI}+\sum\frac{\overline{F}_k F_{kP}}{k_\varphi}=\frac{A_1y_1}{EI}+\frac{A_2y_2}{EI}+\frac{A_3y_3}{EI}+\frac{\overline{F}_k F_{kP}}{k_\varphi}$$

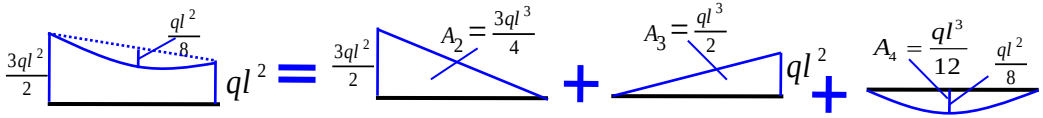
$$=\frac{1}{EI}\times80\times4+\frac{1}{EI}\times160\times4+\frac{1}{EI}\times80\times\frac{8}{3}+\frac{1}{k_\varphi}\times40\times4+=\frac{5440\text{kN}\cdot\text{m}^3}{3EI}(\downarrow)$$

【例题34】（刚架弹簧1-6） 求Δ_{CV}





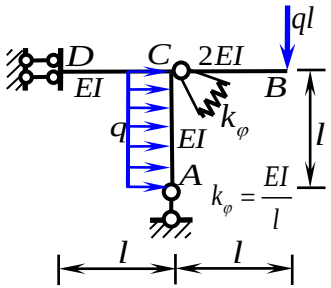


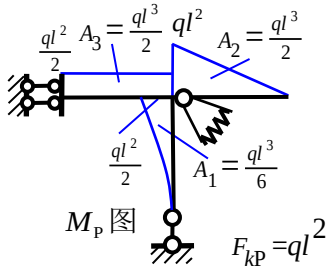


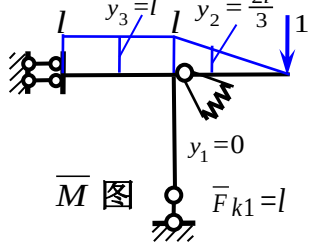
$$\Delta_{CV}=\sum\int\frac{\overline{M}\ M_P}{EI}ds+\sum\frac{\overline{F}_{k1}F_{kP}}{k}=\sum\frac{(\pm)Ay_0}{EI}+\sum\frac{\overline{F}_{k1}F_{kP}}{k}=\frac{A_1y_1}{2EI}+\frac{A_2y_2}{EI}+\frac{A_3y_3}{EI}-\frac{A_4y_4}{EI}+\frac{\overline{F}_{k1}F_{kP}}{k}$$

$$=\frac{1}{2EI}\times\frac{3ql^3}{2}\times l+\frac{1}{EI}\times\frac{3ql^3}{4}\times\frac{2l}{3}+\frac{1}{EI}\times\frac{ql^3}{2}\times\frac{l}{3}-\frac{1}{EI}\times\frac{ql^3}{12}\times\frac{l}{2}+\frac{1}{k}\times ql\times 0=\frac{11ql^4}{8EI}(\downarrow)$$

【例题35】（刚架弹簧1-10） 求Δ_{BV}



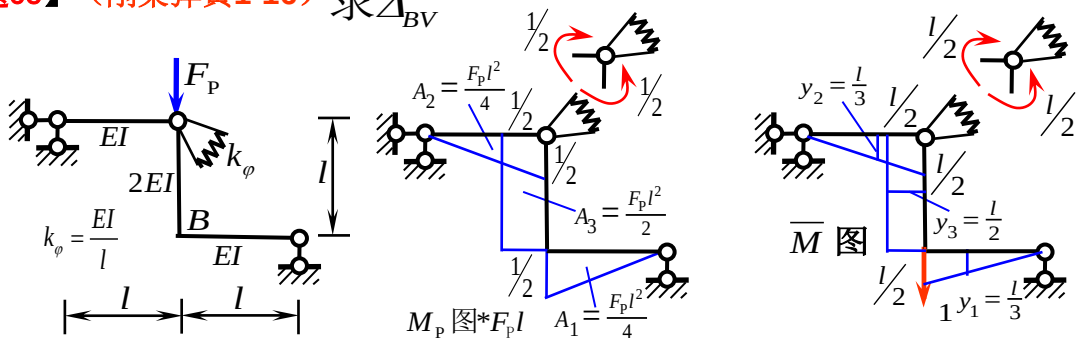




$$\Delta_{BV}=\sum\int\frac{\overline{M}\ M_P}{EI}ds+\sum\frac{\overline{F}_{k1}F_{kP}}{k}=\sum\frac{(\pm)Ay_0}{EI}+\sum\frac{\overline{F}_{k1}F_{kP}}{k}=\frac{A_1y_1}{EI}+\frac{A_2y_2}{2EI}+\frac{A_3y_3}{EI}+\frac{\overline{F}_{k1}F_{kP}}{k_\varphi}$$

$$=\frac{1}{EI}\times\frac{ql^3}{6}\times 0+\frac{1}{2EI}\times\frac{ql^3}{2}\times\frac{2l}{3}+\frac{1}{EI}\times\frac{ql^3}{2}\times l+\frac{1}{k}\times ql^2\times l=\frac{5ql^4}{3EI}(\downarrow)$$

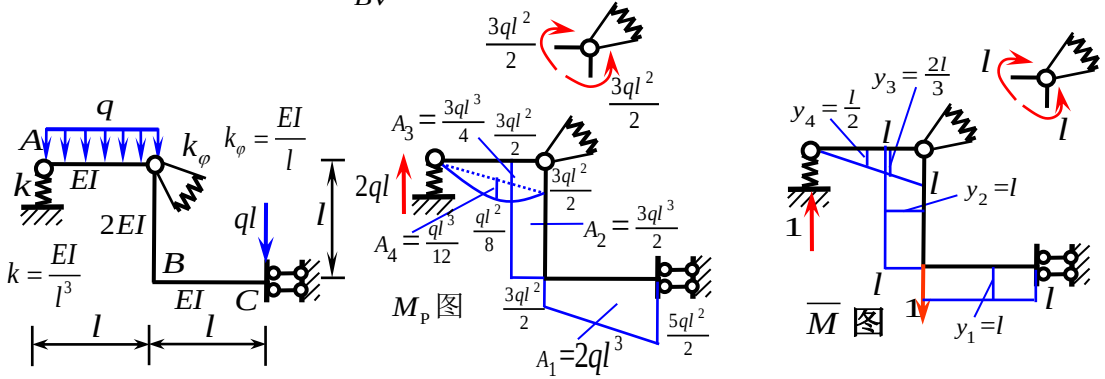
【例题36】（刚架弹簧1-16）求Δ_{BV}



$$\Delta_{BV}=\sum\int\frac{\overline{M}\ M_P}{EI}ds+\sum\frac{\overline{F}_{k1}F_{kP}}{k}=\sum\frac{(\pm)Ay_0}{EI}+\sum\frac{\overline{F}_{k1}F_{kP}}{k}=\frac{A_1y_1}{EI}+\frac{A_2y_2}{EI}+\frac{A_3y_3}{3EI}+\frac{\overline{F}_{k1}F_{kP}}{k}$$

$$=\frac{1}{EI}\times\frac{F_P l^2}{4}\times\frac{l}{3}+\frac{1}{EI}\times\frac{F_P l^2}{4}\times\frac{l}{3}+\frac{1}{2EI}\times\frac{F_P l^2}{2}\times\frac{l}{2}+\frac{1}{k}\times\frac{F_P l}{2}\times\frac{l}{2}=\frac{13F_P l^3}{24EI}(\downarrow)$$

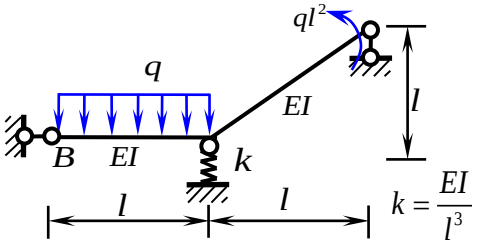
【例题37】（刚架弹簧1-18）求Δ_{BV}

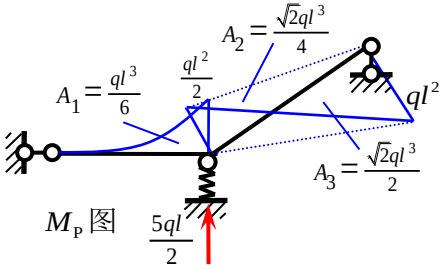


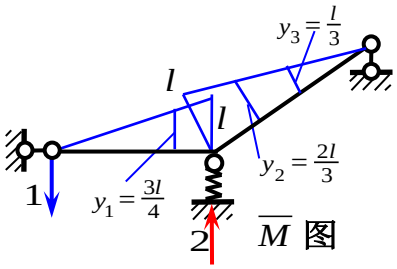
$$\Delta_{BV}=\sum\int\frac{\overline{M}\ M_P}{EI}ds+\sum\frac{\overline{F}_{k1}F_{kP}}{k}=\sum\frac{(\pm)Ay_0}{EI}+\sum\frac{\overline{F}_{k1}F_{kP}}{k}=\frac{A_1y_1}{EI}+\frac{A_2y_2}{2EI}+\frac{A_3y_3}{EI}+\frac{A_4y_4}{EI}+\frac{\overline{F}_{k1}F_{kP1}}{k}+\frac{\overline{F}_{k2}F_{kP2}}{k}$$

$$=\frac{1}{EI}\times2ql^3\times l+\frac{1}{2EI}\times\frac{3ql^3}{2}\times l+\frac{1}{EI}\times\frac{3ql^3}{4}\times\frac{2l}{3}+\frac{1}{EI}\times\frac{ql^3}{12}\times\frac{l}{2}+\frac{1}{k}\times2ql\times1+\frac{1}{k_\varphi}\times\frac{3ql}{2}\times l=\frac{163ql^4}{24EI}(\downarrow)$$

【例题38】（刚架弹簧1-21）求Δ_{BV}



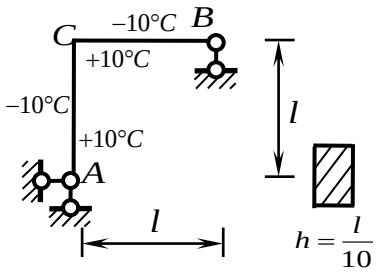


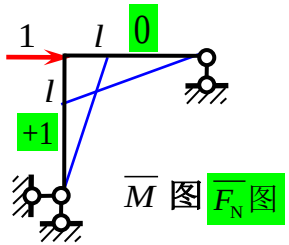


$$\Delta_{BV}=\sum\int\frac{\overline{M}\ M_P}{EI}ds+\sum\frac{\overline{F}_{k1}F_{kP}}{k}=\sum\frac{(\pm)Ay_0}{EI}+\sum\frac{\overline{F}_{k1}F_{kP}}{k}=\frac{A_1y_1}{EI}+\frac{A_2y_2}{EI}-\frac{A_3y_3}{EI}+\frac{\overline{F}_{k1}F_{kP}}{k}$$

$$=\frac{1}{EI}\times\frac{ql^3}{6}\times\frac{3l}{4}+\frac{1}{EI}\times\frac{\sqrt{2}ql^3}{4}\times\frac{2l}{3}-\frac{1}{EI}\times\frac{\sqrt{2}ql^3}{2}\times\frac{l}{3}+\frac{1}{k}\times\frac{5ql^2}{2}\times2=\frac{41ql^4}{8EI}(\downarrow)$$

【例题39】（温度1-2） 求Δ_{CH} 线胀系数α





$t_{0AC}=0°C$ $t_{0CB}=0°C$

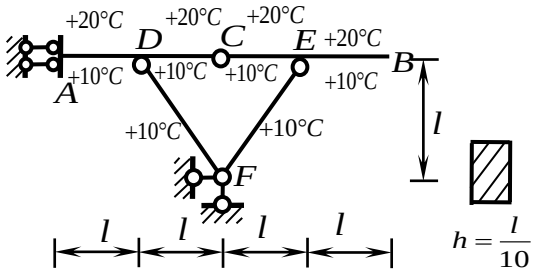
$\Delta t_{AC}=20°C$ $\Delta t_{CB}=20°C$

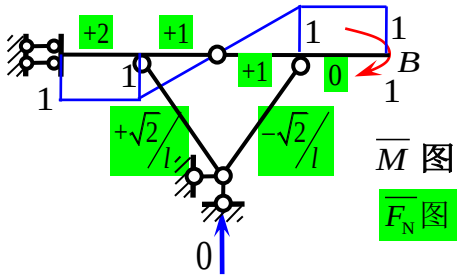
$A_{AC}=\frac{l^2}{2}$ $A_{CB}=\frac{l^2}{2}$

$$\Delta_{CH}=\sum\overline{F}_N du+\sum\overline{M}d\varphi=\sum\overline{F}_N\alpha t_0 l+\sum(\pm)A_{\overline{M}}\frac{\alpha\Delta t}{h}$$

$$=\frac{\alpha}{h}\left(\frac{l^2}{2}\times20+\frac{l^2}{2}\times20\right)=200\alpha l(\rightarrow)$$

【例题40】（温度1-4）求 φ_B　线胀系数 α





$t_{0AD}=15°C$　$t_{0DC}=15°C$　$t_{0CE}=15°C$

$t_{0EB}=15°C$　$t_{0CF}=10°C$　$t_{0EF}=10°C$

$\Delta t_{AD}=10°C$　$\Delta t_{DC}=10°C$　$\Delta t_{CE}=10°C$　$\Delta t_{EB}=10°C$

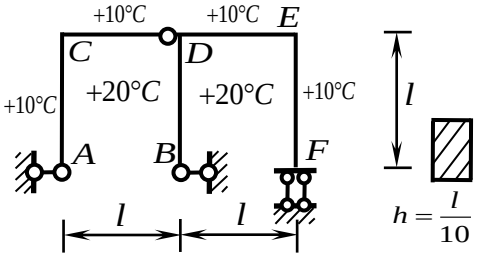
$A_{AD}=l$　$A_{DC}=\frac{l}{2}$　$A_{CE}=\frac{l}{2}$　$A_{EB}=l$

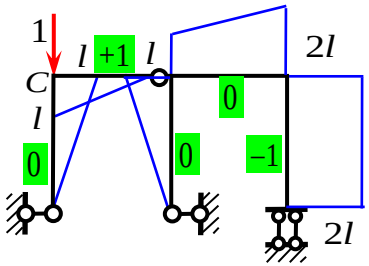
$$\varphi_B=\sum\overline{F}_N du+\sum\overline{M}d\varphi=\sum\overline{F}_N\alpha t_0 l+\sum(\pm)A_{\overline{M}}\frac{\alpha\Delta t}{h}$$

$$=\frac{\sqrt{2}}{l}\times10\alpha l-\frac{\sqrt{2}}{l}\times10\alpha l+2\times15\alpha l+1\times15\alpha l+1\times15\alpha l$$

$$\frac{\alpha}{h}\left(-l\times10+\frac{l}{2}\times10-\frac{l}{2}\times10+l\times10\right)=60\alpha(\text{顺时针})$$

【例题41】（温度1-8）求 Δ_{CV}　线胀系数 α





$t_{0AC}=15°C$　$t_{0CD}=15°C$　$t_{0BD}=20°C$　$t_{0DE}=15°C$　$t_{0EF}=15°C$

$\Delta t_{AC}=10°C$　$\Delta t_{CD}=10°C$　$\Delta t_{BD}=0°C$　$\Delta t_{DE}=10°C$　$\Delta t_{EF}=10°C$

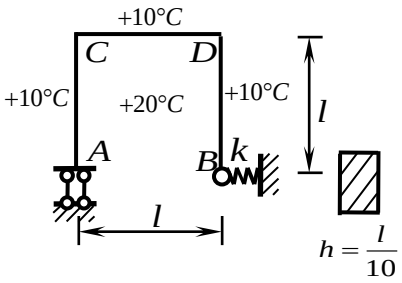
$A_{AC}=\frac{l^2}{2}$　$A_{CD}=\frac{l^2}{2}$　$A_{BD}=\frac{l^2}{2}$　$A_{DE}=\frac{3l^2}{2}$　$A_{EF}=2l^2$

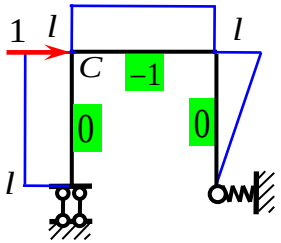
$$\Delta_{CV}=\sum\overline{F}_N du+\sum\overline{M}d\varphi=\sum\overline{F}_N\alpha t_0 l+\sum(\pm)A_{\overline{M}}\frac{\alpha\Delta t}{h}$$

$$=1\times15\alpha l-1\times15\alpha l$$

$$\frac{\alpha}{h}\left(\frac{l^2}{2}\times10+\frac{l^2}{2}\times10-\frac{3l^2}{2}\times10-2l^2\times10\right)=-250\alpha l(\uparrow)$$

【例题42】（温度1-17） 求Δ_{CH} 线胀系数α





$\overline{M}$ 图 $\overline{F}_N$图

$t_{0AC}=15°C$ $t_{0CD}=15°C$ $t_{0BD}=15°C$

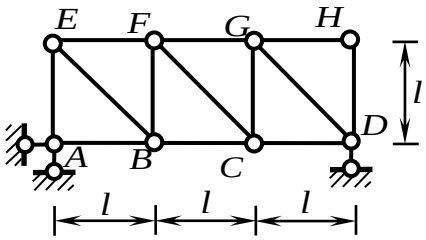
$\Delta t_{AC}=10°C$ $\Delta t_{CD}=10°C$ $\Delta t_{BD}=10°C$

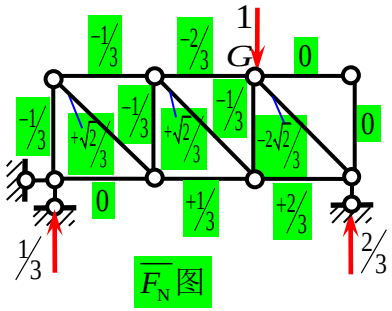
$A_{AC}=l^2$ $A_{CD}=l^2$ $A_{BD}=\frac{l^2}{2}$

$$\Delta_{CH}=\sum\overline{F}_N du+\sum\overline{M}d\varphi=\sum\overline{F}_N\alpha t_0 l+\sum(\pm)A_{\overline{M}}\frac{\alpha\Delta t}{h}$$

$$=-1\times15\alpha l+\frac{\alpha}{h}\left(-l^2\times10-l^2\times10-\frac{l^2}{2}\times10\right)=-265\alpha l(\leftarrow)$$

【例题43】（温度1-2）所有杆件温度升高 $+10°C$ 求Δ_{GV}线胀系数α





$\overline{F}_N$图

所有杆件温度 $t_0=+10°C$

$$\Delta_{GV}=\sum\overline{F}_N du=\sum\overline{F}_N\alpha t_0 l$$

$$=4\times\frac{-1}{3}\times10\alpha l+\frac{-2}{3}\times10\alpha l+\frac{1}{3}\times10\alpha l+2\times\frac{\sqrt{2}}{3}\times10\alpha\times\sqrt{2}l+\frac{-2\sqrt{2}}{3}\times10\alpha\times\sqrt{2}l+\frac{2}{3}\times10\alpha l=-10\alpha l(\uparrow)$$