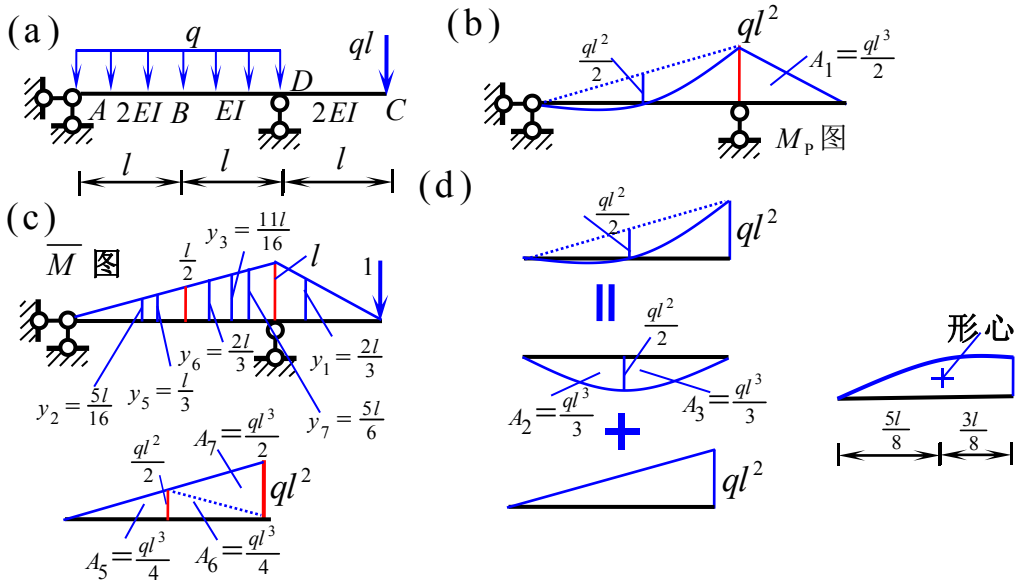
第三章　静定结构位移计算答案

【题 1】 图示体系　　求 Δ_{CV}

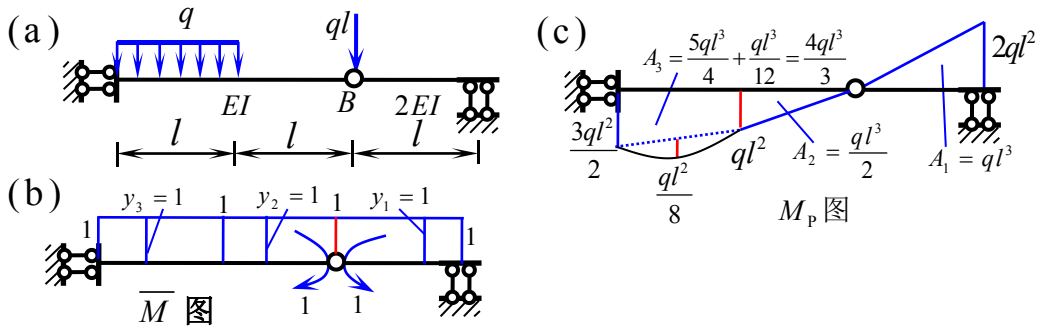


$$\Delta_{CV}=\sum\int\frac{\overline{M}\,M_P}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}=\frac{A_1y_1}{2EI}-\frac{A_2y_2}{2EI}-\frac{A_3y_3}{EI}+\frac{A_5y_5}{2EI}+\frac{A_6y_6}{EI}+\frac{A_7y_7}{EI}$$

$$=\frac{1}{2EI}\times\frac{ql^3}{2}\times\frac{2l}{3}-\frac{1}{2EI}\times\frac{ql^3}{3}\times\frac{5l}{16}+\frac{1}{EI}\times\frac{ql^3}{3}\times\frac{11l}{16}$$

$$+\frac{1}{2EI}\times\frac{ql^3}{4}\times\frac{2l}{3}+\frac{1}{EI}\times\frac{ql^3}{4}\times\frac{2l}{3}+\frac{1}{EI}\times\frac{ql^3}{2}\times\frac{5l}{6}=\frac{97ql^4}{96EI}(\downarrow)$$

【题 2】 图示体系，求铰 B 两侧截面的相对转角。

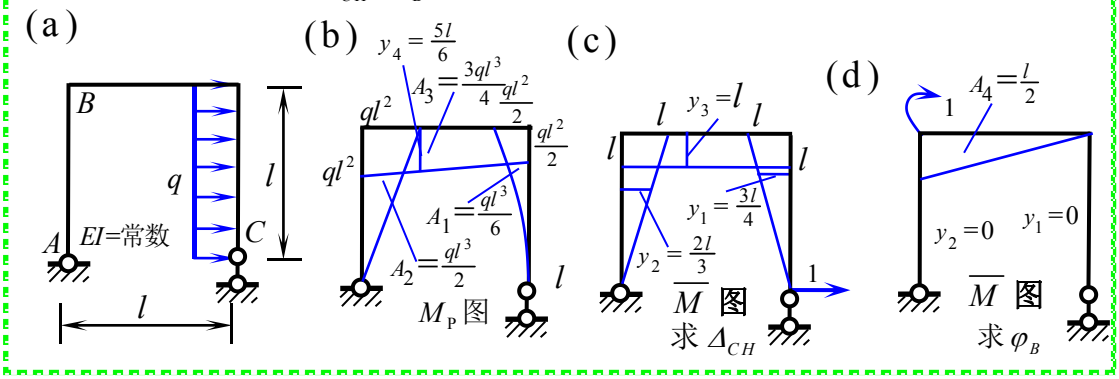


$$\Delta\varphi_B=\sum\int\frac{\overline{M}\,M_P}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}=\frac{A_1y_1}{2EI}-\frac{A_2y_2}{EI}-\frac{A_3y_3}{EI}$$

（位移方向与单位力方向相反）

$$=\frac{1}{2EI}\times ql^3\times1-\frac{1}{EI}\times\frac{ql^3}{2}\times1-\frac{1}{EI}\times\frac{4ql^3}{3}\times1=\frac{-4ql^3}{3EI}$$

【题 3】 图示体系　　求 Δ_{CH}，φ_B



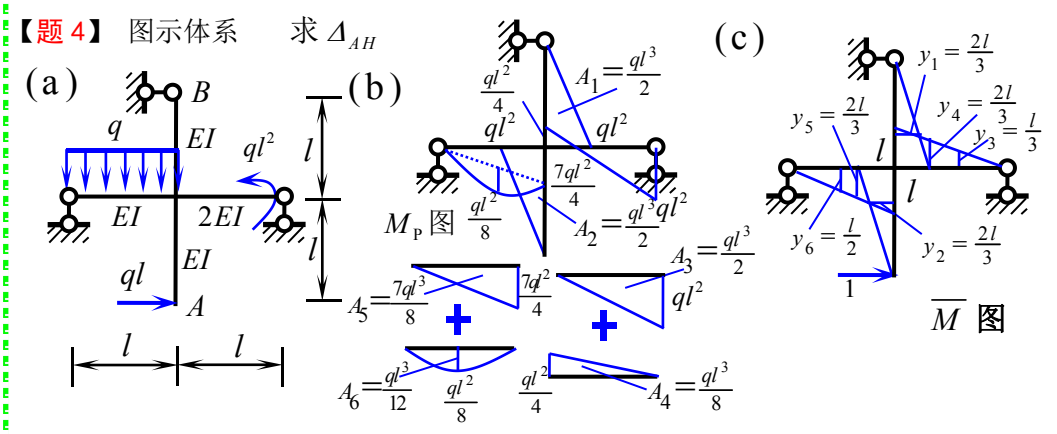
$$\Delta_{CH}=\sum\int\frac{\overline{M}\,M_P}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}=\frac{A_1y_1}{EI}+\frac{A_2y_2}{EI}+\frac{A_3y_3}{EI}$$

$$=\frac{1}{EI}\times\frac{ql^3}{6}\times\frac{3l}{4}+\frac{1}{EI}\times\frac{ql^3}{2}\times\frac{2l}{3}+\frac{1}{EI}\times\frac{3ql^3}{4}\times l=\frac{29ql^4}{24EI}(\rightarrow)$$

$$\varphi_B=\sum\int\frac{\overline{M}\,M_P}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}=\frac{A_1y_1}{EI}+\frac{A_2y_2}{EI}+\frac{A_4y_4}{EI}$$

$$=\frac{1}{EI}\times\frac{ql^3}{6}\times0+\frac{1}{EI}\times\frac{ql^3}{2}\times0+\frac{1}{EI}\times\frac{ql^3}{2}\times\frac{5l}{6}=\frac{5ql^3}{12EI}(\text{顺时针})$$

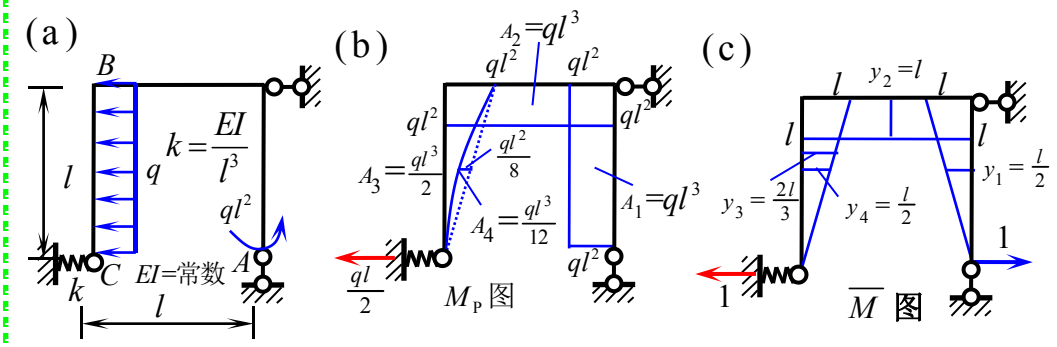
【题 4】 图示体系　　求 Δ_{AH}



$$\Delta_{AH}=\sum\int\frac{\overline{M}\,M_P}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}=\frac{A_1y_1}{EI}+\frac{A_2y_2}{EI}-\frac{A_3y_3}{2EI}+\frac{A_4y_4}{2EI}+\frac{A_5y_5}{EI}+\frac{A_6y_6}{EI}$$

$$=\frac{1}{EI}\times\frac{ql^3}{2}\times\frac{2l}{3}+\frac{1}{EI}\times\frac{ql^3}{2}\times\frac{2l}{3}-\frac{1}{2EI}\times\frac{ql^3}{2}\times\frac{l}{3}+\frac{1}{2EI}\times\frac{ql^3}{8}\times\frac{2l}{3}+\frac{1}{EI}\times\frac{7ql^3}{8}\times\frac{2l}{3}+\frac{1}{EI}\times\frac{ql^3}{12}\times\frac{l}{2}=\frac{5ql^4}{4EI}(\rightarrow)$$

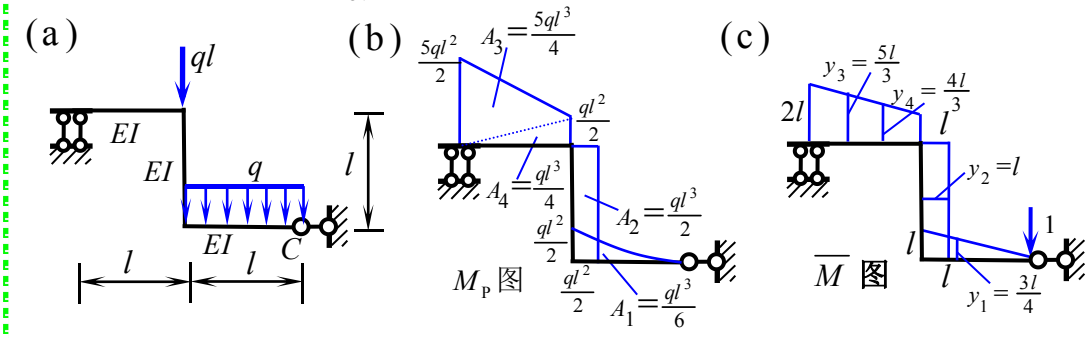
【题 5】 图示体系　　求 Δ_{AH}



$$\Delta_{AH}=\sum\int\frac{\overline{M}\,M_P}{EI}ds+\sum\frac{\overline{F}_{k1}F_{kP}}{k}=\sum\frac{(\pm)Ay_0}{EI}+\sum\frac{\overline{F}_{k1}F_{kP}}{k}=\frac{A_1y_1}{EI}+\frac{A_2y_2}{EI}+\frac{A_3y_3}{EI}-\frac{A_4y_4}{EI}+\frac{\overline{F}_{k1}F_{kP}}{k}$$

$$=\frac{1}{EI}\times ql^3\times\frac{l}{2}+\frac{1}{EI}\times ql^3\times l+\frac{1}{EI}\times\frac{ql^3}{2}\times\frac{2l}{3}-\frac{1}{EI}\times\frac{ql^3}{12}\times\frac{l}{2}+\frac{1}{k}\times\frac{ql}{2}\times1=\frac{55ql^4}{24EI}(\rightarrow)$$

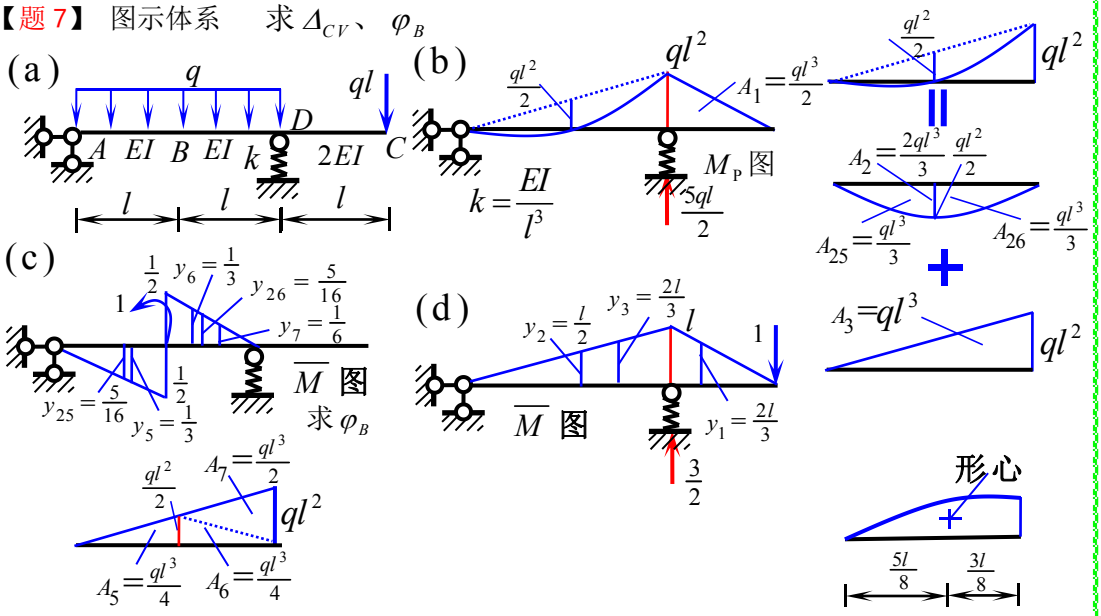
【题 6】 图示体系　　求 Δ_{CV}



$$\Delta_{CV}=\sum\int\frac{\overline{M}\,M_P}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}=\frac{A_1y_1}{EI}+\frac{A_2y_2}{EI}+\frac{A_3y_3}{EI}+\frac{A_4y_4}{EI}$$

$$=\frac{1}{EI}\times\frac{ql^3}{6}\times\frac{3l}{4}+\frac{1}{EI}\times\frac{ql^3}{2}\times l+\frac{1}{EI}\times\frac{5ql^3}{4}\times\frac{5l}{3}+\frac{1}{EI}\times\frac{ql^3}{4}\times\frac{4l}{3}=\frac{73ql^4}{24EI}(\downarrow)$$

【题 7】 图示体系　　求 Δ_{CV}、φ_B



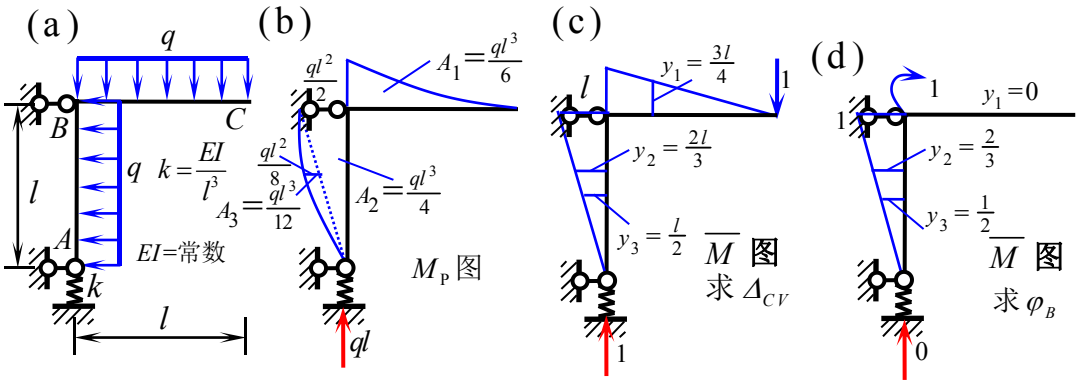
$$\Delta_{DV}=\sum\int\frac{\overline{M}\,M_P}{EI}ds+\sum\frac{\overline{F}_k F_{kP}}{k}=\sum\frac{(\pm)Ay_0}{EI}+\sum\frac{\overline{F}_k F_{kP}}{k}=\frac{A_1y_1}{2EI}-\frac{A_2y_2}{EI}+\frac{A_3y_3}{EI}+\frac{\overline{F}_k F_{kP}}{k}$$

$$=\frac{1}{2EI}\times\frac{ql^3}{2}\times\frac{2l}{3}-\frac{1}{EI}\times\frac{2ql^3}{3}\times\frac{l}{2}+\frac{1}{EI}\times ql^3\times\frac{2l}{3}+\frac{1}{k}\times\frac{5ql}{2}\times\frac{3l}{2}=\frac{17ql^4}{4EI}(\downarrow)$$

$$\varphi_B=\sum\int\frac{\overline{M}\,M_P}{EI}ds+\sum\frac{\overline{F}_k F_{kP}}{k}=\frac{A_{25}y_{25}}{EI}+\frac{-A_{26}y_{26}}{EI}+\frac{-A_5y_5}{EI}+\frac{A_6y_6}{EI}+\frac{A_7y_7}{EI}+\frac{\overline{F}_k F_{kP}}{k}$$

$$=\frac{1}{EI}\times\frac{ql^3}{3}\times\frac{5}{16}+\frac{-1}{EI}\times\frac{ql^3}{3}\times\frac{5}{16}+\frac{-1}{EI}\times\frac{ql^3}{4}\times\frac{1}{3}+\frac{1}{EI}\times\frac{ql^3}{4}\times\frac{1}{3}+\frac{1}{EI}\times\frac{ql^3}{2}\times\frac{1}{6}+\frac{1}{k}\times\frac{5ql}{2}\times\frac{3}{2}=\frac{23ql^3}{6EI}\text{(逆时针)}$$

【题 8】 图示体系　　求 Δ_{CV}，φ_B



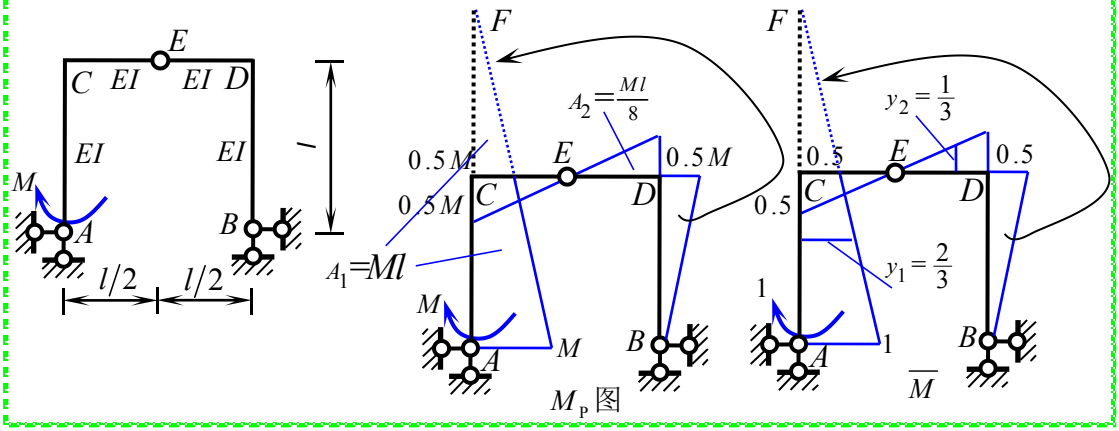
$$\Delta_{CV}=\sum\int\frac{\overline{M}\,M_P}{EI}ds+\sum\frac{\overline{F}_{k1}F_{kP}}{k}=\sum\frac{(\pm)Ay_0}{EI}+\sum\frac{\overline{F}_{k1}F_{kP}}{k}=\frac{A_1y_1}{EI}+\frac{A_2y_2}{EI}+\frac{A_3y_3}{EI}+\frac{\overline{F}_{k1}F_{kP}}{k}$$

$$=\frac{1}{EI}\times\frac{ql^3}{6}\times\frac{3l}{4}+\frac{1}{EI}\times\frac{ql^3}{4}\times\frac{2l}{3}+\frac{1}{EI}\times\frac{ql^3}{12}\times\frac{l}{2}+\frac{1}{k}\times ql\times 1=\frac{4ql^4}{3EI}(\downarrow)$$

$$\varphi_B=\sum\int\frac{\overline{M}\ M_{\mathrm{P}}}{EI}ds+\sum\frac{\overline{F}_{k1}F_{k\mathrm{P}}}{k}=\sum\frac{(\pm)Ay_0}{EI}+\sum\frac{\overline{F}_{k1}F_{k\mathrm{P}}}{k}=\frac{A_1y_1}{EI}+\frac{A_2y_2}{EI}+\frac{A_3y_3}{EI}+\frac{\overline{F}_{k1}F_{k\mathrm{P}}}{k}$$

$$=\frac{1}{EI}\times\frac{ql^3}{6}\times0+\frac{1}{EI}\times\frac{ql^3}{4}\times\frac{2}{3}+\frac{1}{EI}\times\frac{ql^3}{12}\times\frac{1}{2}+\frac{1}{k}\times ql\times0=\frac{5ql^4}{24EI}(\text{顺时针})$$

【题 9】计算图示结构 A 的转角位移。各杆 EI=常数。



●求 A 截面转角：

$$\varphi_A=\sum\int\frac{\overline{M}\ M_{\mathrm{P}}}{EI}ds=\sum\frac{(\pm)Ay_0}{EI}$$

$$=\frac{A_1y_1}{EI}+\frac{2A_2y_2}{EI}$$

$$=\frac{2Ml}{3}+\frac{Ml}{12}=\frac{3Ml}{4}(\text{顺时针})$$