HW 3: EDA

Question 1

a) $m(a+bX) = a + b \cdot m(X)$

$$
\begin{aligned}
m(a+bX) &= E[a+bX] \\
&= E[a] + E[bX] \\
&= a + bE[X] \\
&= a + b \cdot m(X) \quad \checkmark
\end{aligned}
$$

b)

$$
\begin{aligned}
\text{cov}(X, a+bY) &= E[(X-E[X])((a+bY) - \underbrace{E[a+bY]}))] \\
&= E[(X-E[X])(\underbrace{(a+bY)-(a+bE[Y])})] \\
&= E[(X-E[X])(b(Y-E[X]))] \\
&= b \cdot \text{cov}(X, Y) \quad \checkmark
\end{aligned}
$$

c) $\text{cov}(a+bX, a+bX) = b^2 \text{cov}(X, X)$

$$
\begin{aligned}
\text{cov}(a+bX, a+bX) &= E[(b(X-E[X]))(b(X-E[X]))] \\
&= b^2 E[(X-E[X])(X-E[X])] \\
&= b^2 \text{cov}(X, X) \quad \checkmark
\end{aligned}
$$

$$
\begin{aligned}
\text{Var}(X) &= E[(X-E[X])^2] \\
&= E[(X-E[X])(X-E[X])] \\
&= \text{cov}(X, X)
\end{aligned}
$$

$\text{cov}(X, X) = s^2$ ✓ sample variance of X is s^2

d) Yes, if you apply a ~~non-decreasing transformation~~ g to variable X, then the median of $g(X)$ equals g applied to the median of X. All values above the median will stay above and the same applies for values below will stay below. This also applies to any quantile, the IQR, and the range.

e) No, in examples such as non-linear transformations, the mean will not be preserved.

for example:

$$g(x) = x^2 \qquad X = \{1, 2, 3\} \qquad m(x) = 2$$

$$g(x) = \{1, 4, 9\}$$

$$m(g(x)) \approx 4.67 \qquad g(m(x)) = g(2) = 4$$

$$4.67 \neq 4$$

$$m(g(x)) \neq g(m(x))$$