

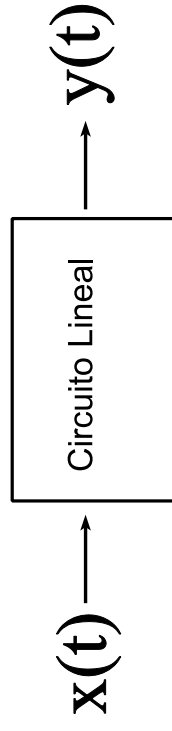
Diagrama de Bode

FFT

Para ver la respuesta en frecuencia de un circuito, se usa el Diagrama de Bode, que es la representación gráfica del módulo (o magnitud) y la fase (o argumento) de la función de transferencia en $s=i\omega$, es decir, particularizando para señales armónicas.

Representa el módulo: $20\log_{10}|T(i\omega)|$ (dB)
Representa la fase: $\arcsin(T(i\omega))$ (radianes o grados)
Frente a la frecuencia: $\log_{10}(\omega)$ ó $\log_{10}(f)$ (rad/s ó Hz)

Util: $20\log(\sqrt{2}) = 3$ dB $20\log(2) = 6$ dB A recordar: $\omega = 2\pi f$
 $20\log\left(\frac{1}{\sqrt{2}}\right) = -3$ dB $20\log\left(\frac{1}{2}\right) = -6$ dB
 $20\log(10) = 20$ dB $20\log(100) = 40$ dB
 $20\log(0,1) = -20$ dB $20\log(0,01) = -40$ dB



Aplicando la Transformada de Laplace a la ecuación diferencial del circuito:

$$D(s) Y(s) - CI(s) = N(s) X(s)$$

$$Y(s) = \frac{N(s)}{D(s)} X(s) + \frac{CI(s)}{D(s)}$$

Respuesta total = a la debida a la entrada + la debida a las condiciones iniciales

Función de transferencia del circuito:

$$T(s) = \frac{N(s)}{D(s)} = \left. \frac{Y(s)}{X(s)} \right|_{CI=0}$$

Circuito lineal => el numerador y el denominador son polinomios

$$T(s) = k \frac{(s-z_1)(s-z_2)...(s-z_m)}{(s-p_1)(s-p_2)...(s-p_n)}$$

Circuito real => las raíces son reales o complejas conjugadas

$$T(s) = k \frac{\prod_i (s-z_i) \prod_k (s^2 + 2\delta_k s\omega_k + \omega_k^2)}{\prod_j (s-p_j) \prod_r (s^2 + 2\delta_r s\omega_r + \omega_r^2)} \quad 0 < \delta < 1$$

La función de transferencia se reduce a esta forma (normalizada):

$$T(s) = k s^n \frac{\prod_i \left(\frac{s}{\omega_i} + 1 \right) \prod_k \left(\frac{s^2}{\omega_k^2} + 2\delta_k \frac{s}{\omega_k} + 1 \right)}{\prod_j \left(\frac{s}{\omega_j} + 1 \right) \prod_r \left(\frac{s^2}{\omega_r^2} + 2\delta_r \frac{s}{\omega_r} + 1 \right)}$$

Se representan los bloques básicos, ya que: $T = T_1 T_2 / T_3$

$$20\log(T) = 20\log(T_1) + 20\log(T_2) - 20\log(T_3)$$

$$\text{fase}(T) = \text{fase}(T_1) + \text{fase}(T_2) - \text{fase}(T_3)$$

$$T(s) =$$

$$k$$

$$T(i\omega) =$$

$$k$$

Diagrama de Bode de los bloques básicos:

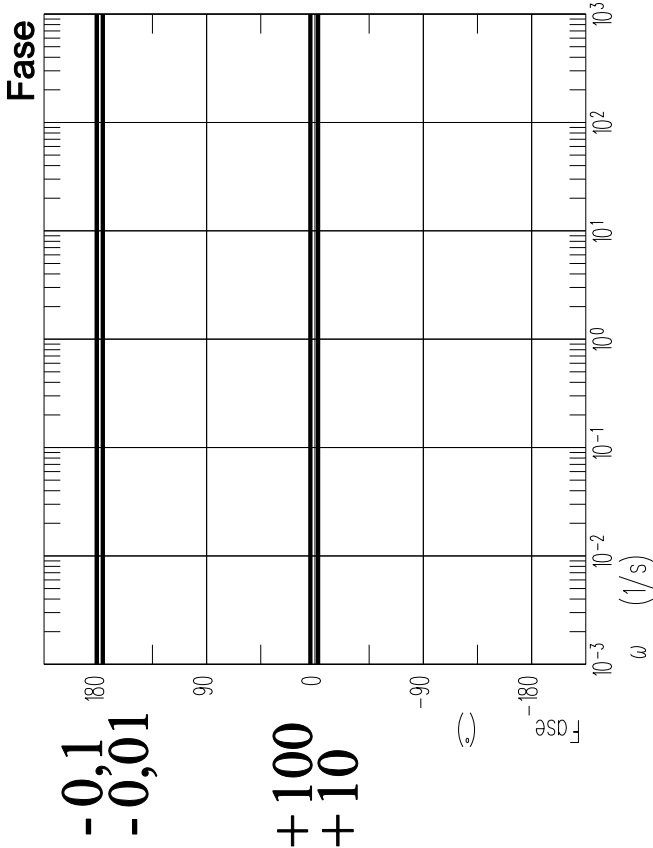
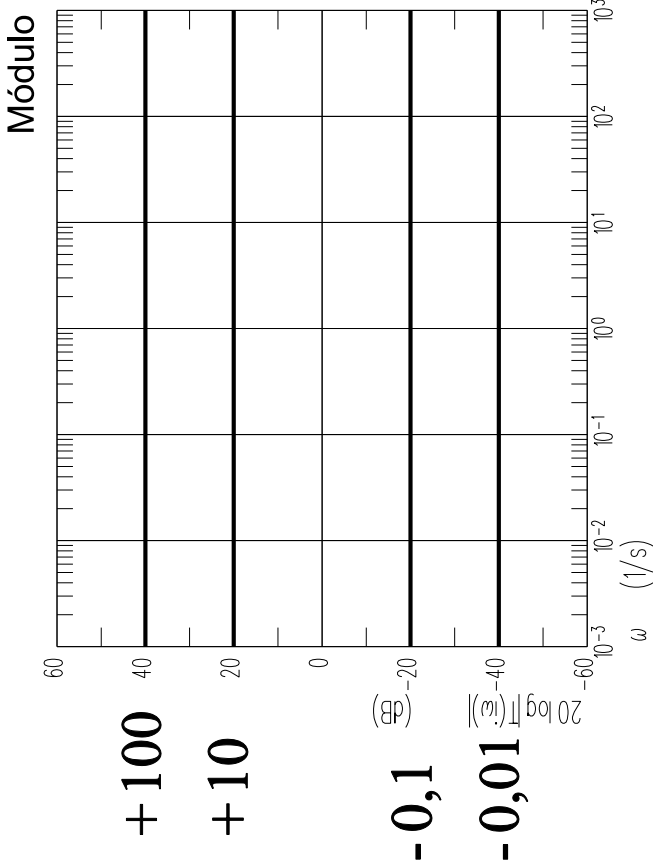
Módulo

Fase

$$20 \log |k|$$

$$0 \text{ si } k > 0$$

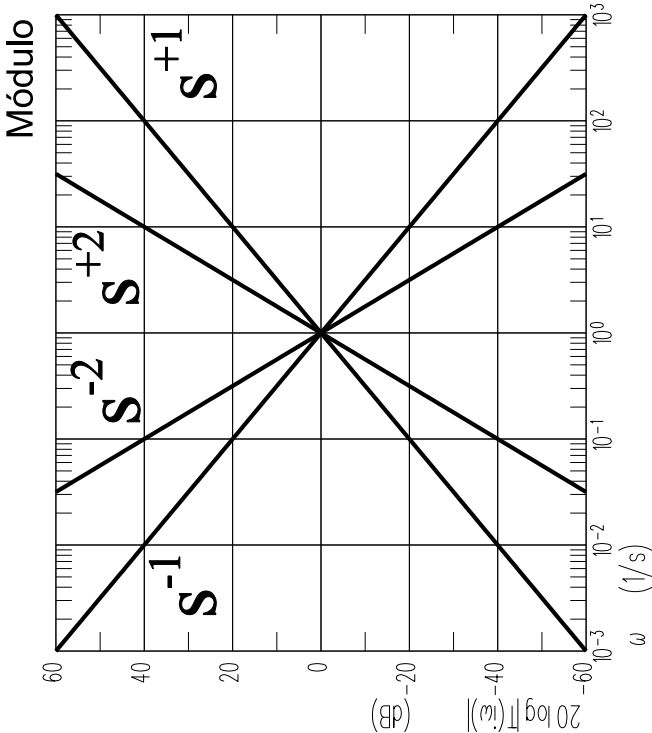
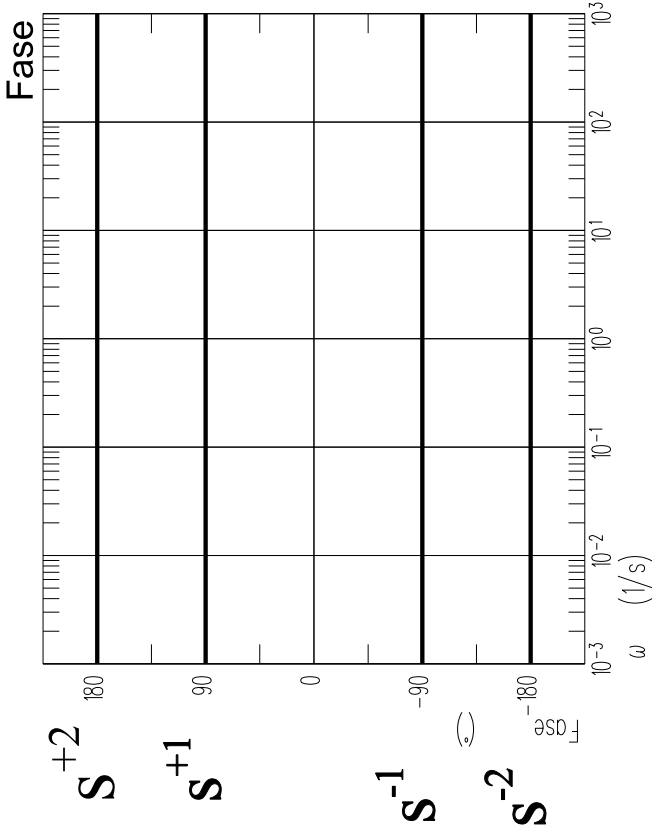
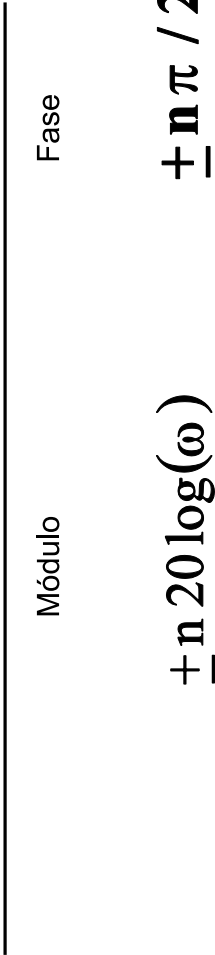
$$\pi \text{ si } k < 0$$



$T(s)$
 $=$

$T(i\omega)$
 $=$

S_{-}^{+n}
 $(i\omega)^{+n}_{-}$



$T(s)$
 $=$

$T(i\omega)$
 $=$

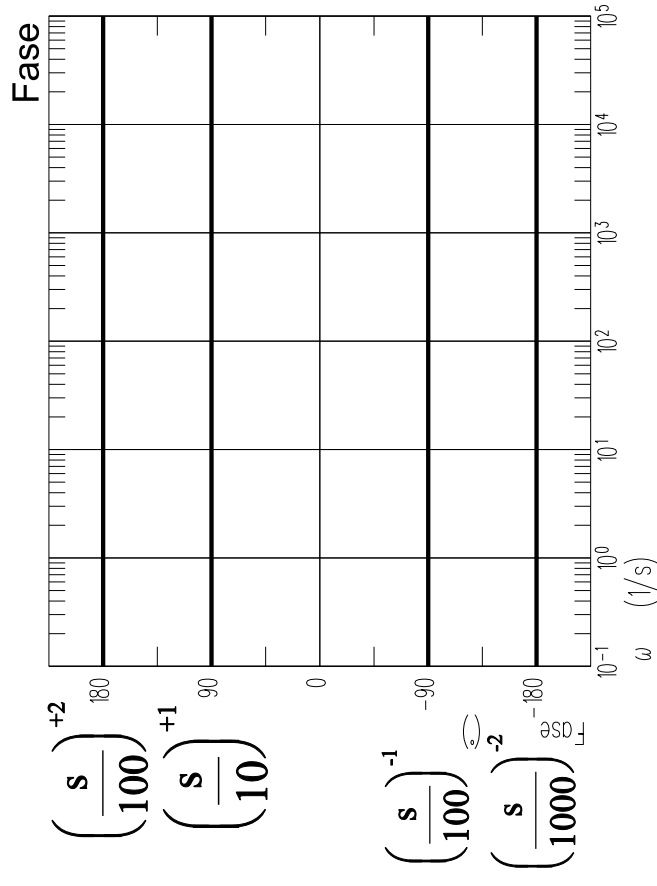
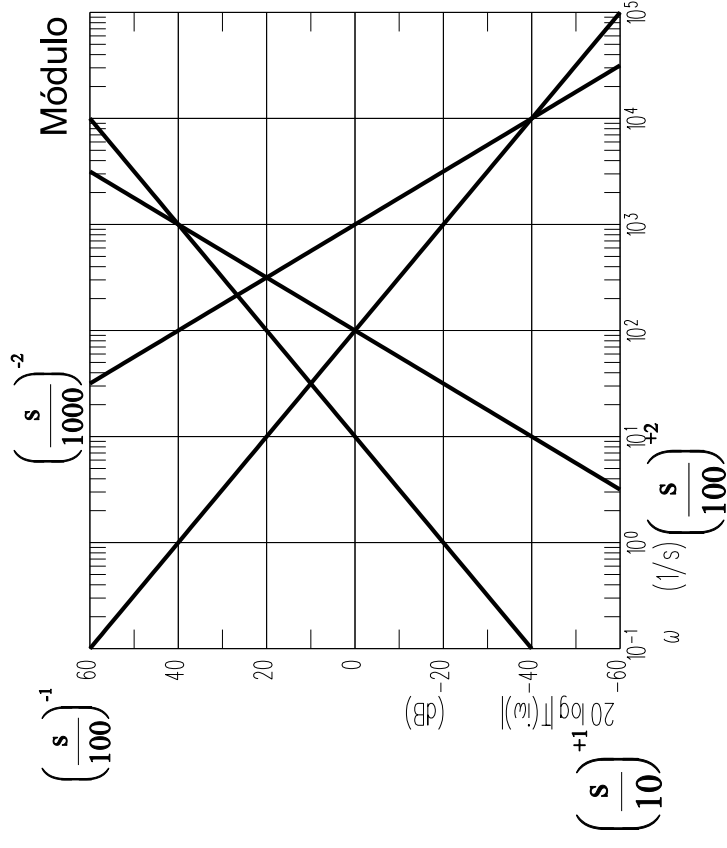
$\left(\frac{s}{\omega_o}\right)^{+n}_{-}$
 $\left(i\frac{\omega}{\omega_o}\right)^{+n}_{-}$

Módulo

Fase

$+n\,20\log(\omega/\omega_o)$

$+n\pi/2$



Ejercicios

$$\frac{s}{0,1} = 10s$$

$$\left(\frac{s}{100}\right)\left(\frac{s}{1000}\right)^{-2}$$

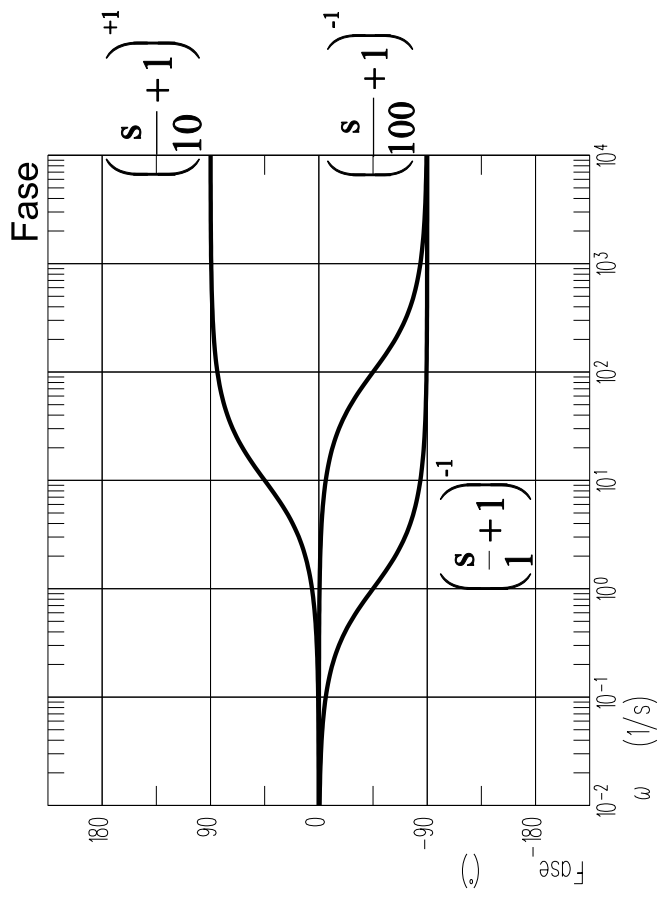
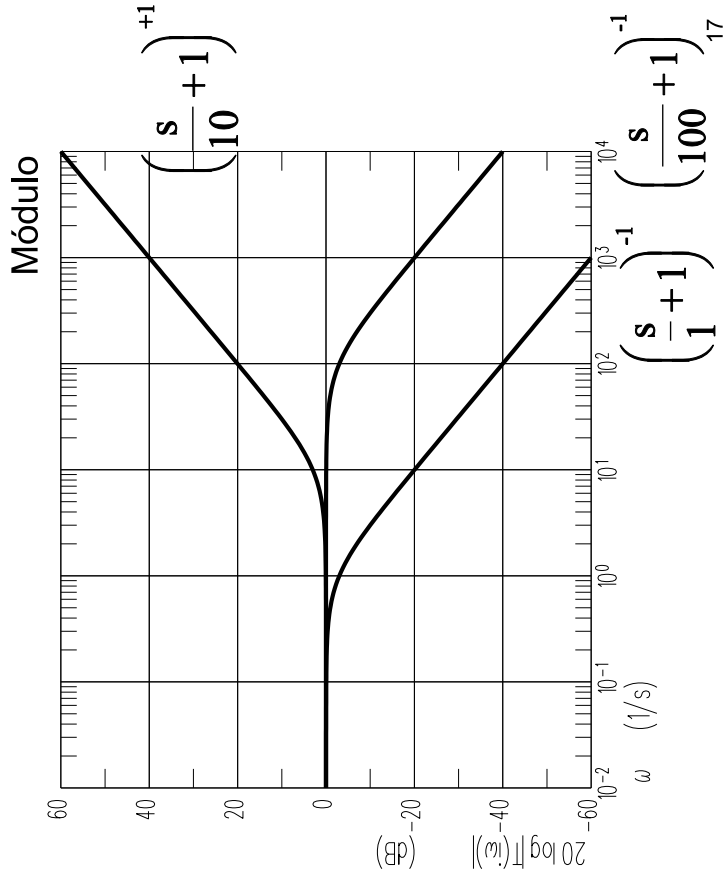
$$\left(\frac{\$}{10}\right) \left(\frac{\$}{100}\right)$$

$$-10\left(\frac{s}{100}\right)^3$$

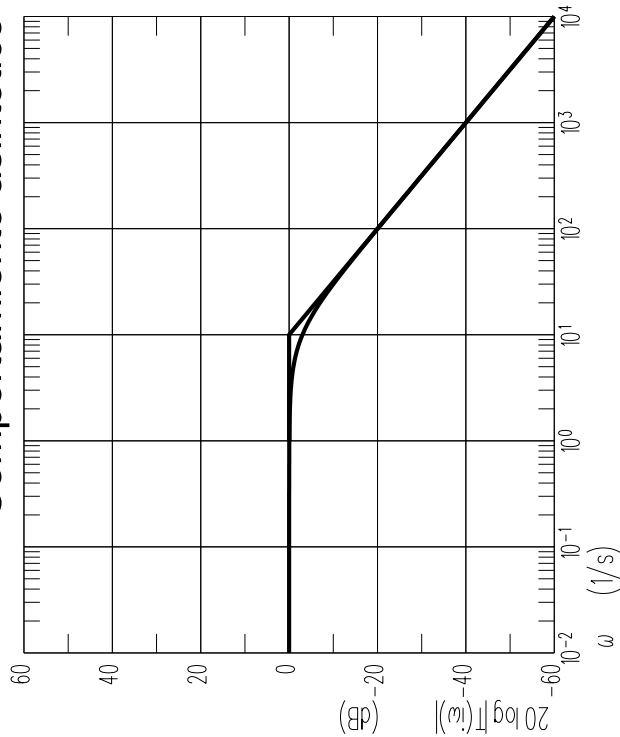
$$\left(\frac{s}{10}\right)^{-2}$$

$$T(s) = \left(\frac{s}{s+1} \right)^{+1}_{-} \quad T(i\omega) = \left(\frac{i\omega}{1+i\frac{\omega}{\omega_0}} \right)^{+1}_{-}$$

	Módulo	Fase
$\omega \ll \omega_o$	0	0
$\omega = \omega_o$	$+20 \log \sqrt{2}$	$+\pi / 4$
$\omega \gg \omega_o$	$+20 \log (\omega / \omega_o)$	$+\pi / 2$



Comportamiento asintótico



Ejercicios

$$(s+10)$$

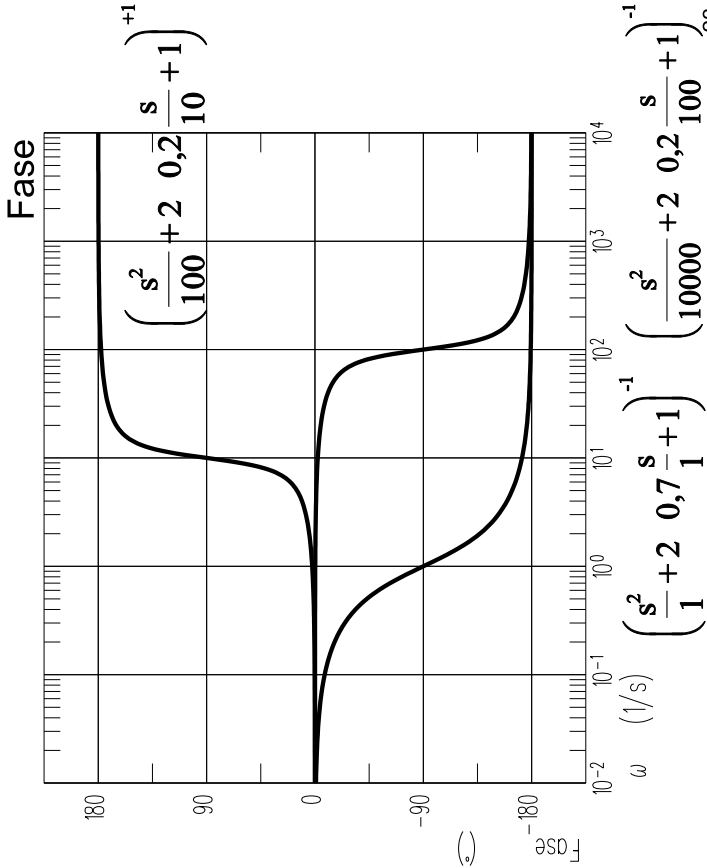
$$-10 \frac{\left(\frac{s}{100}\right)}{\left(\frac{s}{1000} + 1\right)}$$

$$10 \frac{s}{10} \frac{\left(\frac{s}{100} + 1\right)}{\left(\frac{s}{10} + 1\right)}$$

$$T(s) = \left(\frac{s^2}{\omega_o^2} + 2\delta \frac{s}{\omega_o} + 1 \right)^{\pm 1} T(i\omega) = \left[\left(1 - \frac{\omega^2}{\omega_o^2} \right) + i2\delta \frac{\omega}{\omega_o} \right]^{\pm 1}$$

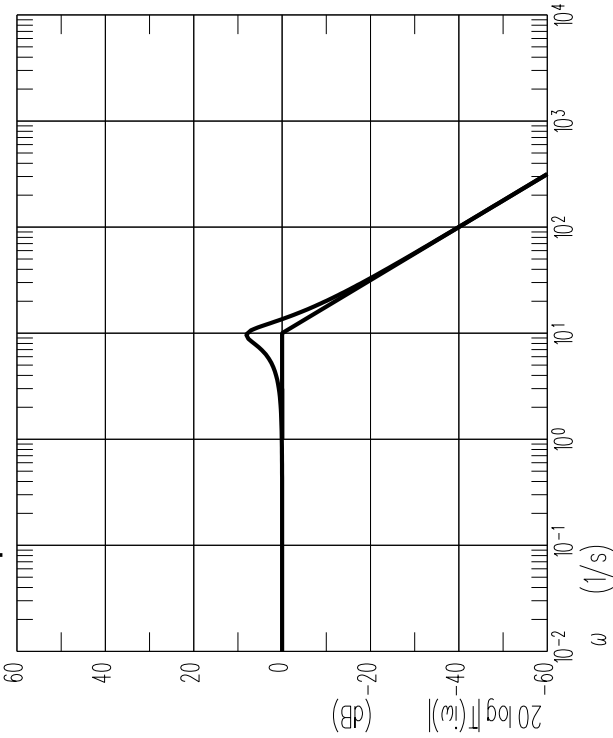
	Módulo	Fase
$\omega \ll \omega_o$	0	0
$\omega = \omega_o$	$+20\log(2\delta)$	$\pm \pi / 2$
$\omega \gg \omega_o$	$+40\log(\omega / \omega_o)$	$\pm \pi$

21

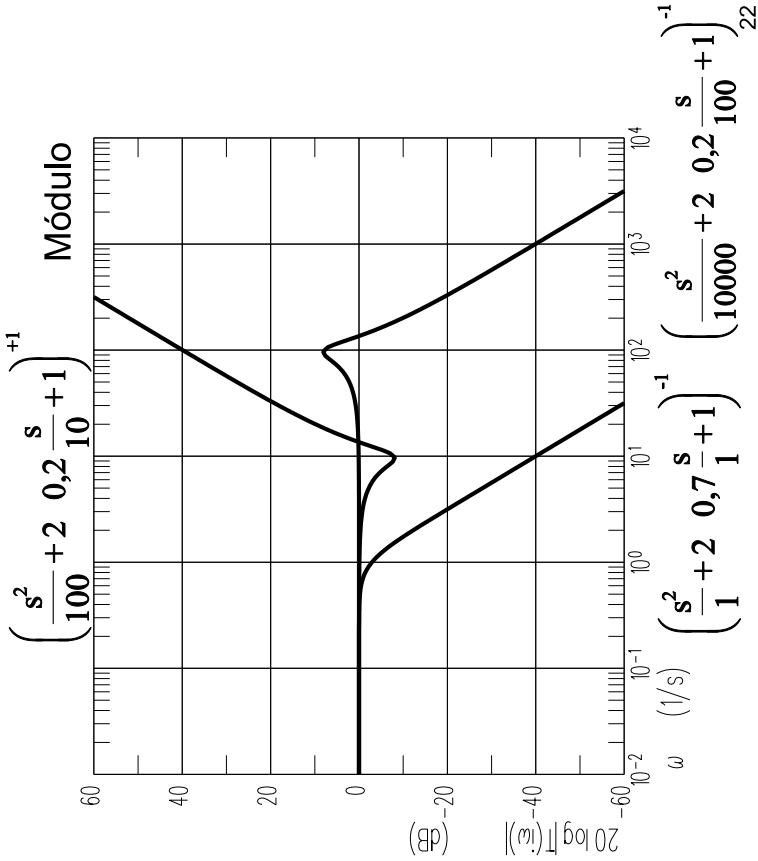


23

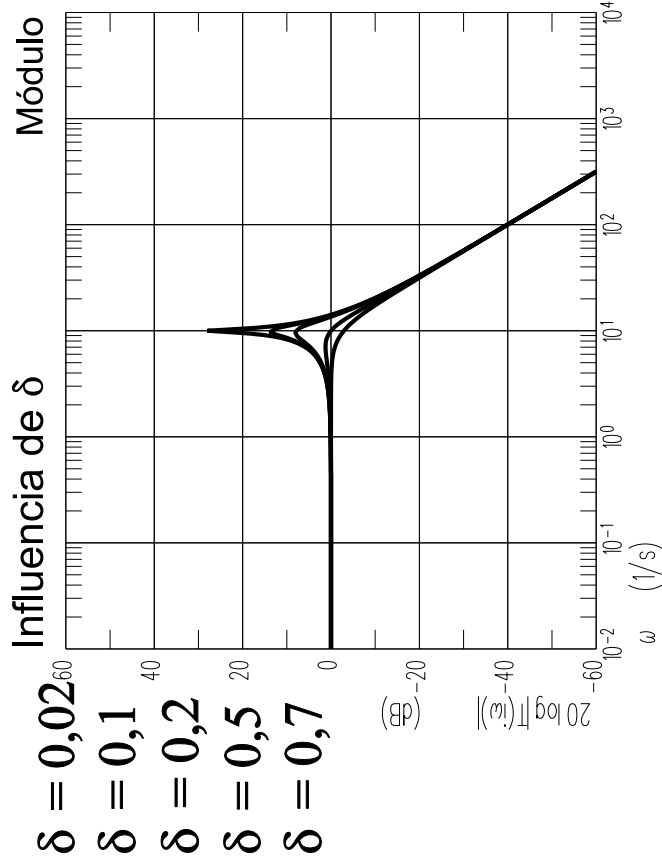
Comportamiento asintótico



24



22



25

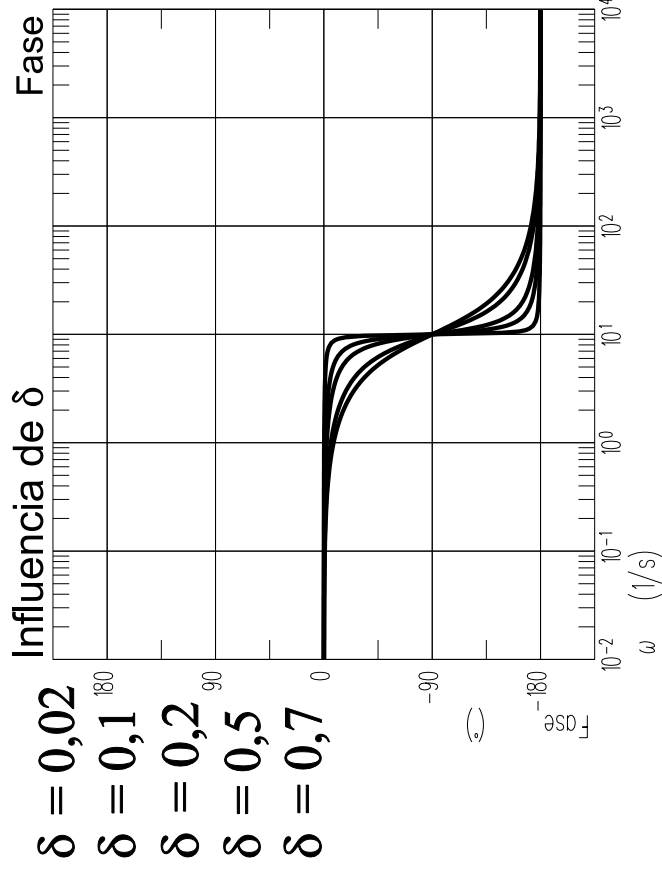
Ejercicios

$$(s^3 + 10s^2 + 100s)$$

$$\frac{(s+10)}{(s^2+s+1)} - \frac{(s^2+2s+100)}{(s^2+s+1)}$$

$$\frac{(s+1)(s+100)}{(s^2+10s+100)} \cdot \frac{(s^2+2s+4)}{\left(\frac{s^2}{100} + 0,2s+4\right)}$$

27



26

Bode en una hoja de cálculo

Se hará un diagrama de Bode con datos similares a los de la práctica 2. Datos en el fichero bd2.prn

Frecuencia de corte :

LP1 = 1000 rad/s

LP2 = 10.000 rad/s

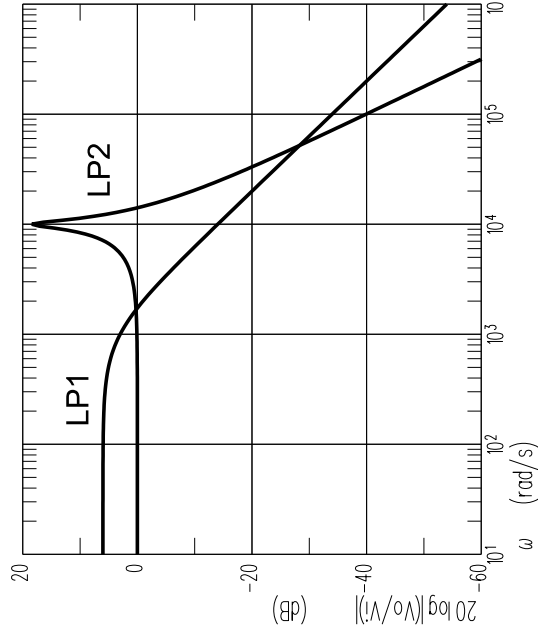
Ganancia :

LP1 = 2 (6 dB)

LP2 = 1 (0 dB)

Coef. amort. δ :

LP2 = 0,06



28

Diagrama de Bode



FFT

Granada granada.net78.net

10-X-2011
S.O.: Win95
Res.: 800x600
Col.: 16bit

FIN