Densidad de carga

FFT

1

Definiciones:

Densidad de carga (ρ):
= carga por unidad de volumen

$$\rho = \frac{q}{v} \qquad (C/\,m^3)$$

Densidad superficial de carga (σ):
= carga por unidad de superficie

$$\sigma = \frac{q}{s} \qquad (C/\,m^2)$$

Densidad lineal de carga (λ):
= carga por unidad de longitud

$$\lambda = \frac{q}{l} \qquad (C/\,m)$$

1º Cálculo de la densidad de carga en varias figuras

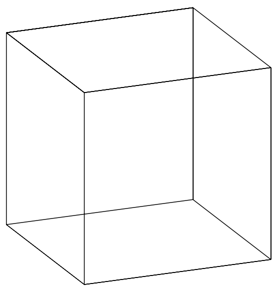
2º Cálculo de la carga en una figura, sabida la densidad de carga

2

1. Cálculo de la densidad de carga en varias figuras

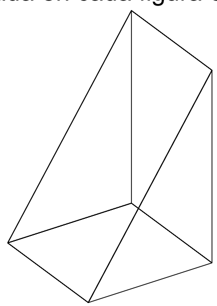
Calcular la densidad de carga ρ

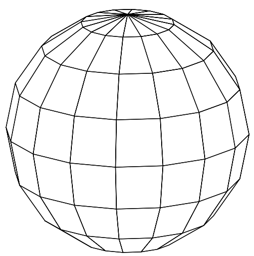
La carga total almacenada en cada figura es q



Cubo:
Lado = a

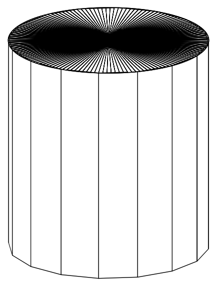
Cuña:
Ancho = a
Largo = b
Altura = h





Esfera:
Radio = R

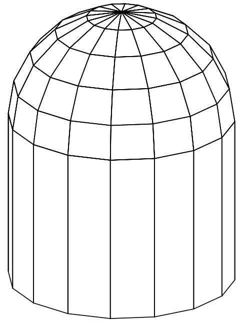
Cilindro:
Radio = R
Altura = h



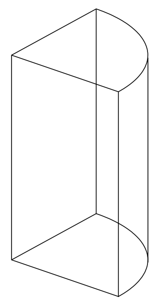
3

Calcular la densidad de carga ρ

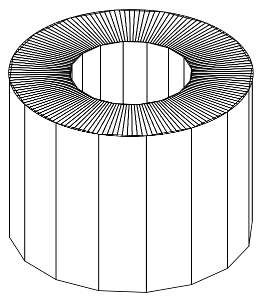
La carga total almacenada en cada figura es q



Radio = R
Altura = h+R



Arco = 90°
Radio = R
Altura = h

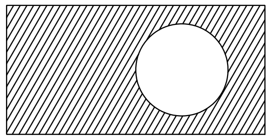


Radio interior = R1
Radio exterior = R2
Altura = h

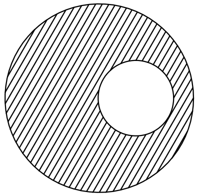
4

Calcular la densidad superficial de carga σ

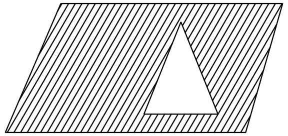
La carga total almacenada en cada figura es q



Radio = R
Base = b
Altura = h

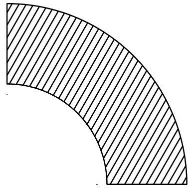


Radio interior = R1
Radio exterior =R2

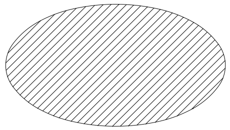


Paralelogramo:
Base = a
Altura = d

Triángulo:
Base = b
Altura = h



Radio interior = R1
Radio exterior =R2

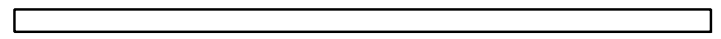


Semieje menor = a
Semieje mayor = b

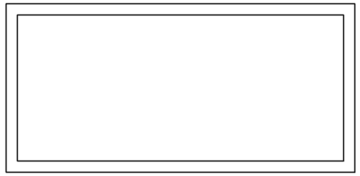
Calcular σ en las figuras de las páginas 3 y 4

Calcular la densidad lineal de carga λ

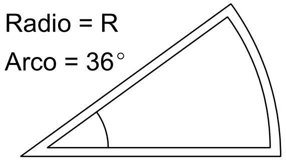
La carga total almacenada en cada figura es q



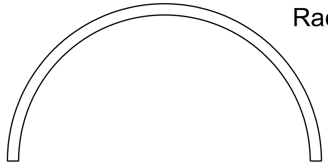
Longitud = l



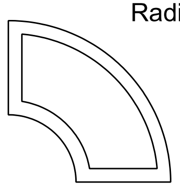
Base = b
Altura = h



Radio = R
Arco = 36°



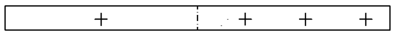
Radio = R



Radio interior = R1
Radio exterior = R2
Arco = 90°

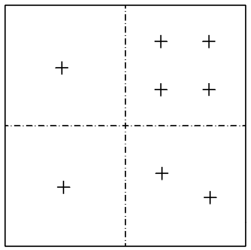
La carga puede no estar distribuida de forma homogénea

Cada "+" equivale a una carga +q



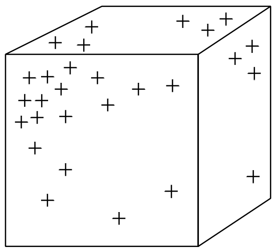
(longitud = l)

Calcular λ:
- en el hilo completo.
- en cada mitad del hilo.



(lado = l)

Calcular σ:
- en la superficie completa.
- en cada cuadrante de la superficie.



En general, distribución de carga no homogénea.

ρ = q/v   sólo es una media ⇒

$$\rho = \frac{\Delta q}{\Delta v} \longrightarrow \rho = \frac{dq}{dv}$$

ρ = carga infinitesimal Δq / volumen infinitesimal Δv

ρ depende de la posición: ρ = ρ(x,y,z)

$$q = \int dq =$$

$$\rho = \frac{dq}{dv} \longrightarrow dq = \rho\, dv \longrightarrow = \int_{vol} \rho\, dv$$

$$\sigma = \frac{dq}{ds} \longrightarrow dq = \sigma\, ds \longrightarrow = \int_{sup} \sigma\, ds$$

$$\lambda = \frac{dq}{dl} \longrightarrow dq = \lambda\, dl \longrightarrow = \int_{lin} \lambda\, dl$$

$$\int_{lin} = \int \qquad \int_{sup} = \iint \qquad \int_{vol} = \iiint$$

Apéndice: Integral

$$\int [\alpha\, f(x) + \beta\, g(x)]\, dx = \alpha \int f(x)\, dx + \beta \int g(x)\, dx$$

$$\int_a^b 1\, dx = x \Big|_a^b = (b - a)$$

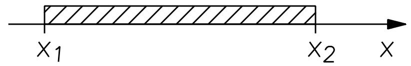
$$\int_a^b x\, dx = \frac{x^2}{2} \Big|_a^b = \left(\frac{b^2}{2} - \frac{a^2}{2} \right)$$

$$\int_a^b \cos(x)\, dx = \mathrm{sen}(x) \Big|_a^b = [\mathrm{sen}(b) - \mathrm{sen}(a)]$$

2. Cálculo de la carga en una figura, sabida la densidad de carga

Calcular la carga q total almacenada en la figura

Conocida la densidad lineal de carga λ



$$\lambda(x) = 2 + 4x$$

$x_1 = 0$

$x_2 = 3$

$$q = \int_{x=0}^{x=3} (2 + 4x)\, dx$$

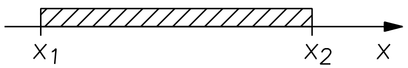
$$= \left(2x + 4\frac{x^2}{2} \right) \Bigg|_0^3$$

$$= 2(3 - 0) + 2(3^2 - 0)$$

$$q = 6 + 18 = 24$$

Calcular la carga q total almacenada en la figura

Conocida la densidad lineal de carga λ



$$\lambda(x) = 5 \cos(x)$$

$x_1 = 1$

$x_2 = 3$

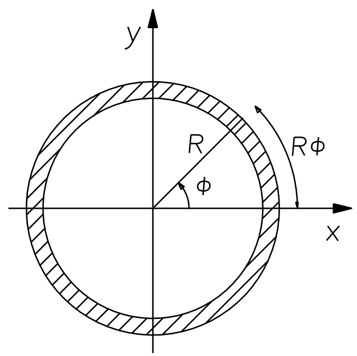
$$q = \int_{x=1}^{x=3} 5 \cos(x)\, dx$$

Hacer ...

$$q = 5\, [\mathrm{sen}(3) - \mathrm{sen}(1)]$$

Calcular la carga q total almacenada en las figura

Conocida la densidad lineal de carga λ



$$\lambda(\phi) = 3$$

Desde $\phi = 0$ a $\phi = 2\pi$

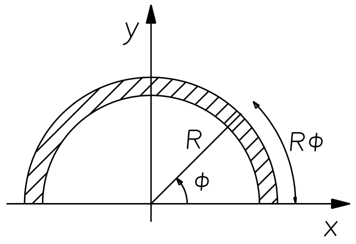
$$q = \int_{\phi=0}^{\phi=2\pi} 3\, R\, d\phi$$

$$= 3R\phi \Big|_0^{2\pi} = 3R(2\pi - 0)$$

$$q = 6\, \pi\, R$$

Calcular la carga q total almacenada en la figura

Conocida la densidad lineal de carga λ



$$q = \int_{\phi=0}^{\phi=\pi} 5\, R\, d\phi$$

$$\lambda(\phi) = 5$$

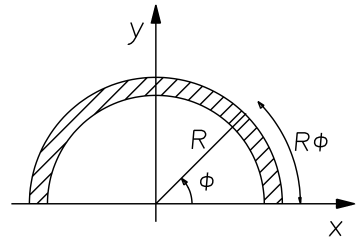
Hacer ...

Desde $\phi = 0$ a $\phi = \pi$

$$q = 5\,\pi\,R$$

Calcular la carga q total almacenada en la figura

Conocida la densidad lineal de carga λ



$$q = \int_{\phi=0}^{\phi=\pi} (3 + 2\phi)\, R\, d\phi$$

$$\lambda(\phi) = 3 + 2\phi$$

$$= 3R\phi\Big|_0^{\pi} + 2R\frac{\phi^2}{2}\Big|_0^{\pi}$$

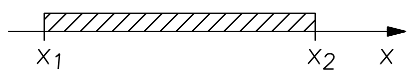
Desde $\phi = 0$ a $\phi = \pi$

$$= 3R(\pi - 0) + R(\pi^2 - 0)$$

$$q = \pi R(3 + \pi)$$

Calcular la carga q total almacenada en la figura

Conocida la densidad lineal de carga λ



$x_1 = 1$
$x_2 = 7$

$$\lambda(x) = 3 + 3x$$

$x_1 = -\pi/2$
$x_2 = +\pi/2$

$$\lambda(x) = 3\cos(x)$$

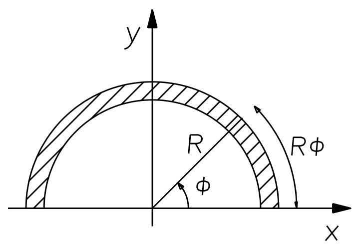
Soluciones:

$q = 90$

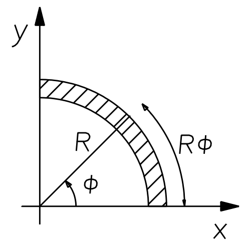
$q = 6$

Calcular la carga q total almacenada en las figuras

Conocida la densidad lineal de carga λ



$$\lambda(\phi) = 5 + \cos(\phi)$$



$$\lambda(\phi) = 3 + 4\phi$$

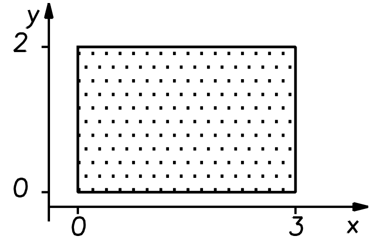
$$\lambda(\phi) = 7 + \cos(\phi)$$

Soluciones:

$q = 5\pi R$ $\qquad q = \frac{3\pi R}{2} + \frac{\pi^2 R}{2}$ $\qquad q = \frac{7\pi R}{2} + R$

Calcular la carga q total almacenada en la figura

Conocida la densidad superficial de carga σ



$$\sigma(x,y) = 2x + 2y$$

$$q = \int_{y=0}^{y=2}\int_{x=0}^{x=3} (2x + 2y)\, dxdy$$

$$= \int_{y=0}^{y=2} \left[2\frac{x^2}{2}\Big|_0^3 + 2yx\Big|_{x=0}^{x=3} \right] dy$$

$$= \int_{y=0}^{y=2} \left[(9-0) + 2y(3-0)\right] dy$$

$$= \int_{y=0}^{y=2} (9 + 6y)\, dy \quad \rightarrow$$

$$\rightarrow \quad = \int_{y=0}^{y=2} (9 + 6y)\, dy$$

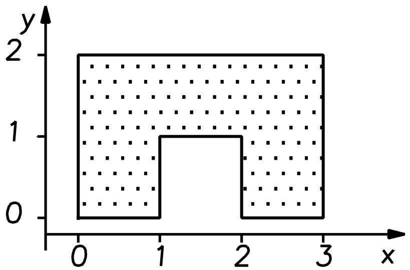
$$= \left[9y\Big|_0^2 + 6\frac{y^2}{2}\Big|_0^2 \right]$$

$$= \left[9(2-0) + 3(4-0)\right]$$

$$q = 18 + 12 = 30$$

Calcular la carga q total almacenada en la figura

Conocida la densidad superficial de carga σ



$$\sigma(x,y) = 2x + 2y$$

$$q_{hueco} = \int_{y=0}^{y=1}\int_{x=1}^{x=2} (2x + 2y)\, dxdy$$

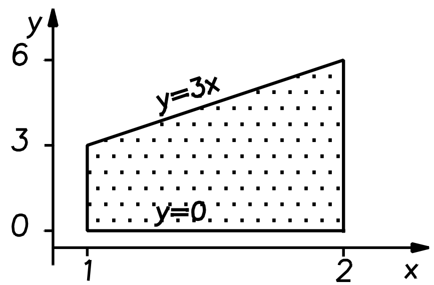
Hacer ...

$$q_{total} = q_{rectangulo} - q_{cuadrado\text{-}hueco}$$

$$q_{total} = 30 - 4 = 26$$

Calcular la carga q total almacenada en la figura

Conocida la densidad superficial de carga σ



$$\sigma(x,y) = 2y - x$$

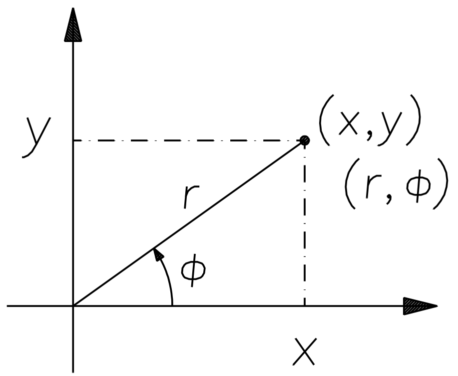
$$q = \int_{x=1}^{x=2}\int_{y=0}^{y=3x} (2y - x)\, dydx$$

$$= \int_{x=1}^{x=2} \left[2\frac{y^2}{2}\Big|_{y=0}^{y=3x} - xy\Big|_{y=0}^{y=3x} \right] dx$$

Hacer ...

$$= \int_{x=1}^{x=2} 6x^2\, dx = \ldots = 14$$

Apéndice: Coordenadas polares



$$\begin{cases} x = r\cos(\phi) \\ y = r\,\mathrm{sen}(\phi) \end{cases}$$

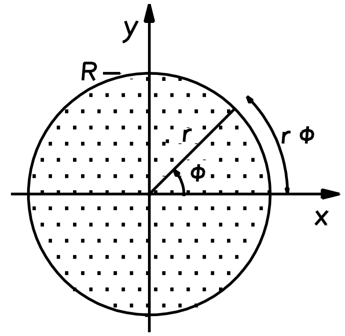
$$\begin{cases} r = \sqrt{x^2+y^2} \\ \phi = \mathrm{arc\,tg}\,\dfrac{y}{x} \end{cases}$$

$$\int_{sup} f\,ds = \int_y\int_x f(x,y)\,dxdy = \int_\phi\int_r f(r,\phi)\,r\,dr\,d\phi$$

$$ds = dxdy = r\,dr\,d\phi$$

Calcular la carga q total almacenada en la figura

Conocida la densidad superficial de carga σ



$$\sigma(r,\phi) = 3$$

Desde $r = 0$ a $r = R$
Desde $\phi = 0$ a $\phi = 2\pi$

$$q = \int_{\phi=0}^{\phi=2\pi}\int_{r=0}^{r=R} 3\ r\,dr\,d\phi$$

$$= \int_0^{2\pi} 3\ \frac{r^2}{2}\Big|_0^R d\phi$$

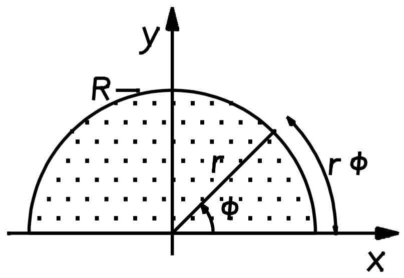
$$= \int_0^{2\pi} 3\ \frac{R^2}{2}\,d\phi$$

$$= 3\ \frac{R^2}{2}\phi\Big|_0^{2\pi}$$

$$q = 3\ \frac{R^2}{2}(2\pi - 0) = 3\pi R^2$$

Calcular la carga q total almacenada en la figura

Conocida la densidad superficial de carga σ



$$q = \int_{\phi=0}^{\phi=\pi}\int_{r=0}^{r=R} 3\ r\,drd\,\phi$$

$$\sigma(r,\phi) = 3$$

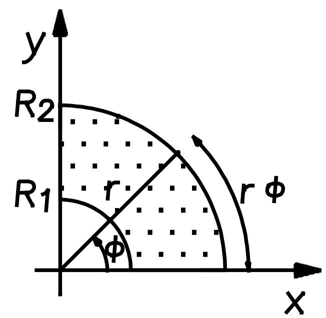
Hacer ...

Desde $r = 0$ a $r = R$
Desde $\phi = 0$ a $\phi = \pi$

$$q = \frac{3}{2}\pi R^2$$

Calcular la carga q total almacenada en la figura

Conocida la densidad superficial de carga σ



$$q = \int_{\phi=0}^{\phi=\pi/2}\int_{r=2}^{r=3} \frac{1}{r}\ r\,drd\,\phi$$

$$\sigma(r,\phi) = \frac{1}{r}$$

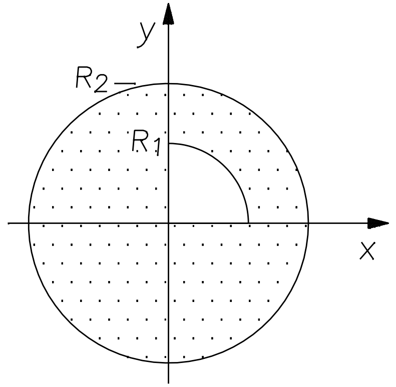
Hacer ...

Desde $r = R_1=2$ a $r = R_2=3$
Desde $\phi = 0$ a $\phi = \pi/2$

$$q = \frac{\pi}{2}$$

Calcular la carga q total almacenada en la figura

Conocida la densidad superficial de carga σ



$$q_{circulo} = \int_{\phi=0}^{\phi=2\pi} \int_{r=0}^{r=R_2} \cos(\phi)\; r\, dr d\phi$$

$$q_{circulo} = \int_{\phi=0}^{\phi=2\pi} \int_{r=0}^{r=R_2} \cos(\phi)\; r\, dr d\phi$$

$$q_{sector} = \int_{\phi=0}^{\phi=\pi/2} \int_{r=0}^{r=R_2} \cos(\phi)\; r\, dr d\phi$$

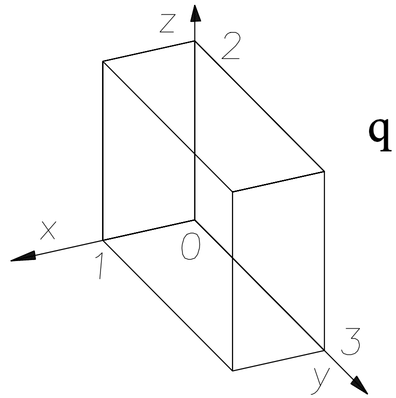
$$\sigma(r,\phi) = \cos\phi$$

$$q_{total} = q_{circulo} - q_{sector\text{-}hueco}$$

$$q_{total} = 0 - \frac{R_1^2}{2}$$

Calcular la carga q total almacenada en la figura

Conocida la densidad de carga ρ



$$q = \int_{z=0}^{z=2} \int_{y=0}^{y=3} \int_{x=0}^{x=1} (1 + 2x)\, y\, dx dy dz$$

$$\int_{z=0}^{z=2} \int_{y=0}^{y=3} 2y\, dy dz$$

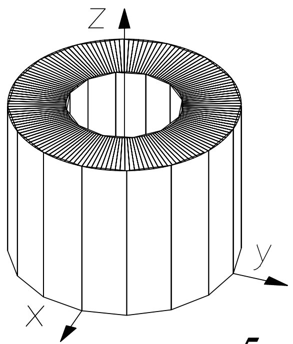
$$\rho(x,y,z) = (1 + 2x)\, y$$

$$\int_{z=0}^{z=2} 9\; dz$$

$$q = 18$$

Calcular la carga q total almacenada en la figura

Conocida la densidad de carga ρ



$$q = \int_{z=0}^{z=3} \int_{\phi=0}^{\phi=2\pi} \int_{r=R_1}^{r=R_2} \frac{5}{r}\; r\, dr\, d\phi dz$$

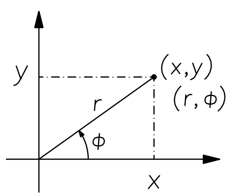
$$\int_{z=0}^{z=3} \int_{\phi=0}^{\phi=2\pi} 5(R_2 - R_1)\; d\phi\, dz$$

$$\rho(r,\phi,z) = \frac{5}{r}$$

Radio interior = R1

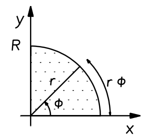
Radio exterior = R2

Altura = 3



$$\int_{z=0}^{z=3} 5(R_2 - R_1) 2\pi\; dz$$

$$q = 3 \cdot 2\pi \cdot 5(R_2 - R_1)$$



Densidad de carga

FFT Granada granada.net78.net

27-IX-2011

S.O.: Win95

Res.: 800x600

Col.: 16bit

FIN