Week 1: Exponential Smoothing (Advanced Material)

- An Operational Decision Problem

- Forecasting with Past Historical Data
- Moving Averages
- Exponential Smoothing

- Thinking about Trends and Seasonality

- Forecasting for new Products
- Fitting distributions

Moving Averages: What data to use?

- If you choose to use moving average method of last 10 data points,
  - all the older data is ignored
    - (e.g. data from 12 periods back is not used at all).
  - all the recent 10 data points are weighed the same.
    - (e.g. yesterday's data has the same weight as the data from a week before).

- You may want to give more weight to more recent data and less weight to older data.

- Exponential smoothing is based on this precise idea.
  - Advanced slides.

Exponential Smoothing Method

- Forecasting method that applies declining weights to past data.

- New Forecast = α(most recent observation) + (1 - α) (last forecast)

$$F_{t+1} = \alpha D_t + (1 - \alpha) F_t$$

  – where 0 < $\alpha < 1$

  – generally is small for stability of forecasts (around .1 to .2)

Assigning recursive weights

- $F_{t+1} = \alpha D_t + (1-\alpha)F_t$

- Now, we can write F_t as α(previous demand) + (1 - α) (last forecast)
  i.e. $F_t = \alpha D_{t-1} + (1-\alpha)F_{t-1}$

- Hence, in $F_{t+1} = \alpha D_t + (1-\alpha)F_t$ we replace F_t with $\alpha D_{t-1} + (1-\alpha)F_{t-1}$.

- $$\begin{aligned} F_{t+1} &= \alpha D_t + (1-\alpha)F_t \\ &= \alpha D_t + (1-\alpha)(\alpha D_{t-1} + (1-\alpha)F_{t-1}) \\ &= \alpha D_t + \alpha(1-\alpha)D_{t-1} + (1-\alpha)^2 F_{t-1} \end{aligned}$$

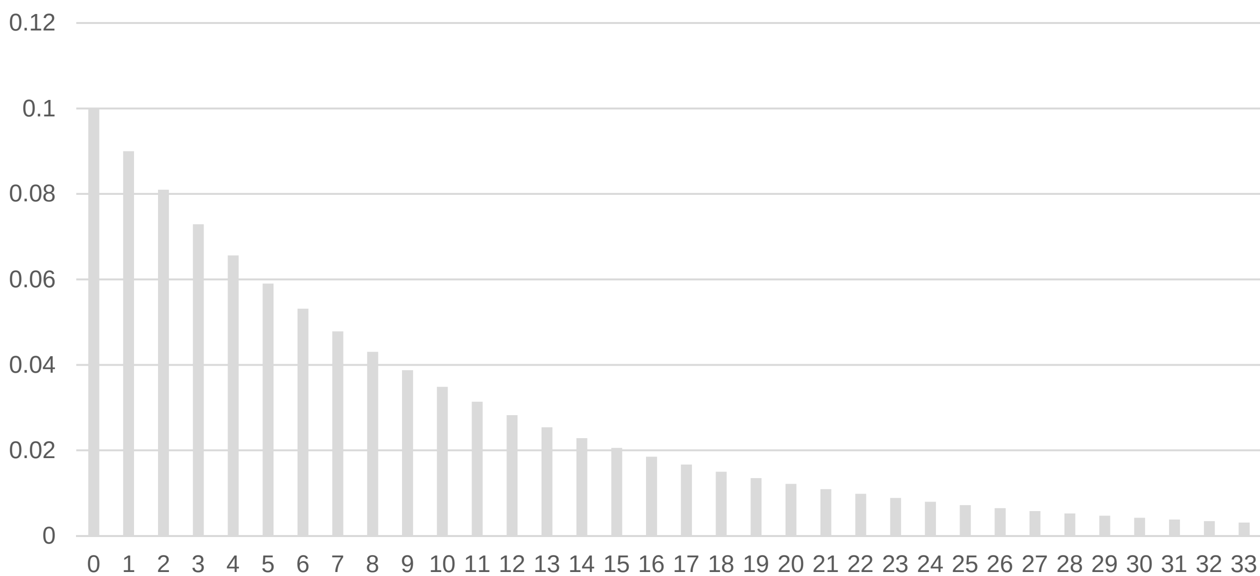
$$= \alpha D_t + \alpha(1-\alpha)D_{t-1} + \alpha(1-\alpha)^2 D_{t-2} + \cdots$$

Exponential Smoothing (cont.)

- Thus, ES assigns a set of exponentially declining weights to past data. (i.e. recent past has more weight than distant past)

- We can show by algebra using a geometric series that the sum of the weights is exactly one.

exponential weights decrease gradually for past data (α=0.1)



Exponential Smoothing vs. Moving Averages

- Similarities:
  - Both methods are appropriate for stationary time series
  - Both methods depend only on a single parameter
  - Both methods lag behind a trend
  - One can achieve the same distribution of forecast error by setting

$$\alpha = 2/ (N + 1).$$

Exponential Smoothing vs Moving Averages

- Differences:
  - ES carries all past history. MA eliminates “bad” data after N periods
  - For implementation, MA requires storing N past data points while ES can be implemented knowing only the last forecast and the last observation.

- Finally, if there is some trend in Data, we can fit trend line and adjust its slope by exponential smoothing.