Contents


```
% DRSP course project
% Ballistic Missile Interception Simulation and Prediction
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% Date: 12/07/2025
```
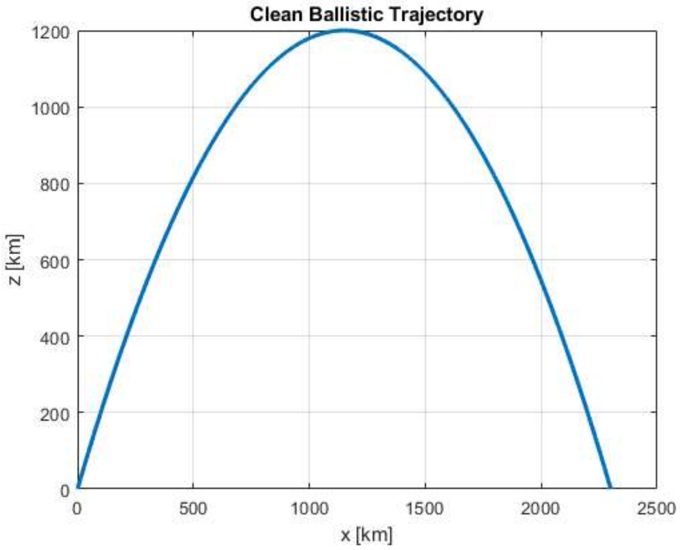
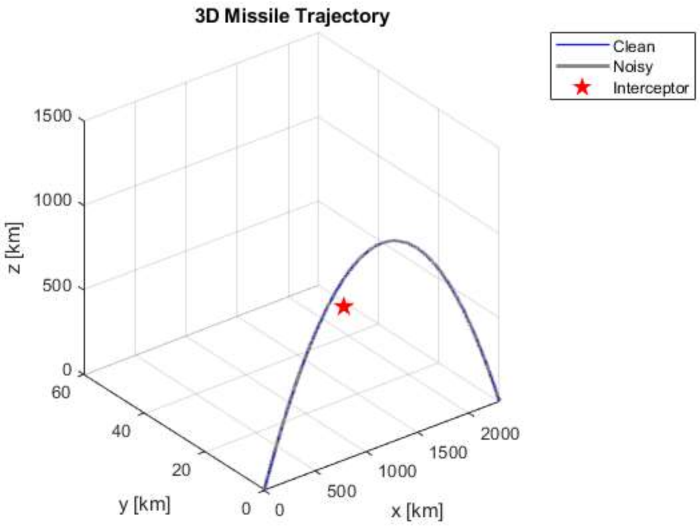

Clear workspace and close all figures

```
clear;
close all;
clc;
```

Step I – Simulation of the ballistic missile trajectory

```
% A. Define parameters and simulate the clean trajectory
xtarget = 2300;            % Expected x-coordinate of impact (in km)
yt = 0;                    % y-coordinate remains 0 (2D trajectory)
zt = 0;                    % z = 0 at impact

x0 = 0;                    % Launch point x (Iran)
y0 = 0;                    % Launch point y
z0 = 0;                    % Launch point z

xa = 2250;                 % Arrow missile (interceptor) x position
ya = 50;                   % Arrow y position
za = 0;                    % Arrow z position

h = 1200;                  % Maximum height of missile (vertex of parabola)
Ttotal = 600;              % Total simulation time in seconds
N = 2048;                  % Number of time samples in the simulation

b = xtarget / 2;           % x-location of the peak height
c = h;                     % z-location of the peak height
a = -h / b^2;              % Coefficient that defines curvature of parabola

T = linspace(0, Ttotal, N);           % Time vector
x = linspace(0, xtarget, N);          % Horizontal position vector

zb = a * (x - b).^2 + c;              % Parabolic formula for the ballistic arc

% B. Simulate AR(1) noise and add it to clean trajectory
sigmaW = 1;                          % Noise standard deviation
whiteNoise = sigmaW * randn(1, N);   % Generate white noise
noise = filter(1, [1, -0.8], whiteNoise);  % Pass white noise through AR(1) filter
zbNoisy = zb + noise;                % Add noise to clean trajectory to simulate measurement

% C. Plot the clean and noisy trajectories in 2D and 3D
figure;
plot(x, zb, 'LineWidth', 2);
xlabel('x [km]');
ylabel('z [km]');
title('Clean Ballistic Trajectory');
grid on;

figure;
plot3(x, zeros(size(x)), zb, 'b', 'LineWidth', 1);
hold on;
plot3(x, zeros(size(x)), zbNoisy, 'Color', [0.5 0.5 0.5],'LineWidth', 2);
plot3(xa, ya, za, 'rp', 'MarkerFaceColor', 'r', 'MarkerSize', 10);
xlabel('x [km]');
ylabel('y [km]');
zlabel('z [km]');
title('3D Missile Trajectory');
grid on;
legend('Clean','Noisy','Interceptor');
```

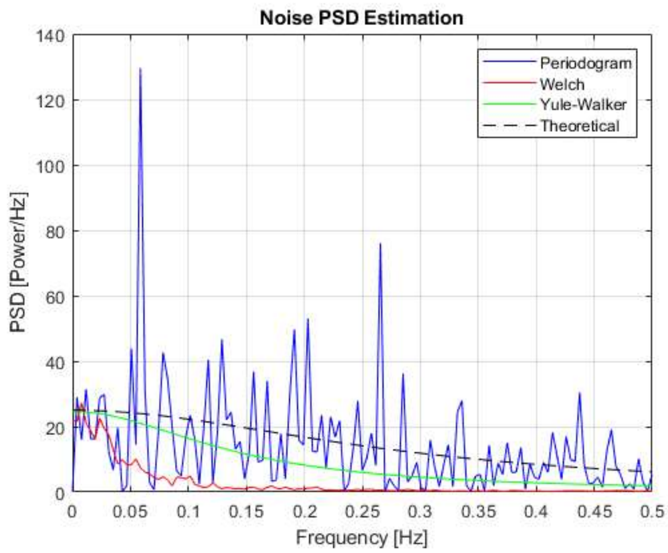




Step II – Spectral analysis and modeling of the noise you have simulated

```
% A. Estimate the power spectral density of the noise using the periodogram
periodogramPSD = zeros(1, N);
for k = 1:N
    S = sum(noise .* exp(-1j*2*pi*(k-1)*(0:N-1)/N));
    periodogramPSD(k) = (1/N) * abs(S)^2;
end
f = (0:N-1)/N;

% Estimate PSD using the Welch method (Corrected Welch: 50% overlap + Hamming window)
M = 256;
D = M/2;
w = hamming(M)';
U = sum(w.^2);
K = floor((N - M) / D) + 1;

PxWelch = zeros(1, M);
for k = 0:K-1
    idx = k*D + (1:M);
    segment = noise(idx) .* w;
    Pseg = abs(fft(segment)).^2;
    PxWelch = PxWelch + Pseg;
end
PxWelch = PxWelch / (K * U);
fWelch = (0:M-1)/M;

% Estimate PSD using the Yule-Walker method with xcorr for autocorrelation
p = 1;
[rxx, lags] = xcorr(noise, p, 'unbiased');
rxx = rxx(lags >= 0);
RYw = toeplitz(rxx(1:p));
rVec = rxx(2:p+1);

aYw = -RYw \ rVec;
aCoeff = [1; aYw];

% Estimate innovation variance sigma^2 using the Yule-Walker equation
sigmaYW = rxx(1) + aYw * rxx(2);
```

```
% Evaluate PSD using direct formula, not freqz
fYw = linspace(0, 0.5, 512);
omegaYw = 2 * pi * fYw;
PYw = sigmaYW ./ abs(1 + aYw * exp(-1j * omegaYw)).^2;

omega = 2 * pi * f;
Ptheory = sigmaW^2 ./ abs(1 - 0.8 * exp(-1j * omega)).^2;

% B. Theoretical PSD, and the estimators PSD's on the same set of axes
% Align all vectors to common plotting length
minLen = min([length(fYw), floor(N/2)+1, floor(M/2)+1]);
f1 = linspace(0, 0.5, minLen);
PxPer1 = periodogramPSD(1:minLen);
fWelch1 = fWelch(1:minLen);
PxWelch1 = PxWelch(1:minLen);
PYw1 = PYw(1:minLen);
Ptheory1 = Ptheory(1:minLen);

figure;
plot(f1, PxPer1, 'b', fWelch1, PxWelch1, 'r', f1, PYw1, 'g', f1, Ptheory1, 'k--');
xlabel('Frequency [Hz]'); ylabel('PSD [Power/Hz]');
title('Noise PSD Estimation'); grid on;
legend('Periodogram','Welch','Yule-Walker','Theoretical');
```

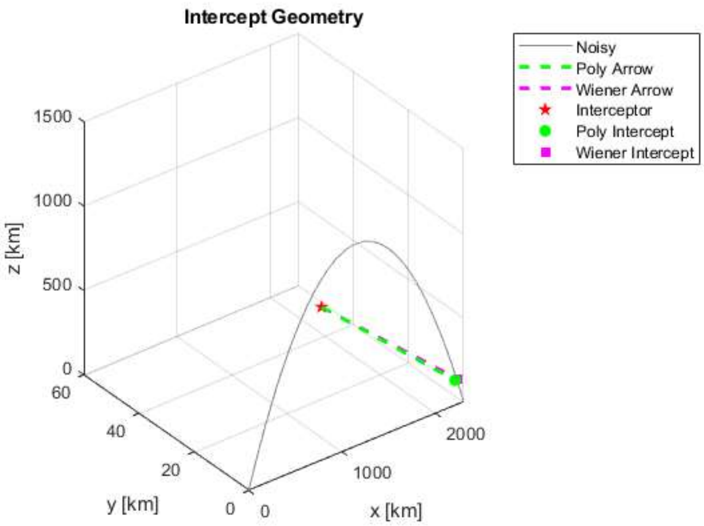


Step III – Trajectory prediction The missile

```
% a. Use a matched filter on the noisy signal to find the beginning of the descent phase
segmentLength = round(N/4);
matchedTemplate = zb(end - segmentLength + 1:end) - mean(zb(end - segmentLength + 1:end)); % subtract the mean to reduce DC bias - center the signal and to imp
matchedFilter = conv(zbNoisy - mean(zbNoisy), flip(matchedTemplate), 'same'); % flip the template for time-reversal
% signal is real-valued, conjugation is redundant
[~, descentIndex] = max(matchedFilter);
tDescent = T(descentIndex);

% b. Polynomial fit of the trajectory after descent
segmentSamples = round(15 * N / Ttotal);
fitIndices = descentIndex : min(descentIndex + segmentSamples - 1, N);
xFit = x(fitIndices)';
zFit = zbNoisy(fitIndices)';

polyCoeffs = polyfit(xFit, zFit, 2);

xPolyPred = x(fitIndices(end):end)';
zPolyPred = polyval(polyCoeffs, xPolyPred);

% b. Wiener FIR prediction, alpha seconds ahead
alpha = round(5 * N / Ttotal);
pw = 20;
L = pw + alpha;
rWiener = zeros(L+1,1);
for k = 0:L
    rWiener(k+1) = sum(zbNoisy(descentIndex:end-k) .* zbNoisy(descentIndex+k:end)) / (N - descentIndex + 1);
end
Rw = toeplitz(rWiener(1:pw));
rc = rWiener(alpha+1 : alpha+pw);
hWiener = Rw \ rc;

zWienerPred = nan(1, N);
for n = descentIndex + pw : N - alpha
    % Predicts alpha steps ahead using past pw samples of the noisy signal
    % The delay alpha affects how far into the future we attempt to predict, creating a lag in real-time application
    zWienerPred(n + alpha) = hWiener' * zbNoisy(n-1:-1:n-pw)';
end

% c. Calculate intercept location and time at 150 km altitude
interceptAlt = 150;
```

```
idxPolyIntercept = find(zPolyPred <= interceptAlt, 1);
validWien = (descentIndex + alpha):N;
idxWienIntercept = validWien(find(zWienerPred(validWien) <= interceptAlt, 1));

xInterceptPoly = xPolyPred(idxPolyIntercept);
zInterceptPoly = zPolyPred(idxPolyIntercept);
xInterceptWien = x(idxWienIntercept);
zInterceptWien = zWienerPred(idxWienIntercept);

% d. Calculate required missile launch time
arrowSpeed = 2.5; % Arrow missile speed in km/s
tInterceptPoly = T(fitIndices(end) + idxPolyIntercept - 1);
tInterceptWien = T(idxWienIntercept);

polyDistance = hypot(xInterceptPoly - xa, zInterceptPoly - za);
wienDistance = hypot(xInterceptWien - xa, zInterceptWien - za);

launchTimePoly = tInterceptPoly - polyDistance / arrowSpeed;
launchTimeWien = tInterceptWien - wienDistance / arrowSpeed;

% Print results for c and d sections
% These values indicate when the descent starts, the estimated intercept points and times,
% and when the interceptor must be launched to successfully meet the missile.
% These are critical for evaluating system performance and responsiveness.
disp('Step III Results:');
fprintf('Descent detected at t = %.2f s\n', tDescent);
fprintf('Polynomial intercept: x=%.1f z=%.1f km, t=%.2f s, launch=%.2f s\n', xInterceptPoly, zInterceptPoly, tInterceptPoly, launchTimePoly);
fprintf('Wiener intercept:    x=%.1f z=%.1f km, t=%.2f s, launch=%.2f s\n', xInterceptWien, zInterceptWien, tInterceptWien, launchTimeWien);

% e. Add to 3D plot the arrow interception paths
figure;
plot3(x, zeros(size(x)), zbNoisy, 'Color', [0.5 0.5 0.5]); hold on;
plot3([xa, xInterceptPoly], [ya, 0], [za, zInterceptPoly], 'g--', 'LineWidth', 2);
plot3([xa, xInterceptWien], [ya, 0], [za, zInterceptWien], 'm--', 'LineWidth', 2);
plot3(xa, ya, za, 'rp', 'MarkerFaceColor', 'r');
plot3(xInterceptPoly, 0, zInterceptPoly, 'go', 'MarkerFaceColor', 'g');
plot3(xInterceptWien, 0, zInterceptWien, 'ms', 'MarkerFaceColor', 'm');
xlabel('x [km]');
ylabel('y [km]');
zlabel('z [km]');
title('Intercept Geometry');
grid on;
legend('Noisy','Poly Arrow','Wiener Arrow','Interceptor','Poly Intercept','Wiener Intercept');
```

```
Step III Results:
Descent detected at t = 524.96 s
Polynomial intercept: x=2209.0 z=148.2 km, t=576.26 s, launch=514.74 s
Wiener intercept:    x=2237.1 z=147.5 km, t=583.59 s, launch=524.37 s
```



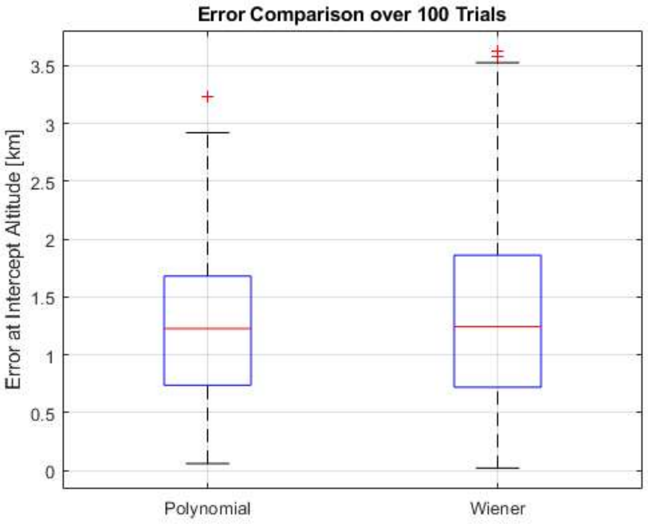
Step IV – Comparison and Evaluation

Display numerical comparison between methods

```
% Evaluate Accuracy over 1 Trials

fprintf('\nStep IV - Comparison:\n');
fprintf('Polynomial method error at intercept (z): %.2f km\n', abs(zInterceptPoly - interceptAlt));
fprintf('Wiener method error at intercept (z): %.2f km\n', abs(zInterceptWien - interceptAlt));

% Evaluate Accuracy over 100 Trials

nTrials = 100;
errorsPoly = nan(1, nTrials);
errorsWien = nan(1, nTrials);
```

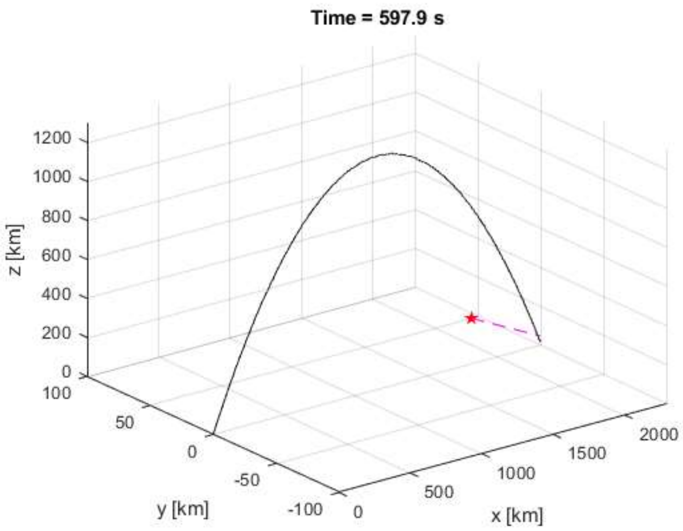
```matlab
for trial = 1:nTrials
    % Generate new noise + noisy trajectory
    wn = sigmaW * randn(1, N);
    noiseTrial = filter(1, [1, -0.8], wn);
    zbTrial = zb + noiseTrial;

    % Matched filter for descent detection
    matchedTemplate = zb(end - segmentLength + 1:end) - mean(zb(end - segmentLength + 1:end));
    matchedFilter = conv(zbTrial - mean(zbTrial), flip(matchedTemplate), 'same');
    [~, descentIdx] = max(matchedFilter);

    % Polynomial method
    fitIdx = descentIdx : min(descentIdx + segmentSamples - 1, N);
    xFit = x(fitIdx)';
    zFit = zbTrial(fitIdx)';
    coeffs = polyfit(xFit, zFit, 2);
    xPolyPred = x(fitIdx(end):end)';
    zPolyPred = polyval(coeffs, xPolyPred);
    idxPoly = find(zPolyPred <= interceptAlt, 1);

    if ~isempty(idxPoly)
        errPoly = abs(zPolyPred(idxPoly) - interceptAlt);
    else
        errPoly = NaN; % No intercept found
    end

    % Wiener method
    rWien = zeros(L+1,1);
    for k = 0:L
        rWien(k+1) = sum(zbTrial(descentIdx:end-k) .* zbTrial(descentIdx+k:end)) / (N - descentIdx + 1);
    end
    Rw = toeplitz(rWien(1:pw));
    rc = rWien(alpha+1 : alpha+pw);
    hWien = Rw \ rc;

    zWienPred = nan(1, N);
    for n = descentIdx + pw : N - alpha
        zWienPred(n + alpha) = hWien' * zbTrial(n-1:-1:n-pw)';
    end
    validWien = (descentIdx + alpha):N;
    idxWien = validWien(find(zWienPred(validWien) <= interceptAlt, 1));

    if ~isempty(idxWien)
        errWien = abs(zWienPred(idxWien) - interceptAlt);
    else
        errWien = NaN;
    end

    % Store errors
    errorsPoly(trial) = errPoly;
    errorsWien(trial) = errWien;
end

% Compute stats ignoring NaNs
meanPoly = mean(errorsPoly, 'omitnan');
meanWien = mean(errorsWien, 'omitnan');
varPoly = var(errorsPoly, 'omitnan');
varWien = var(errorsWien, 'omitnan');

fprintf('\nStep VI – Accuracy Evaluation:\n');
fprintf('Polynomial: Mean Error = %.2f km, Variance = %.2f\n', meanPoly, varPoly);
fprintf('Wiener:     Mean Error = %.2f km, Variance = %.2f\n', meanWien, varWien);

% Plot
figure;
boxplot([errorsPoly', errorsWien'], 'Labels', {'Polynomial', 'Wiener'});
ylabel('Error at Intercept Altitude [km]');
title('Error Comparison over 100 Trials');
grid on;


% differences in accuracy, delay, reliability
fprintf('Discussion:\n');
fprintf('- Polynomial fit is more stable but depends heavily on the chosen fitting segment.\n');
fprintf('- Wiener prediction is better for forecasting but more sensitive to noise and estimation errors.\n');
fprintf('- In real-time, Wiener filter may be more adaptable but riskier unless tuned well.\n');
```

```
Step IV – Comparison:
Polynomial method error at intercept (z): 1.77 km
Wiener method error at intercept (z): 2.52 km

Step VI – Accuracy Evaluation:
Polynomial: Mean Error = 1.26 km, Variance = 0.49
Wiener:     Mean Error = 1.33 km, Variance = 0.70
Discussion:
- Polynomial fit is more stable but depends heavily on the chosen fitting segment.
- Wiener prediction is better for forecasting but more sensitive to noise and estimation errors.
- In real-time, Wiener filter may be more adaptable but riskier unless tuned well.
```



Step V – Simulation Animation

```
% Animate the missile and interceptor paths over time
% We animate every 20 time steps to balance visual clarity and computational efficiency
% This shows missile motion and interceptor attempt visually over the full flight duration
figure;
for i = 1:20:N
    clf;
    plot3(x(1:i), zeros(1,i), zbNoisy(1:i), 'k');
    hold on;
    if i >= descentIndex + pw + alpha && ~isnan(zWienerPred(i))
        plot3(xa, ya, za, 'rp', 'MarkerFaceColor', 'r');
        plot3([xa, x(i)], [ya, 0], [za, zWienerPred(i)], 'm--');
    end
    xlim([0 xtarget]);
    ylim([-100 100]);
    zlim([0 1300]);
    xlabel('x [km]');
    ylabel('y [km]');
    zlabel('z [km]');
    title(sprintf('Time = %.1f s', T(i)));
    grid on;
    drawnow;
end
```



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