## January 18, 2018

## Abstract

## 1 Problem

Aqui

$$\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{3,4} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \\ a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} \end{pmatrix} \begin{pmatrix} y_{1,1} & y_{1,2} & y_{1,3} & y_{1,4} \\ y_{2,1} & y_{2,2} & y_{2,3} & y_{2,4} \\ y_{3,1} & y_{3,2} & y_{3,3} & y_{3,4} \\ y_{4,1} & y_{4,2} & y_{4,3} & y_{4,4} \end{pmatrix} \begin{pmatrix} y_{1,1} & y_{1,2} & y_{1,3} & y_{1,4} \\ y_{2,1} & y_{2,2} & y_{2,3} & y_{2,4} \\ y_{3,1} & y_{3,2} & y_{3,3} & y_{3,4} \\ y_{4,1} & y_{4,2} & y_{4,3} & y_{4,4} \end{pmatrix} \begin{pmatrix} 1_{1,1} & 1_{1,2} & 1_{1,3} & 1_{1,4} \\ 0 & t_{2,2} & t_{2,3} & t_{2,4} \\ 0 & 0 & t_{3,3} & t_{3,4} \\ 0 & 0 & t_{3,3} & t_{3,4} \\ 0 & 0 & t_{3,3} & t_{3,4} \end{pmatrix} \begin{pmatrix} d_{1,1} & d_{1,2} & d_{1,3} & d_{1,4} \\ d_{2,1} & d_{2,2} & d_{2,3} & d_{2,4} \\ d_{3,1} & d_{3,2} & d_{3,3} & d_{3,4} \\ d_{3,1} & d_{3,2} & d_{3,3} & d_{3,4} \\ d_{3,1} & d_{3,2} & d_{3,3} & d_{3,4} \end{pmatrix} \begin{pmatrix} 1_{1,1} & t_{1,2} & t_{1,3} & t_{1,4} \\ t_{1,1} & t_{1,2} & t_{1,3} & t_{1,4} \\ t_{2,1} & t_{2,2} & t_{2,3} & t_{2,4} \\ t_{2,1} & t_{2,2} & t_{2,3} & t_{2,4} \\ t_{3,1} & d_{3,2} & d_{3,3} & d_{3,4} \\ t_{3,1} & d_{3,2} & d_{3,3} & d_{3,4} \\ t_{3,1} & d_{3,2} & d_{3,3} & d_{3,4} \end{pmatrix} \begin{pmatrix} 1_{1,1} & t_{1,2} & t_{1,3} & t_{1,4} \\ t_{2,1} & t_{2,2} & t_{2,3} & t_{2,4} \\ t_{3,1} & t_{3,2} & t_{3,3} & t_{3,4} \\ t_{3,1} & t_{3,2} & t_{3,3} & t_$$

$$\begin{pmatrix} y_{1,1} & y_{1,2} & y_{1,3} & y_{1,4} \\ y_{2,1} & y_{2,2} & y_{2,3} & y_{2,4} \\ y_{3,1} & y_{3,2} & y_{3,3} & y_{3,4} \\ y_{4,1} & y_{4,2} & y_{4,3} & y_{4,4} \end{pmatrix} \begin{pmatrix} t_{1,1} & t_{1,2} & t_{1,3} & t_{1,4} \\ 0 & t_{2,2} & t_{2,3} & t_{2,4} \\ 0 & 0 & t_{3,3} & t_{3,4} \\ 0 & 0 & t_{4,3} & t_{4,4} \end{pmatrix}$$
(2)

$$AY_{\cdot,3} - \begin{pmatrix} y_{1,i}t_{i,3} + \dots + y_{1,3}t_{3,3} + y_{1,4}t_{4,3} \\ y_{2,i}t_{i,3} + \dots + y_{2,3}t_{3,3} + y_{2,4}t_{4,3} \\ y_{3,i}t_{i,3} + \dots + y_{3,3}t_{3,3} + y_{3,4}t_{4,3} \\ y_{4,i}t_{i,3} + \dots + y_{4,3}t_{3,3} + y_{4,4}t_{4,3} \end{pmatrix} = D_{\cdot,3} \quad (3)$$

$$AY_{\cdot,3} - \sum_{i=1}^{2} Y_{\cdot,i} t_{i,3} - Y_{\cdot,3} t_{3,3} - Y_{\cdot,4} t_{4,3} = D_{\cdot,3}$$
 (4)

$$AY_{\cdot,4} - \begin{pmatrix} y_{1,i}t_{i,4} + \dots + y_{1,3}t_{3,4} + y_{1,4}t_{4,4} \\ y_{2,i}t_{i,4} + \dots + y_{2,3}t_{3,4} + y_{2,4}t_{4,4} \\ y_{3,i}t_{i,4} + \dots + y_{3,3}t_{3,4} + y_{3,4}t_{4,4} \\ y_{4,i}t_{i,4} + \dots + y_{4,3}t_{3,4} + y_{4,4}t_{4,4} \end{pmatrix} = D_{\cdot,4} \quad (5)$$

$$AY_{\cdot,4} - \sum_{i=1}^{2} Y_{\cdot,i} t_{i,4} - Y_{\cdot,3} t_{3,4} - Y_{\cdot,4} t_{4,4} = D_{\cdot,4}$$
 (6)

$$AY_{\cdot,3} - \sum_{i=1}^{2} Y_{\cdot,i} t_{i,3} - Y_{\cdot,3} t_{3,3} - Y_{\cdot,4} t_{4,3} = D_{\cdot,3}$$
 (7)

$$AY_{\cdot,4} - \sum_{i=1}^{2} Y_{\cdot,i} t_{i,4} - Y_{\cdot,3} t_{3,4} - Y_{\cdot,4} t_{4,4} = D_{\cdot,4}$$
 (8)

$$AY_{\cdot,3} - K - Y_{\cdot,3}t_{3,3} - Y_{\cdot,4}t_{4,3} = D_{\cdot,3}$$
(9)

$$AY_{\cdot,4} - S - Y_{\cdot,3}t_{3,4} - Y_{\cdot,4}t_{4,4} = D_{\cdot,4}$$
(10)

$$K = \sum_{i=1}^{2} Y_{\cdot,i} t_{i,3} \tag{11}$$

$$S = \sum_{i=1}^{2} Y_{\cdot,i} t_{i,4} \tag{12}$$

$$AY_{\cdot,3} - Y_{\cdot,3}t_{3,3} - Y_{\cdot,4}t_{4,3} = D_{\cdot,3} + K \tag{13}$$

$$AY_{\cdot,4} - Y_{\cdot,3}t_{3,4} - Y_{\cdot,4}t_{4,4} = D_{\cdot,4} + S \tag{14}$$

$$AY_{\cdot,3} - Y_{\cdot,3}t_{3,3} - Y_{\cdot,4}t_{4,3} = D_k \tag{15}$$

$$AY_{\cdot,4} - Y_{\cdot,3}t_{3,4} - Y_{\cdot,4}t_{4,4} = D_s \tag{16}$$

$$(A - t_{3,3}I)Y_{\cdot,3} - Y_{\cdot,4}t_{4,3} = D_k \tag{18}$$

$$(A - t_{3,3}I)Y_{\cdot,3} - D_k = Y_{\cdot,4}t_{4,3}$$
(19)

$$\frac{(A - t_{3,3}I)Y_{\cdot,3}}{t_{4,3}} - \frac{D_k}{t_{4,3}} = Y_{\cdot,4} \tag{20}$$

$$\frac{(A - t_{3,3}I)}{t_{4,3}}Y_{\cdot,3} - \frac{D_k}{t_{4,3}} = Y_{\cdot,4}$$
 (21)

$$P = \frac{D_k}{t_{4,3}} \tag{22}$$

$$R = \frac{(A - t_{3,3}I)}{t_{4,3}} \tag{23}$$

$$RY_{\cdot,3} - P = Y_{\cdot,4} \tag{24}$$

$$AY_{\cdot,4} - Y_{\cdot,3}t_{3,4} - Y_{\cdot,4}t_{4,4} = D_s \tag{25}$$

(26)

$$RY_{\cdot,3} - P = Y_{\cdot,4} \tag{27}$$

$$A(RY_{\cdot,3} - P) - Y_{\cdot,3}t_{3,4} - (RY_{\cdot,3} - P)t_{4,4} = D_s$$
(28)

$$ARY_{\cdot,3} - AP - Y_{\cdot,3}t_{3,4} - RY_{\cdot,3}t_{4,4} + Pt_{4,4} = D_s$$
 (29)

$$(AR - t_{3,4}I - Rt_{4,4})Y_{\cdot,3} - AP + Pt_{4,4} = D_s$$
(30)

$$(AR - t_{3,4}I - Rt_{4,4})Y_{\cdot,3} = D_s + AP - Pt_{4,4}$$
(31)

$$Z = AR - t_{3,4}I - Rt_{4,4} (32)$$

$$W = D_s + AP - Pt_{4,4} (33)$$

$$ZY_{\cdot,3} = W \tag{34}$$

(35)

$$AY_{\cdot,4} - Y_{\cdot,3}t_{3,4} - Y_{\cdot,4}t_{4,4} = D_s \tag{36}$$

$$AY_{\cdot,4} - Y_{\cdot,4}t_{4,4} - D_s = Y_{\cdot,3}t_{3,4} \tag{37}$$

$$\frac{AY_{\cdot,4} - Y_{\cdot,4}t_{4,4} - D_s}{t_{3,4}} = Y_{\cdot,3} \tag{38}$$

(39)

$$AY_{\cdot,3} - Y_{\cdot,3}t_{3,3} - Y_{\cdot,4}t_{4,3} = D_{\cdot,3} + K \tag{40}$$

$$AY_{\cdot,4} - Y_{\cdot,3}t_{3,4} - Y_{\cdot,4}t_{4,4} = D_{\cdot,4} + S \tag{41}$$

(42)

$$AY_{\cdot,2} - Y_{\cdot,1}t_{1,2} - Y_{\cdot,2}t_{2,2} - Y_{\cdot,3}t_{3,2} = D_2$$

$$\tag{43}$$

$$AY_{\cdot,3} - Y_{\cdot,1}t_{1,3} - Y_{\cdot,2}t_{2,3} - Y_{\cdot,3}t_{3,3} = D_3$$

$$\tag{44}$$

$$(A - t_{2,2}I)Y_{\cdot,2} - Y_{\cdot,1}t_{1,2} - Y_{\cdot,3}t_{3,2} = D_2$$

$$\tag{45}$$

$$AY_{\cdot,3} - Y_{\cdot,1}t_{1,3} - Y_{\cdot,2}t_{2,3} - Y_{\cdot,3}t_{3,3} = D_3$$

$$\tag{46}$$

$$(A - t_{2,2}I)Y_{\cdot,2} - Y_{\cdot,1}t_{1,2} - D_2 = Y_{\cdot,3}t_{3,2}$$

$$\tag{47}$$

$$AY_{\cdot,3} - Y_{\cdot,1}t_{1,3} - Y_{\cdot,2}t_{2,3} - Y_{\cdot,3}t_{3,3} = D_3$$

$$\tag{48}$$

$$Y_{\cdot,3} = \frac{(A - t_{2,2}I)Y_{\cdot,2} - Y_{\cdot,1}t_{1,2} - D_2}{t_{3,2}}$$
(49)

$$AY_{\cdot,3} - Y_{\cdot,1}t_{1,3} - Y_{\cdot,2}t_{2,3} - Y_{\cdot,3}t_{3,3} = D_3$$
(50)

$$Y_{\cdot,3} = \frac{(A - t_{2,2}I)Y_{\cdot,2}}{t_{3,2}} - \frac{Y_{\cdot,1}t_{1,2}}{t_{3,2}} - \frac{D_2}{t_{3,2}}$$
 (51)

$$Y_{\cdot,3} = KY_{\cdot,2} - S \tag{52}$$

$$K = \frac{(A - t_{2,2}I)}{t_{3,2}} \tag{53}$$

$$S = \frac{Y_{\cdot,1}t_{1,2}}{t_{3,2}} - \frac{D_2}{t_{3,2}} \tag{54}$$

(55)

$$AY_{\cdot,3} - Y_{\cdot,1}t_{1,3} - Y_{\cdot,2}t_{2,3} - Y_{\cdot,3}t_{3,3} = D_3$$
(56)

$$A(KY_{\cdot,2} - S) - Y_{\cdot,1}t_{1,3} - Y_{\cdot,2}t_{2,3} - (KY_{\cdot,2} - S)t_{3,3} = D_3$$
(57)

$$AKY_{\cdot,2} - AS - Y_{\cdot,1}t_{1,3} - Y_{\cdot,2}t_{2,3} - KY_{\cdot,2}t_{3,3} - St_{3,3} = D_3$$
 (58)

$$(AK - Kt_{3,3} - t_{1,3}I)Y_{\cdot,2} - AS - Y_{\cdot,1}t_{1,3} - St_{3,3} = D_3$$
(59)

$$(AK - Kt_{3,3} - t_{1,3}I)Y_{\cdot,2} = D_3 + AS + Y_{\cdot,1}t_{1,3} + St_{3,3}$$

$$(60)$$

(61)