Models of Computation Assessed Coursework I

Ben Brannick

November 24, 2014

Question 1:

a)

$$\text{E.ADD}\ \frac{(\dagger) \qquad (\ddagger) \qquad 0 = 0 \underline{+} 0}{\langle(\texttt{newpair.fst}) + (\texttt{newpair.snd}), \emptyset, \emptyset\rangle \Downarrow_e \langle(0, \emptyset, (1 \mapsto 0, 2 \mapsto 0, 5 \mapsto 0, 6 \mapsto 0)\rangle}$$

$$(\dagger) \equiv \text{E.FST}\ \frac{\text{E.NEW}\ \dfrac{1 \notin \text{dom}(h) \qquad (1+1) \notin \text{dom}(h)}{\langle newp, \emptyset, \emptyset\rangle \Downarrow_e \langle \ulcorner 1 \urcorner, \emptyset, (1 \mapsto 0, 2 \mapsto 0)\rangle}}{\langle \texttt{newpair.fst}, \emptyset, \emptyset\rangle \Downarrow_e \langle 0, \emptyset, (1 \mapsto 0, 2 \mapsto 0)\rangle}$$

$$(\ddagger) \equiv \text{E.SND}\ \frac{\text{E.NEW}\ \dfrac{5 \notin \text{dom}(h) \qquad (5+1) \notin \text{dom}(h)}{\langle \texttt{newpair}, \emptyset, (1 \mapsto 0, 2 \mapsto 0)\rangle \Downarrow_e \langle \ulcorner 5 \urcorner, \emptyset, (1 \mapsto 0, 2 \mapsto 0, 5 \mapsto 0, 6 \mapsto 0)\rangle}}{\langle \texttt{newpair.snd}, \emptyset, (1 \mapsto 0, 2 \mapsto 0)\rangle \Downarrow_e \langle 0, \emptyset, (1 \mapsto 0, 2 \mapsto 0, 5 \mapsto 0, 6 \mapsto 0)\rangle}$$

b)

The semantics is not deterministic, as there is more than one choice for the value of a in `E.NEW`. This means one program could choose a value of 1 for a, whereas another could choose 2.

c)

The semantics is not total, as not all expressions will evaluate to an answer. For example, the expression $1.\texttt{snd}$ is a valid expression, however it requires $\langle 1, s, h\rangle \Downarrow_e \langle \ulcorner a \urcorner, s', h'\rangle$ for some a, which is not possible as according to rule `E.NUM`, $\langle 1, s, h\rangle \Downarrow_e \langle 1, s, h\rangle$

d)

As mentioned above, it is not the case that, for any value v or state s, $\langle 1.\texttt{snd}, s, h\rangle \Downarrow_e \langle v, s', h'\rangle$.

Question 2:

a)

$$\textsc{b.true}\ \frac{}{\langle \texttt{true}, s, h\rangle \Downarrow_b \langle \texttt{true}, s, h\rangle} \qquad \textsc{b.false}\ \frac{}{\langle \texttt{false}, s, h\rangle \Downarrow_b \langle \texttt{false}, s, h\rangle}$$

$$\textsc{b.and.true}\ \frac{\langle B_1, s, h\rangle \Downarrow_b \langle \texttt{true}, s', h'\rangle \qquad \langle B_2, s', h'\rangle \Downarrow_b \langle \texttt{true}, s'', h''\rangle}{\langle B_1 \ \&\ B_2, s, h\rangle \Downarrow_b \langle \texttt{true}, s'', h''\rangle}$$

$$\textsc{b.and.leftfalse}\ \frac{\langle B_1, s, h\rangle \Downarrow_b \langle \texttt{false}, s', h'\rangle \qquad \langle B_2, s', h'\rangle \Downarrow_b \langle \texttt{true}, s'', h''\rangle}{\langle B_1 \ \&\ B_2, s, h\rangle \Downarrow_b \langle \texttt{false}, s'', h''\rangle}$$

$$\textsc{b.and.rightfalse}\ \frac{\langle B_1, s, h\rangle \Downarrow_b \langle \texttt{true}, s', h'\rangle \qquad \langle B_2, s', h'\rangle \Downarrow_b \langle \texttt{false}, s'', h''\rangle}{\langle B_1 \ \&\ B_2, s, h\rangle \Downarrow_b \langle \texttt{false}, s'', h''\rangle}$$

$$\textsc{b.and.bothfalse}\ \frac{\langle B_1, s, h\rangle \Downarrow_b \langle \texttt{false}, s', h'\rangle \qquad \langle B_2, s', h'\rangle \Downarrow_b \langle \texttt{false}, s'', h''\rangle}{\langle B_1 \ \&\ B_2, s, h\rangle \Downarrow_b \langle \texttt{false}, s'', h''\rangle}$$

$$\textsc{b.not.true}\ \frac{\langle B, s, h\rangle \Downarrow_b \langle \texttt{false}, s', h'\rangle}{\langle \neg B, s, h\rangle \Downarrow_b \langle \texttt{true}, s', h'\rangle}$$

$$\textsc{b.not.false}\ \frac{\langle B, s, h\rangle \Downarrow_b \langle \texttt{true}, s', h'\rangle}{\langle \neg B, s, h\rangle \Downarrow_b \langle \texttt{false}, s', h'\rangle}$$

b)

The expression (`newpair`) could cause

$$\langle E = E, s, h\rangle \Downarrow_e \langle \texttt{false}, s', h'\rangle,$$

as the memory address chosen for `newpair`is not constrained to a single choice. This can be proven using the derivation:

$$\textsc{b.eq.false}\ \frac{(\dagger) \qquad (\ddagger) \qquad \ulcorner 1 \urcorner \neq \ulcorner 3 \urcorner}{\langle (\texttt{newpair} = \texttt{newpair}), s, h\rangle \Downarrow_b \langle \texttt{false}, s'', h''\rangle}$$

$$(\dagger) \equiv \ \textsc{e.new}\ \frac{1 \notin \mathrm{dom}(h) \qquad (1+1) \notin \mathrm{dom}(h)}{\langle \texttt{newpair}, s, h\rangle \Downarrow_e \langle \ulcorner 1 \urcorner, s, h[1 \mapsto 0, 2 \mapsto 0]\rangle}$$

$$(\ddagger) \equiv \ \textsc{e.new}\ \frac{3 \notin \mathrm{dom}(h) \qquad (3+1) \notin \mathrm{dom}(h)}{\langle \texttt{newpair}, s, h\rangle \Downarrow_e \langle \ulcorner 3 \urcorner, s, h[3 \mapsto 0, 4 \mapsto 0]\rangle}$$

Question 3

a)

$$\text{C.SEQ}\ \frac{(\dagger) \qquad (\ddagger)}{\langle x := c.\texttt{fst}; \texttt{fst}[c] \leftarrow y; c := c.\texttt{snd}, s, h\rangle \Downarrow_c \langle s'', h'\rangle}$$

$$(\dagger) \equiv \text{C.ASSIGN}\ \frac{\text{E.FST}\ \dfrac{\text{E.VAR}\ \dfrac{c \in \mathrm{dom}(s)}{\langle c, s, h\rangle \Downarrow_e \langle \ulcorner 5 \urcorner, s, h\rangle}}{\langle c.\texttt{fst}, s, h\rangle \Downarrow_e \langle 3, s, h\rangle}}{\langle x := c.\texttt{fst}, s, h\rangle \Downarrow_c \langle s[x \mapsto 3], h\rangle}$$

$$(\ddagger) \equiv \text{C.SEQ}\ \frac{(\ddagger') \qquad (\ddagger'')}{\langle \texttt{fst}[c] \leftarrow y; c := c.\texttt{snd}, s', h\rangle \Downarrow_c \langle s'', h'\rangle}$$

$$(\ddagger') \equiv \text{C.STOR.FST}\ \frac{\text{E.VAR}\ \dfrac{c \in \mathrm{dom}(s)}{\langle c, s', h\rangle \Downarrow_e \langle \ulcorner 5 \urcorner, s', h\rangle} \qquad \text{E.VAR}\ \dfrac{y \in \mathrm{dom}(s)}{\langle y, s', h\rangle \Downarrow_e \langle 12, s', h\rangle} \qquad 5 \in \mathrm{dom}(h)}{\langle \texttt{fst}[c] \leftarrow y, s', h\rangle \Downarrow_c \langle s', h[5 \mapsto 12]\rangle}$$

$$(\ddagger'') \equiv \text{C.ASSIGN}\ \frac{\text{E.SND}\ \dfrac{\text{E.VAR}\ \dfrac{c \in \mathrm{dom}(s)}{\langle c, s', h'\rangle \Downarrow_e \langle \ulcorner 5 \urcorner, s', h'\rangle}}{\langle c.\texttt{snd}, s', h\rangle \Downarrow_e \langle \ulcorner 3 \urcorner, s', h'\rangle}}{\langle c := c.\texttt{snd}, s', h'\rangle \Downarrow_c \langle s'[c \mapsto \ulcorner 3 \urcorner], h'\rangle}$$

Where:

$$s' \equiv s[x \mapsto 3],$$
$$s'' \equiv s'[c \mapsto \ulcorner 3 \urcorner],$$
$$h' \equiv h[5 \mapsto 12]$$

b)

$$\langle C, s, h\rangle \Downarrow_c \langle s', h'\rangle$$

Where:

$$s' \equiv (hd \mapsto \ulcorner 1 \urcorner, c \mapsto 0, x \mapsto \ulcorner 1 \urcorner, y \mapsto 3)$$
$$h' \equiv (1 \mapsto 7, 2 \mapsto \ulcorner 5 \urcorner, 3 \mapsto 3, 4 \mapsto 0, 5 \mapsto 7, 6 \mapsto \ulcorner 3 \urcorner)$$

Question 4

a)

i. $h \Vdash \ulcorner 3 \urcorner : ((nat, (nat, nat)), nat)$

ii. There is no type τ such that $h \Vdash \ulcorner 9 \urcorner : \tau$

iii. As the type τ_2 (Where $h \Vdash \ulcorner 1 + 1 \urcorner : \tau_2$) relies on the type τ, where $\tau \equiv (\tau_1, \tau_2)$, there is a cycle on type reliance, so there is no type τ' such that $h \Vdash \ulcorner 1 \urcorner : \tau'$

b)

i. $\Gamma; s; h \vdash \mathsf{well\text{-}typed}$, as both Γ and $s \& h$ agree on the types of $x \& z$

ii. Here, it is not the case that $\Gamma; s; h \vdash \mathsf{well\text{-}typed}$, as $\Gamma \vdash x : (nat, (nat, nat))$, but in $s; h$, $x \mapsto nat$

iii. Again, $\Gamma; s; h \vdash \mathsf{well\text{-}typed}$

c) SEE ATTACHED PAPER