

Models of Computation Assessed Coursework I

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Question 1:

a)

$$\begin{aligned}
 & \text{E.ADD} \frac{(\dagger) \quad (\ddagger) \quad 0 = 0 \pm 0}{\langle (\text{newpair.fst}) + (\text{newpair.snd}), \emptyset, \emptyset \rangle \Downarrow_e \langle (0, \emptyset, (1 \mapsto 0, 2 \mapsto 0, 5 \mapsto 0, 6 \mapsto 0)) \rangle} \\
 & (\dagger) \equiv \text{E.FST} \frac{\text{E.NEW} \frac{1 \notin \text{dom}(h) \quad (1+1) \notin \text{dom}(h)}{\langle \text{newp}, \emptyset, \emptyset \rangle \Downarrow_e \langle \ulcorner 1 \urcorner, \emptyset, (1 \mapsto 0, 2 \mapsto 0) \rangle}}{\langle \text{newpair.fst}, \emptyset, \emptyset \rangle \Downarrow_e \langle 0, \emptyset, (1 \mapsto 0, 2 \mapsto 0) \rangle} \\
 & (\ddagger) \equiv \text{E.SND} \frac{\text{E.NEW} \frac{5 \notin \text{dom}(h) \quad (5+1) \notin \text{dom}(h)}{\langle \text{newpair}, \emptyset, (1 \mapsto 0, 2 \mapsto 0) \rangle \Downarrow_e \langle \ulcorner 5 \urcorner, \emptyset, (1 \mapsto 0, 2 \mapsto 0, 5 \mapsto 0, 6 \mapsto 0) \rangle}}{\langle \text{newpair.snd}, \emptyset, (1 \mapsto 0, 2 \mapsto 0) \rangle \Downarrow_e \langle 0, \emptyset, (1 \mapsto 0, 2 \mapsto 0, 5 \mapsto 0, 6 \mapsto 0) \rangle}
 \end{aligned}$$

b)

The semantics is not deterministic, as there is more than one choice for the value of a in **E.NEW**. This means one program could choose a value of 1 for a , whereas another could choose 2.

c)

The semantics is not total, as not all expressions will evaluate to an answer. For example, the expression `1.snd` is a valid expression, however it requires $\langle 1, s, h \rangle \Downarrow_e \langle \ulcorner a \urcorner, s', h' \rangle$ for some a , which is not possible as according to rule **E.NUM**, $\langle 1, s, h \rangle \Downarrow_e \langle 1, s, h \rangle$

d)

As mentioned above, it is not the case that, for any value v or state s , $\langle 1.\text{snd}, s, h \rangle \Downarrow_e \langle v, s', h' \rangle$.

Question 2:

a)

$$\begin{array}{c}
\text{B.TRUE} \frac{}{\langle \mathbf{true}, s, h \rangle \Downarrow_b \langle \mathbf{true}, s, h \rangle} \quad \text{B.FALSE} \frac{}{\langle \mathbf{false}, s, h \rangle \Downarrow_b \langle \mathbf{false}, s, h \rangle} \\
\text{B.AND.TRUE} \frac{\langle B_1, s, h \rangle \Downarrow_b \langle \mathbf{true}, s', h' \rangle \quad \langle B_2, s', h' \rangle \Downarrow_b \langle \mathbf{true}, s'', h'' \rangle}{\langle B_1 \ \& \ B_2, s, h \rangle \Downarrow_b \langle \mathbf{true}, s'', h'' \rangle} \\
\text{B.AND.FALSE} \frac{\langle B_1, s, h \rangle \Downarrow_b \langle v_1, s', h' \rangle \quad \langle B_2, s', h' \rangle \Downarrow_b \langle v_2, s'', h'' \rangle \quad v_1 \neq \mathbf{true} \vee v_2 \neq \mathbf{true}}{\langle B_1 \ \& \ B_2, s, h \rangle \Downarrow_b \langle \mathbf{false}, s'', h'' \rangle} \\
\text{B.NOT.TRUE} \frac{\langle B, s, h \rangle \Downarrow_b \langle \mathbf{false}, s', h' \rangle}{\langle \neg B, s, h \rangle \Downarrow_b \langle \mathbf{true}, s', h' \rangle} \\
\text{B.NOT.FALSE} \frac{\langle B, s, h \rangle \Downarrow_b \langle \mathbf{true}, s', h' \rangle}{\langle \neg B, s, h \rangle \Downarrow_b \langle \mathbf{false}, s', h' \rangle}
\end{array}$$

b)

The expression (**newpair**) could cause

$$\langle E = E, s, h \rangle \Downarrow_e \langle \mathbf{false}, s', h' \rangle,$$

as the memory address chosen for **newpair** is not constrained to a single choice. This can be proven using the derivation:

$$\begin{array}{c}
\text{B.EQ.FALSE} \frac{(\dagger) \quad (\dagger) \quad \ulcorner 1 \urcorner \neq \ulcorner 3 \urcorner}{\langle (\mathbf{newpair} = \mathbf{newpair}), s, h \rangle \Downarrow_b \langle \mathbf{false}, s'', h'' \rangle} \\
(\dagger) \equiv \text{E.NEW} \frac{1 \notin \text{dom}(h) \quad (1 + 1) \notin \text{dom}(h)}{\langle \mathbf{newpair}, s, h \rangle \Downarrow_e \langle \ulcorner 1 \urcorner, s, h[1 \mapsto 0, 2 \mapsto 0] \rangle} \\
(\dagger) \equiv \text{E.NEW} \frac{3 \notin \text{dom}(h) \quad (3 + 1) \notin \text{dom}(h)}{\langle \mathbf{newpair}, s, h \rangle \Downarrow_e \langle \ulcorner 3 \urcorner, s, h[3 \mapsto 0, 4 \mapsto 0] \rangle}
\end{array}$$

Question 3

a)

$$\text{C.SEQ} \frac{(\dagger) \quad (\ddagger)}{\langle x := c.\mathbf{fst}; \mathbf{fst}[c] \leftarrow y; c := c.\mathbf{snd}, s, h \rangle \Downarrow_c \langle s'', h' \rangle}$$

$$(\dagger) \equiv \text{C.ASSIGN} \frac{\text{E.VAR} \frac{c \in \text{dom}(s)}{\langle c, s, h \rangle \Downarrow_e \langle \ulcorner 5 \urcorner, s, h \rangle} \quad \text{E.FST} \frac{}{\langle c.\mathbf{fst}, s, h \rangle \Downarrow_e \langle 3, s, h \rangle}}{\langle x := c.\mathbf{fst}, s, h \rangle \Downarrow_c \langle s[x \mapsto 3], h \rangle}$$

$$(\ddagger) \equiv \text{C.SEQ} \frac{(\dagger') \quad (\ddagger'')}{\langle \mathbf{fst}[c] \leftarrow y; c := c.\mathbf{snd}, s', h \rangle \Downarrow_c \langle s'', h' \rangle}$$

$$(\dagger') \equiv \text{C.STOR.FST} \frac{\text{E.VAR} \frac{c \in \text{dom}(s)}{\langle c, s', h \rangle \Downarrow_e \langle \ulcorner 5 \urcorner, s', h \rangle} \quad \text{E.VAR} \frac{y \in \text{dom}(s)}{\langle y, s', h \rangle \Downarrow_e \langle 12, s', h \rangle} \quad 5 \in \text{dom}(h)}{\langle \mathbf{fst}[c] \leftarrow y, s', h \rangle \Downarrow_c \langle s', h[5 \mapsto 12] \rangle}$$

$$(\ddagger'') \equiv \text{C.ASSIGN} \frac{\text{E.VAR} \frac{c \in \text{dom}(s)}{\langle c, s', h' \rangle \Downarrow_e \langle \ulcorner 5 \urcorner, s', h' \rangle} \quad \text{E.SND} \frac{}{\langle c.\mathbf{snd}, s', h \rangle \Downarrow_e \langle \ulcorner 3 \urcorner, s', h' \rangle}}{\langle c := c.\mathbf{snd}, s', h' \rangle \Downarrow_c \langle s'[c \mapsto \ulcorner 3 \urcorner], h' \rangle}$$

Where

$$\begin{aligned} s' &\equiv s[x \mapsto 3], \\ s'' &\equiv s'[c \mapsto \ulcorner 3 \urcorner], \\ h' &\equiv h[5 \mapsto 12] \end{aligned}$$