Models of Computation Assessed Coursework I

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Question 1:

a)

$$\text{E.ADD}\ \frac{(\dagger) \qquad (\ddagger) \qquad 0 = 0 \underline{+} 0}{\langle (\texttt{newpair.fst}) + (\texttt{newpair.snd}), \emptyset, \emptyset \rangle \Downarrow_e \langle (0, \emptyset, (1 \mapsto 0, 2 \mapsto 0, 5 \mapsto 0, 6 \mapsto 0) \rangle}$$

$$(\dagger) \equiv \text{E.FST}\ \frac{\text{E.NEW}\ \dfrac{1 \notin \mathrm{dom}(h)(1+1) \notin \mathrm{dom}(h)}{\langle newp, \emptyset, \emptyset \rangle \Downarrow_e \langle \ulcorner 1 \urcorner, \emptyset, (1 \mapsto 0, 2 \mapsto 0) \rangle}}{\langle \texttt{newpair.fst}, \emptyset, \emptyset \rangle \Downarrow_e \langle 0, \emptyset, (1 \mapsto 0, 2 \mapsto 0) \rangle}$$

$$(\ddagger) \equiv \text{E.SND}\ \frac{\text{E.NEW}\ \dfrac{5 \notin \mathrm{dom}(h)(5+1) \notin \mathrm{dom}(h)}{\langle \texttt{newpair}, \emptyset, (1 \mapsto 0, 2 \mapsto 0) \rangle \Downarrow_e \langle \ulcorner 5 \urcorner, \emptyset, (1 \mapsto 0, 2 \mapsto 0, 5 \mapsto 0, 6 \mapsto 0) \rangle}}{\langle \texttt{newpair.snd}, \emptyset, (1 \mapsto 0, 2 \mapsto 0) \rangle \Downarrow_e \langle 0, \emptyset, (1 \mapsto 0, 2 \mapsto 0, 5 \mapsto 0, 6 \mapsto 0) \rangle}$$

b)

The semantics is not deterministic, as there is more than one choice for the value of a in `E.NEW`. This means one program could choose a value of 1 for a, whereas another could choose 2.

c)

The semantics is not total, as not all expressions will evaluate to an answer. For example, the expression $1.\texttt{snd}$ is a valid expression, however it requires $\langle 1, s, h \rangle \Downarrow_e \langle \ulcorner a \urcorner, s', h' \rangle$ for some a, which is not possible as according to rule `E.NUM`, $\langle 1, s, h \rangle \Downarrow_e \langle 1, s, h \rangle$

d)

As mentioned above, it is not the case that, for any value v or state s, $\langle 1.\texttt{snd}, s, h \rangle \Downarrow_e \langle v, s', h' \rangle$.

Question 2:

a)

$$\textsc{b.true}\ \frac{}{\langle \texttt{true}, s, h\rangle \Downarrow_b \langle \texttt{true}, s, h\rangle} \qquad \textsc{b.false}\ \frac{}{\langle \texttt{false}, s, h\rangle \Downarrow_b \langle \texttt{false}, s, h\rangle}$$

$$\textsc{b.and.true}\ \frac{\langle B_1, s, h\rangle \Downarrow_b \langle \texttt{true}, s', h'\rangle \qquad \langle B_2, s', h'\rangle \Downarrow_b \langle \texttt{true}, s'', h''\rangle}{\langle B_1 \ \&\ B_2, s, h\rangle \Downarrow_b \langle \texttt{true}, s'', h''\rangle}$$

$$\textsc{b.and.false}\ \frac{\langle B_1, s, h\rangle \Downarrow_b \langle v_1, s', h'\rangle \qquad \langle B_2, s', h'\rangle \Downarrow_b \langle v_2, s'', h''\rangle \qquad v_1 \neq \texttt{true} \vee v_2 \neq \texttt{true}}{\langle B_1 \ \&\ B_2, s, h\rangle \Downarrow_b \langle \texttt{false}, s'', h''\rangle}$$

$$\textsc{b.not.true}\ \frac{\langle B, s, h\rangle \Downarrow_b \langle \texttt{false}, s', h'\rangle}{\langle \neg B, s, h\rangle \Downarrow_b \langle \texttt{true}, s', h'\rangle}$$

$$\textsc{b.not.false}\ \frac{\langle B, s, h\rangle \Downarrow_b \langle \texttt{true}, s', h'\rangle}{\langle \neg B, s, h\rangle \Downarrow_b \langle \texttt{false}, s', h'\rangle}$$

b) The expression `newpair` could