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1.

(a) imagine we can find A "k", that can make $k \cdot n^2 > \frac{n(n-1)}{2}$ work for all $n \geq m$. And if it establishes, the algorithm is $O(n^2)$

eg: $k = 4$ $\quad 2n > n$ for all $n > 0$

$m = 0$

(b) ① $0 < n < 10$, $\quad n^3 < 10n^2$

but we can find A "k", the make the inequality $k \cdot n^3 > 10n^2$

for example: $k = 11$ $\quad 11 \cdot n^3 > 10n^2$

② $n \geq 10$ $\quad n^3 \geq 10n^2$

So obviously $\quad 11 \cdot n^3 > 10n^2$

Above all for all n, $n > 0$

$11 \cdot n^3 > 10n^2$ And we can say the algorithm is $O(n^3)$

(c) $n^{k+1} = n^k + n^k + \cdots + n^k \quad (n \cdot n^k)$

$\sum_{i=1}^{n} i^k = 1^k + 2^k + \cdots + n^k$

$$\lim_{n \to \infty} \frac{1^k + 2^k + \cdots + n^k}{n^k + n^k + \cdots + n^k} \leq \frac{n \cdot n^k}{n \cdot n^k} = 1$$

So $\sum_{i=1}^{n} i^k$ is $O(k^{n+1})$ for integer k

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(d) If $P(x)$ is any k^{th} degree polynomial with a positive leading coefficient, so $P(x) = \sum_{n=0}^{k} a_n x^n$

$$\therefore \lim_{n\to\infty} \frac{P(n)}{n^k} = a_0\left(\frac{1}{n^k}\right) + a_1\left(\frac{1}{n^{k-1}}\right) + \cdots + a_n$$

$= a_n$ it's a positive number

So $P(n)$ is $O(n^k)$

2. (a): $n^{\log n} = e^{\log n \cdot \log n}$; $(\log n)^n = e^{n \log(\log n)}$

let $g(x) \cdot (\log x)^2 = x \cdot \log(\log x)$

when $x \to \infty$

$$\frac{(\log x)^2}{x \cdot \log(\log x)} = \frac{2\log x}{x\log(\log x) + \frac{x}{\log x}} = \frac{2(\log x)^2}{x\log x \log(\log x) + x}$$

$= 0$

Thus, when $x \to \infty$ $n\log^n < (\log n)^n$

$\Rightarrow$ $(\log n)^n$ grow faster.

(b) $\log n^k = k\log n$.

let use x replace the $\log n$.

$\Rightarrow$ ~~x^k~~ when can compare kx with x^k.

$kx < x^k$ $(k>0, x>0)$

x^k grow faster

Thus $(\log n)^k$ grow faster

(c) I know the function: $\log(n!) = n\log n$.

So ~~we~~ we can compare the $\log n^{\log\log\log n}$ with $\log(\log n)!$

$$\log n^{\log\log\log n} = \log\log\log n \cdot \log n.$$

$$\log(\log n)! = \log n \log\log n$$

when $n \to \infty$

$$\frac{\log\log\log n \cdot \log n}{\log n \cdot \log\log n} = 0.$$

Thus, $(\log n)!$ grow faster.

(d) when $n \to \infty$

$$\frac{n!}{n^n} = \frac{n(n-1)\cdots 1}{n \cdot n \cdot \cdots, n} = 0$$

$\Rightarrow$ Thus, n^n is faster.

3. for $f_1(n) = O(g_1(n))$, there exists c_1 such that $f_1(n) \le c_1 g_1(n)$, when $n \ge n_0$.
for $f_2(n) = O(g_2(n))$, there exists c_2 such that $f_2(n) \le c_2 g_2(n)$, when $n \ge n_2$.
so when ~~$n \ge n_0$~~ $n \ge max(n_1, n_2)$, and f_1, f_2 are positive functions of n.

$$f_1(n) + f_2(n) \le c_1 g_1(n) + c_2 g_2(n) \le c_1 g_k(n) + c_2 g_k(n) \le (c_1 + c_2) g_k(n)$$

where $g_k(n) = max(g_1(n), g_2(n))$, $k = \{1, 2\}$

make $C = c_1 + c_2$,

$f_1(n) + f_2(n) \le C \cdot g_k(n)$

according to the definition of Big Oh.

$f_1(n) + f_2(n)$ is $O(max(g_1(n), g_2(n)))$

4. Prove or disprove: Any positive n is $O(\frac{n}{2})$
Assuming $f(n) = n = O(\frac{n}{2})$
Choose $k = 1$
Assuming $n > 1$

$$\frac{f(n)}{g(n)} \le \frac{Cg(n)}{g(n)} = \frac{n}{\frac{n}{2}} = 2$$

This shows that $n > 1$ implies $f(n) \le Cg(n)$,
So any positive n is $O(\frac{n}{2})$

5. Prove or disprove: 3^n is $O(2^n)$

$3^n \ge C \cdot 2^n$

$(\frac{3}{2})^n \ge C$

$n \ge \log \frac{2c}{3}$

For every $n \ge \log \frac{2c}{3}$, $3^n \ge 2^n$

Therefore, 3^n is not $O(2^n)$