1 Singular Control Problem

1.1 Maximization Problem

The **value function** of the maximization problem for $j = 0, w, b$ (corresponding to the three portfolios: no option, writer, buyer) is defined as:

$$V^j(t, b, y, s) = \sup_{L,M} \mathbb{E}\left[\mathcal{U}\left(\mathcal{W}_T^j\right) \;\middle|\; B_t = b, Y_t = y, S_t = s \right], \tag{1}$$

where $\mathcal{U} : \mathbb{R} \to \mathbb{R}$ is a concave increasing **utility function**. The **exponential utility** is what we are looking for:

$$\mathcal{U}(x) := 1 - e^{-\gamma x} \qquad \gamma > 0. \tag{2}$$

1.2 Indifference Pricing

The writer (buyer) option price is defined as the amount of cash to add (subtract) to the bank account, such that the maximal expected utility of wealth of the writer (buyer) is the same as he could get with the zero-option portfolio.

- The **writer price** is the value $p^w > 0$ such that

$$V^0(t, b, y, s) = V^w(t, b + p^w, y, s), \tag{3}$$

- The **buyer price** is the value $p^b > 0$ such that

$$V^0(t, b, y, s) = V^b(t, b - p^b, y, s). \tag{4}$$