SDE and Seasonal Function OLS

Average temperature t-th day:

$$T_t = \frac{T_{\max_t} + T_{\min_t}}{2}$$

following an Ornstein–Uhlenbeck (OU) process:

$$dT_t = a(T_m - T_t)\,dt + \sigma_t\,dW_t$$

Seasonality with a time-dependent mean that accounts for this seasonality implies -

$$\frac{dT_t^m}{dt} = B + \omega \cdot C \cdot \cos(\omega t + \phi)$$

0.1 SDE -

$$dT_t = \left\{ \frac{dT_t^m}{dt} + a(T_t^m - T_t) \right\} dt + \sigma_t\,dW_t$$

Distribution of differences $\Delta T_t = T_t - T_{t-1}$ has mean and variance :

$$\overline{\Delta T_t} = \frac{1}{n}\sum_{t=1}^{n} \Delta T_t$$

$$\frac{1}{n}\sum_{t=1}^{n} (\Delta T_t - \overline{\Delta T_t})^2.$$

Calibration

Following Alaton as well as Bibby and Sorensen in a *martingale estimation method* for drift and diffussion coefficients. This efficient estimator can be obtained by finding the value of $\hat{a}_n$ such that:

$$G_n(a) = 0 = \sum_{i=1}^{n} \frac{\partial_a b(T_{i-1}; a)}{\sigma_{i-1}^2} \left(T_i - \mathbb{E}[T_i | T_{i-1}]\right)$$

Discretized SDE based variance estimator -

$$\hat{\sigma}_u^2 = \frac{1}{N_u - 2} \sum_{j=0}^{N_u - 1} \left(T_j - \hat{a} T_{j-1}^m - (1 - \hat{a}) T_{j-1}\right)^2$$

0.2 OLS to be fitted -

$$T_t^m = a_1 + a_2 t + a_3 \sin(\omega t) + a_4 \cos(\omega t)$$

where:

- T_t^m: Modeled temperature at time t
- ω: Angular frequency, typically set to $\frac{2\pi}{365}$ for daily data with an annual cycle

such that -

$$T_t^m = A + Bt + C \sin(\omega t + \phi)$$

- $A = a_1$, $B = a_2$, $C = \sqrt{a_3^2 + a_4^2}$
- $\phi = \arctan\left(\frac{a_4}{a_3}\right) - \pi$