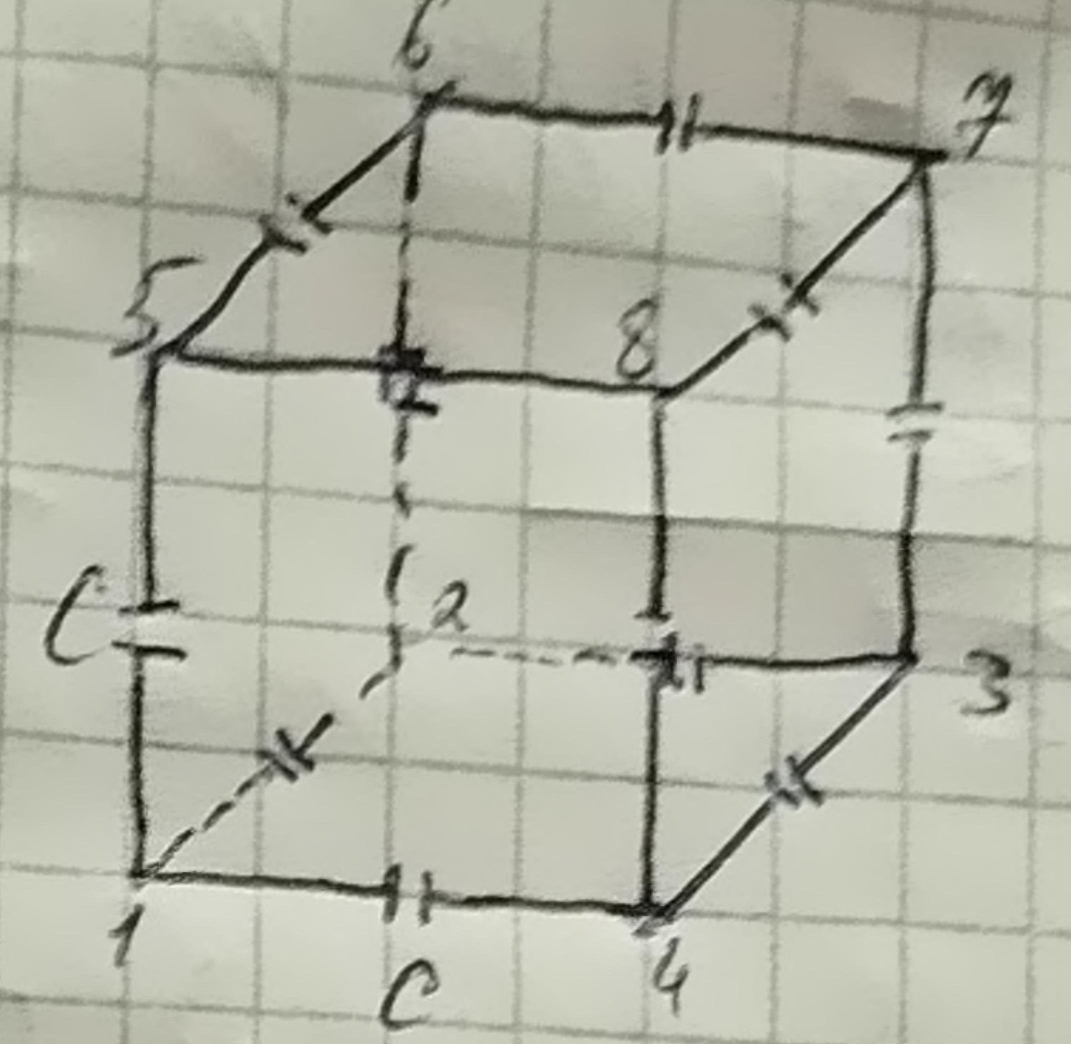
$$\Rightarrow \sigma = \frac{\varepsilon_0 d(\varepsilon_1 - \varepsilon_2)}{\varepsilon_2 d_1 + \varepsilon_1 d_2}.$$

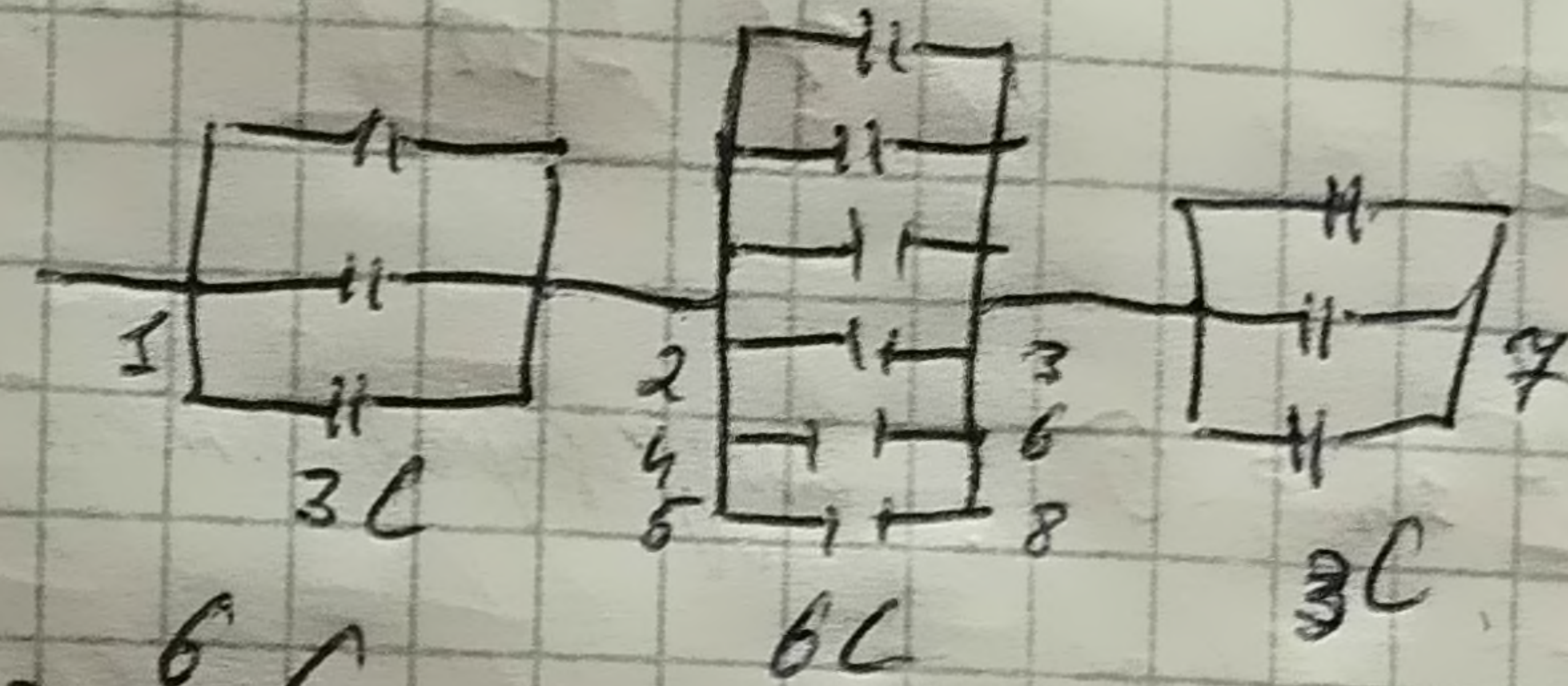
4.3. Дано: конденсаторы.

$C_{17} = ?$



Решение:

$\varphi_2 = \varphi_4 = \varphi_5$; $\varphi_3 = \varphi_6 = \varphi_8$.



$$\frac{1}{C_{17}} = \frac{2}{3C} + \frac{1}{6C} = \frac{5}{6C} \Rightarrow C_{17} = \frac{6}{5}C.$$

Ответ: $C_{17} = \frac{6}{5}C$.

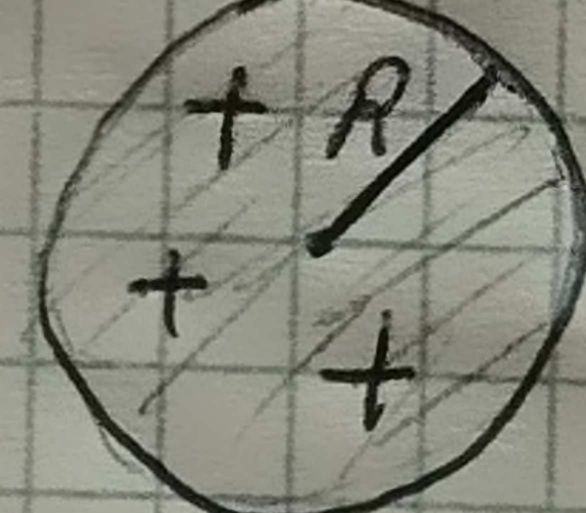
Задача 4.4.

Дано: Шар.

q, R, $\varepsilon = 1$.

а) W - ?

б) $\frac{W_1}{W_2}$ - ?



Решение:

$$W = \frac{1}{8\pi\varepsilon_0} \int_V E^2 dV$$

$r \le R$:

$$q_r = q\left(\frac{r^3}{R^3}\right).$$

$$E(r) = \frac{q_r}{r^2} \cdot \frac{1}{4\pi\varepsilon_0} = \frac{q}{4\pi\varepsilon_0 R^3} \cdot r$$

$$W_1 = \frac{1}{8\pi\varepsilon_0} \int^R E(r)^2 \cdot 4\pi r^2 dr =$$

$$= \frac{1}{8\pi\varepsilon_0} \int_0^R \left(\frac{q r}{4\pi\varepsilon_0 R^3}\right)^2 4\pi r^2 dr =$$

$$= \frac{q^2}{32\pi^2\varepsilon_0^2 R^6} \int_0^R r^4 dr =$$

$$= \frac{q^2}{32\pi^2\varepsilon_0^2 R^6} \cdot \frac{R^5}{5} = \frac{q^2}{160\pi\varepsilon_0 R}$$

$r > R$:

$$E(r) = \frac{q}{4\pi\varepsilon_0 r^2}.$$

$$W_2 = \frac{1}{8\pi\varepsilon_0} \int_R^\infty E(r)^2 \cdot 4\pi r^2 dr =$$

$$= \frac{1}{8\pi\varepsilon_0} \int_R^\infty \left(\frac{q}{4\pi\varepsilon_0 r^2}\right)^2 4\pi r^2 dr =$$

$$= \frac{q^2}{32\pi^2\varepsilon_0^2} \cdot \int_R^\infty \frac{1}{r^2} dr =$$

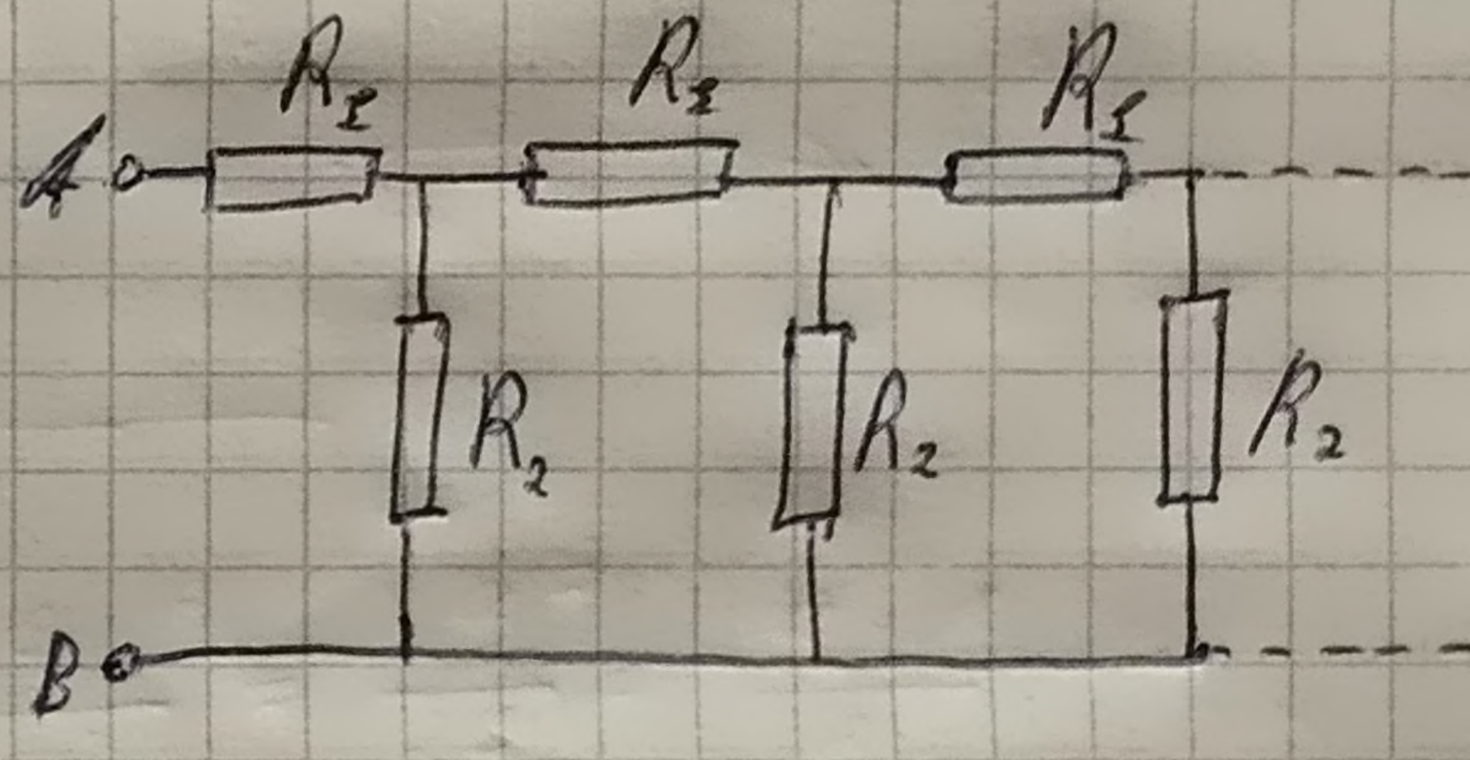
$$= \frac{q^2}{32\pi^2\varepsilon_0^2} \cdot \frac{1}{R} = \frac{q^2}{32\pi^2\varepsilon_0^2}.$$

$$W = W_1 + W_2 = \frac{3 q^2}{80\pi\varepsilon_0 R}.$$

$$\frac{W_1}{W_2} = \frac{1}{5}.$$

Ответ: $W = \frac{3 q^2}{80\pi\varepsilon_0 R}$; $\frac{W_1}{W_2} = \frac{1}{5}$.

Задача 4.5.



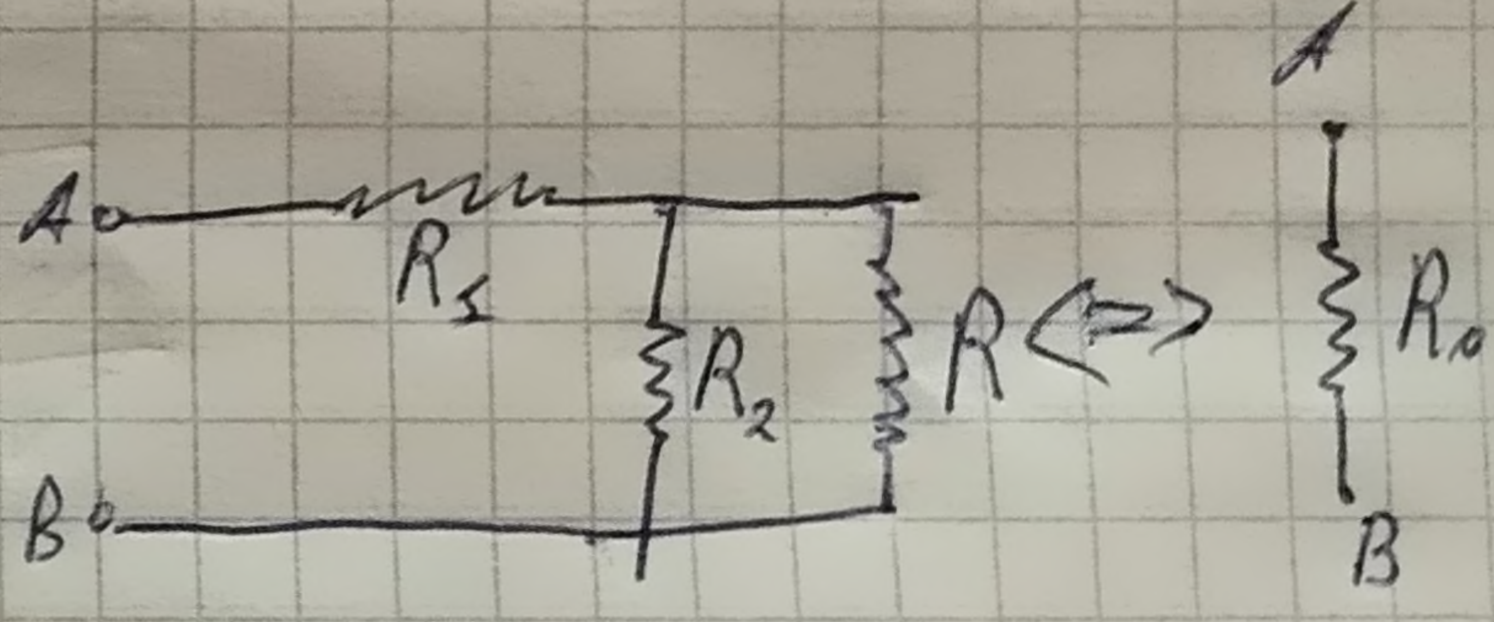
Дано: Цепь.

$R_1 = 4$ [Ом].

$R_2 = 3$ [Ом].

R_{AB} – ?

Решение:



$$R = R_1 + \frac{R_2 R}{R_2 + R} \quad \Big| \cdot (R_2 + R).$$

$$R^2 - R_1 R - R_1 R_2 = 0.$$

$$R = \frac{R_1 + \sqrt{R_1^2 + 4R_1 R_2}}{2} = \frac{R_1}{2}\left(1 + \sqrt{1 + 4\frac{R_2}{R_1}}\right).$$

Ответ: $R = 6$ [Ом]

4.5 для C.

Аналогично, только для паралл.: $C = C_1 + C_2 \ldots$

послед.: $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \ldots$

Тогда: ~~[illegible]~~

для одного звена:

$$C_{пар.} = C_2 + C;$$

и $C = C_1 + C_{пар.} \Rightarrow C = C_1 + (C_2 + C)$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2 + C} \quad | \cdot C(C_2 + C)$$

$$C^2 = CC_2 + C_1C_2 + C_1C$$

$$C^2 - (C_2 + C_1)C - C_1C_2 = 0.$$

$$D = (C_2 + C_1)^2 + 4(C_1C_2) = C_2^2 + 2C_1C_2 + C_1^2 + 4C_1C_2 = C_1^2 + 6C_1C_2 + C_2^2.$$

Ответ: $C = \frac{(C_1 + C_2) + \sqrt{C_1^2 + 6C_1C_2 + C_2^2}}{2}.$