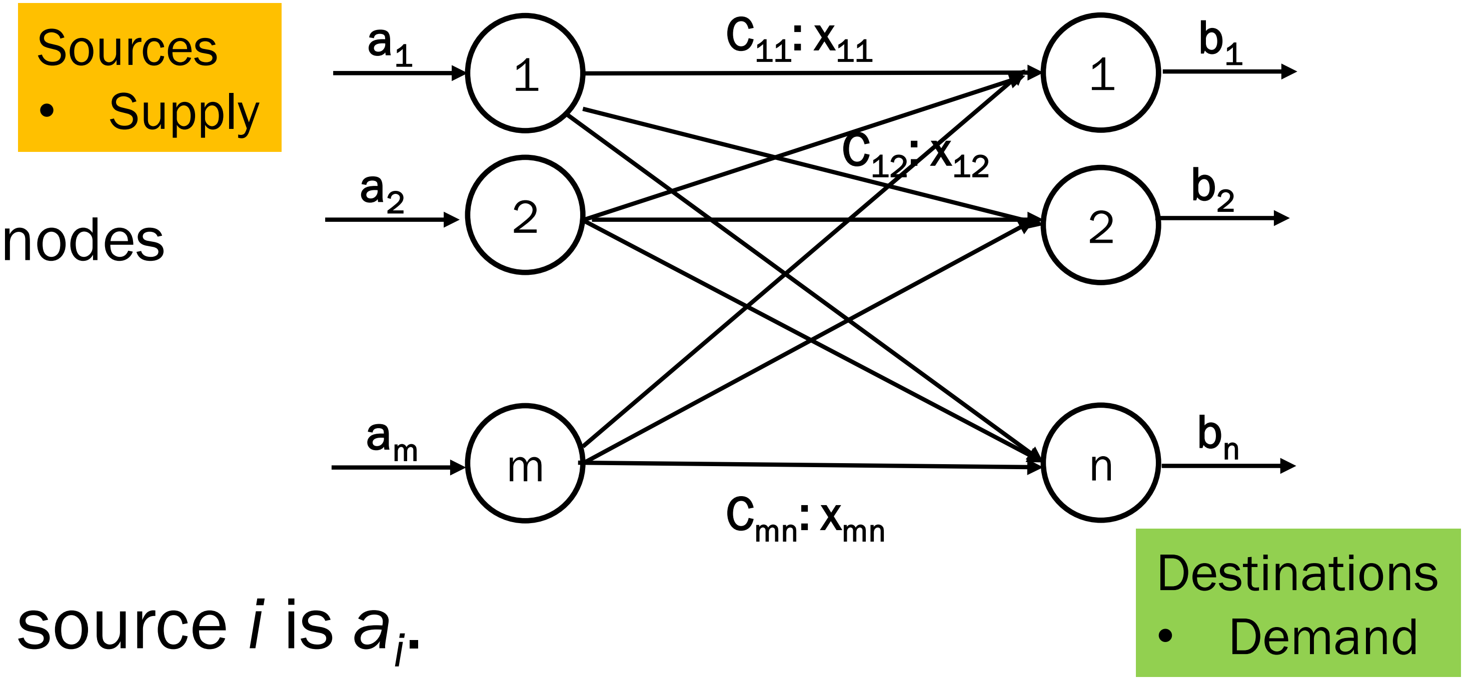
# Lectures 14, 15 and 16: Transportation Model and Its Variants

# 1. Transportation Model

- A special class of linear programs that deals with shipping a commodity from source (e.g., factories) to destinations (e.g., warehouses).
- The objective is to minimize the total shipping cost while satisfying supply and demand limits.
- More applications like **inventory control, employment scheduling, personnel assignment** etc.
- Steps of transportation model are precisely those of the simplex method.

# Definition of The Transportation model

- Sources: m
- Destinations: n
  - Both are represented by nodes
- Arc (i, j)
  - Transportation cost, $c_{ij}$,
  - Amount shipped, $x_{ij}$.
- The amount of supply at source *i* is $a_i$.
- The amount of demand at destination *j* is $b_j$.
- The objective of the model is to determine the unknowns that will minimize the transportation cost while satisfying all the supply and demand restrictions.



# Example

MG Auto has three plants in Los Angeles, Detroit, and New Orleans, and two major distribution centers in Denver and Miami. The capacities of the three plants during the next quarter are 1000, 1500, and 1200 cars. The quarterly demands at the two distribution centers are 2300 and 1400 cars. The mileage chart between the plants and the distribution centers is given in Table 5.1.

The trucking company in charge of transporting the cars charges 8 cents per mile per car. The transportation costs per car on the different routes, rounded to the closest dollar, are given in Table 5.2.

**TABLE 5.1** Mileage Chart

| | Denver | Miami |
|---|---|---|
| Los Angeles | 1000 | 2690 |
| Detroit | 1250 | 1350 |
| New Orleans | 1275 | 850 |

**TABLE 5.2** Transportation Cost per Car

| | Denver (1) | | Miami (2) | | Supply |
|---|---|---|---|---|---|
| Los Angeles (1) | \$80 | $x_{11}$ | \$215 | $x_{12}$ | 1000 |
| Detroit (2) | \$100 | $x_{21}$ | \$108 | $x_{22}$ | 1500 |
| New Orleans (3) | \$102 | $x_{31}$ | \$68 | $x_{32}$ | 1200 |
| Demand: | 2300 | | 1400 | | |

# LP Model

$$\text{Minimize } z = 80x_{11} + 215x_{12} + 100x_{21} + 108x_{22} + 102x_{31} + 68x_{32}$$

$$\begin{aligned}
x_{11} + x_{12} &= 1000 \quad \text{(Los Angeles)} \\
x_{21} + x_{22} &= 1500 \quad \text{(Detroit)} \\
+ x_{31} + x_{32} &= 1200 \quad \text{(New Oreleans)} \\
x_{11} + x_{21} + x_{31} &= 2300 \quad \text{(Denver)} \\
x_{12} + x_{22} + x_{32} &= 1400 \quad \text{(Miami)} \\
x_{ij} \geq 0, i = 1, 2, 3, j &= 1, 2
\end{aligned}$$

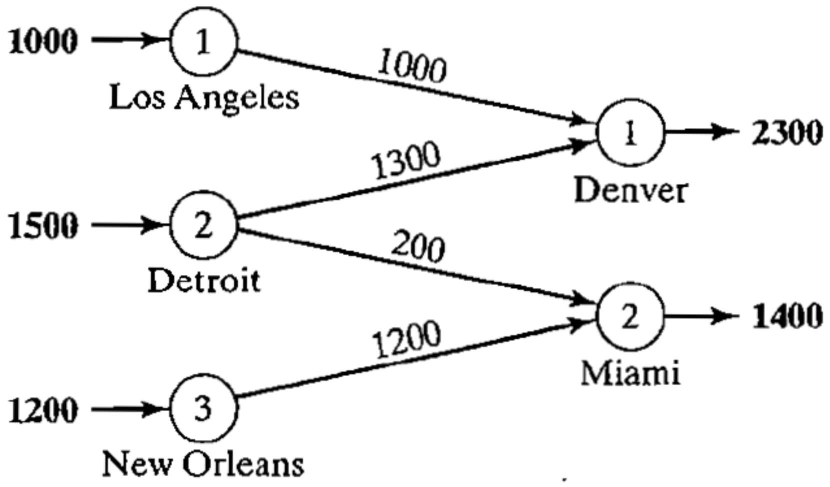
- Problem can be solved using the simplex method
- More conveniently can be solved using transportation tableau.

# Example

TABLE 5.3 MG Transportation Model

| | Denver | Miami | Supply |
|---|---|---|---|
| Los Angeles | 80<br>$x_{11}$ | 215<br>$x_{12}$ | **1000** |
| Detroit | 100<br>$x_{21}$ | 108<br>$x_{22}$ | **1500** |
| New Orleans | 102<br>$x_{31}$ | 68<br>$x_{32}$ | **1200** |
| Demand | **2300** | **1400** | |

- The optimal solution using simplex method.
  - Min. transportation cost is $313,200



# Balancing the Transportation Model

- The transportation algorithm is based on the assumption that the model is balanced.
  - Total supply = total demand
- If model is unbalanced, we can add dummy source or dummy destination

# Example

- In the MG model, suppose that the Detroit's plant capacity is 1300 cars (instead of 1500).
  - Total supply is 3500 cars and total demand is 3700 cars
  - Unbalanced model
- Balanced model with the optimal solution

TABLE 5.4 MG Model with Dummy Plant

| | Denver | Miami | Supply |
|---|---|---|---|
| Los Angeles | 80<br>**1000** | 215 | **1000** |
| Detroit | 100<br>**1300** | 108 | **1300** |
| New Orleans | 102 | 68<br>**1200** | **1200** |
| Dummy Plant | 0 | 0<br>**200** | **200** |
| Demand | **2300** | **1400** | |

**Demand at Denver is now 1900 cars only.**

TABLE 5.5 MG Model with Dummy Destination

| | Denver | Miami | Dummy | |
|---|---|---|---|---|
| Los Angeles | 80<br>**1000** | 215 | 0 | **1000** |
| Detroit | 100<br>**900** | 108<br>**200** | 0<br>**400** | **1500** |
| New Orleans | 102 | 68<br>**1200** | 0 | **1200** |
| Demand | **1900** | **1400** | **400** | |

# 2. Nontraditional Transportation Models

- Production-Inventory Control

Boralis manufactures backpacks for serious hikers. The demand for its product occurs during March to June of each year. Boralis estimates the demand for the four months to be 100, 200, 180, and 300 units, respectively. The company uses part-time labor to manufacture the backpacks and, accordingly, its production capacity varies monthly. It is estimated that Boralis can produce 50, 180, 280, and 270 units in March through June. Because the production capacity and demand for the different months do not match, a current month's demand may be satisfied in one of three ways.

1. Current month's production.
2. Surplus production in an earlier month.
3. Surplus production in a later month (backordering).

In the first case, the production cost per backpack is $40. The second case incurs an additional holding cost of $.50 per backpack per month. In the third case, an additional penalty cost of $2.00 per backpack is incurred for each month delay. Boralis wishes to determine the optimal production schedule for the four months.

# Production-Inventory Control

| Transportation | Production-inventory |
| --- | --- |
| 1. Source $i$ | 1. Production period $i$ |
| 2. Destination $j$ | 2. Demand period $j$ |
| 3. Supply amount at source $i$ | 3. Production capacity of period $i$ |
| 4. Demand at destination $j$ | 4. Demand for period $j$ |
| 5. Unit transportation cost from source $i$ to destination $j$ | 5. Unit cost (production + inventory + penalty) in period $i$ for period $j$ |

- Transportation cost from period *i* to period *j*:

$$c_{ij} = \begin{cases} \text{Production cost in } i, i = j \\ \text{Production cost in } i + \text{holding cost from } i \text{ to } j, i < j \\ \text{Production cost in } i + \text{penaty cost from } i \text{ to } j, i > j \end{cases}$$

- Example

$$c_{11} = \$40.00$$

$$c_{24} = \$40.00 + (\$.50 + \$.50) = \$41.00$$

$$c_{41} = \$40.00 + (\$2.00 + \$2.00 + \$2.00) = \$46.00$$

# Production-Inventory Control

TABLE 5.12 Transportation Model for Example 5.2-1

| | 1 | 2 | 3 | 4 | Capacity |
|---|---|---|---|---|---|
| 1 | \$40.00 | \$40.50 | \$41.00 | \$41.50 | 50 |
| 2 | \$42.00 | \$40.00 | \$40.50 | \$41.00 | 180 |
| 3 | \$44.00 | \$42.00 | \$40.00 | \$40.50 | 280 |
| 4 | \$46.00 | \$44.00 | \$42.00 | \$40.00 | 270 |
| Demand | 100 | 200 | 180 | 300 | |

# Production-Inventory Control

- The optimal solution is $31, 455.