Forecasting

- A forecast is a prediction of what will occur in the future.
- The process of analyzing current and historical data to determine future trends.
- Forecasting is the art of specifying meaningful information about the future.
- Examples: who uses forecasting in their jobs ?
  - Meteorologists forecast the weather
  - Gamblers predict the winners of football games
  - Managers of business firms attempt to predict how much of their product will be desired in the future
- A forecast of product demand is the basis for most important management planning decisions.
  - scheduling, inventory, process, facility layout and design, work force, material purchasing, etc.

Forecasting

Management generally hopes to forecast demand with as much as **accuracy** as possible, which is becoming increasingly difficult to do.

Characteristics of forecasts:

- They are almost always going to be wrong
- A good forecast also gives some measure of error
- Forecasting aggregate units is generally easier that forecasting individual units.
- A forecasting technique should not be used to the exclusion of known information.

Forecasting Horizons

Short range:

- Time frame from one day to three months.
- Used for day to day production: scheduling, inventory planning, workforce planning, etc.

Medium-range Forecast:

- Time frame from three months to three years.
- Used for production and layout planning, sales and marketing planning, cash budget planning and capital budget planning.

Long range Forecast:

- Time frame of more than three years.
- Used for strategic planning in terms of capacity planning, new product planning, expansion planning, etc.

Forecasting Methods

Qualitative method : It is based on human judgment. Qualitative method is used if historical data are not available or not relevant for making forecasts. This method is used for long range forecasting, like strategic planning, development of new product.

Causal forecasting method: It uses explanatory variables to predict the future.

Time series method or analysis : are statistical techniques that make use of historical data.

Qualitative method:

a) Sales force composites
b) Customer surveys or market research
c) The Delphi method

Qualitative methods

Sales force composites: In forecasting product demand, a good source of subjective information is the company sales force or customer service area. The sale force has direct contact with consumers and is therefore in a good position to see changes in their preferences. Sales managers would then be responsible for aggregating individual estimates to arrive at overall forecast for each geographical region or product group.

Customer surveys or market research: is an organized approach using surveys to determine what product or services customers want and will purchase. It is normally conducted by the marketing department or private marketing firms. Although it can provide accurate and useful forecasts of product demand, it must be skillfully and correctly conducted and it can be expensive. The survey can be conducted through mailings, telephone or personal interviews and a sufficient response must be obtained. Designed of questionnaires, numbers of responses and sampling plans or scheme should be accurate.

The Delphi method: It is especially useful for forecasting technological change and advances. It is a structured communication technique or method relies on a panel of experts. In this method, expert opinions are collected for preparing forecasts.

Causal forecasting method

It uses explanatory variables to predict the future. It employs the cause-effect relationship between the product of interest (demand) and the factors affecting it.

For example: let say that the demand of TV of a locality is proportional to the number of housing sales in that region. Then what we can do, we can make a relation between the demand of TV and number of newly constructed House.

$$D(TV) = \alpha_0 + \alpha_1 N$$

Or we can make it more complicated like

$$D(Car) = \alpha_0 + \alpha_1 N + \alpha_2 i$$

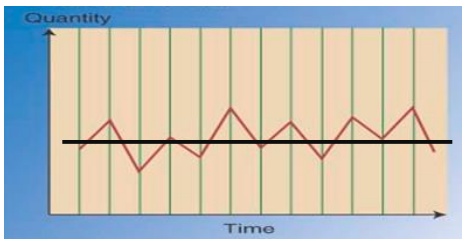
This technique is used for forecasting economic phenomena like gross national product (GNP), gross domestic product (GDP).

Time series methods

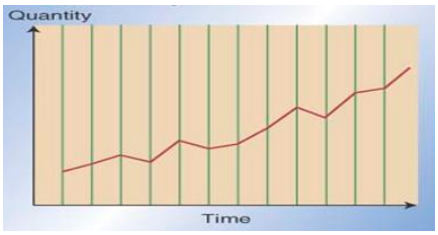
- Time series methods are statistical techniques that make use of historical data (*i.e.* Historical actual demand data).
- These methods assume that what has occurred in the past will continue to occur in the future.

- Moving average; Exponential smoothing; Trend analysis; Seasonality

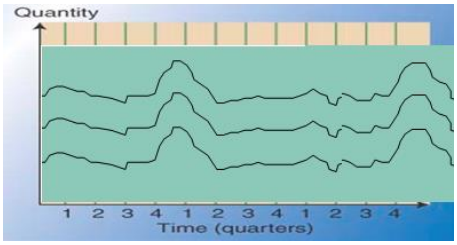
Demand Behavior



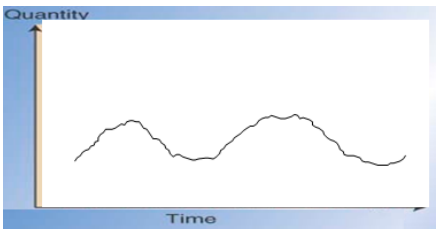
Horizontal pattern



Trend Pattern: A trend is a gradual long-term up or down movement of demand



Seasonal Pattern: Short-term regular variations in data



Cycle: wavelike variations of more than one year's duration

Time Series Models

Simple moving average: in this method forecast is obtained by averaging the data points over a desired number of past periods.

$$F_t = \frac{\sum_{j=1}^{n} D_{t-j}}{n}$$

Where, D_j is the demand during period j

i.e, for N =3; $F_t = \frac{D_{t-1} + D_{t-2} + D_{t-3}}{3}$

Month	Demand (D_t)	3-period moving average (Ft)	Six- Month Moving average (Ft)
January	450	--	-
February	440	--	-
March	460	--	-
April	510	450	-
May	520	470	-
June	495	496.7	-
July	475	508.3	479.2
August	560	496.7	483.3
September	510	510	503.3
October	520	515	511.7
November	540	530	513.3
December	550	523.3	516.7
January, 2021	???	537.7	525.8

Time Series Models

Weighted Moving Average : in this method, higher weight is assigned to the most recent data.

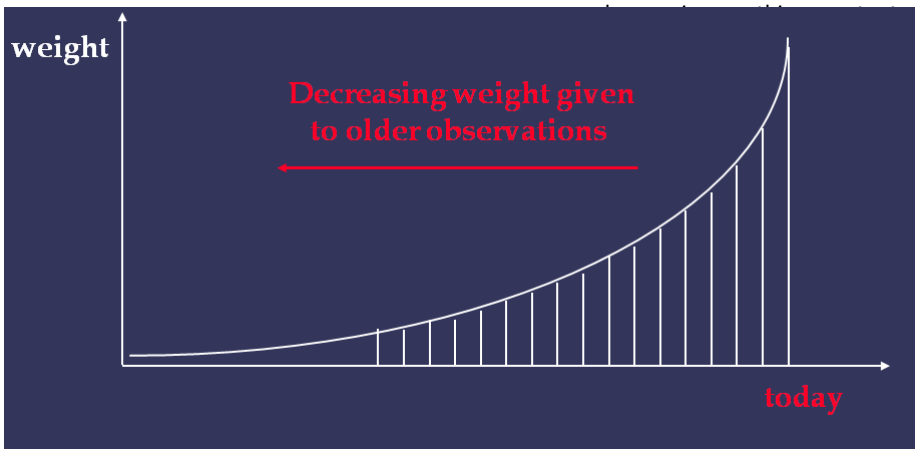
$$F_t = \sum_{t=1}^{n} C_t D_t \qquad \text{Where,} \quad \sum_{t=1}^{n} C_t = 1$$

Month	Demand (D_t)	3-month Weighted Moving average (Ft)
January	450	--
February	440	--
March	460	--
April	510	452.5
May	520	480
June	495	502.5
July	475	505
August	560	491.3
September	510	522.5
October	520	513.8
November	540	527.5
December	550	527.5
January, 2021	???	540

$$F_{April} = 0.5 D_{March} + 0.25 D_{Feb} + 0.25 D_{Jan}$$

Time Series Models

Exponential Smoothing: $F_{t+1} = \alpha D_t + (1-\alpha)F_t$



Time Series Models

Exponential Smoothing: $F_{t+1} = \alpha D_t + (1-\alpha)F_t$

Month	Demand (D_t)	Forecasted value(Ft)
January	450	Equal to the demand of this period
February	440	450
March	460	442
April	510	457.6
May	520	498.1
June	495	508.5
July	475	490.8
August	560	475.6
September	510	543.5
October	520	506.8
November	540	515.4
December	550	532.3
January, 2021	???	541.9

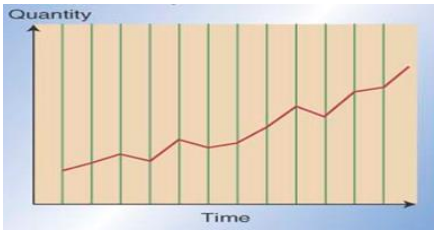
$F_{Feb} = 0.8D_{Jan} + (1-0.8)F_{Jan}$

Time Series Models

Holt trend model or Double Exponential Smoothing :

$$F_{t+1} = \alpha D_t + (1-\alpha)(F_t + T_t)$$
$$T_t = \beta^*(F_t - F_{t-1}) + (1-\beta)T_{t-1}$$
where $0<\beta<1$,

where T_t is called trend.



Trend Pattern: A trend is a gradual long-term up or down movement of demand

Initial Values:

$F_1 = D_1$

$T_1 = D_2 - D_1$ Method-1

$T_1 = [(D_2 - D_1) + (D_3 - D_2) + (D_4 - D_3)]/3$ Method-2

$T_1 = (D_n - D_1)/(n - 1)$

Time Series Models

Holt trend model or Double Exponential Smoothing :

For α =0.2 and β =0.2

$$F_{t+1} = \alpha D_t + (1-\alpha)(F_t + T_t)$$
$$T_t = \beta^*(F_t - F_{t-1}) + (1-\beta)T_{t-1}$$

Month	Demand (D_t)
January	450
February	440
March	460
April	510
May	520
June	495
July	475
August	560
September	510
October	520
November	540
December	550
January, 2021	?

Time Series Models

Holt trend model or Double Exponential Smoothing :

For α =0.2 and β =0.2

$F_{t+1} = \alpha D_t + (1-\alpha)(F_t + T_t)$

$T_t = \beta^*(F_t - F_{t-1}) + (1-\beta)T_{t-1}$

Month	Demand (D_t)	Forecast (F_t)	Trend (T_t)
January	450	450	9.1
February	440		
March	460		
April	510		
May	520		
June	495		
July	475		
August	560		
September	510		
October	520		
November	540		
December	550		
January, 2021	?		

$F_1 = D_1$

$T_1 = (D_{12} - D_1)/(12-1) = (550-540)/11$
$= 9.09090 \approx 9.1$ (Method-3)

$F_2 = 0.2 \times 450 + 0.8 \times (450+9) = 457.3$

$T_2 = 0.2 \times (457.2 - 450) + 0.8 \times 9 = 8.7$

Time Series Models

Trend Projection by Linear Regression: In this method, a linear trend line relates demand with time. Y=a+bX where, X is time and Y is forecasted value for period X

$$b = \frac{\sum XY - n\overline{X}\,\overline{Y}}{\sum X^2 - n\overline{X}^2} \qquad a = \overline{Y} - b\overline{X}$$

Weeks (X)	Sales (Y)
1	150
2	157
3	162
4	166
5	177
6	??

a= 143.5
b=6.3

Y(6)=143.5+6.3×6= 181.3

Time Series Models

Trend Projection by Linear Regression: In this method, a linear trend line relates demand with time. Y=a+bX where, X is time and Y is forecasted value for period X

$$b = \frac{\sum XY - n\overline{X}\,\overline{Y}}{\sum X^2 - n\overline{X}^2} \qquad a = \overline{Y} - b\overline{X}$$

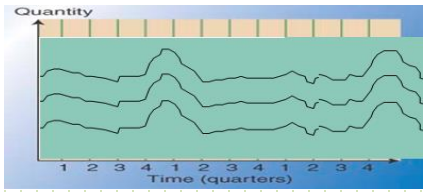
Week (X)	Sales (Y)	Forecast value (F_t)
1	150	150
2	157	156
3	162	162
4	166	169
5	177	175
6	???	181

a= 143.5
b=6.3

Y(6)=143.5+6.3×6= 181.3

Time Series Models

Forecasting Seasonality:

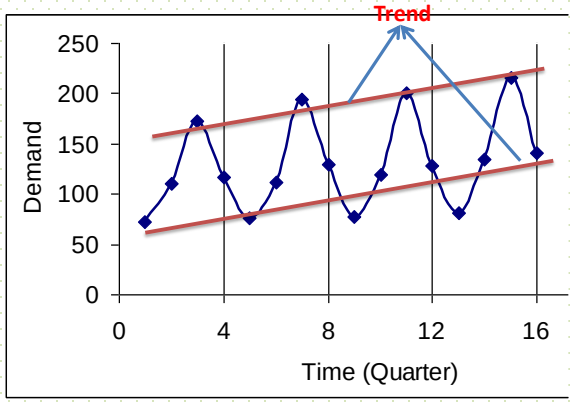


Seasonal Pattern: Short-term regular variations in data

- Calculate the average demand per season
  - *E.g.:* average quarterly demand
- Calculate a seasonal index for each season of each year:
  - Divide the actual demand of each season by the average demand per season for that year
- Average the indexes by season
  - *E.g.*: take the average of all Spring indexes, then of all Summer indexes, ...

Time Series Models

Forecasting Seasonality:



Year	Quarter	Demand (Ft)
1	1	72
	2	110
	3	172
	4	117
2	1	76
	2	112
	3	194
	4	130
3	1	78
	2	119
	3	201
	4	128
4	1	81
	2	134
	3	216
	4	141

Time Series Models

1. Forecast trend pattern and seasonality pattern separately.
2. Combine the forecasts

Trend forecast: Decompose the past data (filter out the seasonal influence from original data) using averaging.

Year	Average yearly demand
1	117.75
2	128.00
3	131.50
4	143.00

Year	Trend Forecast
1	118.175
2	126.100
3	134.025
4	141.950
5	149.875

$y = a + b*x$

$a = 110.25$; $b = 7.925$

Time Series Models

Forecasting Seasonality:

Seasonal index

Quarter \ Year	1	2	3	4
Q1	72	76	78	81
Q2	110	112	119	134
Q3	172	194	201	216
Q4	117	130	128	141

Time Series Models

Forecasting Seasonality:

Seasonal index

Quarter \ Year	1	2	3	4	Average	Seasonal Index
Q1	72	76	78	81	76.75	0.590101
Q2	110	112	119	134	118.75	0.913023
Q3	172	194	201	216	195.75	1.505046
Q4	117	130	128	141	129	0.991831
				Total average	130.0625	

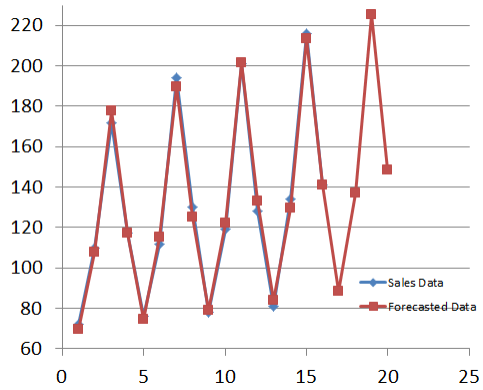
$$Seasonal\ Index\ (SI_i) = \frac{Avg.\ \ Q_i}{Total\ Avg.}$$

Forecasting of demand patterns with trend and seasonality components

Year	Quarter	Trend Forecast	Seasonal Index	Combine forecast
1	1	118.1751	0.590101	**69.7352**
	2		0.913023	**107.8965**
	3		1.505046	**177.8589**
	4		0.991831	**117.2097**
2	1	126.1001	0.590101	**74.4118**
	2		0.913023	**115.1322**
	3		1.505046	**189.7860**
	4		0.991831	**125.0700**
3	1	134.0251	0.590101	**79.0883**
	2		0.913023	**122.3679**
	3		1.505046	**201.7139**
	4		0.991831	**132.9302**
4	1	141.9501	0.590101	**83.76488**
	2		0.913023	**129.6036**
	3		1.505046	**213.6414**
	4		0.991831	**140.7905**

Forecasting of demand patterns with trend and seasonality components

Year	Quarter	Trend Forecast	Seasonal Index	Combine forecast
5	1	149.875	0.590101	**88.44137**
	2		0.913023	**136.8393**
	3		1.505046	**225.5687**
	4		0.991831	**148.6506**



Forecasting Performance

How good is the forecast?

- Mean Absolute Deviation (MAD)
  - measures the total error in a forecast without regard to sign

$$\mathbf{MAD} = \frac{\sum |\mathbf{actual} - \mathbf{forecast}|}{\mathbf{n}}$$

- Cumulative Forecast Error (CFE)
  - Measures any bias in the forecast

$$\mathbf{CFE} = \sum (\mathbf{actual} - \mathbf{forecast})$$

- Mean Square Error (MSE)
  - Penalizes larger errors

$$\mathbf{MSE} = \frac{\sum (\mathbf{actual} - \mathbf{forecast})^2}{\mathbf{n}}$$

- Tracking Signal
  - Measures if your model is working

$$\mathbf{TS} = \frac{\mathbf{CFE}}{\mathbf{MAD}}$$