4 Iterative Computation of Transportation Algorithm

- Determine the starting basic feasible solution
  - Northwest corner method
  - Least cost method
  - Vogel's approximation method
- **Step 1** : Use **primal simplex optimality condition** to determine the entering variable as the current nonbasic variable that can improve the solution. If the optimality condition is satisfied, stop. Otherwise, go to step 2.
- **Step 2**: Determine leaving variable using **primal simplex feasibility condition.** Change the basis, and return to step 1.

Transportation Algorithm: Entering Variable (It#1)

		V1=10	V2=2	V3=4	V4=15	
		1	2	3	4	Supply
U1=0	1	10 5	2 10	20 -16	11 4	15
U2=5	2	12 3	7 5	9 15	20 5	25
U3=3	3	4 9	14 -9	16 -9	18 10	10
	Demand	5	15	15	15	

Cost is $520

Entering variable is x_{31}

- **Determine the entering variable.**
- For basic variables, $U_i + V_j = c_{ij}$. Let's assign U1 = 0 arbitrarily, and find values of other U_i and V_j. For x_{11}, U1+V1 = 10 → V1 = 10. For x_{12}, U1 + V2 = 2→ V2 = 2. For x_{22}, U2 + V2 = 7 → U2 = 5. For x_{23}, U2 + V3 = 9 → V3 = 4. For x_{24}, U2 + V4 = 20 → V4 = 15. For x_{34}, U3 + V4 = 18 → U3 = 3.
- Now calculate $U_i + V_j - c_{ij}$ for all nonbasic variables. For minimization problem, find most positive value.

Transportation Algorithm: Leaving Variable (It#1)

		V1=10	V2=2	V3=4	V4=15	
		1	2	3	4	Supply
U1=0	1	10 5	2 10	20	11	15
U2=5	2	12	7 5	9 15	20 5	25
U3=3	3	4 θ	14	16	18 10	10
	Demand	5	15	15	15	

Entering variable is x_{31}

θ: maximum amount to be shipped. Two conditions

1. Supply limits and demand requirements remain satisfied.
2. Shipment through all routes remain nonnegative.

Transportation Algorithm: Leaving Variable (It#1)

		V1=10	V2=2	V3=4	V4=15	
		1	2	3	4	Supply
U1=0	1	10 $5 - \Theta$	2 $10 + \Theta$	20	11	15
U2=5	2	12	7 $5 - \Theta$	9 15	20 $5 + \Theta$	25
U3=3	3	4 Θ	14	16	18 $10 - \Theta$	10
	Demand	5	15	15	15	

Entering variable is x_{31}

Θ: maximum amount to be shipped.

1. Construct a close loop that starts and end at the entering variable. Except for the entering variable, each corner of the closed loop must coincide with a basic variable.
2. The loop consists of only horizontal and vertical segments only (no diagonals are allowed.)

3. Satisfy nonnegativity condition. Maximum Θ is 5 (tie) and leaving variable is x_{11} or x_{22}. Choose arbitrarily.

Transportation Algorithm: Entering and leaving variables (It#2)

1. For basic variables,
$U_i + V_j = c_{ij}$.
Start with U1 = 0

		V1=1	V2=2	V3=4	V4=15	
		1	2	3	4	Supply
U1=0	1	10 0 -9	2 15 - Θ	20 -16	11 Θ 4	15
U2=5	2	12 -6	7 0 + Θ	9 15	20 10 - Θ	25
U3=3	3	4 5	14 -9	16 -9	18 5	10
	Demand	5	15	15	15	

Entering variable is x_{14}

2. Calculate $U_i + V_j - c_{ij}$ for all nonbasic variables. For minimization problem, find most positive value.

3. Determine leaving variable. **Θ:** maximum amount to be shipped to entering variable.

4. Satisfy nonnegativity condition. Maximum **Θ** is 10 and leaving variable is x_{24}.

Cost is \$520 - \$9X5 = \$475

Transportation Algorithm: Entering and leaving variables (It#3)

1. For basic variables,
   $U_i + V_j = c_{ij}$.
   Start with U1 = 0

		V1=-3	V2=2	V3=4	V4=11	
		1	2	3	4	Supply
U1=0	1	10 -13	2 5	20 -16	11 10	15
U2=5	2	12 -10	7 10	9 15	20 0 -4	25
U3=7	3	4 5	14 -5	16 -5	18 5	10
	Demand	5	15	15	15	

2. Calculate $U_i + V_j - c_{ij}$ for all nonbasic variables. For minimization problem, find most positive value.

3. No positive value. Stop.

Cost is $435

5 Assignment Model

- "Best person for the job"
- Assignment of workers with varying degree of skill to jobs.
- Objective of the model is to minimum-cost assignment of workers to jobs.

	Jobs					
Workers		1	2	...	n	
	1	c_{11}	c_{12}		c_{1n}	1
	2	c_{21}	c_{22}	...	c_{2n}	1
	. . .	. . .	. . .	. . .	. . .	. . .
	n	c_{n1}	c_{n2}	...	c_{nn}	1
		1	1	1	1	

- Cij: cost of assigning worker *i* to job *j*.

Assignment Model

- Assignment model is a special case of the transportation model.
  - Workers → sources
  - Jobs → destinations
  - The supply (demand) amount at each source (destination) exactly equals 1.
  - Cost of "transporting" worker *i* to job *j* is c_{ij}.
- Simple model as supply and demand amounts equal to 1.
- Hungarian Method

Hungarian Method

Joe Klyne's three children, John, Karen, and Terri, want to earn some money to take care of personal expenses during a school trip to the local zoo. Mr. Klyne has chosen three chores for his children: mowing the lawn, painting the garage door, and washing the family cars. To avoid anticipated sibling competition, he asks them to submit (secret) bids for what they feel is fair pay for each of the three chores. The understanding is that all three children will abide by their father's decision as to who gets which chore. Table 5.32 summarizes the bids received. Based on this information, how should Mr. Klyne assign the chores?

TABLE 5.32 Klyne's Assignment Problem

	Mow	Paint	Wash
John	\$15	\$10	\$9
Karen	\$9	\$15	\$10
Terri	\$10	\$12	\$8

- Solve this problem using Hungarian method

Hungarian Method

- **Step 1.** For the original cost matrix, identify each row's minimum, and subtract it from all the entries of the row.
- **Step 2.** For the matrix resulting from step 1, identify each column's minimum, and subtract it from all the entries of the column.
- **Step 3.** Indentify the optimal solution as the feasible assignment associated with the zero elements of the matrix obtained in step 2.

Example: Hungarian method

Step 1 of Hungarian method			
	Mow	Paint	Wash
John	15	10	9
Karen	9	15	10
Terri	10	12	8

Step 1. For the original cost matrix, identify each row's minimum, and subtract it from all the entries of the row.

Example: Hungarian method

Step 1 of Hungarian method				
	Mow	Paint	Wash	Row minimum
John	15	10	9	P1 = 9
Karen	9	15	10	P2 = 9
Terri	10	12	8	P3 = 8

Row1. 15-9 = 6; 10-9=1; 9-9=0
Row2. 9-9 = 0; 15-9=6; 10-9=1
Row3. 10-8 = 2; 12-8=4; 8-8=0

Step 1 of Hungarian method				
	Mow	Paint	Wash	Row minimum
John	6	1	0	P1 = 9
Karen	0	6	1	P2 = 9
Terri	2	4	0	P3 = 8

Step 2. For the matrix resulting from step 1, identify each column's minimum, and subtract it from all the entries of the column.

Example: Hungarian method

Step 2 of Hungarian method				
	Mow	Paint	Wash	Row minimum
John	6	1	0	P1 = 9
Karen	0	6	1	P2 = 9
Terri	2	4	0	P3 = 8
Column minimum	Q1 = 0	Q2 = 1	Q3 = 0	

Subtract minimum column from all the entries of the column

Step 2 of Hungarian method				
	Mow	Paint	Wash	Row minimum
John	6	0	0	P1 = 9
Karen	0	5	1	P2 = 9
Terri	2	3	0	P3 = 8
Column minimum	Q1 = 0	Q2 = 1	Q3 = 0	

Example: Hungarian method

Step 3 of Hungarian method				
	Mow	Paint	Wash	Row minimum
John	6	0	0	P1 = 9
Karen	0	5	1	P2 = 9
Terri	2	3	0	P3 = 8
Column minimum	Q1 = 0	Q2 = 1	Q3 = 0	

- Cell with **0** is the optimal solution.
- John will get painting
- Karen will get Mowing
- Terri will get washing

TABLE 5.32 Klyne's Assignment Problem

	Mow	Paint	Wash
John	\$15	\$10	\$9
Karen	\$9	\$15	\$10
Terri	\$10	\$12	\$8

- Total cost is 9 + 10 + 8 = \$27 =(P1+P2+P3+Q1+Q2+Q3)

Example 2: Hungarian method

- Extending the previous problem in four children and four chores.

		Chores				
		1	2	3	4	Row minimum
Child	1	$1	$4	$6	$3	
	2	$9	$7	$10	$9	
	3	$4	$5	$11	$7	
	4	$8	$7	$8	$5	
Column minimum						

Example 2: Hungarian method

Chores						
Child		1	2	3	4	Row minimum
	1	0	3	5	2	P1=1
	2	2	0	3	2	P2=7
	3	0	1	7	3	P3=4
	4	3	2	3	0	P4=5
Column minimum						

Example 2: Hungarian method

Chores						
Child		1	2	3	4	Row minimum
	1	0	3	2	2	P1=1
	2	2	0	0	2	P2=7
	3	0	1	4	3	P3=4
	4	3	2	0	0	P4=5
Column minimum		Q1=0	Q2=0	Q3=3	Q4=0	

- If we assign Chore 1 to child 1, then child 3 has no entry zero.

Example 2: Hungarian method

Step 2a: (i) Draw minimum number of horizontal and vertical lines in the last reduced matrix that will cover all the zero entries.

		1	2	3		Row minimum
Child	1	0	3	2	2	
	2	2	0	0	2	
	3	0	1	4	3	
	4	3	2	0	0	
Column minimum						

Example 2: Hungarian method

Step 2a: (ii) Select smallest undiscovered entry, subtract it from uncovered entry, then add it to every entry at the intersection of two lines.

Child		1	2	3		Row minimum
	1	0	2	1	1	P1=1
	2	3	0	0	2	P2=7
	3	0	0	3	2	P3=4
	4	4	2	0	0	P4=5
Column minimum		Q1=0	Q2=0	Q3=3	Q4=0	

Example 2: Hungarian method

Step 2a: (iii) If no feasible assignment can be found among the resulting zeros, repeat step 2a. Otherwise, go to step 3 to determine the optimal assignment.

		1	2	3		Row minimum
Child	1	0	2	1	1	P1=1
	2	3	0	0	2	P2=7
	3	0	0	3	2	P3=4
	4	4	2	0	0	P4=5
Column minimum		Q1=0	Q2=0	Q3=3	Q4=0	

Optimal cost is 1+10+5+5 =21