Lectures 14, 15 and 16: Transportation Model and Its Variants

3. The Transportation Algorithm

SunRay Transport Company ships truckloads of grain from three silos to four mills. The supply (in truckloads) and the demand (also in truckloads) together with the unit transportation costs per truckload on the different routes are summarized in the transportation model in Table . The unit transportation costs, c_{ij}, (shown in the northeast corner of each box) are in hundreds of dollars. The model seeks the minimum-cost shipping schedule x_{ij} between silo i and mill j ($i = 1, 2, 3; j = 1, 2, 3, 4$).

	Mill 1	Mill 2	Mill 3	Mill 4	Supply
Silo 1	10 x_{11}	2 x_{12}	20 x_{13}	11 x_{14}	15
Silo 2	12 x_{21}	7 x_{22}	9 x_{23}	20 x_{24}	25
Silo 3	4 x_{31}	14 x_{32}	16 x_{33}	18 x_{34}	10
Demand	5	15	15	15	

- Steps of transportation algorithm

Step 1. Determine a *starting* basic feasible solution, and go to step 2.

Step 2. Use the optimality condition of the simplex method to determine the *entering variable* from among all the nonbasic variables. If the optimality condition is satisfied, stop. Otherwise, go to step 3.

Step 3. Use the feasibility condition of the simplex method to determine the *leaving variable* from among all the current basic variables, and find the new basic solution. Return to step 2.

Determine Starting Basic Feasible Solution

- In transportation model,
  - m sources
  - n destinations
  - m+n constraints
  - Sum of supply = sum of demand
  - One constraint is redundant
  - Model has m+n-1 independent constraint equations.
- Three methods
  - Northwest-corner method
  - Least-cost method
  - Vogel approximation method

Northwest Corner Method

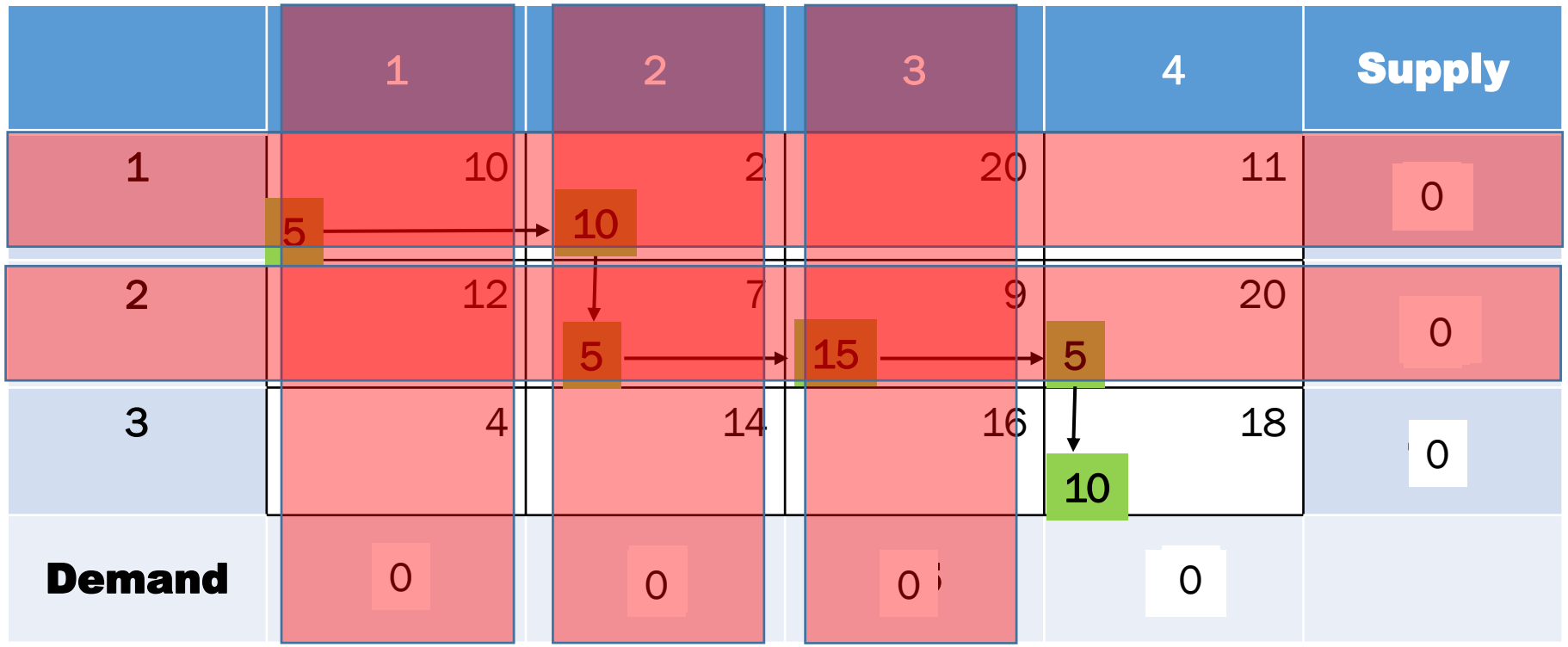
Northwest-Corner Method. The method starts at the northwest-corner cell (route) of the tableau (variable x_{11}).

Step 1. Allocate as much as possible to the selected cell, and adjust the associated amounts of supply and demand by subtracting the allocated amount.

Step 2. Cross out the row or column with zero supply or demand to indicate that no further assignments can be made in that row or column. If both a row and a column net to zero simultaneously, *cross out one only*, and leave a zero supply (demand) in the uncrossed-out row (column).

Step 3. If *exactly one* row or column is left uncrossed out, stop. Otherwise, move to the cell to the right if a column has just been crossed out or below if a row has been crossed out. Go to step 1.

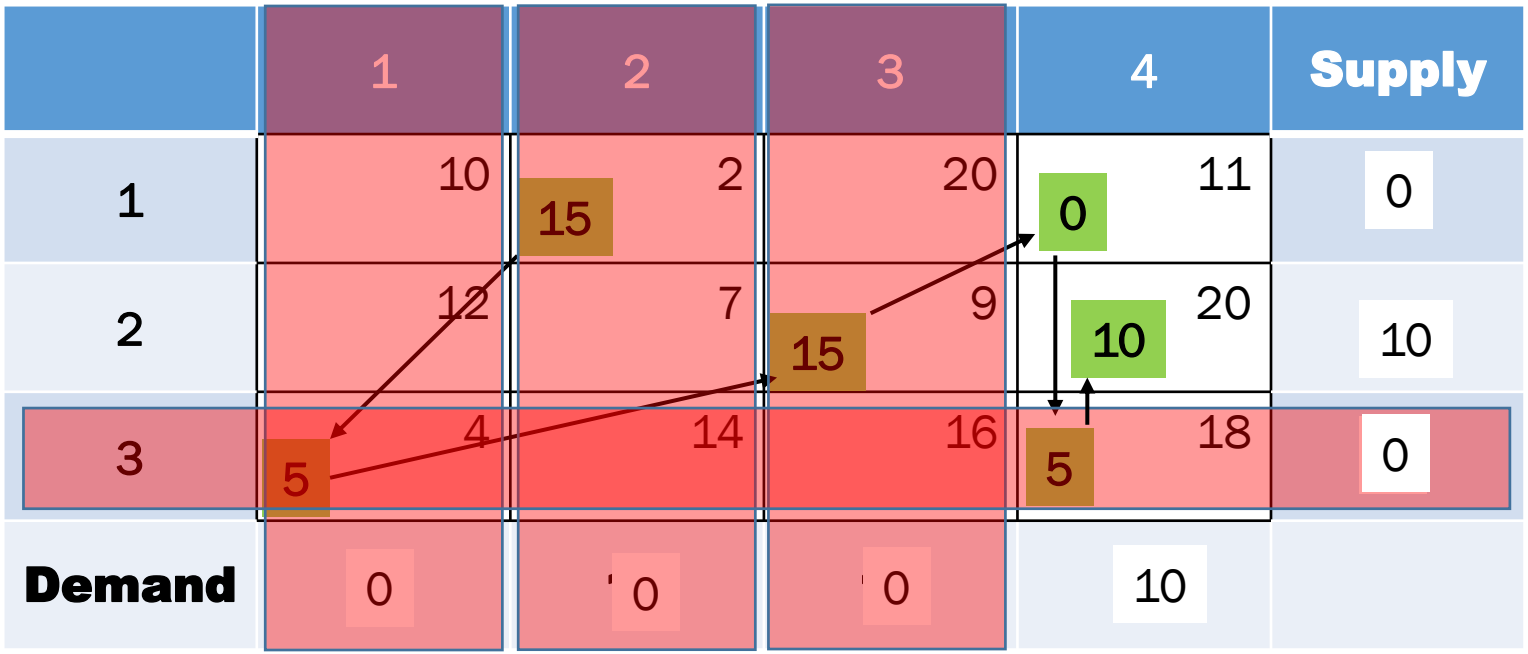
Northwest Corner Method



- Cost = (5*10) + (10*2)+(5*7)+(15*9)+(5*20)+(10*18) = $520

Least-Cost Method

- Method assign as much as possible to the cell with the smallest unit cost.
- Next, the satisfied row or column is crossed and the amounts of supply and demand are adjusted accordingly.
- Look for the uncrossed-out cell with the smallest unit cost and repeat the process.



$$z = 15 \times 2 + 0 \times 11 + 15 \times 9 + 10 \times 20 + 5 \times 4 + 5 \times 18 = \$475$$

Vogel Approximation Method

Step 1. For each row (column), determine a penalty measure by subtracting the *smallest* unit cost element in the row (column) from the *next smallest* unit cost element in the same row (column).

Step 2. Identify the row or column with the largest penalty. Break ties arbitrarily. Allocate as much as possible to the variable with the least unit cost in the selected row or column. Adjust the supply and demand, and cross out the satisfied row *or* column. If a row and a column are satisfied simultaneously, only one of the two is crossed out, and the remaining row (column) is assigned zero supply (demand).

Step 3.

(a) If exactly one row or column with zero supply or demand remains uncrossed out, stop.

(b) If one row (column) with *positive* supply (demand) remains uncrossed out, determine the basic variables in the row (column) by the least-cost method. Stop.

(c) If all the uncrossed out rows and columns have (remaining) zero supply and demand, determine the *zero* basic variables by the least-cost method. Stop.

(d) Otherwise, go to step 1.

Example-Vogel Approximation Method

	1	2	3	4	Supply	Row Penalty
1	10	2 15	20	11	0	
2	12	7	9 15	20 10	10	
3	4 5	14	16	18 5	5	
Demand	0	0	0	**15**		
Column penalty						

Cost = 15X2 + 0X11 + 15X9 + 10X20 + 5X4 + 5X18 = $475