## Basic Stats

Probability

1. Introduction to Basic Terms
2. Variables
3. Random Variables
4. Population, Sample, Population Mean, Sample Mean
5. Pop Distribution, Sample Distribution, Sampling Distribution
6. Mean, Median, Mode - Measure of Central Tendency
7. Range
8. Measure of Dispersion
9. Variance
10. Std. Deviation
11. Gaussian / Normal Distribution

## Intermediate Stats

12. Std. Normal Distribution
13. Z score
14. Prob. Density Function
15. Cumulative Distribution function
16. Hypothesis Testing
17. Many different plotting graphs
18. Kernel Density Estimation
19. Central Limit Theorem
20. Skewness of Data
21. Covariance
22. Pearson Correlation Coefficient
23. Spearman Rank Correlation
24. Hypothesis Testing

## Adv. Stats

25. Q-Q Plot
26. Chebyshev's Inequality
27. Discrete & Continuous Distribution
28. Bernoulli & Binomial Distribution
29. Log Normal Distribution
30. Power Law Distribution
31. Box Cox Transform
32. Poisson Distribution
33. Application of Non Gaussian Distribution

# V. Mean, Median, Mode – Measure of

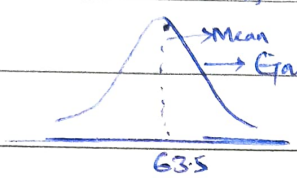
## (i) Mean :- Measure of Central Tendency.

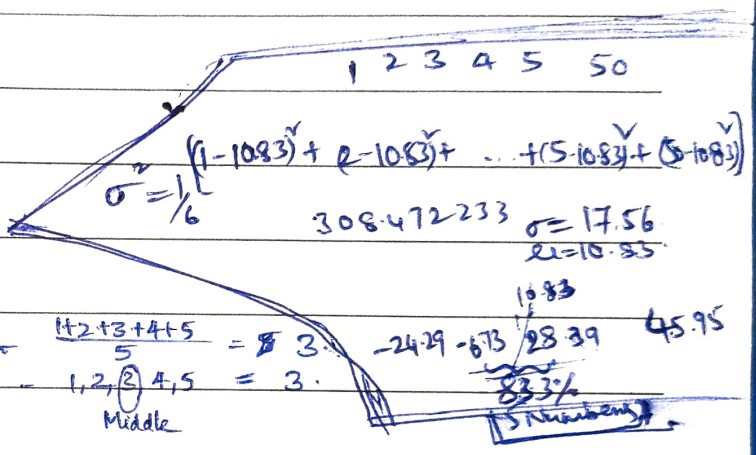
$$\mu = \sum_{i=1}^{n} [x_i] * \frac{1}{n}$$

Sample of Height = {168, 170, 150, 160, 182, 140, 175}.

Ex,

$$\mu = \frac{168 + \dots + 175}{7} = 63.5$$




## (ii) Median :-

[1, 2, 3, 4, 5] → Mean = $\frac{1+2+3+4+5}{5} = 3$

Median = 1, 2, 3, 4, 5 = 3.
(Middle)

Outlier Added [1, 2, 3, 4, 5, 50] → Mean = $\frac{1+2+3+4+5+50}{6} = 10.83$ (64%)

Median 1, 2, 3, 4, 5, 50 = $\frac{3+4}{2} = 3.5$

Write in Ascending Order to find Median.

Size :- Odd No – Middle.
Even No – $\frac{m_1 + m_2}{2}$

Age
23
24
27
32
35
21
– } Data Missing.

If I remove the missing Data. I may loose some information.
In that case, I will fill Data using Mean, Median, Mode

Mean – If no outliers are present – Prefer Mean.
Median/Mode – More Outliers are present – Prefer Median/Mode.

## (iii) Mode –

I.

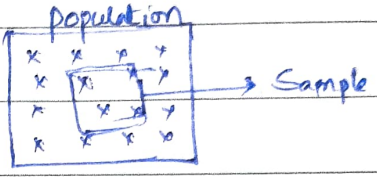
1. Population :-

Question : Average Height of all people in a state

Solution :-

$$\mu = \frac{1}{10L} \sum_{i=1}^{10L} x_i \Rightarrow \boxed{\mu = \frac{1}{N} \sum_{i=1}^{N} x_i}$$

$N \rightarrow$ Population.
$n \rightarrow$ Sample.
$\mu \rightarrow$ Population Mean
$\bar{x} \rightarrow$ Sample Mean




2. Sample :-

To get Data from all people is difficult (Population).

So, We consider Sample.

$$\bar{x} = \frac{1}{10000} \sum_{i=1}^{10000} x_i \Rightarrow \boxed{\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i}$$

$n << N$

When Sample Size increases $n \cong N$

II.

1. Random Variables :- A random Variable which stores Data. Types < Numerical R.V. / Categorical R.V.

Numerical R.V :-

(i) Discrete Variables - Whole Number.

eg:- No. of Bank Accounts a person has?

Ans:- 2, 5, 7..  Can't be 2.5 (decimals)

(ii) Continuous Random Variables :- Within a range, any Value.

10.1, 10.2, 10.368....

Depends on Scaling eg:- Weight of a Person. 63Kg. / 63.2 Kg. / 63.269 Kg. (depends on scaling)
Interest Rate
Salary.

Categorical Variable :- for each record it is continuous repeated.

Gender is Categorical R.V.

eg:- Demographic Statistics

| House Owner | Age | Gender M/F | No. of People in House | Height of House Owner |
|---|---|---|---|---|
| | Numerical C.R.V / D.R.V | Categorical | Numerical D.R.V. | Numerical C.R.V. |

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# Distributions.

## 1. Gaussian / Normal Distribution :

$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i$

Mean S.D.

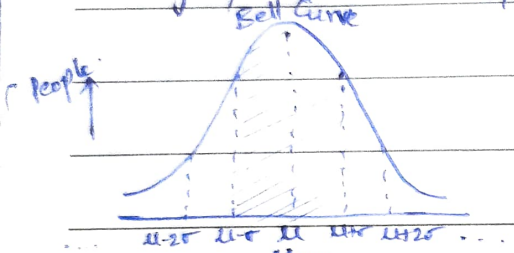
$X \sim GD(\mu, \sigma)$ or $N(\mu, \sigma)$

Var $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$

$GD(\mu, \sigma^2)$ or $N(\mu, \sigma^2)$ (Variance)

Std. deviation $\sigma = \sqrt{Var}$

Prob. Density Fun / Histogram




Std. deviation :- From Mean How far the other element is distributed / deviated (Right/Left distance)

Emperical formula :-

$P_r(\mu - \sigma \le X \le \mu + \sigma) = 68\%$

$P_r(\mu - 2\sigma \le X \le \mu + 2\sigma) = 95\%$

$P_r(\mu - 3\sigma \le X \le \mu + 3\sigma) \cong 99.7\%$

(Probability of Having Data points in first σ deviation)

$X = x_e$

Bell Curve & Symmetrical.

Dataset eg:- (i) Distribution of Height
(ii) IRIS Dataset - Petal length.

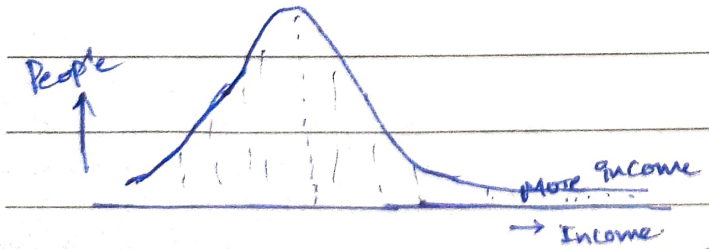
Std S.N.D $\Rightarrow \mu = 0 \quad \sigma = 1$

## 2. Log Normal Distribution

$X \sim$ Log Normal Distribution if $\ln(X)$ is Normally Distributed.

$X = \{x_1, x_2, x_3 \cdots x_n\}$ $\ln(x_1), \ln(x_2) \cdots \ln(x_n)$

$\ln(X) \sim N(\mu, \sigma)$




Similar to G.D, but Right Hand side goes more flatten.
⇒ Right Skewed.

Dataset eg:- (i) Income of the people.
(ii) Amazon Product Review Description Length

More income people are very less.

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Why we use all types of this distributions?

Money Spent on

| | R&D | Marketing | Profit States Company |
|---|---|---|---|
| 1 | 10300/- | 100000/- | |
| 2 | 1- | 1- | |
| 3 | 1- | 1- | |
| 4 | | | |
| 5 | | | |
| 6 | | | |

Gaussian — Log Normal

As per Domain Knowledge, If we know R&D is Gaussian Distribution. & Marketing is Log Normal Distribution.

R&D → Scaling to Std. Normal Distribution ($\mu=0, \sigma=1$)

$$\frac{\bar{x}_i - \mu}{\sigma}$$

Marketing →

| Marketing | Log (marketing) |
|---|---|
| 1500 | ln(1500) |
| | ln( ) |
| | ln( ) |

Log → will follow Normal Distribution.

ln ≈ Gaussian ($\mu=0, \sigma=1$)

↓

S.N.D $= \frac{x_i - \mu}{\sigma}$ Scaling done.

It is called Log Normalization.

Now after Scaling both R&D, Marketing (comparing) → Analyzation with More Accuracy.

~~For~~

~~Conclusion~~:

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1.

**Covariance :-** Useful in Data Preprocessing.

| Size | Price |
|---|---|
| 1200 sqm | 100/- |
| 1500 sqm | 200/- |
| 1800 sqm | 300/- |

Relation b/w Size & Price

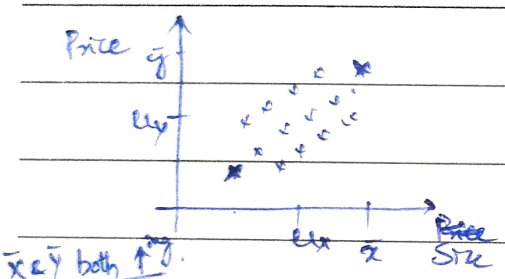
S↑ → P↑ या S↓ तो P↓

$$Cov(X,Y) \quad Cov(\underset{x}{Size}, \underset{y}{Price}) = \frac{1}{n}\sum_{i=1}^{n}(x_i-\mu_x)(y_i-\mu_y)$$

$$Cov(X,X) = Var(X)$$

X↑ & Y↑ ⇒ Cov is +ve

X↑ & Y↓ ⇒ Cov is -ve.



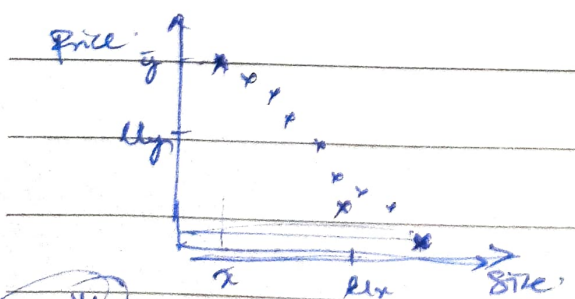

$\bar{x} < \bar{y}$ both ↑

Here Covariance doesn't tell about how much covariance b/w 2 Quantities.

Thats why we use Pearson Correlation Coef.

$\bar{x} > \mu_x$
$\bar{y} > \mu_y$ Cor. = +ve . +ve = +ve.

$\bar{x} < \mu_x$
$\bar{y} < \mu_y$ Cor. = -ve . -ve = +ve.




$\bar{x}$ ↑ $\bar{y}$ ↓ or Vice Versa.

$\bar{x} > \mu_x$
$\bar{y} < \mu_y$ Cor. = +ve . -ve = -ve.

$\bar{x} < \mu_x$
$\bar{y} > \mu_y$ cor = -ve . +ve = -ve.

VI. Central Limit Theorem:-

$X \not\approx G.D(\mu, \sigma^2)$ $\quad n \geq 30$
(R.V.)

This R.V. may/may not belong to Gaussian Distribution

Taking sample from R.V. X (30 data points Randomly selected)

| $\mu$ | $\sigma^2$ | | Mean |
|---|---|---|---|
| | | $S_1$ $x_1\ x_2 \cdots x_{30}$ | $= \bar{x}_1$ |
| | | $S_2$ . . . . . | $= \bar{x}_2$ (some other Random datapoints) |
| | | $S_3$ $x_1$ . . . . . | $= \bar{x}_3$ |
| | | ⋮ | |
| | | $S_{100}$ . . . . . | $= \bar{x}_{100}$ |

$\bar{x} \approx \mu$

According to Theorem ~~Mean~~ $\bar{X} \approx G.D(\mu, \frac{\sigma^2}{n})$ $\quad n \geq 30$.
Mean follows G.D.

VII. Chebyshev's Inequality

For suppose R.V. X following G.D
$X \approx G.D(\mu, \sigma)$

$Pr(\mu - \sigma \leq x \leq \mu + \sigma) \approx 68\%$
$Pr(\mu - 2\sigma \leq x \leq \mu + 2\sigma) \approx 95\%$
$Pr(\mu - 3\sigma \leq x \leq \mu + 3\sigma) \approx 99.7\%$

For suppose R.V. Y not following G.D.
$Y \not\approx G.D(\mu, \sigma)$
What % of datapoints in Y falling within range of first Std. deviation?

↓

Chebyshev's Inequality.

$Pr(\mu - K\sigma < x < \mu + K\sigma) > 1 - \frac{1}{K^2}$

$K=2$ 2nd Std. deviation. $Pr(\mu - 2\sigma < x < \mu + 2\sigma) > 1 - \frac{1}{2^2} \Rightarrow Pr(\mu - 2\sigma < x < \mu + 2\sigma) \geq \frac{3}{4}$
$\Rightarrow Pr(\mu - 2\sigma < x < \mu + 2\sigma) > 75\%$

$K=3$ 3rd Std. deviation $Pr(\mu - 3\sigma < x < \mu + 3\sigma) > 1 - \frac{1}{3^2} \Rightarrow 8/9 \times 100 > 88.9\%$

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Linear

Wikipedia

# Pearson Correlation Coef.

Useful for feature selection

$$\text{Covariance} = \text{Cov}(X,Y) = \frac{1}{n}\sum_{i=1}^{n}(x_i-\mu_x)*(y_i-\mu_y)$$

$x\uparrow$ $y\uparrow$ = +ve

$x\uparrow$ $y\downarrow$ = -ve

Here Covariance gives only Direction of Relationship.

But doesn't know about Strength.

↓

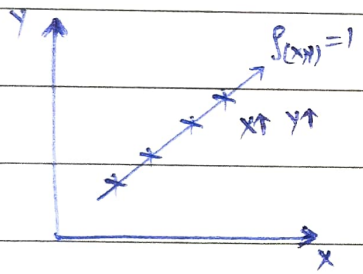
$$\text{Pearson C.C.} = \rho_{(x,y)} = \frac{\text{Cov}(X,Y)}{\sigma_x \cdot \sigma_y}$$

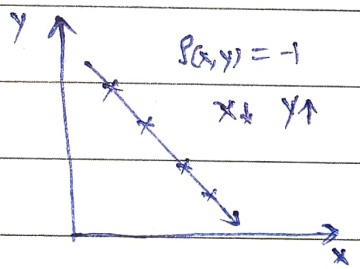
| x | y |
|---|---|
| Height | Weight |

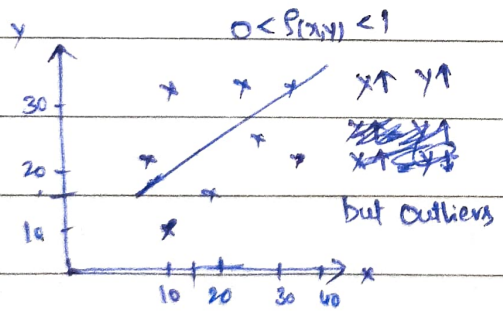
$x\uparrow \Rightarrow y\uparrow^{es}$.

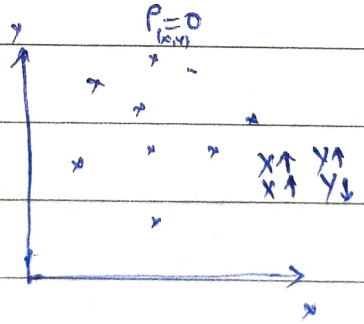
We get How much positivity/Negativity ie. Strength

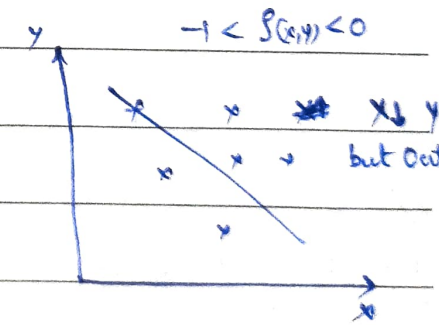
Value Ranges : $\rho_{(x,y)}$ : $\boxed{-1 \le \rho \le 1}$

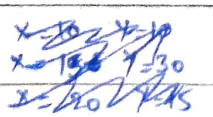


Suppose $\rho_{(x,y)} = 1$, ie $X\uparrow$ $Y\uparrow$ we can drop One Variable either X or Y for feature selection in M.L. as both features are same.

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Spearman's Rank Correlation Coef:- Wikipedia.

* HeatMap use this Spearman's Correlation.

* $\gamma_s = \rho_{rg_x, rg_y} = \frac{cov(rg_x, rg_y)}{\sigma_{rg_x} \cdot \sigma_{rg_y}}$ Here we considering Ranks.

Rank of x, Rank of y

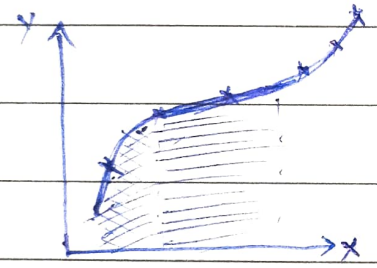
$\rho \rightarrow$ Pearson Correlation Coef., but applied to Rank Variables.

$cov(rg_x, rg_y) \rightarrow$ Covariance of Rank Variables

$\sigma_{rg_x}, \sigma_{rg_y} \rightarrow$ Std. deviations of Rank Variables

$$\gamma_s = 1 - \frac{6 \sum d_i^2}{n(n^2-1)}$$

$d_i = rg(x_i) - rg(y_i) \Rightarrow$ Diff. b/w 2 Ranks of Each Observation.




In this graph,

Initially a small increase in X ⇒ Small increase in Y.

Large increase in X ⇒ Small increase in Y.

So, Non Linear we have consider this difference also.

X↑, Y↑

Pearson Correlation = 0.88

Spearman correlation = 1

Question

| IQ of Students $X_i$ | Hours of TV Per Week $Y_i$ |
|---|---|
| 106 | 7 |
| 100 | 27 |
| 86 | 2 |
| 101 | 50 |
| 99 | 28 |
| 103 | 29 |
| 97 | 20 |
| 113 | 12 |
| 112 | 6 |
| 110 | 17 |

Answer:-

Step1 :- Sort the Order ($X_i$)

Sorted

| $X_i$ | $Y_i$ | rank $x_i$ | rank $y_i$ | $d_i$ rank($x_i$ $y_i$) | $d_i^2$ |
|---|---|---|---|---|---|
| 86 | 2 | 1 | 1 | 0 | 0 |
| 97 | 20 | 2 | 6 | -4 | 16 |
| 99 | 28 | 3 | 8 | -5 | 25 |
| 100 | 27 | 4 | 7 | -3 | 9 |
| 101 | 50 | 5 | 10 | -5 | 25 |
| 103 | 29 | 6 | 9 | -3 | 9 |
| 106 | 7 | 7 | 3 | 4 | 16 |
| 110 | 17 | 8 | 5 | 3 | 9 |
| 112 | 6 | 9 | 2 | 7 | 49 |
| 113 | 12 | 10 | 4 | 6 | 36 |

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Pearson Correlation :- Cov(X,Y) → diff. of X & diff. of Y Independently.

Spearman Correlation :- Rank(X,Y) → diff. of X & Y dependently.

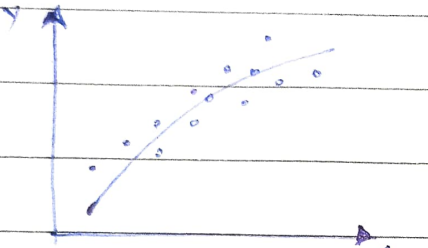
(9)

$$\rho = 1 - \frac{6 \times 194}{10(10^2-1)} = \frac{-29}{165} = -0.17575757\ldots$$

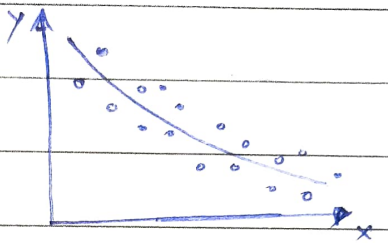
Coef. is very low.

No Relation b/w IQ & Hours per T.V

(p value 0.627 t-distribution)

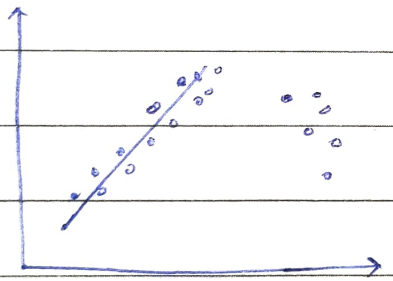



Spearman Correlation = 0.92




Spearman Correlation = −0.91

Outliers



Spearman Correlation = 0.84

Pearson Correlation = 0.67

Pearson Correlation focus on Linear but Spearman helps on Nonlinear also.

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# Finding Outliers in Dataset using Z-score and IQR

**Outlier:**

An Outlier is a data point in a dataset that is distant from all other observations.
A data point that lies outside the overall distribution of the dataset.

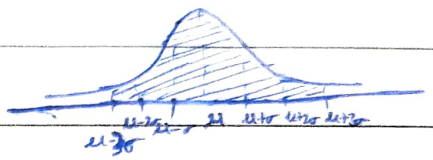
**Z-score:**

$$Z = \frac{X - \mu}{\sigma}$$

$$X = \mu + Z\sigma$$

$$Z = \frac{X - \mu}{\sigma} > 3 \quad \text{[Outliers]}$$

Out of 3rd Std Deviations.



Anything falls after $\mu + 3\sigma$ is an Outlier

$Z \geq 3$.
Outlier.

**I.Q.R** Inter Quartile Range

dataset = [5, 6, 7, 1, 2, 8, 10, 3, 4, 7, 9]

~~for~~ ~~suppose~~

10% 20% 30% 40% 50% 60% 70% 80% 90% 100%?

Here Calculation Wrong.

Ansr Sort the dataset = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]

1 → ~~1~~ 1 → 0 numbers are less than 1 → 0%

2 → 10% of Total Numbers are less than 2 ie. 1
(before 2 = 1 Number only) i.e. $\frac{1 \text{ Number}}{\text{Total Numbers}} \times 100\% = \frac{1}{10} \times 100 = 10\%$

10%

*** 3 → 1, 2, 3, 4 . . . 10

2 Numbers are less than 3. ie. $2/10 \times 100 = 20\%$

4 → 30%

In InterQuartile Range, Most Numbers are Concentrated at 25% 75%

I.Q.R = 75% − 25% value

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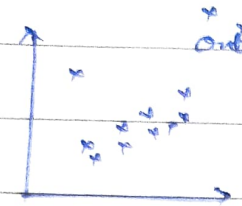
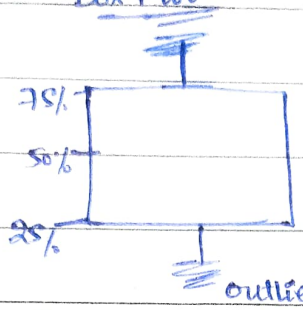
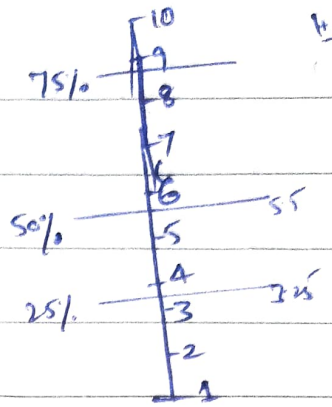
Scatter Plot




Box Plot




Below 25% & Above 75%




$\frac{1+10}{2} = 5.5$

$\frac{1+5.5}{2} = 3.25$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
17 18 19 20 21 22 27 24 25 26 21 28 29 30 31 32 33

S = [10, 10, 10, 10, 10, 11, 11, 12, 12, 12, 12, 12, 12, 12, 13, 13, 13, 13,
16, 16, 16, 16, 16, 16, 15, 15, 15, 15, 15, 17, 19, 102, 107, 108]
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33

Sorted the List -

0% 2.9% 5.8% 8.8 11.7 14.7 17.6 20.5 23.5 26.4 29.4 32.5 35.2 38.2 41.1 44 47 50

S = [10, 10, 10, 10, 10, 11, 11, 12, 12, 12, 12, 12, 12, 12, 13, 13, 13, 13,
14, 14, 14, 14, 14, 14, 15, 15, 15, 15, 15, 17, 19, 102, 107, 108]
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33
52.9 55 58.8 61.7 64.7 67.6 70.5 73.5 76.4

Total Elements = 34

Percentile :-

1st Element '0' Index : '10' - $\frac{0}{34} \times 100\% = 0\%$

2nd Element 1 Index : '10' - $\frac{1}{34} \times 100\% = 2.9\%$

6th Element 5 Index '11' - $\frac{5}{34} \times 100\% = 14.7\%$ (5 elements are there before 11)

Lower $\frac{x}{34} \times 100 = 25\%$

$x = 8.5$ index.

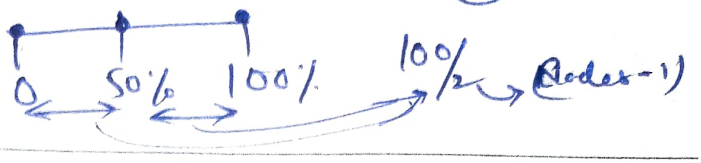
Upper $\frac{y}{34} \times 100 = 75\%$

$y = 25.5$ index.

9

25

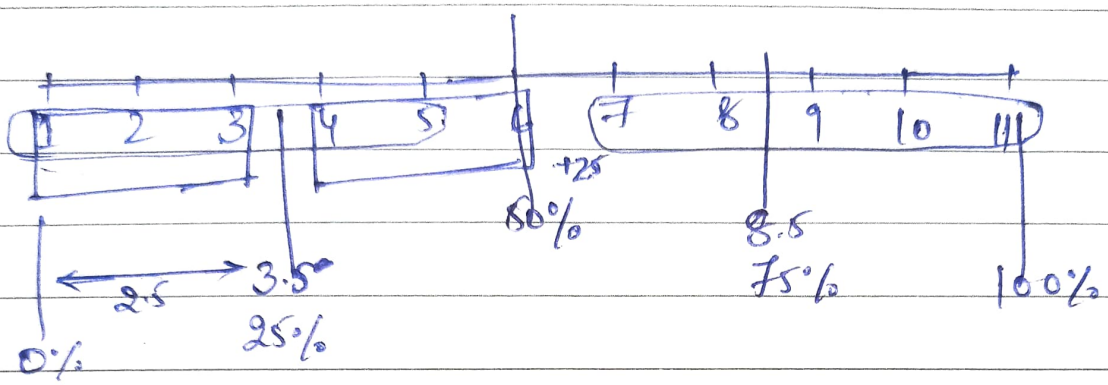
24




C&S electric

# IQR Practice

## 11 Numbers




~~(Total No. Count + 1) P%~~ P% Value.

25% (11+1) 25% = (11+1) × 1/4 = 3

~~1/4 × 100 = 25%~~

11 nodes = Total No. Count.

Each is Segment ~~100/11-1~~ 100/11-1 = 10%

1 → 0%

2 → 1/(11-1) × 100% = 10%

3 → 2/(11-1) × 100% = 20%

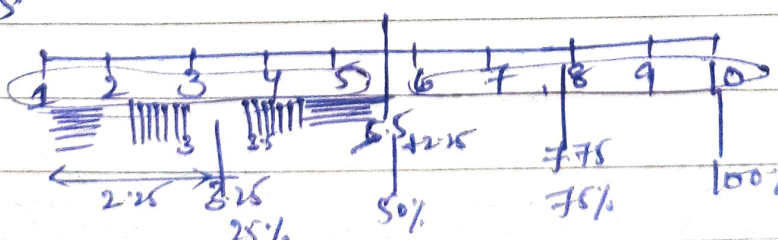
Before Node 3 → There are 2 Numbers

Each 25% is.

$x = \frac{25\%}{100\%} \times 10 = 2.5$

~~25% Value = 25% (Total Count)~~

## 10 Numbers




25% (10-1) = 9/4 = 2.25

P% Value = P% (Total Count-1)

# Interquartile range

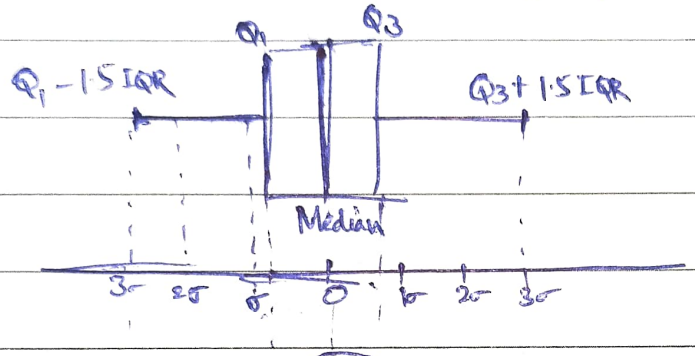
H-Spread , Q-Q , middle 50%

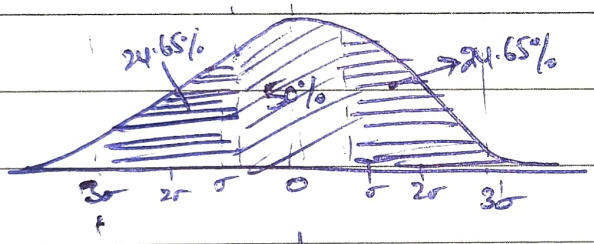
Measure of Statistical dispersion, diff b/w 75th and 25th percentiles

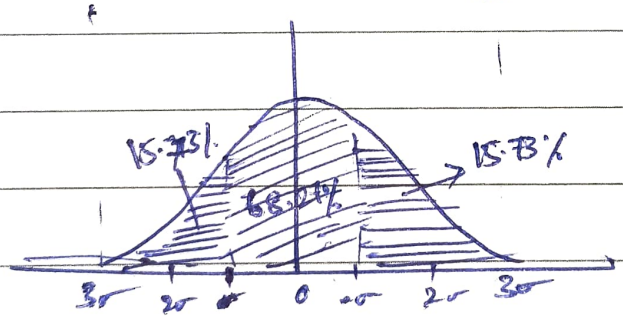
$$IQR = Q_3 - Q_1$$

Here Median is Corresponding Measure of Central Tendency

IQR - Used in Outliers Identifying




Order 7, 7, 31, 37, 47, 75, 87, 115, 116, 119, 119, 155, 177

31 → $Q_1$ Median of first Half

$Q_2$ (Median of whole Table)

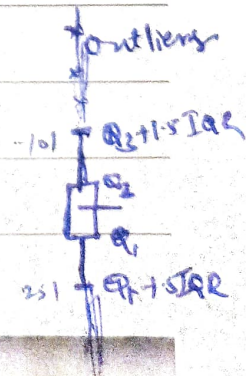
119 → $Q_3$ Median of last Half

Avg

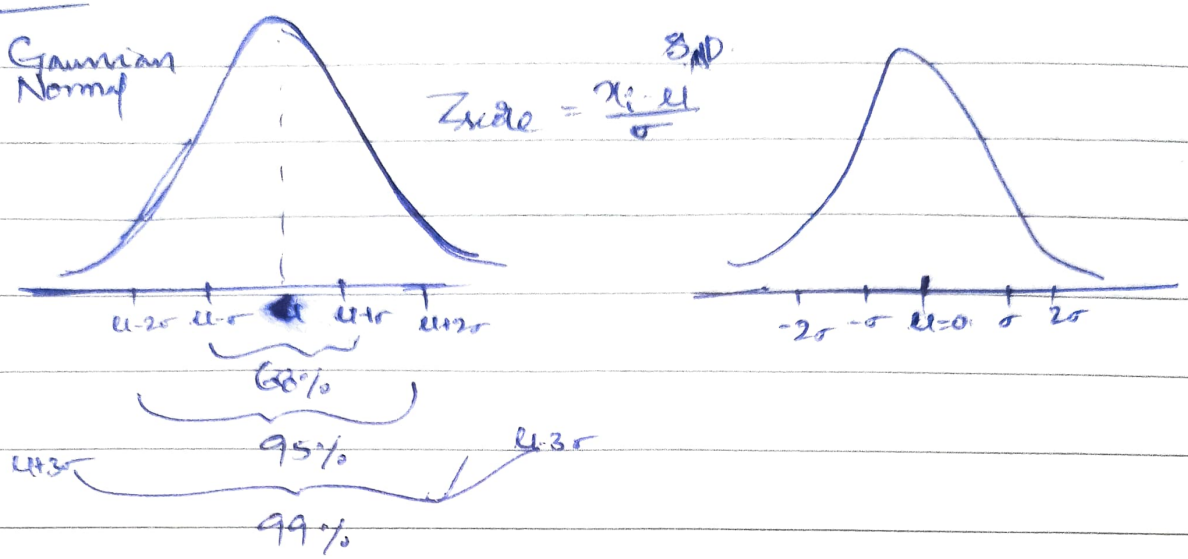
$IQR = Q_3 - Q_1 = 119 - 31 = 88$

Range (L.B, U.B) $= (Q_1 - 1.5IQR, Q_3 + 1.5IQR)$

$= (-101, 251)$



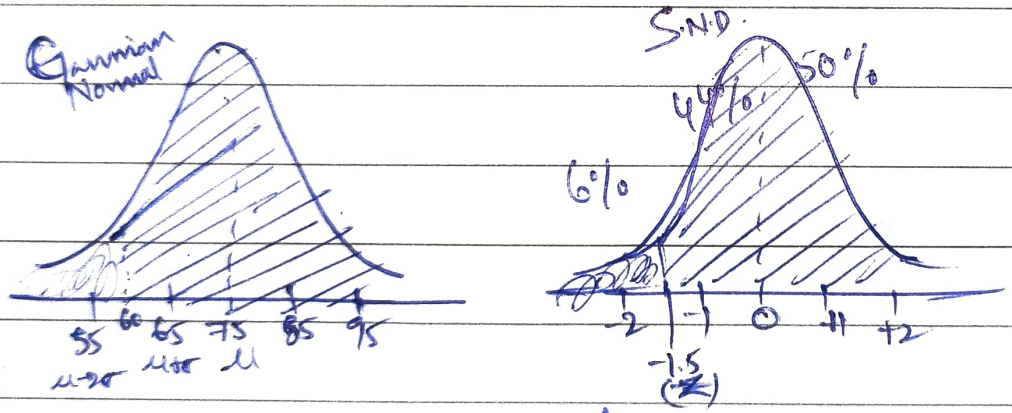
# Z-score :-




$\mu = 75$

$\sigma = 10$

$P(x > 60)$




Z-score Table gives Area of left Handside (-ve)

1.500

= -0.06681 i.e. 6.681%

100 - 6.681 = 93.319

Remaining Area (Probability) = 94%

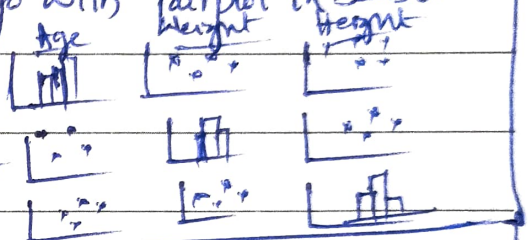
## * Univariate, Bivariate & Multivariate Analysis

| Height cm | Weight Kg | O/p |
|---|---|---|
| 180 | 90 | Obesity |
| 160 | 50 | Slim |
| 170 | 78 | Fit |
| 190 | 90 | Fit |
| 175 | 85 | Slim. |
| : | : | : |

**Multivariate**

More than 3 features - Age, Weight, Height

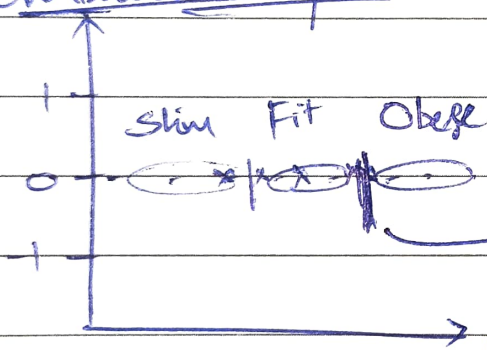
We go with Pairplot in SeaBorn.




Gives concept of Correlation.

A

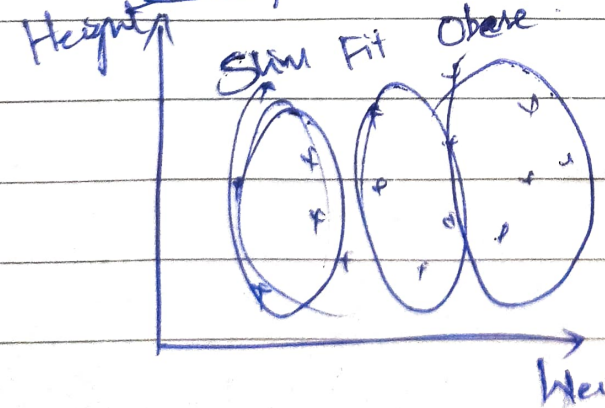
### Univariate Analysis




We ~~can~~ maybe 100% Correct. Overlap point will be there

We go for Bivariate/MultiVariate

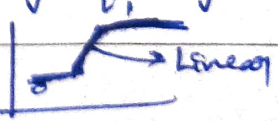
### Bivariate Analysis




~~Like~~

If We are able to classify point properly, We can go for Logistic Regression

Logistic Regression :-



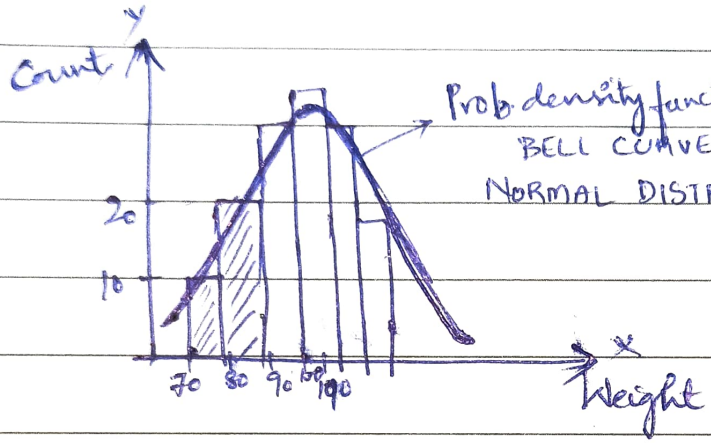

If Overlapping Occurs, We go with (Non Linear) Decision Tree / XG Boost / Random forest / SVM / Adaboost / KNN Algo.

## Histograms:-

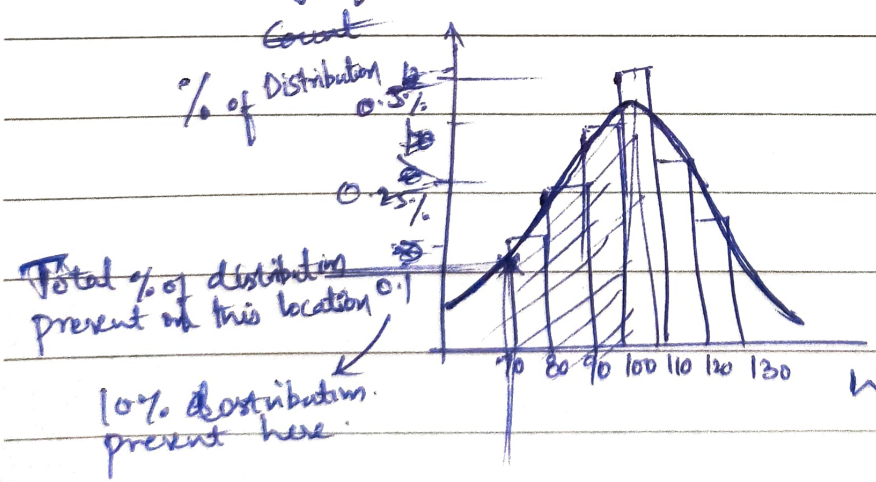
~~The given range~~

Gives Information of How many No. of points in a Given range (bins)

i.e => Builds Bargraph.

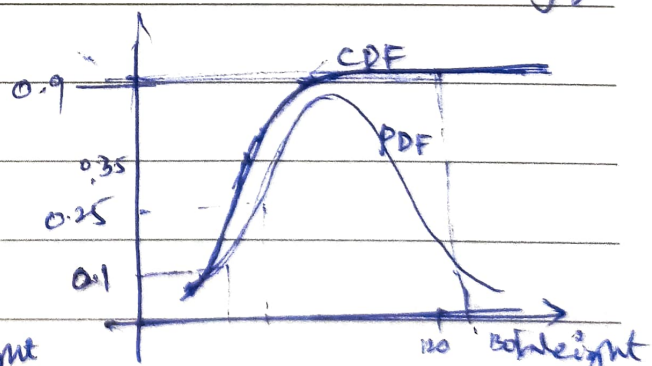



## Prob. density function PDF




## CDF

Cumulative ~~Distri~~ Density function




0.1 to 0.25

90% < 130 kgs

C&S electric

# 21. Standardization Vs Normalization

Important topic for Feature Scaling which is an integral part of feature Engg.

For Data, We analyse features < Independent / Dependent

* Using Independent Variables, We analyse the Dependent Variables.

Features → We Consider ~~not~~ Magnitude & Units.

| | Age | Weight | Height |
|---|---|---|---|
| Units | (No ~~of items~~) 25 | 60 (kgs) | 162 (cms) |
| Magnitude | 25 No. of Years | Kgs | Cms |

Two scaling Techniques are Standardization & Normalization.

(i) Normalization :- (min-max Normalization)

Helps to scale down the feature b/w 0 to 1.

~~min~~

$$X_{Norm} = \frac{X - X_{min}}{X_{max} - X_{min}}$$

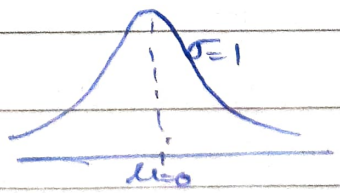
(ii) Standardization :- (Z-score Normalization)

Helps to scale down the feature ~~or~~ based on Std. Normal Distribution.

with Mean $\mu = 0$
Std. Deviation $\sigma = 1$

$$Z = \frac{x - \mu}{\sigma}$$

~~Mean is~~



## Normalization (MinMax)

* M.L Algorithm which involves Euclidean distance.
* D.L Algorithms where Gradient Descent is involved.
* Gradient Descent is nothing but a Parabola Curve where we need to find Global Minimal point.
* We use Min-max Normalization
* Algorithms like KNN
  K means Neighbour
  K means Clustering
  ANN Artificial Neural N/w
  CNN Convolutional Neural N/w } Scale down 0 to 1. Images [0 to 255]
  Linear Regression
  Logistic Regression.

## Standardization (StandardScaler)

* Many of the use cases Standardization Technique is used. and performs well

* Some Algorithms, We won't perform Scaling like Decision Tree, RandomForest, XGBoost and all boosting techniques.

  No use of Scaling Techniques here.

ANN - Tensor Flow, Keras require Inputs b/w 0 to 1 to analyse weights quickly.