Project V

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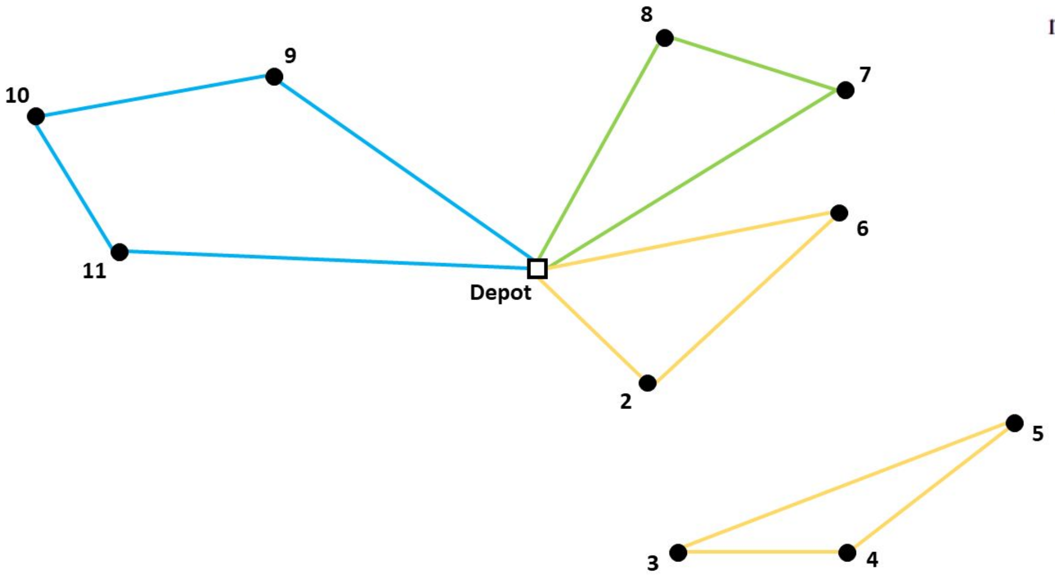
Initial Approach...

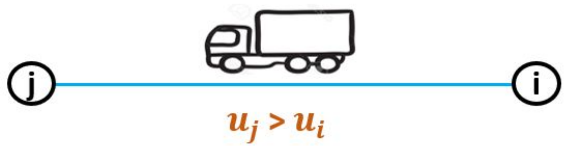
- n is the number of clientes
- N is set of clients, with $N = \{1, 2, \ldots, n\}$
- V is set of vetices (or nodes), with $V = \{0\} \cup N$
- A is set of arcs, with $A = \{(i, j) \in V^2 : i \neq j\}$
- c_{ij} is cost of travel over arc $(i, j) \in A$
- Q is the vehicle capacity
- q_i is the amount that has to be delivered to customer $i \in N$

$$
\begin{array}{lll}
\min & \displaystyle\sum_{i,j \in A} c_{ij} x_{ij} & \\
\text{s.t.} & \displaystyle\sum_{j \in V, j \neq i} x_{ij} = 1 & i \in N \\
& \displaystyle\sum_{i \in V, i \neq j} x_{ij} = 1 & j \in N \\
& \text{if } x_{ij} = 1 \ \Rightarrow \ u_i + q_j = u_j & i, j \in A : j \neq 0, i \neq 0 \\
& q_i \leq u_i \leq Q & i \in N \\
& x_{ij} \in \{0, 1\} & i, j \in A
\end{array}
$$

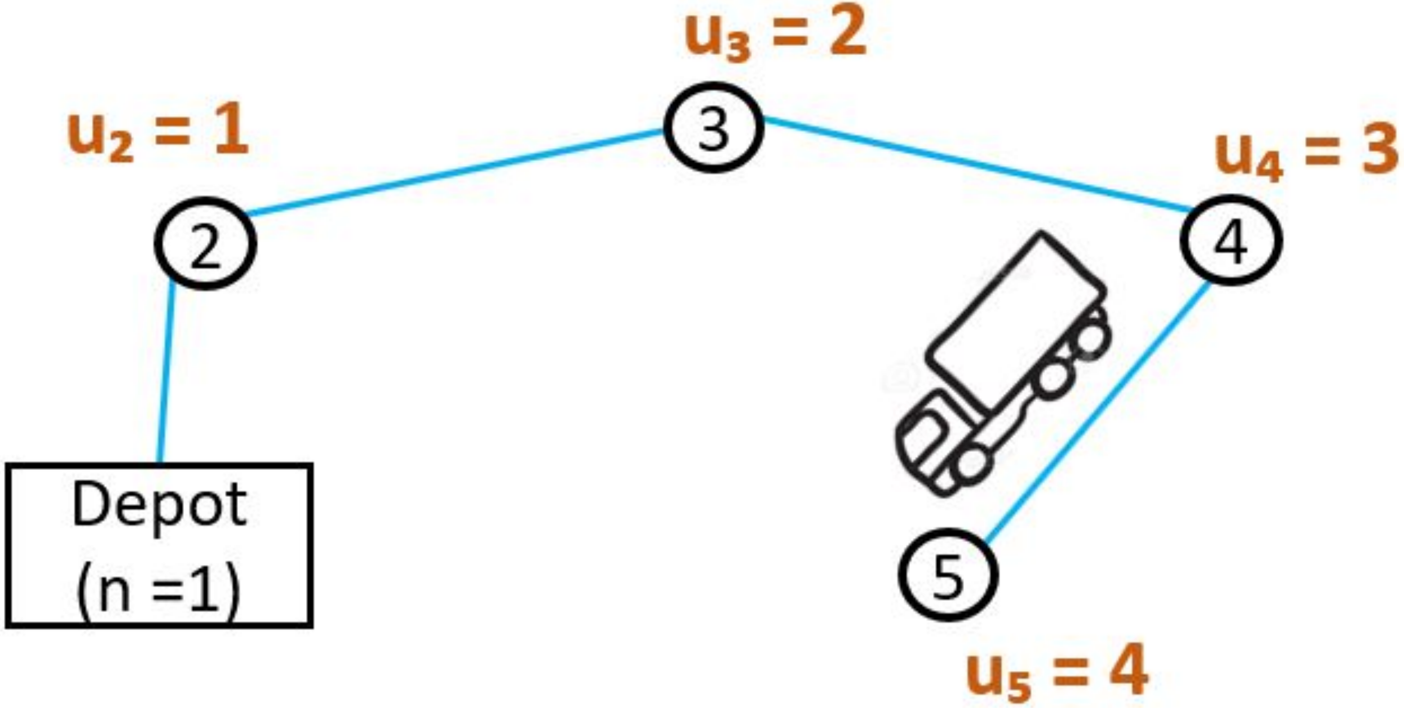
Subtour Problem



$$
\begin{aligned}
\min \quad & \sum_{i,j \in A} c_{ij} x_{ij} & \\
\text{s.t.} \quad & \sum_{j \in V, j \neq i} x_{ij} = 1 & i \in N \\
& \sum_{i \in V, i \neq j} x_{ij} = 1 & j \in N \\
& \boxed{\text{if } x_{ij} = 1 \;\Rightarrow\; u_i + q_j = u_j \qquad i,j \in A : j \neq 0, i \neq 0} & \\
& q_i \leq u_i \leq Q & i \in N \\
& x_{ij} \in \{0, 1\} & i,j \in A
\end{aligned}
$$



Miller-Tucker-Zemlin (Subtour Constraint)



Performance

	Nodes	IDFJ	MTZ	Best Solution
Data31	40	683	691	**IDFJ**
Data32	40	651	651	**-**
Data33	40	571	612	**IDFJ**
Data34	60	936	813	**MTZ**
Data35	60	687	687	**-**
Data36	60	924	864	**MTZ**
Data37	80	920	839	**MTZ**
Data38	80	1249	1027	**MTZ**
Data39	80	1201	978	**MTZ**
Data40	100	1314	912	**MTZ**
Data41	100	1386	1246	**MTZ**

Nodes = Nodes in instance
IDFJ = Implicit Dantzig-Fulkerson-Johnson formulation
MTZ = Miller-Tucker-Zemlin formulation

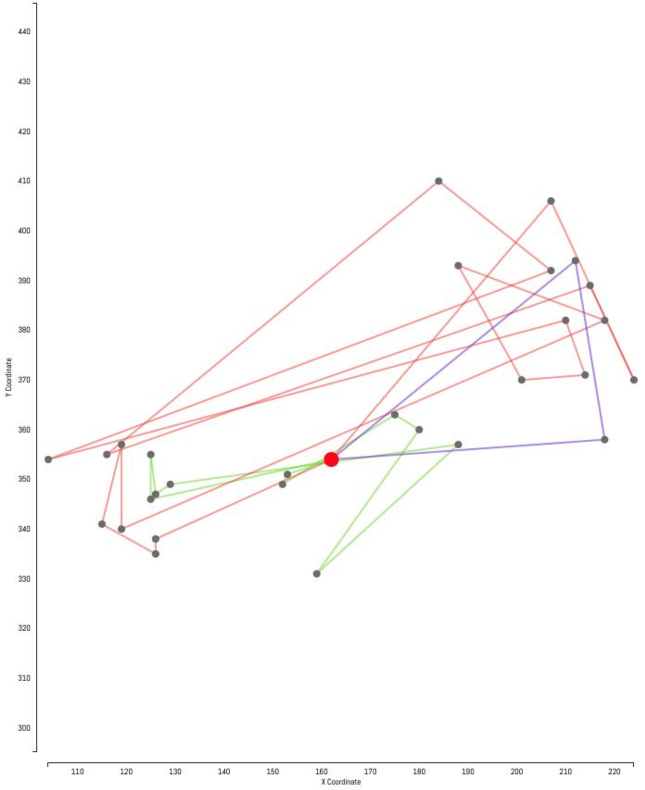
Transitioning to Simulated Annealing

The first mathematical formulation and algorithm for the solution of the CVRP was proposed by Dantzig and Ramser [2] in 1959 and five years later, Clarke and Wright [1] proposed the first heuristic for this problem. To date, many solution methods for the CVRP have been published. General surveys can be found in Toth and Vigo [11] and Laporte [7]. The CVRP belongs to the category of NP hard problems that can be exactly solved only for small instances of the problem. Therefore, researchers have concentrated on developing heuristic algorithms to solve this problem (for example [6], [3]).

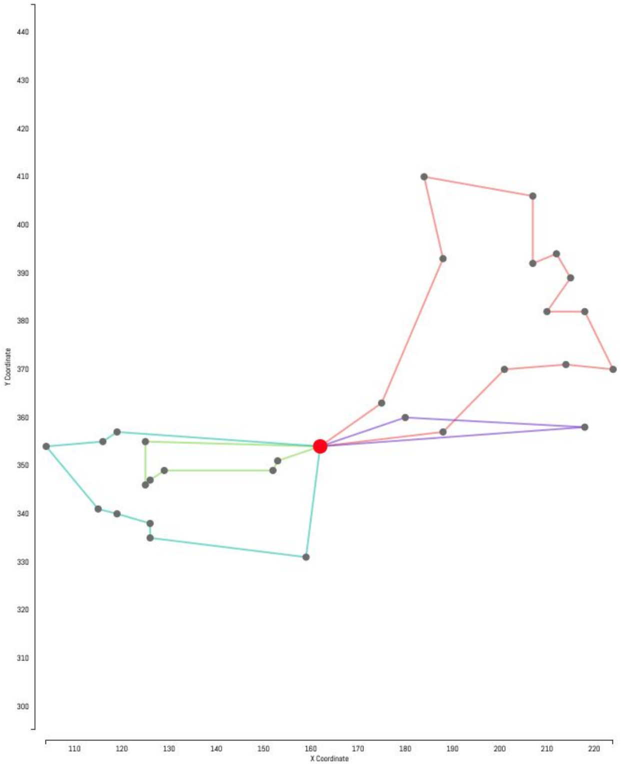
Simulated Annealing

1. Get starting feasible solution (greedy bin-packing)
2. For each route in solution, solve TSP via Simulated Annealing
    a. Neighborhood does random swaps between points
3. Overall Simulated Annealing
    a. Neighborhood = Replace Highest Average
4. Repeat until 10,000 iterations
5. Random restart 4 times

Simulated Annealing



Simulated Annealing



Lessons Learned

- Always read literature before diving in
- CPLEX can't solve everything
- Having a good neighborhood function is important for S.A.
- Starting solution very important for ensuring quality results form S.A.
- Simulated-Annealingception!