Finite Symmetric Zero-Sum Win Loss Draw Games

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1 Introduction

This article explores a class of games defined by the following properties:

1. There are 2 players.
2. Each player can choose from the same set of moves $\{x_1, x_2, \ldots, x_n\}$.
3. Neither player knows what the other will play before making their choice.
4. Either one player wins and the other loses, leading to payoffs of 1 and -1 respectively, or they draw and each receive a payoff of 0.

Now we establish some notation:

P **is the payoff function. I.e if** $P(x_i, x_j) = p$ **then that means that when player** A **plays** x_i **and player** B **plays** x_j **then player** A **receives payoff p. Note that, because our games are zero sum,** $P(x_i, x_j) = -P(x_j, x_i) \forall i, j \in (1, \ldots, n)$**. A game** G **is fully defined by its payoff function** P_G**.**

S **is the set of possible mixed strategies. A mixed strategy is a probability distribution over available moves. A player playing a mixed strategy will select their move randomly using the probability distribution. If** G **has moves** $\{x_1, x_2, \ldots, x_n\}$ **then** $S = \{(x_1, x_2, \ldots, x_n) \in \mathbb{R}^n \mid \sum_{i=1}^{n} x_i = 1, x_i \in [0, 1] \forall i\}$**.**

We can now extend P**. We define** $P : S \times S \mapsto [0, 1]$ **such that** $P(s_1, s_2)$ **is equal to the expected payoff for a player using strategy** s_1 **against another player using** s_2**. It is simple to prove that** $P(s_1, s_2) = -P(s_2, s_1)$**.**

We call $n \in S$ an equilibrium strategy if the scenario when both players play n is a Nash equilibrium. This means that neither player can gain an advantage by changing their strategy. Formally, n is an equilibrium strategy if $P(n, n) \geq P(s, n) \forall s \in S$. The Equilibrium space $N \subset S$ is the set of equilibrium strategies within S.

2 Basic properties of the equilibrium space

Theorem 2.1. *The equilibrium space N is non empty.*

Proof. placeholder $\square$