Final Report

Engineering Project V

Project 2: Motor Control System

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Introduction

This report is the document outlining the outcomes of the second project which consisted of the modelling, simulation and implementation of a DC motor and servo-amplifier based on a closed-loop system designed with an Op-amp control system.

Modelling the Permanent-Magnet Direct-Current (PMDC) Motor

To design and model the motor, MATLAB was used with the Simulink utility. The motor used was a DPP240-29V48 by ElectroCraft Brushed Motor. The parameters for the motor were based on values and measurements already given by the datasheet of the motor.

Preparation

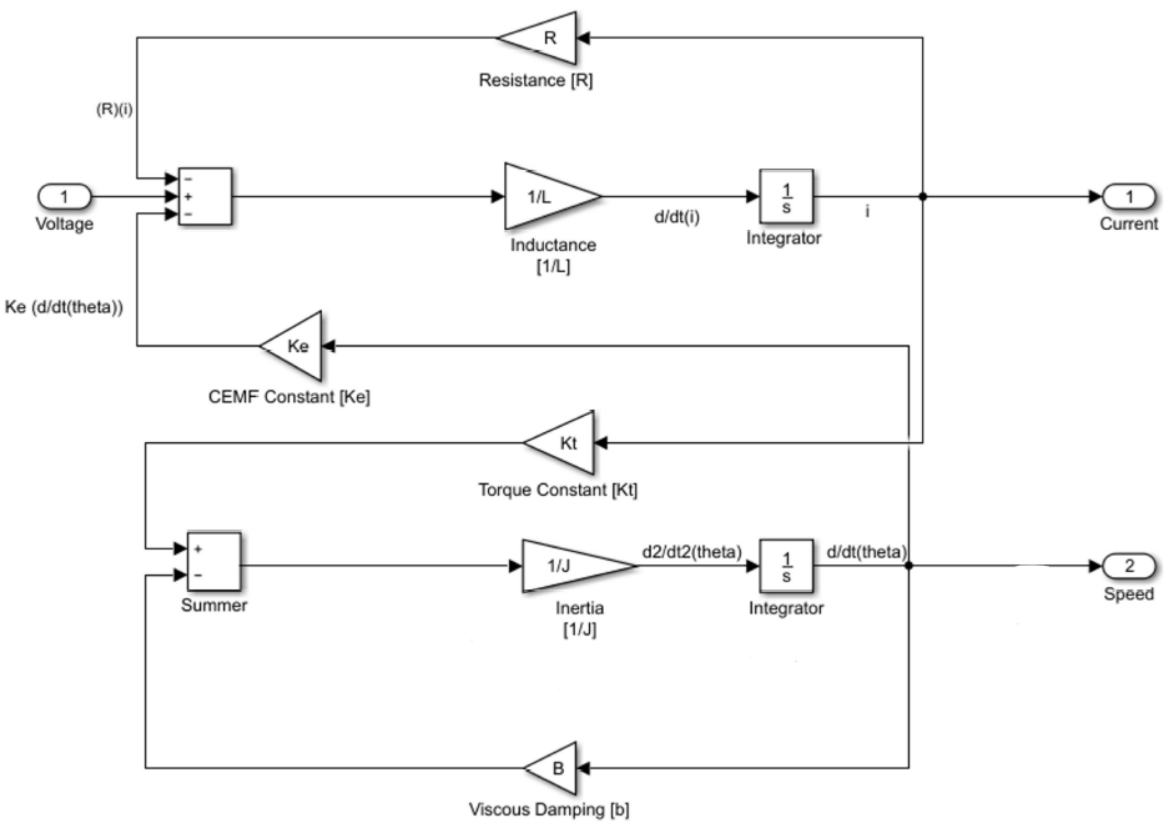


Figure 1 Motor Model simulated on Simulink

Seen above is the model of the motor where the input is a voltage and the output is speed in radians per second. The model shows the marriage between the mechanical portion and the electrical portion of the system using the Kt and Ke values. Every unit is modelled according to SI units.

Kt is the torque constant (no units)

Ke is the electrical constant (CEMF constant) (no units)

J is the moment of inertia of the tachometer ($kg \cdot m^2$)

R is the resistance of the winding within the motor (Ohms)

L is the inductance of the winding within the motor (Henries)

b is the friction coefficient of the motor (Nm/rads/sec)

The DC motor transfer function was derived using the differential equation in figure 2 below:

$$\frac{d^2\theta}{dt^2} = \frac{1}{J}(K_t i - b\frac{d\theta}{dt}) \qquad \frac{di}{dt} = \frac{1}{L}(V - Ri - K_e\frac{d\theta}{dt})$$

$$\boxed{\text{let} \quad \omega = \frac{d\theta}{dt}}$$

$$\boxed{\frac{d\omega}{dt}} = \frac{1}{J}(K_t i - b\boxed{\omega}) \qquad \frac{di}{dt} = \frac{1}{L}(V - Ri - K_e\boxed{\omega})$$

Figure 2 Differential equations. Left side equations is the Mechanical equation (note the torque constant) and the right equation is the electrical (note the electrical constant).

Solving both equations, the system will have a differential equation of the system expressed below:

$$\boxed{G(s) = \frac{\omega(s)}{V(s)} = \frac{\frac{K_t}{LJ}}{s^2 + \frac{(RJ + bL)}{LJ}s + \frac{Rb + K_e K_t}{LJ}}}$$

Figure 3 Motor System equation

Initially, values given in the datasheet were used to simulate the Simulink models. In SI units, they are listed below:

L = 0.0048 Henry

R = 3.2 Ohms

Ke = 10.9

Kt = 10.3

J = 2.825 kg * m^2

b (dampening ratio) = Assumption of 0.1

From here, the Advanced Controls Amplifier was disconnected from the motor and the motor inputs were used to measure the inductance, resistance and the torque constant of the motor. A Digital MultiMeter was used at the leads of the positive and negative power lines that powered the motor:

L = 0.00524 Henries

R = 4 Ohms

For Measuring Torque, the two leads were attached onto a DC power supply and powered up to 10 V with an amperage of 100 mA. A torque meter was attached to the spindle and once the motor was moving, the torque meter measured the torque for every measure of current. The current was increased by 100 mA a total of 8 times and the torque was measured. The results were put into an excel document and the slope was determined.

Torque (N· cm)	Current (A)	Slope
2.12	0.1	7.858333
2.33	0.2	
2.81	0.3	
3.99	0.4	
4.45	0.5	
5.41	0.6	
6.39	0.7	
7.47	0.8	

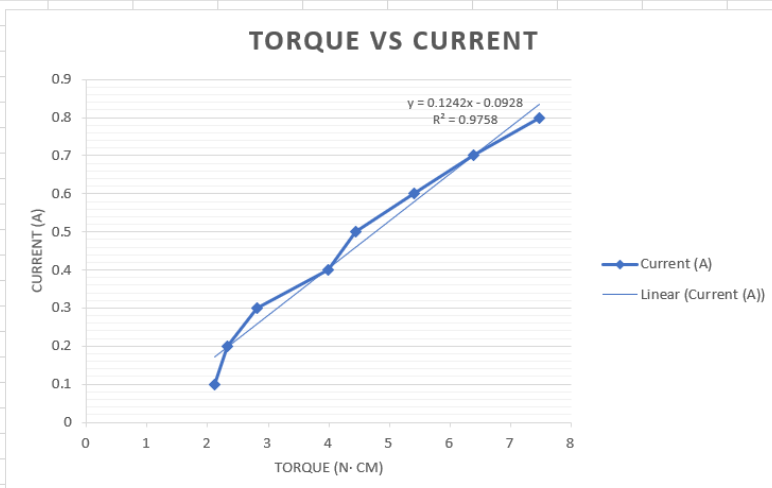


Figure 4 Current vs Torque

The value of the slope was the value of Ke and Kt, which came to be approximately 7.8583x10^{-2} V/rad/second and Nm/A, respectively. These values were the same hence they were the marriage between the electrical portion of the system and the mechanical portion of the system.

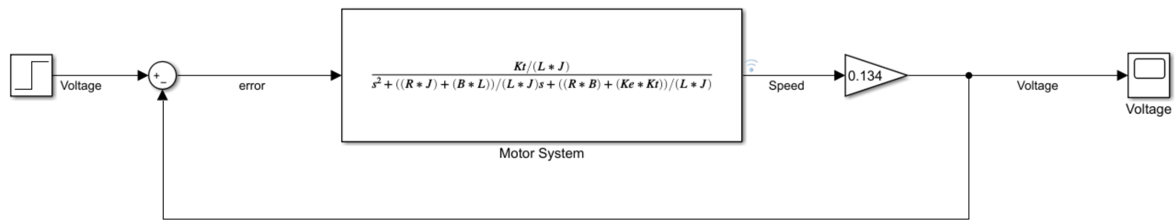


Figure 4 Initial Control Loop, with Unity Feedback

The Initial closed-loop control was implemented on Simulink shown in figure 4. It was tested with a step input of 5 V and was measured at the output with a Digital MultiMeter. The voltage constant of this particular model was noted to be 10.9 V/kRPM according to the datasheet of the model.

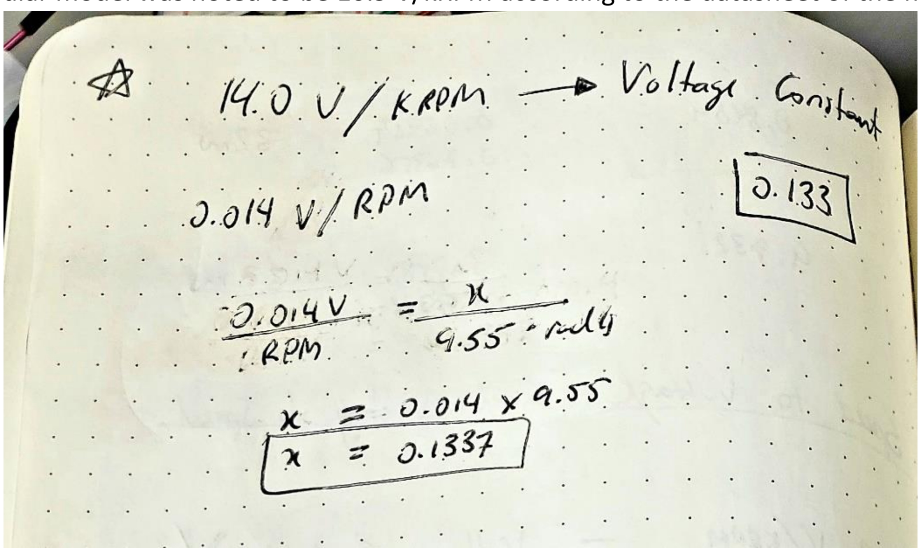


Figure 5 Voltage Constant calculation

The Voltage constant of the motor was converted from V/kRPM into V/RPM and was used in a conversion equation to convert it into a constant. The 9.55 rad/sec is the conversion unit from Voltage/RPM to Radians/second. This constant, as denoted with the term ‘x’ in figure 5, was used to convert the compensated output shown in the simulation in figure 6.

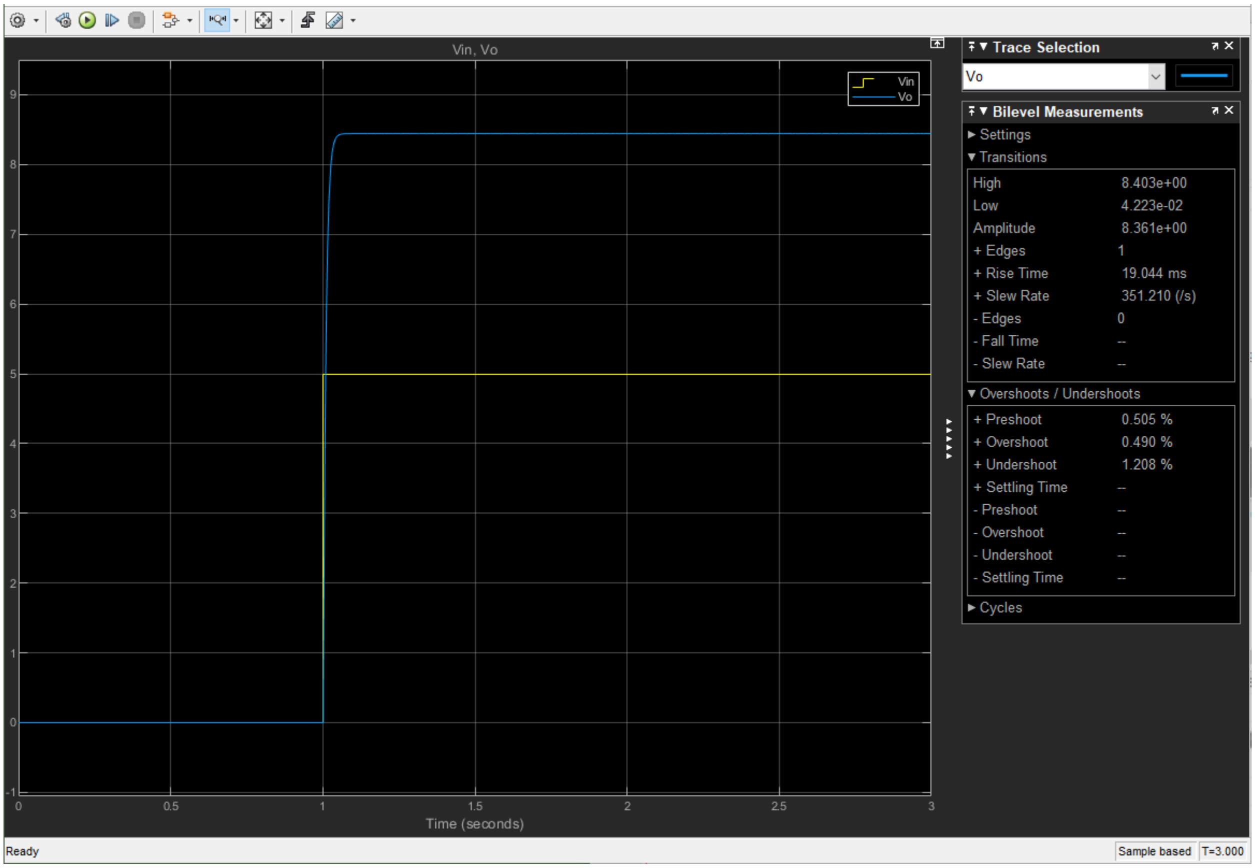


Figure 6 Simulated Step response

With a 5 V step input, the output yields 8.4 V with a T_r of 19.04 ms (Rise time).

The values of B and J were adjusted using MATLAB. According to figure 3, the J value adjusts the pass-band gain and the B value adjusts the damping ratio. The scope shot below shows the initial response before tuning the model to match the motor.

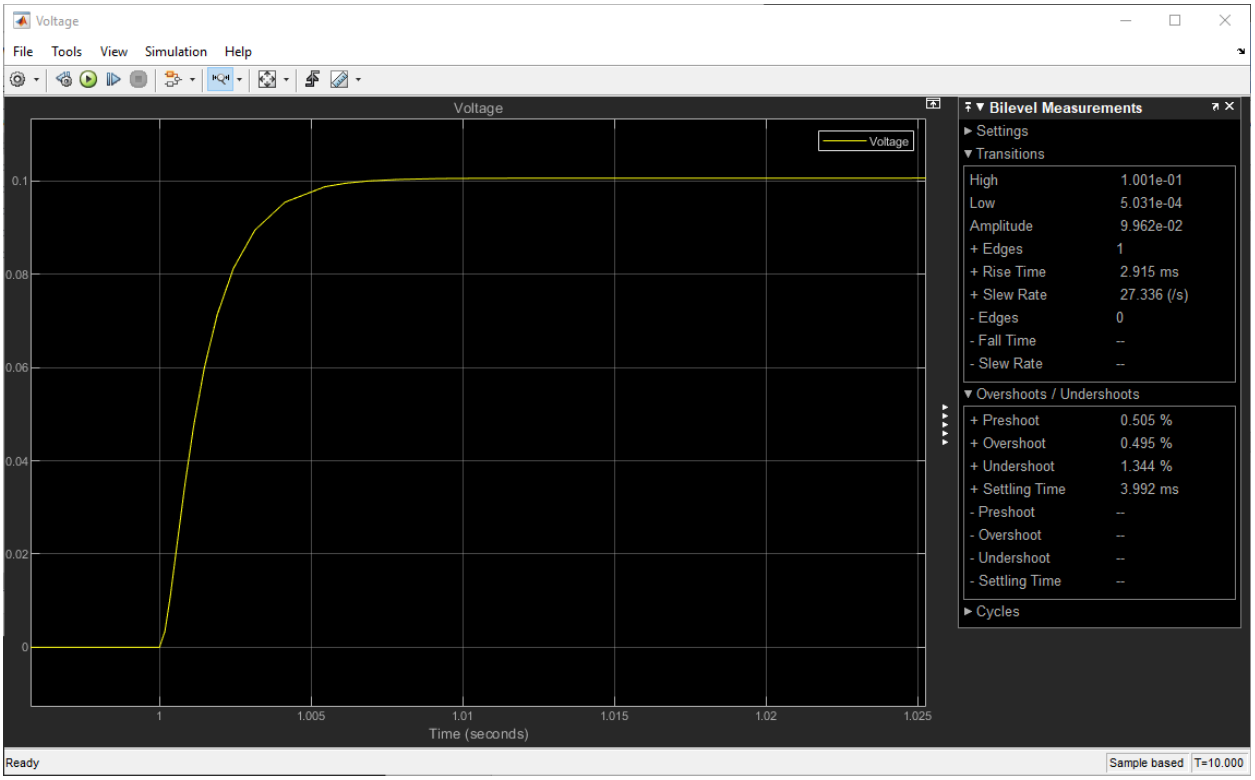


Figure 6 Pre-tuned response

The final values of the model were found as:

	Initial Estimate	*Final Tuned Value*
Kt	10.3	7.858333
Ke	10.9	7.858333
J	2.83E-05	2.83E-05
L	0.0048	0.00524
R	3.2	4
B	0.1	0.7811

The final transfer function is the same transfer function used in the onset of the project, but with the final tuned values:

$$G(s)=\frac{\omega(s)}{V(s)}=\frac{\frac{K_t}{LJ}}{s^2+\frac{(RJ+bL)}{LJ}s+\frac{Rb+K_eK_t}{LJ}}$$

Figure 7 Final Transfer Function

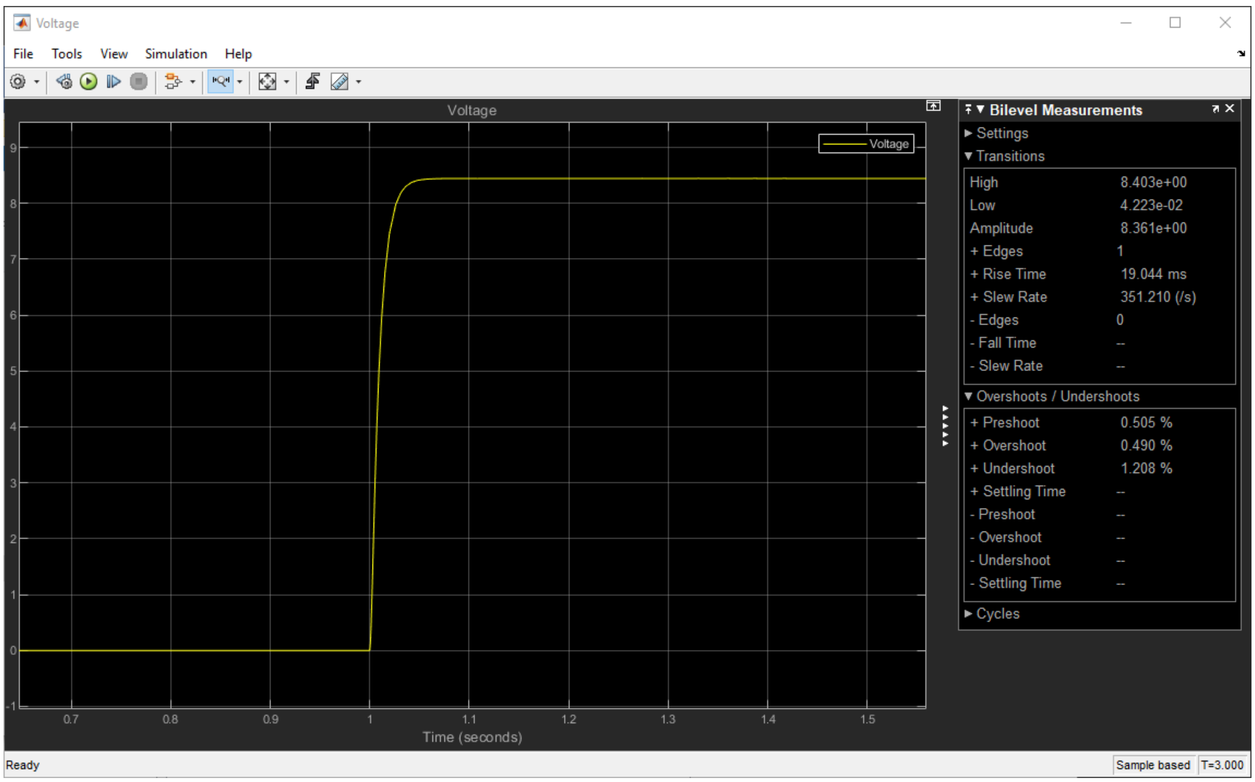


Figure 8 Tuned Response

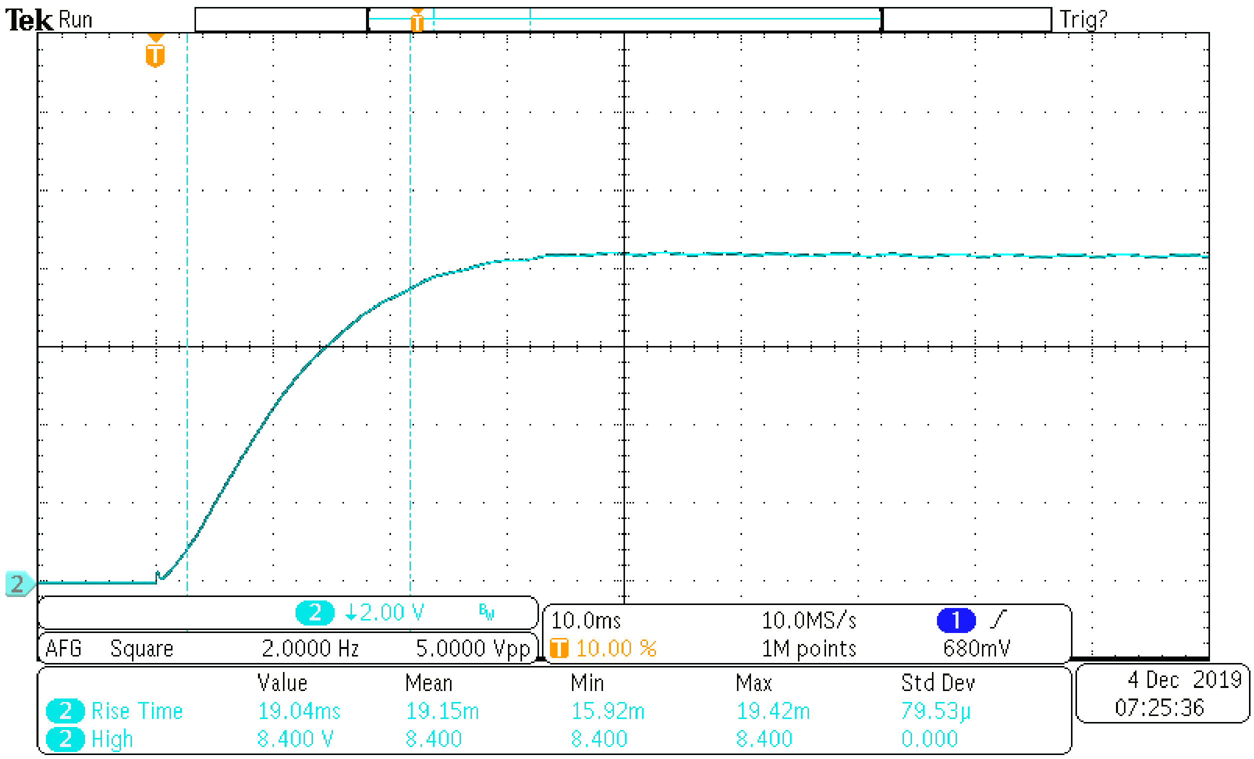


Figure 9 Actual measured motor response

Both have a 5 V step input to measure the response. The actual measured motor response, as seen in figure 9, has a Rise time close to the simulated response in the MATLAB simulation at 19.04 ms, observed in figure 8. The High value of 8.4 V measured in the MATLAB is on point with the actual measured response.

Design and Implement a Closed-Loop Speed Control With PI Compensator

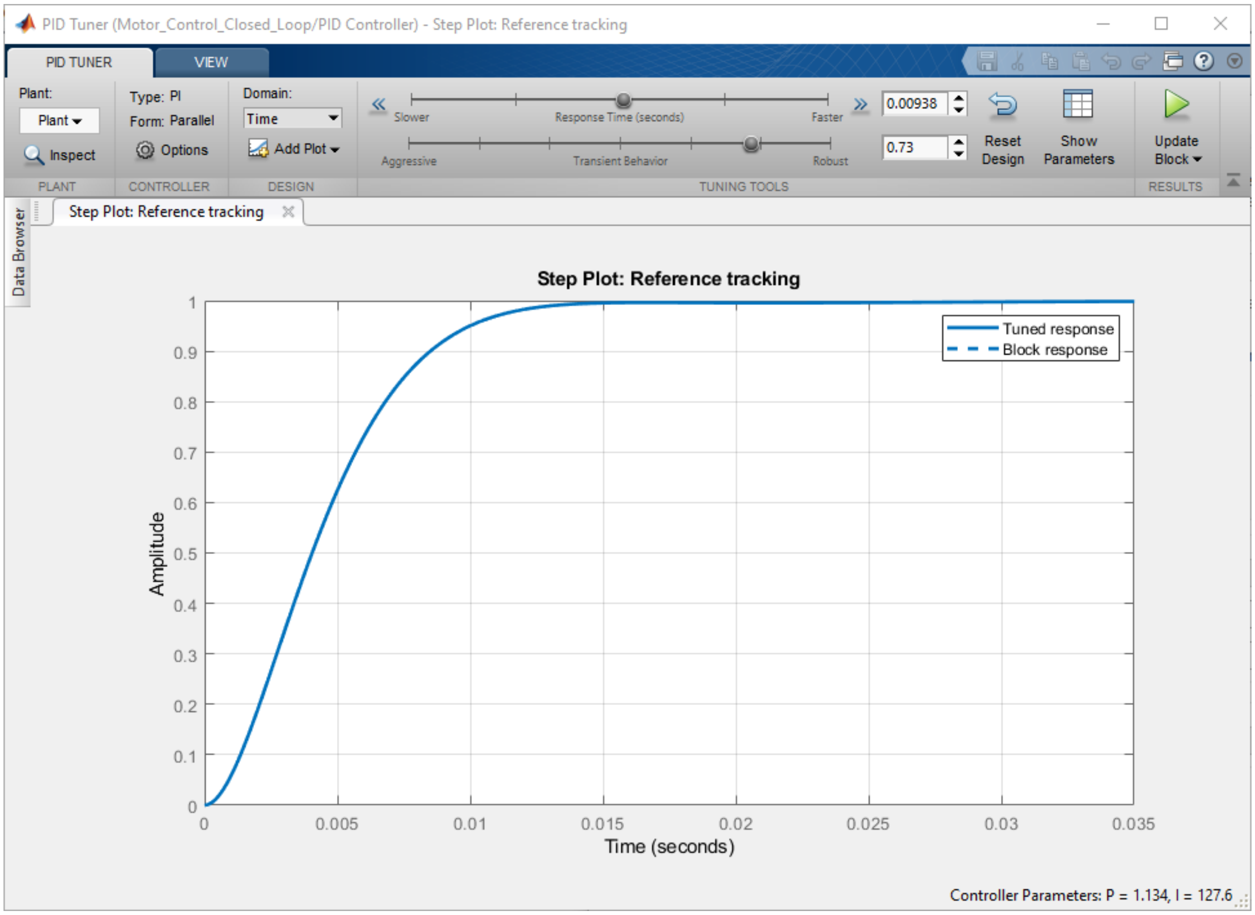


Figure 10 PID Tool with P and I value

TABLE 28.1 The effects of adding the P, I, and D terms to a controller.

Control Term	Rise Time	Overshoot	Settling Time	Steady-State Error
K_p	Decreases	Increases	No Change	Decreases
K_i	Decreases	Increases	Increases	Eliminates
K_d	No Change	Decreases	Decreases	No Change

Figure 11 Source: Carryer et al document

The Design of the PI control is outlined in the following figures:

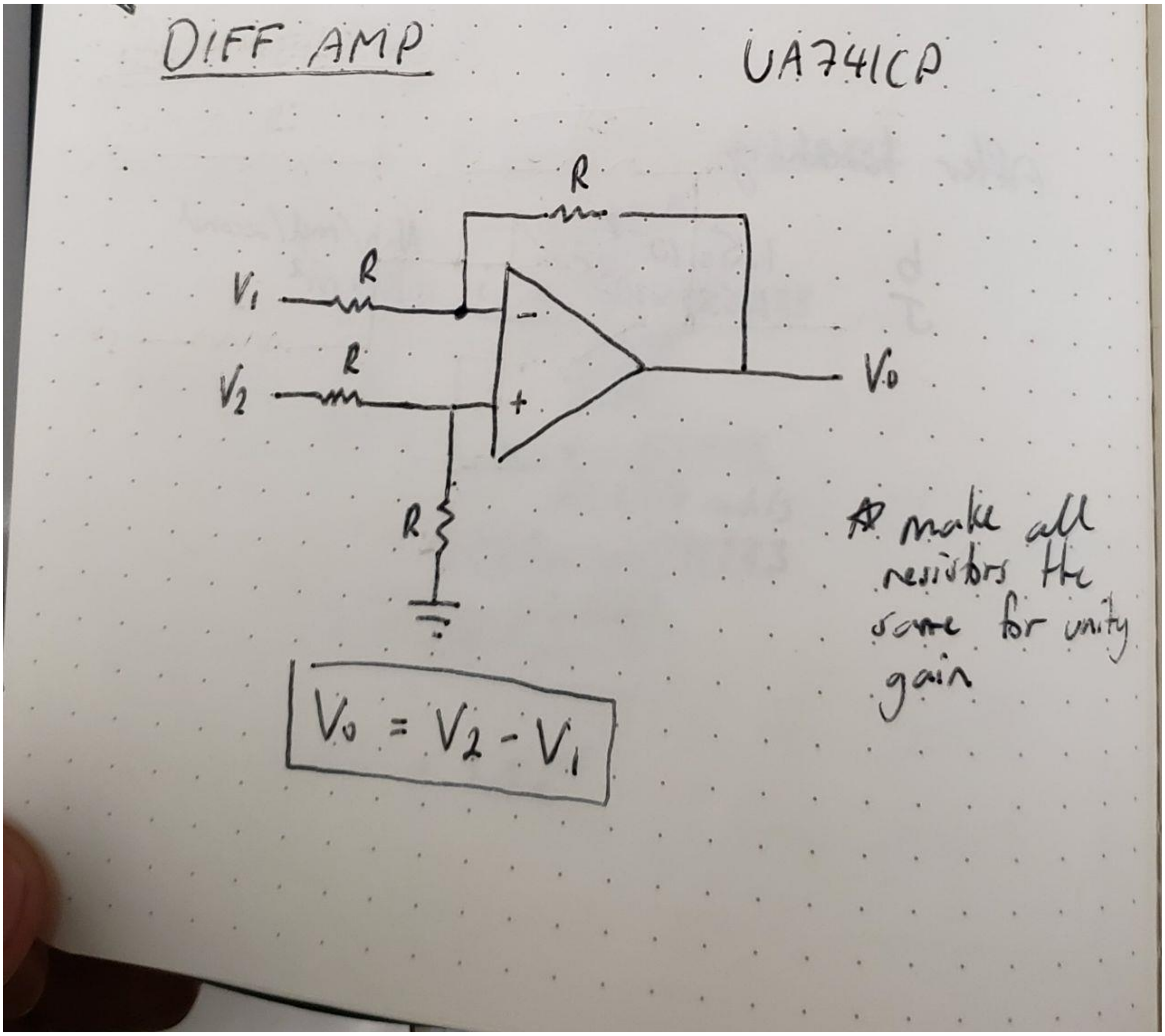


Figure 12 Difference Amplifier Design

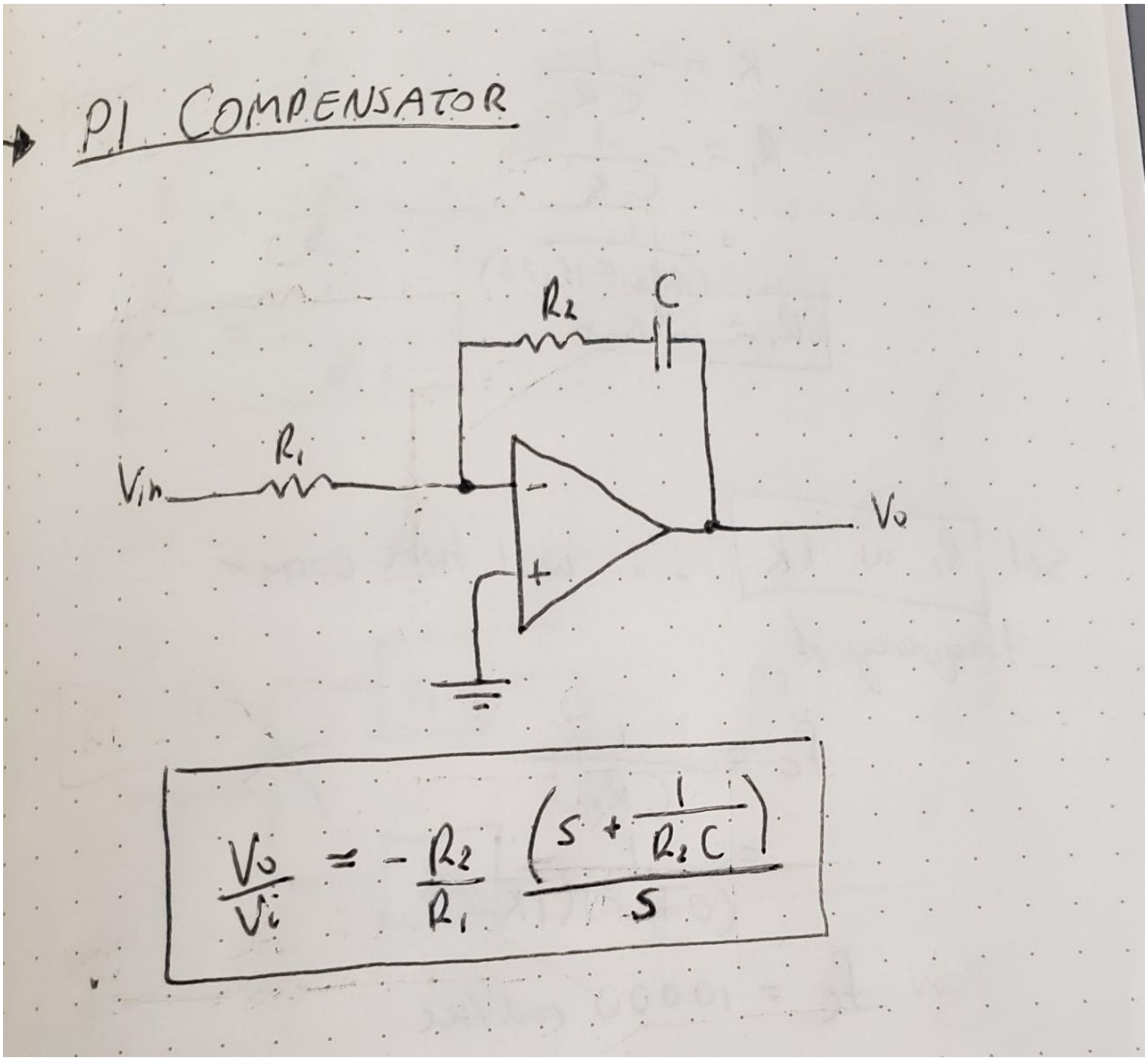


Figure 13 PI compensator Design

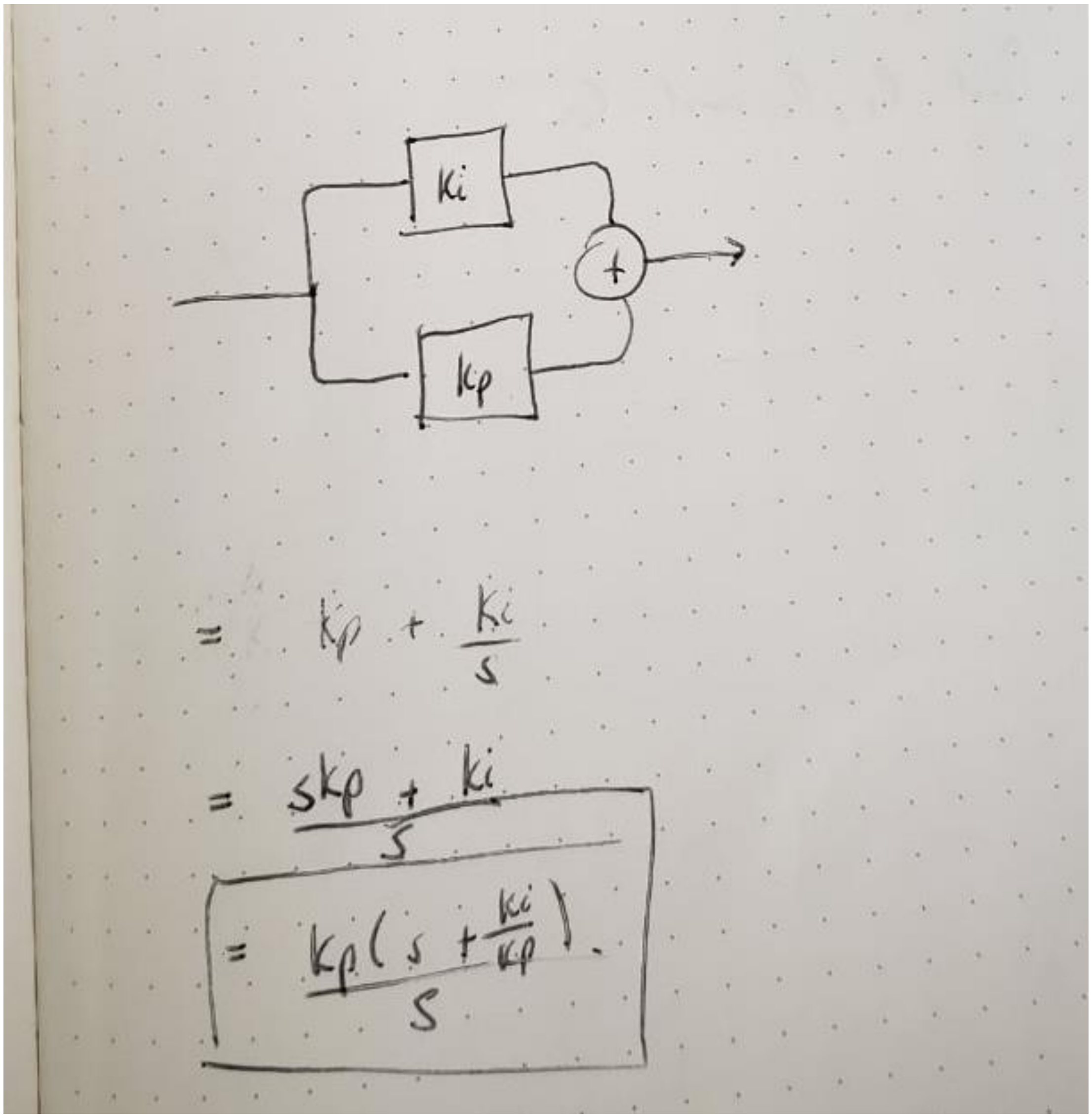


Figure 14 Implementation of the Ki and Kp

$P = 1.134$

$I = 127.6$

$$= \frac{1.134\left(s + \frac{127.6}{1.134}\right)}{s}$$

$$\boxed{PI(s) = 1.134\ \frac{(s + 112.52)}{s}}$$

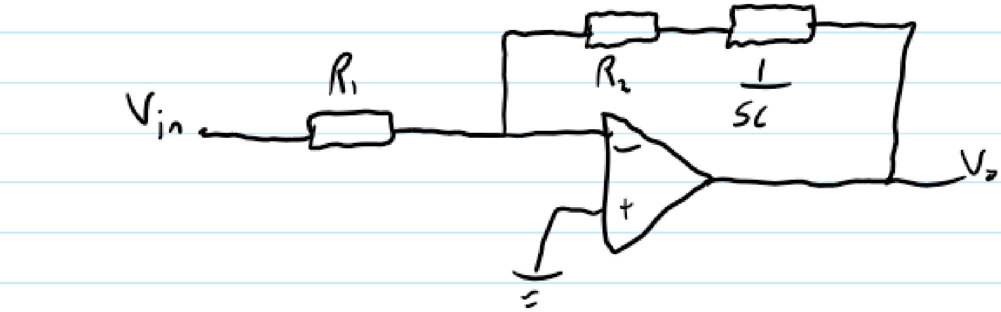
✓ Transfer Function

$$\boxed{\frac{V_o}{V_i} = -\frac{R_2}{R_1}\ \frac{\left(s + \frac{1}{R_2 C}\right)}{s}}$$

find R_1, R_2 and C

Figure 15 Implementation of the I and P values

Project 2: PI Control Design



$$H(s) = \frac{V_o}{V_i} = \frac{-R_2}{R_1} \frac{\left(s + \frac{1}{R_2 C}\right)}{s}$$

$$PI(s) = 1.134 \frac{(s + 112.52)}{s}$$

Design $H(s)$ to be $PI(s)$

① $1.134 = \frac{R_2}{R_1}$

② $112.52 = \frac{1}{R_2 C}$

Assume $C = 0.1\mu F$

Using ②: $R_2 = \frac{1}{112.52(0.1\mu)} = 88.873 k\Omega$

from ① $R_1 = \frac{(88.873k)}{1.134} = 78.371 k\Omega$

Figure 16 Overall PI design outline

Design of the Op amp circuit to realize the compensator and difference amplifier:

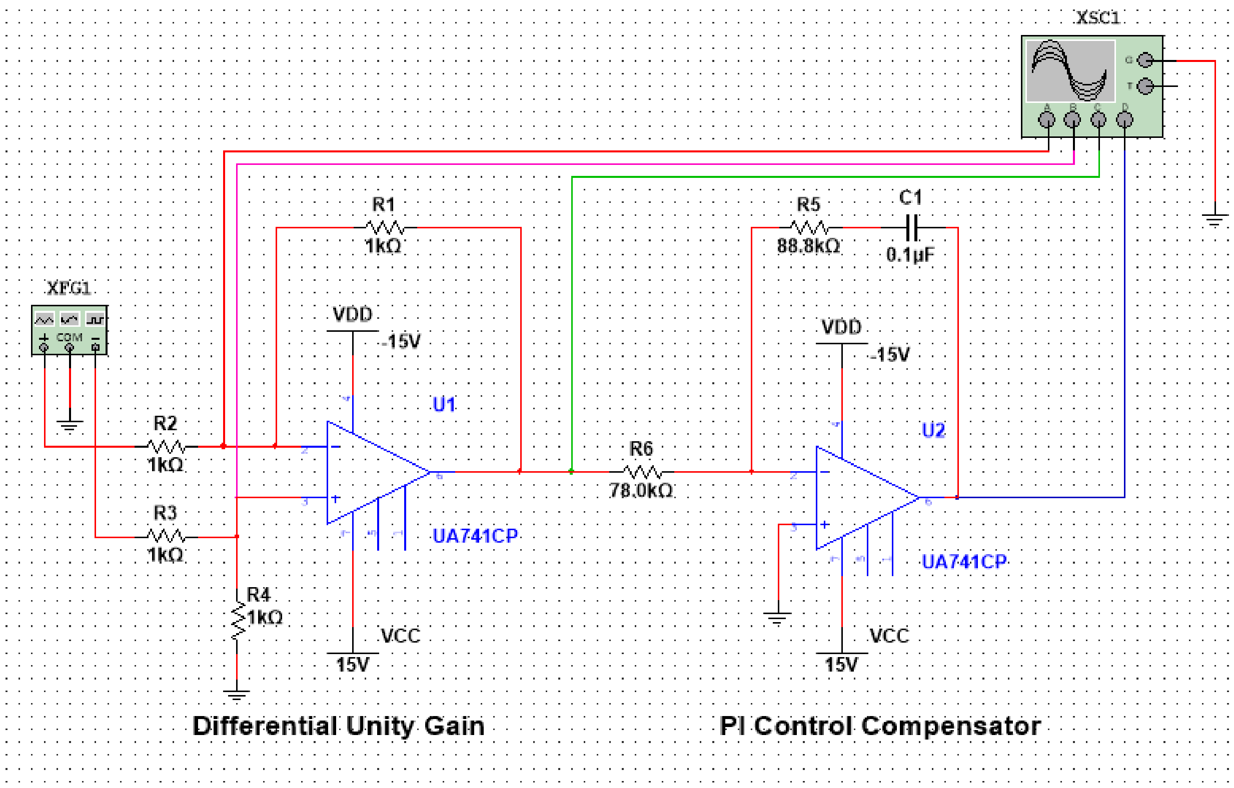


Figure 17 PI control Schematic without motor

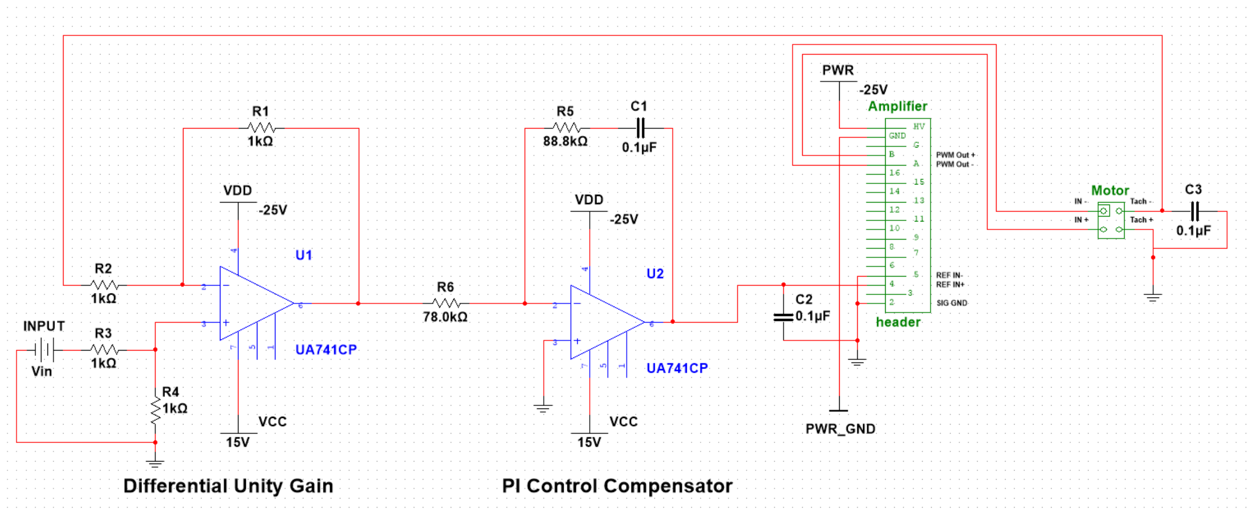


Figure 18 Full PI Schematic with motor and Amplifier modelled

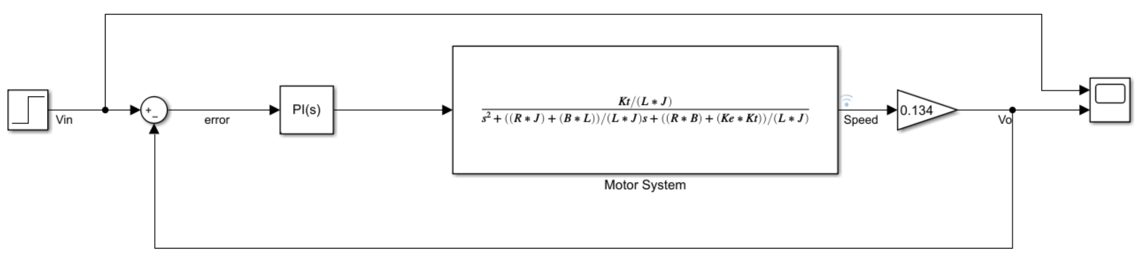


Figure 19 Simulink with PI controller

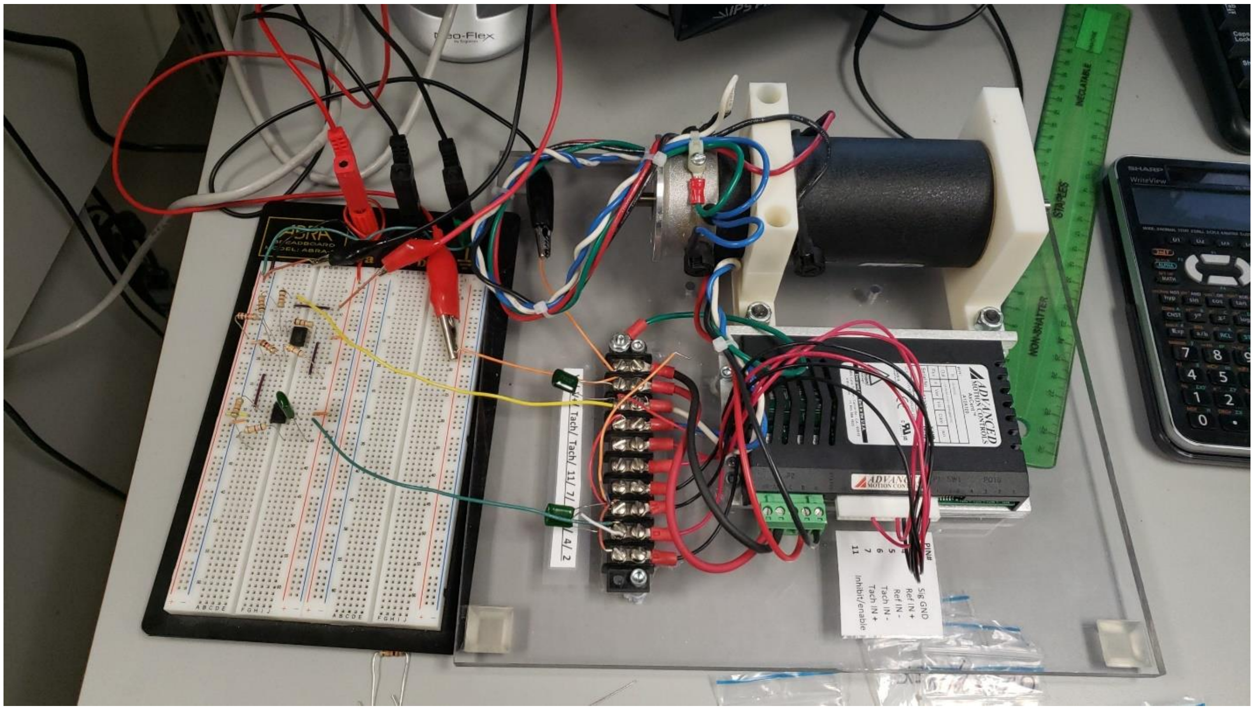


Figure 20 Final Working Motor Topology

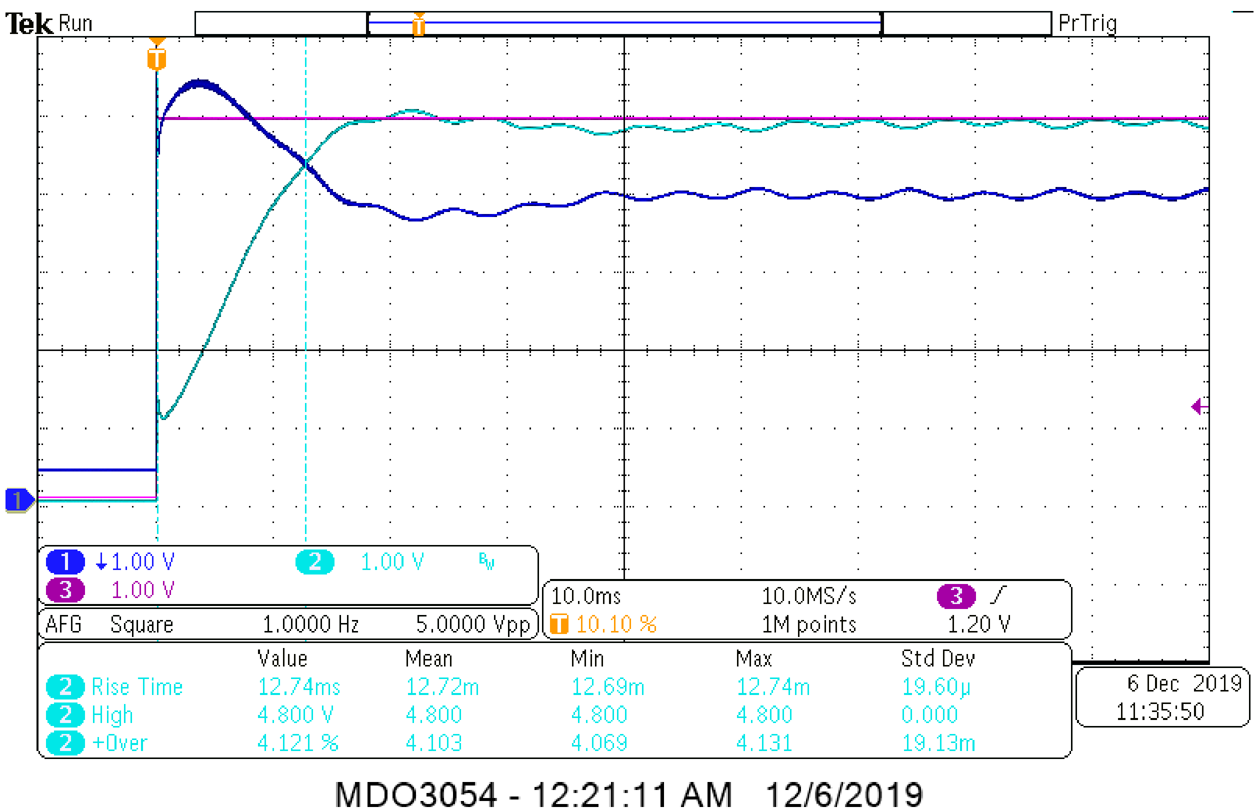


Figure 21 Steady State Error; Final result

The expected output in the actual Steady State Error must be the Pink, at 5 V step input. However, the output we got, coloured in Cyan in Figure 16, was at 4.8 V which was 0.2 V off, and that was our Steady State Error.

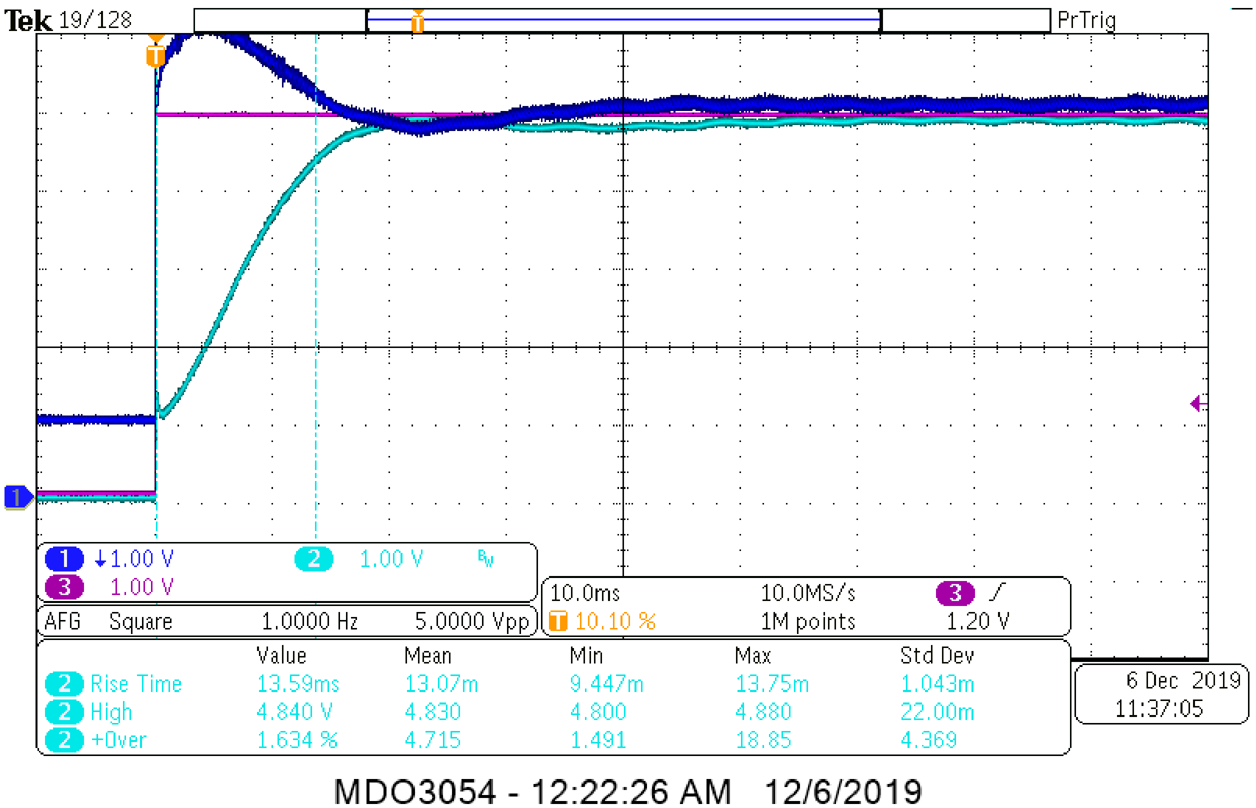


Figure 22 Steady State Error with load

Compared to the 5 V step with a load added, the compensator output, the dark blue line in Figure 17, shows that the compensator adds more current to keep the cyan output steady at 4.8V. In other words, the compensator is compensating for the addition of the load on the motor by increasing output to keep the motor output steady near the 5 V output.

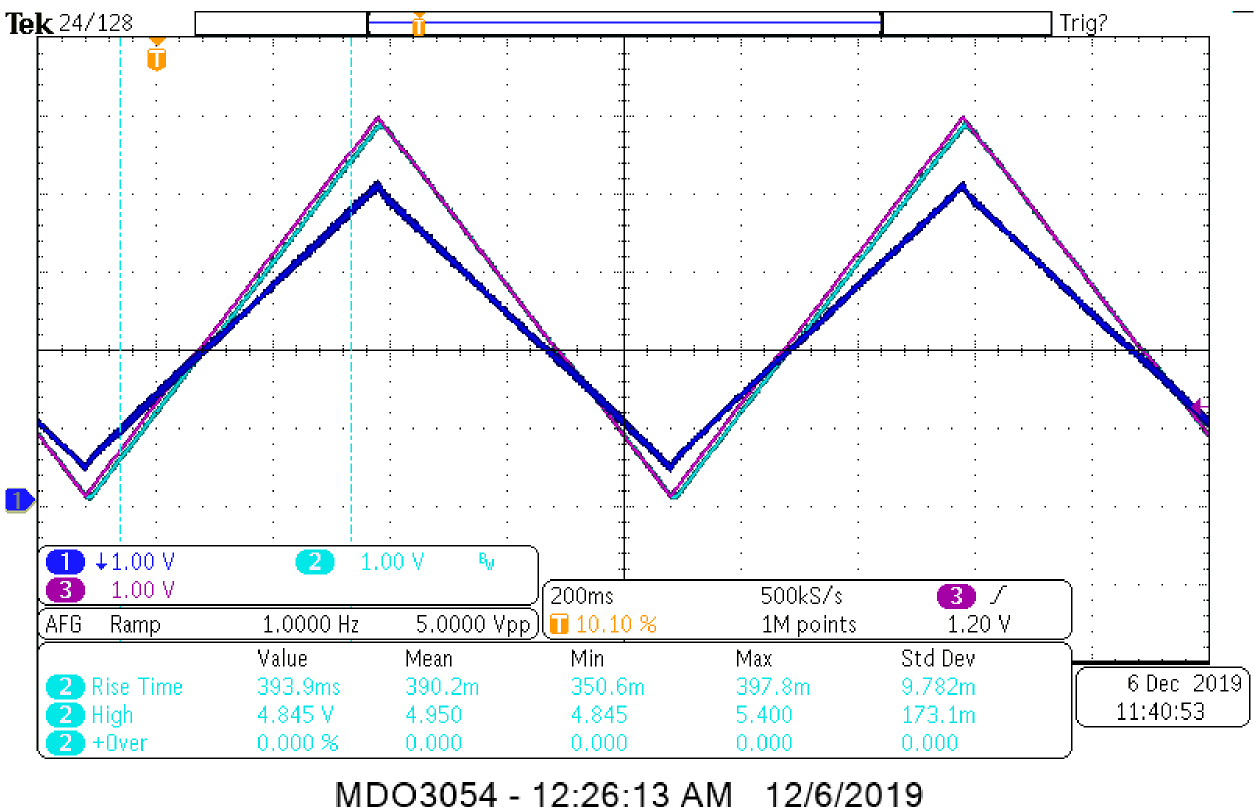


Figure 23 Tracking a ramp change

Shown in Figure 18, Pink is the 5 V step input. The compensator, in dark blue, is compensating the load on the motor to keep the output of the motor, in cyan colour, close to the 5 V input, but like the previous experimentation results, it is actually closer to 4.8 V.

Conclusions

We successfully and implemented an op-amp based PI controller. With our measuring and calculations in our design and simulation, were able to model the real controller with the simulated results on MATLAB and SIMULINK.