Relaxation in Globular Clusters

Jacob Kalnins

Numerical N-body simulation of stellar systems began in the 1960s[2] and can offer valuable insights into Globular Clusters, groups of a few hundred thousand stars with a radius of ~1-10pc[1]. The simulation calculates the force between each pair of particles then applies a Leapfrog integration scheme to update particle positions and velocities over a given timestep. This system allows up to ~1000 particles to be simulated over useful timescales (dependent on mass & configuration) which means globular clusters can be simulated well.

Initial Conditions

The initial conditions involve particles distributed according to a 3D Poisson Disk sampling[3] method. The particles begin at rest, and the simulation timestep is set to balance error with simulation time. The system initially collapses due to its excess gravitational potential energy (see **Fig. 1**). The system then settles over time to an approximate equilibrium state. It was assumed that the energy peak linearly correlates to relaxation time, t_{rel}.

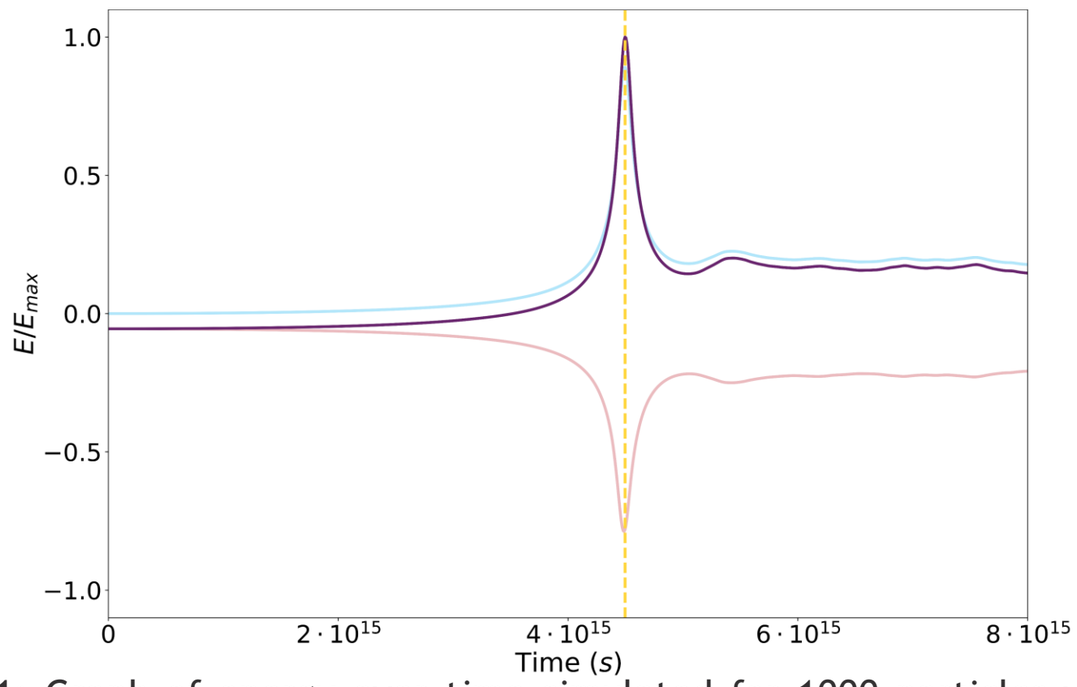

Fig 1: Graph of energy over time simulated for 1000 particles with blue and red lines showing kinetic and gravitational potential energy respectively. Yellow line marks minimum mean radius at $t = 4.504 \times 10^{15}$ s. Purple line shows virial energy, $2K_T + U_T$.

Modelling

We can express relaxation time using $\sigma = \sqrt{\langle v^2 \rangle / 3}$ and ρ the particle mass density, as[3]:

$$t_{rel} = \frac{\alpha \sigma^3}{G^2 m \rho \ln \Lambda} \quad (1)$$

Where α and Λ are dimensionless factors whose calculation complicates the problem. In particular, $\ln \Lambda$ is the Coulomb logarithm. Considering the half-mass relaxation time yields:

$$t_{rh} \approx \frac{\gamma N t_{dyn}}{\ln \lambda N} \quad (2)$$

Here, $t_{dyn} = r_{h.mass}^{3/2} / \sqrt{Gm}$ is the dynamical time and λN approximates Λ. The dependence of t_{rh} on particle mass m, and N was tested. The assumption was made that within the half-mass radius the particle density is constant, and outside the half-mass radius is negligible.

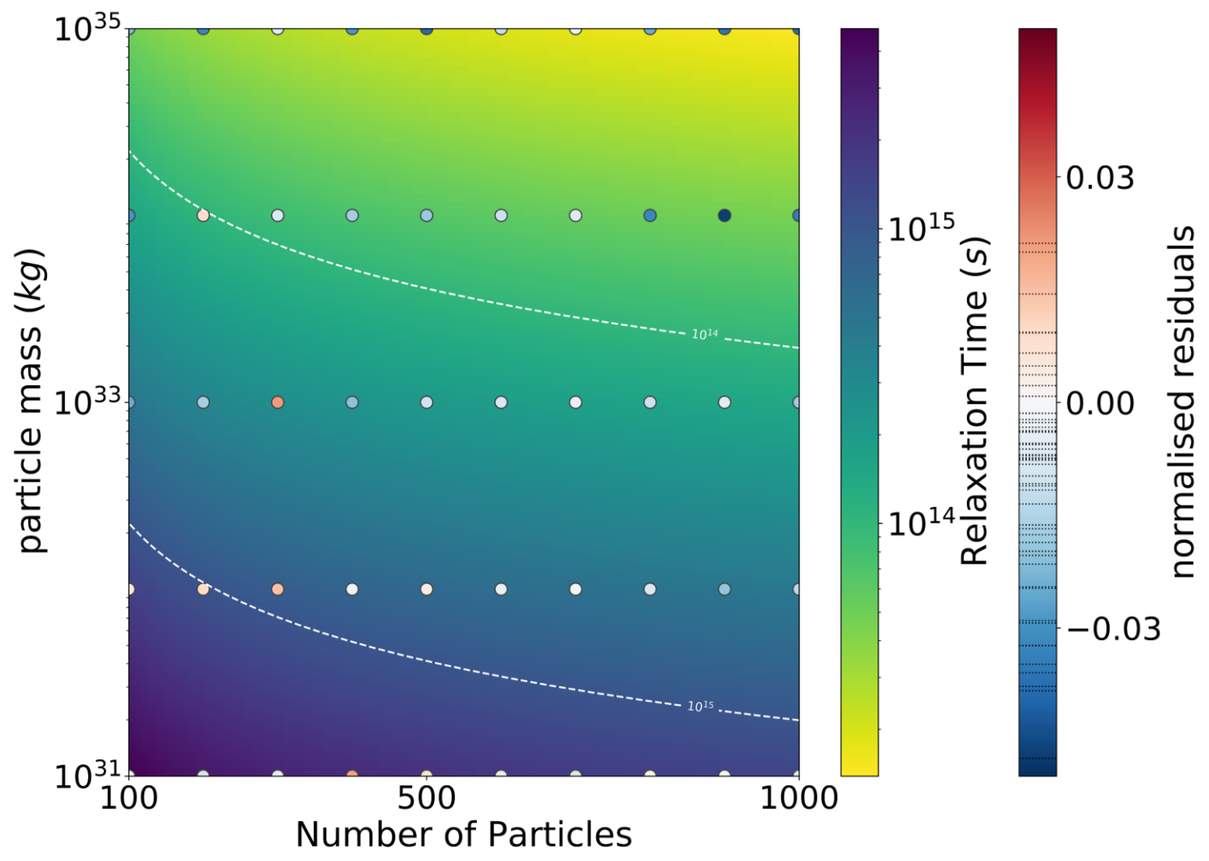

Fig 2: Heatmap of collapse time using observation model. Markers show simulated points, and their colours demonstrate the normalised residuals of the fit. Contour lines show lines of constant collapse time. The model minimised χ_ν^2 at $t_c = 2.0(2) \times 10^{31} m^{-0.502(1)} n^{-0.530(1)}$ with a mean normalised residual of 0.0146.

Results & Errors

The simulations were fitted to two models. The first was suggested observationally to be of the form $t = am^{-b}n^{-c}$ (Model I). The second followed **Eq. 2**: $t = am^{-b}n / \ln cn$ (Model II). Model I (**Fig 2**) was accurate, with $\left(\chi^2_{\nu,min}\right)_I = 4.63$. Model II was found to be inaccurate with large $\left(\chi^2_{\nu,min}\right)_{II}$. However, Giersz & Heggie's (1994)[5] value of λ was found to be accurate ($\lambda_{II} = 0.113(2)$). Although χ_I^2 is small, the model fit may be inaccurate for high masses. To keep conditions constant, the timestep was not varied, which became an issue at high mass & particle number due to the smaller number of steps needed to simulate the system, so uncertainty in these parameters may be underestimated.

Top image: Globular Cluster NGC 1866 – ESA/Hubble and NASA
[1] Talpur, J., 1997, A Guide To Globular Clusters, Keele University, retrieved 4th Feb 2020
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[3] Cook, R.L., 1986. Stochastic sampling in computer graphics. ACM Transactions on Graphics (TOG), 5(1), pp.51-72.
[4] Meiron, Y. and Kocsis, B., 2018. Diffusion and mixing in globular clusters. The Astrophysical Journal, 855(2), p.87.
[5] Giersz, M. and Heggie, D.C., 1994. Statistics of N-body simulations–I. Equal masses before core collapse. M.N.R.A.S., 268(1), pp.257-275.