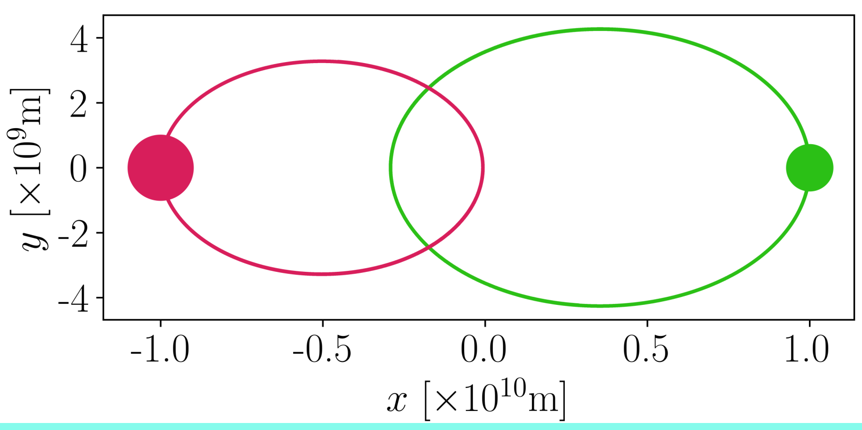
N-body Simulations of Gravitational Systems Using a Leapfrog Integration Method



2-body simulation of similar masses orbiting a common barycentre. (Solvable)

With N>2, there is no analytical solution for the movement of a system acting under gravity. Therefore we must use numerical methods to study system properties.

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Loop over n timesteps:
   Loop over n particles:
      Calculate force from
      all other particles
      Integrate to find
      position and velocity
      Calculate energy
```

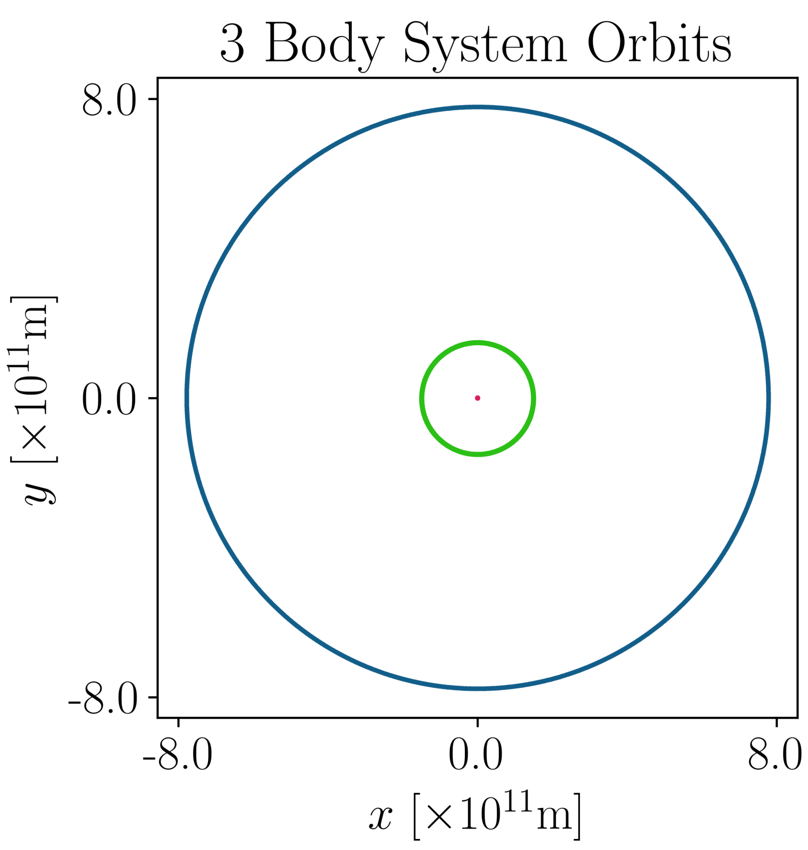
The code can be optimised due to equal & opposite force, but still requires $O(n^2)$ time. Optimisation requires complex computation (e.g. the Barnes-Hut[1] method).

Force on a particle:

$$\boldsymbol{F}_i = -G\sum_{j\neq i} m_i m_j \frac{\boldsymbol{r}_i - \boldsymbol{r}_j}{|\boldsymbol{r}_i - \boldsymbol{r}_j|^3}$$

Velocity & Position Updater:

Velocity is calculated out of step with position (at $t_0 \pm dt/2$); this is called a Leapfrog integration method. It's a 2nd order method



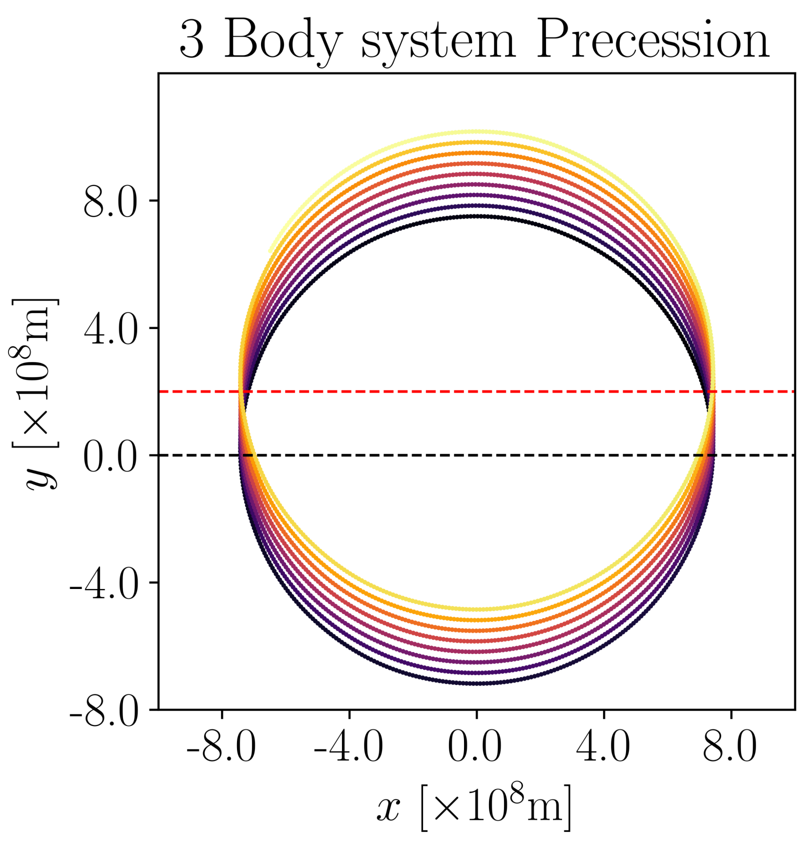
Sun-Earth-Jupiter system orbits over 100yrs.

Sun / Earth / Jupiter

- Simulating the 3-body system of the Earth, Sun, and Jupiter over 100yrs with a timestep of 1 day
- Energy conservation was slightly better than 1%

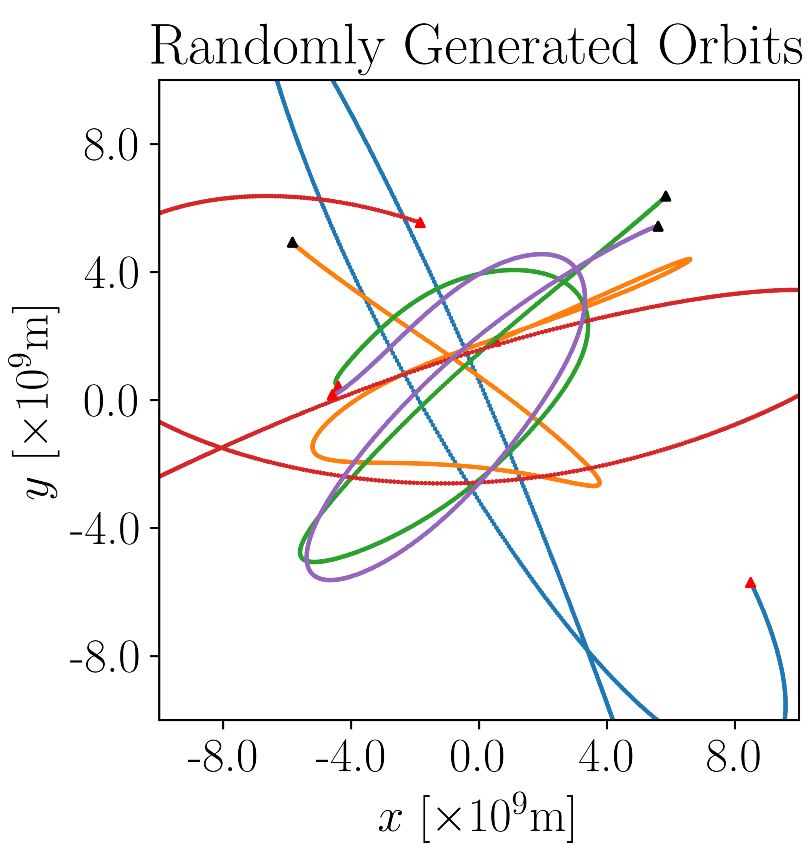
Precession of orbits can be observed, as system is not stable over longer time periods

- Initial conditions calculated as if Sun and Jupiter were a 2-body system



Precession of Solar orbit (100yrs). Black line shows starting position, red line shows end position.

Extension



Particles generated in 2D plane with equal masses simulated for 10^7 s

[1] Barnes, J., Hut, P. A hierarchical O(N log N) force-calculation algorithm. Nature 324, 446–449 (1986) doi:10.1038/324446a0

[2] Dehnen, W. (2001). Towards optimal softening in three-dimensional N-body codes - I. Minimizing the force error. Monthly Notices of the Royal Astronomical Society, 324(2), pp.273-291.