Relaxation in Globular Clusters

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Globular clusters are stellar clusters of a few hundred thousand stars with a half-light radius of ~1-10pc[1]. Numerical (N-body) simulation of these clusters began in the 1960s[2] and offers valuable insights into stellar systems, dependent on initial conditions. The simulation calculates the force between each pair of particles then applies a 2nd-order Leapfrog integration scheme to update particle positions and velocities. This format of the simulation allows up to ~1000 particles to be run over useful timescales, dependent on mass & initial configuration, which means globular clusters can be simulated well.

Initial Conditions

The simulation is begun with bodies at rest in a sphere, distributed according to a 3D Poisson Disk sampling[3] method. The system is evolved over time, with a timestep set so that the error term is small. As the system is not in equilibrium, it will collapse due to its excess gravitational potential energy (see **Fig. 1**). The system then quickly settles to an approximate equilibrium state. The relationship between the particle mass & number and this energy peak (and the corresponding mean radius minimum) was investigated.

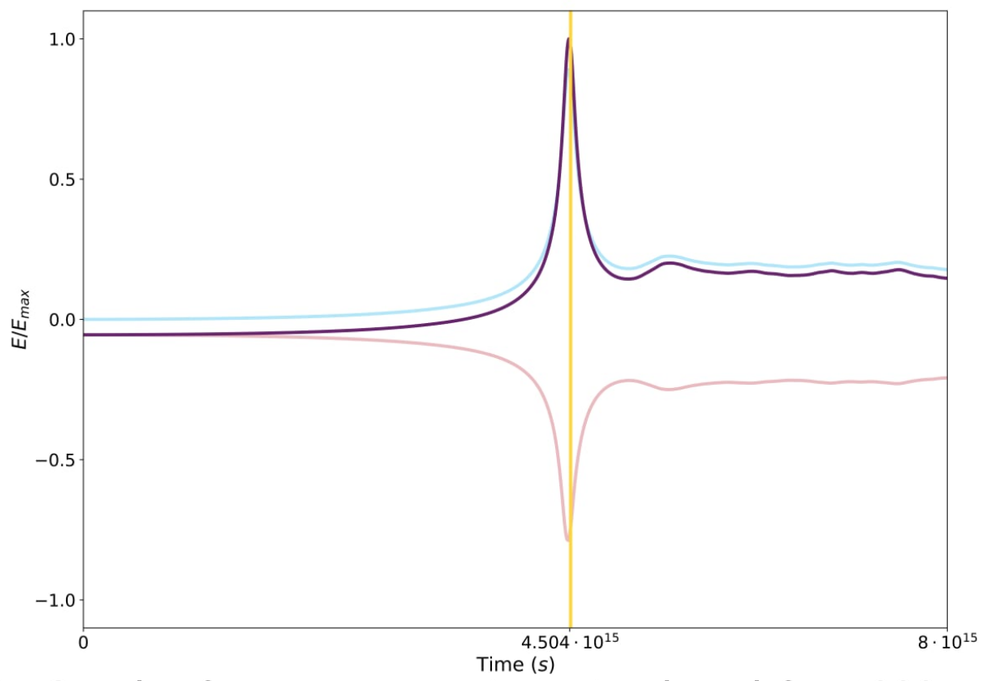

Fig 1: Graph of energy over time simulated for 1000 particles with blue and red lines showing kinetic and gravitational potential energy respectively. Yellow line marks minimum mean radius. Purple line shows virial energy, $2K_T + U_T$.

Models

By making several assumptions, we can approximate relaxation time, using $\sigma = \sqrt{\langle v^2 \rangle / 3}$ and ρ the particle mass density[3]:

$$t_{rel} = \frac{\alpha \sigma^3}{G^2 m \rho \ln \Lambda} \quad (1)$$

Where α and Λ are dimensionless factors which complicate the problem. In particular, $\ln \Lambda$ is the Coulomb logarithm. Considering the half-mass relaxation time yields:

$$t_{rh} = \frac{\gamma N t_{dyn}}{\ln \lambda N} \quad (2)$$

Here, $t_{dyn} = r_{half}^{3/2} / \sqrt{Gm}$ is the dynamical time and λ is a factor . Simulations of varying particle mass and number tested this formula. The assumption was made that within the half-mass radius the particle density is constant, and outside the half mass radius is negligible. The energy peak was assumed to correlate with collapse time.

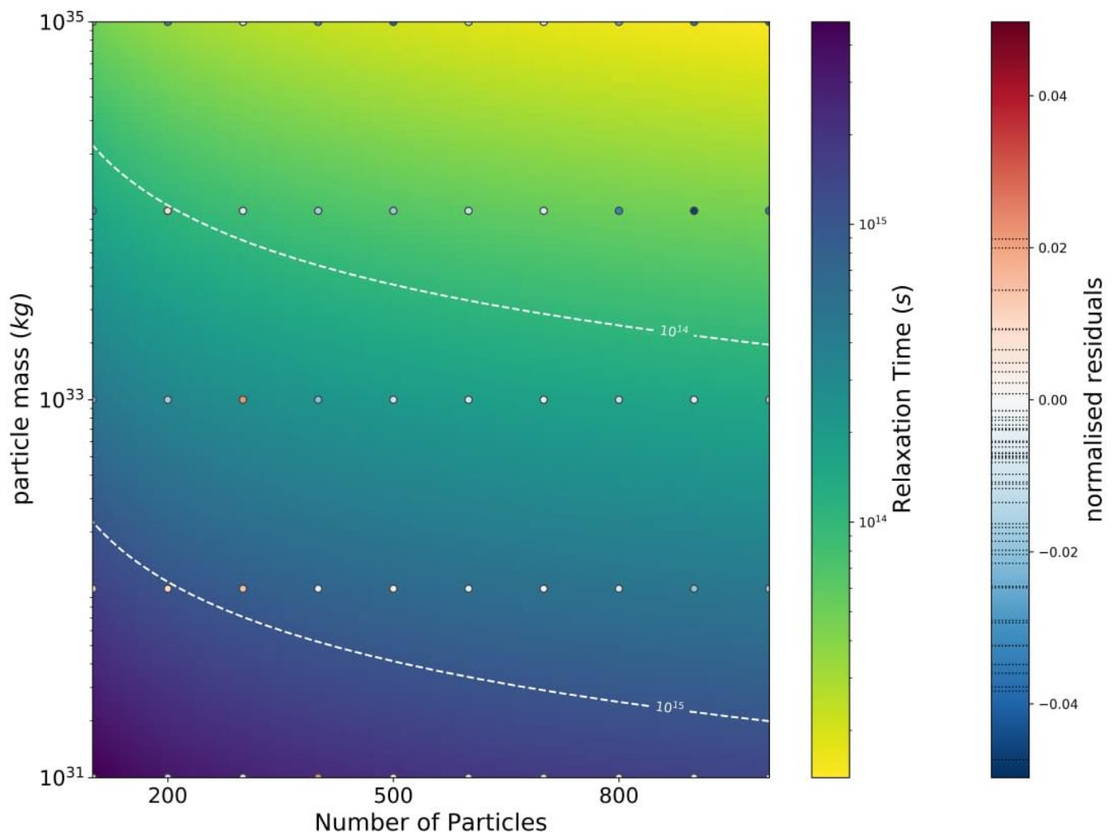

Fig 2: Heatmap of collapse time using observation model. Markers show simulated points, and their colours demonstrate the normalised residuals of the fit. Contour lines show lines of constant collapse time. The model minimised χ_ν^2 at $t_c = 2.0(2) \times 10^{31} m^{-0.502(1)} n^{-0.530(1)}$ with a mean normalised residual of 0.0146.

Results & Errors

The simulations were fitted to two models. The first was suggested observationally to be of the form $t = am^{-b}n^{-c}$ (Model I). The second followed **Eq. 2**: $t = am^{-b}n / \ln cn$ (Model II). I (**Fig 2**) was accurate, with $\left(\chi^2_{\nu,min}\right)_I = 4.63$. II was found to be inaccurate with large $\left(\chi^2_{\nu,min}\right)_{II}$. However, Giersz & Heggie's (1994)[5] value of λ was found to be accurate ($\lambda_{II} = 0.113(2)$).

Although χ_I^2 is small, the model fit may be inaccurate for high masses. To keep conditions constant, the timestep was not varied, which became an issue at high mass & particle number due to the smaller number of steps needed to simulate the system, so uncertainty in these parameters may be underestimated.

Top image: Globular Cluster NGC 1866 – ESA/Hubble and NASA

[1] Talpur, J., 1997, A Guide To Globular Clusters, Keele University, retrieved 4th Feb 2020

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[3] Cook, R.L., 1986. Stochastic sampling in computer graphics. ACM Transactions on Graphics (TOG), 5(1), pp.51-72.

[4] Meiron, Y. and Kocsis, B., 2018. Diffusion and mixing in globular clusters. The Astrophysical Journal, 855(2), p.87.

[5] Giersz, M. and Heggie, D.C., 1994. Statistics of N-body simulations–I. Equal masses before core collapse. M.N.R.A.S., 268(1), pp.257-275.