Pitch Class Sets

Paul Nelson - 2/27/2003

1 Pitch Class Sets

1.1 Pitches

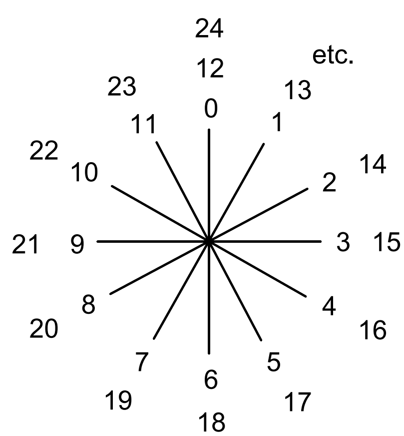
- A "pitch" is any note that we hear.
- The standard piano can play 88 pitches: A0 to C8
- C4 = middle-C
- The notes above middle-C: C4 (B♯3), C♯4(D♭4), D4, D♯4(E♭4), E4(F♭4), F4(E♯4), F♯4(G♭4), G4, G♯4(A♭4), A4, A♯4(B♭4), B4(C♭5)

1.2 Pitch Classes

- Used to discuss pitches *independent of octave displacement and enharmonic spelling*
- Any two pitches which sound the same or are only different due to octave displacement are said to belong to the same "pitch class"
- For example: C0, C1, C2, C3, C4 . . . and B#4, B#5, . . . and D♭♭4, D♭♭5 . . . etc. all belong to the same class of pitches: "Pitch Class C"
- There are only 12 pitch classes
- Pitch classes can also be numbered: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11
  (also called a "Pitch Class Representative")
- 0 = Pitch Class **C** (i.e. "fixed Do")
- C = 0, C# = 1, D = 2, D# = 3, E = 4, F = 5, F# = 6, G = 7, G# = 8, A = 9, A# = 10, B = 11
- The numbers represent the number of half-steps each pitch class is away from C
- Sometimes the letter 'T' or 'A' is used instead of the number 10, and 'E' or 'B' instead of 11

1.3 "Clock" Math or Modulo Math

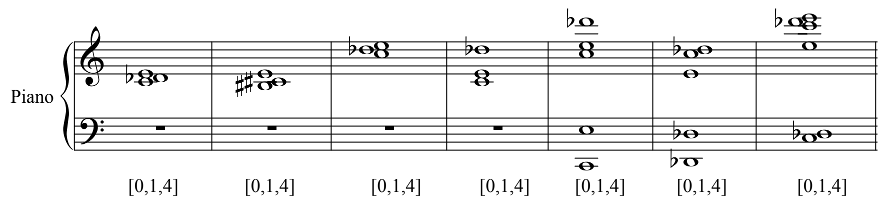
- The "modulo" operator takes the remainder of an integer divided by some other integer
- For example: 19 modulo 12 = 7
  (i.e. 12 goes into 19 once, with 7 left over)
- Pitch class sets use "modulo 12" a lot
- Can be visualized using a clock face
- Some interesting characteristics of the clock face:
  - A tritone is made up of two notes which are opposite of each other (for example: C = 0 and F# = 6)
  - The notes of a cross make up a doubly-diminished 7th chord (for example: C = 0, D# = 3, F# = 6, A = 9)
  - An augmented triad (C = 0, E = 4, G# = 8) is also pleasingly symmetric



1.4 Pitch Class Sets

- Is simply a list of pitch class numbers: [0, 4, 7, 10]
  (note the square brackets)
- Also called: "PC Set"
- C minor triad: [0, 3, 7]
- G major triad: [7, 11, 2]
- Note: Octave doublings and displacements are ignored:
  - [0, 3, 7, 12] => [0, 3, 7]
  - [14, 7, 11] => [2, 7, 11]

- For example, all of the following can be described with Pitch Class Set [0, 1,4]



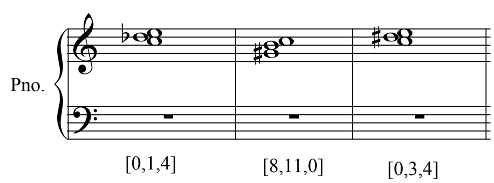
- Sometimes shown without the commas: [037]
  (here is where A=T=10 and B=E=11 comes in handy, for example: [0,4,7,10] = [047T] = [047A]

1.5 Transposing Pitch Class Sets

- To transpose a pitch class set, add (or subtract) the same number to all elements of the list:
  [0,1,4] => (transpose up a major third) [0+4, 1+4, 4+4] => [4,5,8]
- Remember to use "Module 12" when numbers are greater than or equal to 12:
  [0,1,4] => (transpose up a major 7th) [0+11, 1+11, 4+11] => [11, 12, 15] => [11, 0, 3]

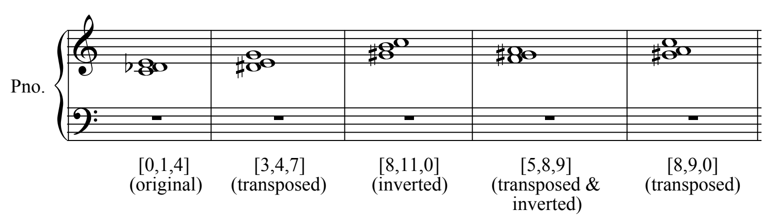
1.6 Inverting Pitch Class Sets

- To invert a PC Set, subtract each element of the list from 12:
  [0,1,4] => [12 - 0, 12 - 1, 12 - 4] => [12, 11, 8] => [0, 11, 8]
- By convention, a simple inversion is always around Pitch Class C (0)
- Very often you will want to invert and transpose at the same time:
  [0,1,4] => [(12-0) + 4, (12-1) + 4, (12-4) + 4] => [16, 15, 12] => [4, 3, 0]
  This has a special notation: T4I (invert and then transpose up 4 half steps)



1.7 Similar Pitch Class Sets: Set Classes & Prime Forms

- Some pitch class sets are very similar, for example: [0,1,4] => [3,4,7] (transposition)
  [8,11,0] (inversion) [5,8,9] (transposition and inversion) [8,9,0] (transposition)

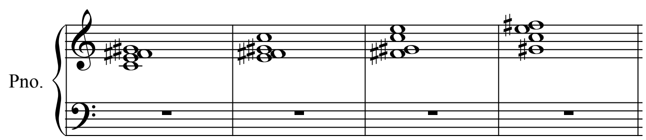


- A group of similar PC Sets like these is called a "Pitch Class Set Class"
  Or more simply, a "Set Class"
- If two PC Sets differ only by transposition or inversion, then they belong in the same Set Class.
- There are only 208 different Set Classes!
- Each one is represented by a "Prime Form" PC Set. For example:
  [0,1,4]; [3,4,7]; [0,3,4]; [5,8,9]; and [8,9,0] all belong to the Prime Form: (0,1,4)
- Note that parenthesis are used to denote Prime Forms
  (Note: There is disagreement on this syntax)

1.8 Determining The Prime Form

- Goal: Find the most reduced version of the PC Set. "Most reduced" means that it is packed as tightly as possible, and as far to the left as possible.
- Example: What is the prime form of [8,0,4,6] ?
- Step 1: Put the Pitch Classes in numerical order => [0,4,6,8]
- Step 2: Determine which "Rotation" of the PC Set has the minimum distance between the first and last numbers in the PC Set. To rotate a PC Set, simply move the first number to the end and add 12 to it (i.e. shift it up an octave).

  The Rotations of [0,4,6,8] are:
  - `[0,  4,  6,  8]  =>  ( 8 - 0) =  8`
  - `[4,  6,  8, 12]  =>  (12 - 4) =  8`
  - `[6,  8, 12, 16]  =>  (16 - 6) = 10`
  - `[8, 12, 16, 18]  =>  (18 - 8) = 10`



[0,4,6,8] [4,6,8,12] [6,8,12,16] [8,12,16,18]

(note: numbers over twelve shown to demonstrate rotation, these should be reduced to 0-11 with "Modulo 12")

  There is a tie! Versions [0,4,6,8] and [4,6,8,12] both have a minimum distance between first and last of 8

- Step 3: If there is a tie, use the rotation which has a minimum distance between the first and second-to-last numbers:

  Distances between the first and second numbers:
  - `[0,  4,  6,  8]  =>  ( 6 - 0) =  6`
  - `[4,  6,  8, 12]  =>  ( 8 - 4) =  4`

- So, in our example, [4,6,8,12] is preferred.
- Step 4: If there is still a tie, then look for the minimum distance between the first and third-to-last numbers. (and so on)
- **The PC Set at this point is in "Normal" Form**
- Step 5: Transpose the pitch class set so that the first number is zero:
  [4 - 4, 6 - 4, 8 - 4, 12 - 4] => [0, 2, 4, 8]
- Step 6: Invert the pitch class set and reduce it using steps 1-5 above.
  - Invert [0,2,4,8] => [12-0, 12-2, 12-4, 12-8] => [12, 10, 8, 4] => [0, 10, 8, 4]
  - Put in numerical order: [0, 4, 8, 10]
  - Find the best rotation:

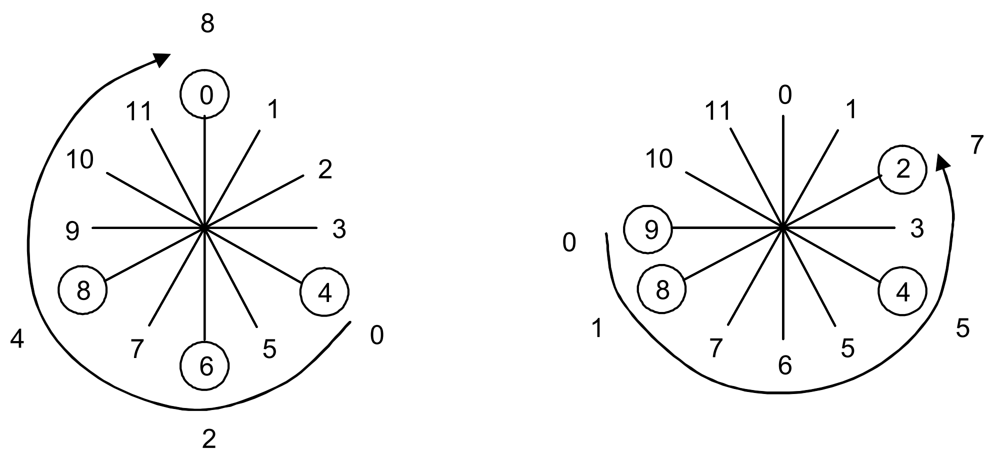
```
PC Set          (last-first)   (third-first)
[0,  4,  8, 10]      10              8
[4,  8, 10, 12]       8              6
[8, 10, 12, 16]       8              4      << Preferred
[10,12, 16, 20]      10              6
```

  - Transpose down: [8 - 8, 10 - 8, 12 - 8, 16 - 8] => [0, 2, 4, 8]
- Step 7: Which form, the original or the inverted, is most packed to the left (has the smallest numbers)? That will be the Prime Form.
  In this case, both forms produced the same Prime Form (the original chord was "inversionally symmetric"), and so the Prime Form is (0, 2, 4, 8)

1.9 Determining the Prime Form: Easier Ways

- **Option 1:** Figure it out on the piano
- Step 1: Keep rotating your chord until it is the minimum distance from the bottom note to the top note.
- Step 2: If there are ties, then use the rotation that has the notes most packed towards the bottom.
- Step 3: Check to see if the inversion is better packed.
- **Option 2:** Use the "Simplified Set List" at the back of *Post Tonal Theory* by Joseph N. Straus.
- **Option 3:** Use a MAX/MSP patch which displays the Prime form of a chord you play on your MIDI keyboard. See the URL: http://www.music.richmond.edu/amber/?handler=mumble .
- **Option 4:** Find the interval vector first, then look it up in the table (see below).
- **Option 5:** Don't bother with checking for the inversion (steps 6 & 7 above). Instead, just look for the inverted form in the table of prime and inverted pitch class sets.
- **Option 6:** Visualize the Pitch Class Set on a clock face and locate the prime form visually

Goals: 1) The shortest distance traveled around the clock.
2) Numbers packed as close to the starting point as possible.



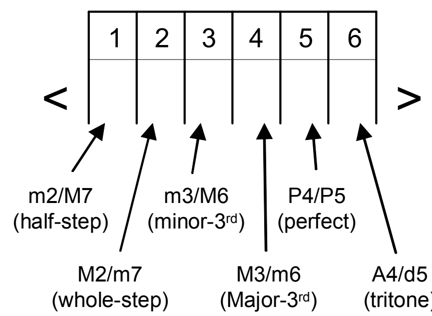
2 Interval Vectors

2.1 Pitch Intervals

- Is the standard definition of an interval between any two pitches.
- Ordered Intervals:
  - A3 to D5 is a Perfect 11^{th} (ascending); also +17 half steps
  - D5 to A3 is a Perfect 11^{th} (descending); also -17 half steps
- Un-ordered Intervals:
  - Is strictly a measure of distance.
  - The distance between A3 and D5 is a Perfect 11^{th}; also 17 half steps wide

2.2 Interval Classes

- Ordered Intervals:
  - Can only be from 0 to 11. Since Pitch Classes are independent of octave, there are no negative numbers.
  - Pitch Class A (9) to Pitch Class D (2) => (2+12) - 9 => 5 half steps
  - Pitch Class D (2) to Pitch Class A (9) => 9 - 2 => 7 half steps
- Unordered Intervals:
  - Is the minimum distance between two Pitch Classes
  - From Pitch Class A (9) to Pitch Class D (2) the minimum distance is 5 half steps
  - Can only be from 0-6.
  - Also called an "Interval Class"
  - P1 => 0 M3 / m6 => 4
  - m2 / M7 => 1 P4 / P5 => 5
  - M2 / m7 => 2 Tritone => 6
  - m3 / M6 => 3



2.3 Interval Vectors

- Is a measure of all of the interval content in a Pitch Class Set
- A count of all (unordered, pitch-class) intervals between all pairs of notes
- How to compute:
  - Step 1: Determine the distance between all pairs of pitch classes in the PC Set
  - Step 2: For each pair, increment the corresponding slot of the interval vector

Number of PCs	Number of Intervals		Number of PCs	Number of Intervals
2	1	|	7	21
3	3	|	8	28
4	6	|	9	36
5	10	|	10	45
6	15	|	11	55

  Formula: Number of Intervals = (N*(N-1)) / 2

11	10	9	8	7	
1	2	3	4	5	6
1	1	0	1	2	1

- Example: [0, 2, 6, 7]
  - 0, 2 => 2 0, 6 => 6 0, 7 => 5
  - 2, 6 => 4 2, 7 => 5
  - 6, 7 => 1

  Number of 1 Intervals: 1 Number of 4 Intervals: 1
  Number of 2 Intervals: 1 Number of 5 Intervals: 2
  Number of 3 Intervals: 0 Number of 6 Intervals: 1

- Total Interval Vector: <110121>
- No standard punctuation for interval vectors. Some use angle brackets, some simply list 6 digits (with no punctuation)

3 The Table of All Prime PC Sets

3.1 Structure of the table

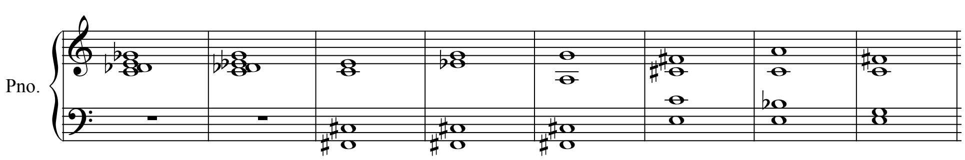
- Column 1: The interval vector
- Column 2: The count of PC Sets which reduce to the prime form
- Column 3: The Forte code (see below)
- Column 4: The Prime Form PC Set, as seen above
- Column 5: Inverted form (if different than the Prime Form)

- Grouped into 13 sections, one each for pitch class sets of a common number of elements (e.g. all sets with 5 pitch classes are grouped together)
- Within each group the list is sorted by interval vector. Interval vectors with the most half-step intervals are listed first, then vectors with the most whole-step intervals, etc.
- Z-related forms are listed together, one after the other (see below)
- Commonly known pitch class sets (e.g. well-known chord qualities, types of scales, etc.) are labeled inside of {curly braces}
- With the exception of the sets of 6 Pitch Classes, each set is listed opposite of its "complement". For example, set 4-16, (0,1,5,7) is listed to the left of set 8-16, (0,1,2,3,5,7,8,9). A set and its complement share many similar properties (see below for a discussion of Pitch Class Set complements).
- A = 10, B = 11, C = 12

3.2 Forte Names and Z-Related Sets

- Allen Forte's book, *The Structure of Atonal Music*, published the first version of this table. In his table, he labeled each prime form of the PC Set with a unique designation, such as 6-Z25.
- The first number (6-) is the number of pitch classes in the set.
- The second number (25) is a unique number given to the prime form, which was simply sequentially assigned.
- When analyzing PC Sets, many people will use the Forte designation, although simply writing out the prime form (e.g. (0,2,3,6,7,9) or (023679)) is becoming more common.

- The 'Z' indicates that the prime form is one of two which produce the same interval vector. 'Z' doesn't mean anything, it is just an identifier.
- Z related prime forms produce the same interval vector, but one can not be reduced to the other by inversion or transposition.
- Z-related sets are "close cousins" to one another, called a "Z Correspondent". They sound similar to each other, but not as similar as sets related by transposition or inversion.



4-Z15 [0,1,4,6] 4-Z29 [0,1,3,7] 4-Z15 [0,1,4,6] 4-Z15 [1,3,6,7] 4-Z29 [6,7,9,1] 4-Z15 [0,1,4,6] 4-Z29 [9,10,0,4] 4-Z29 [0,4,6,7]

3.3 Other Comments

- Surprisingly small number of interval vectors (200), prime forms (208) and chord qualities (351).
- Some famous PC Sets (how would you describe these chords otherwise?)

7-32: (0,1,3,4,6,8,9) Stravinsky: Rite of Spring

7-Z36: (0,1,2,3,5,6,8) Stravinsky: Rite of Spring

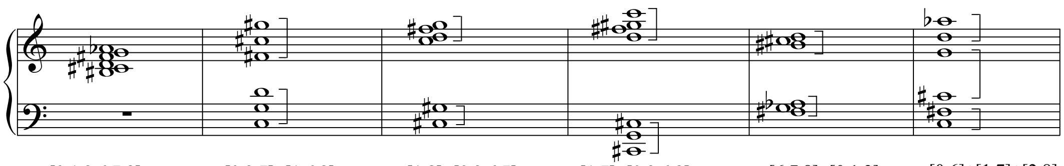
4-19: (0,1,4,8) Bernard Herrmann: "Psycho" Prelude

4 Subsets and Supersets

- Supersets: Combining two PC Sets to make a larger, more complex PC Set
- Subsets: Removing pitch classes from a larger PC Set to make smaller PC Sets
- Not much mathematics to help, but **an extremely useful compositional technique**
- Example:

Super set: (0,1,2,6,7,8)	= very dissonant
=> [0,2,7] + [1,6,8]	= quite consonant - two quintal/quartal chords
=> [0,2,6,7] + [1,8]	= unresolved Tt + 5^{th} (a dominant+tonic sound) + a simple fifth
=> [0, 2, 6, 8] + [1,7]	= whole-tone-scale-ish sound + tritone
=> [0,1,2] + [6,7,8]	= two chromatic clusters
=> [0,6] + [1,7] + [2,8]	= sets of tritones

- Things to experiment with:
  - Use sub-sets for growth; i.e. restrict sections of your music to use only portions of your larger PC set and then grow the PC set over time
  - Put the sub-sets in different registers to emphasize their unique sounds (see examples below)
  - Construct melodies from sub-sets



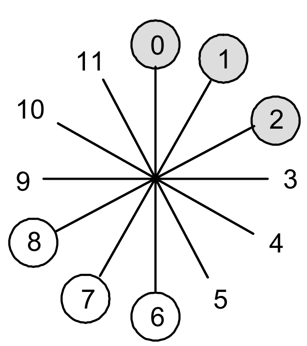
[0,1,2,6,7,8] [0,2,7]+[1,6,8] [1,8]+[0,2,6,7] [1,7]+[0,2,6,8] [6,7,8]+[0,1,2] [0,6]+[1,7]+[2,8]

4.1 Definition: Transpositional Combination

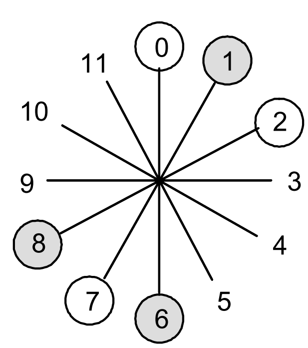
- When a superset is created by combining two copies of the same subset where one is transposed.
- Example: [0,1,2] + [6,7,8] => [0,1,2,6,7,8]
  => both [0,1,2] and [6,7,8] belong to the same prime form: (0,1,2)
- Second example: [0,2,7] + [1,6,8] => [0,1,2,6,7,8]
  => [0,2,7] and [1,6,8] belong to the same prime form: (0,2,7) and [1,6,8] = [0,2,7] + 6 half-steps

4.2 Definition: Inversional Combination

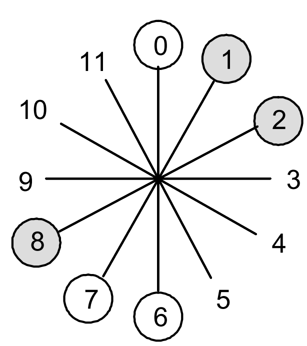
- When a superset is the combination of two subsets where one is inverted (and possibly transposed)
- Example: [0,6,7] + [1,2,8] => [0,1,2,6,7,8]
  => both [0,6,7] and [1,2,8] belong to the same prime form: (0,1,6) -- [0,6,7] is an inversion of [1,2,8]
- The result will always be "inversionally symmetric" (see below for a discussion of inversional symmetry)



Transpositional Combination



Transpositional Combination



Inversional Combination

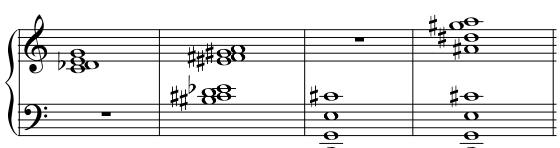
5 PC Set Complements

5.1 Defined

- Literal Complement: When one PC Set contains all of the Pitch Classes not in some other PC Set.
  Example: [0,1,4,7] and [2,3,5,6,8,9,10,11] are literal complements of each other
- Abstract Complement: When two PC Sets would be complements of each other, except that one is transposed or inverted from the other.
  Example: (0,1,4,7) and (0,1,2,3,5,6,8,9) are abstract complements of each other
- The prime forms of abstract complements are listed next to each other in the PC Set table (except for the sets of 6 pitch classes).
- Note that the Forte designation for a PC Set and it's complement will always have the same sub-number (after the dash). For example, 4-18 and 8-18 are complements.

5.2 And interval vectors

- The pitch class set and its complement have very similar interval vectors
- In fact, there is a simple formula for computing the interval vector of a complement:
  - How many **more** pitch classes does the complement have? Call this 'N'.
    - Note: If the original PC set has X pitch classes,
    - it's complement will have (12-X) pitch classes, and
    - the difference between the two will be: N = (12-X)-X = (12-X*2)
  - If the interval vector for the original PC Set is $<I_1, I_2, I_3, I_4, I_5, I_6>$
  - Then the interval vector for the complement will be:
    $<I_1+N,\ I_2+N,\ I_3+N,\ I_4+N,\ I_5+N,\ I_6+ (N/2) >$
  - Note that, because of the symmetry of the tritone, it is N/2
  - Also note that N will always be an even number {0, 2, 4, 6, 8, 10}, and so N/2 will always be an integer number (never a fraction).
- Example:
  - 4-18:(0,1,4,7) has 4 pitch classes and an interval vector of <102111>
  - It's complement is 8-18:(0,1,2,3,5,6,8,9)
  - 8 - 4 = N = 4
  - The complement's interval vector is: <1+4, 0+4, 2+4, 1+4, 1+4, 1+(4/2)> = <546553>

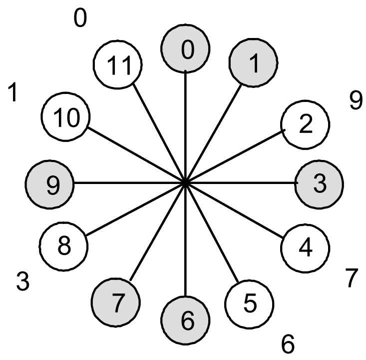


(0,1,4,7) (0,1,2,3,5,6,8,9) (0,1,4,7) (0,1,2,3,5,6,8,9)

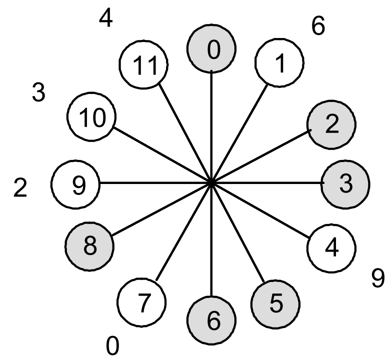
- Some famous complements:
  - Pentatonic Scale (5 PCs) : <032140> ⇔ Diatonic Scale (7 PCs) : <254361>
  - Octatonic Scale (8 PCs) : <448444> ⇔ doubly-diminished 7^{th} chord (4 PCs) : <004002>

5.3 6-note complements

- The complement of a set with 6 Pitch Classes will itself have 6 Pitch Classes
- Therefore, the difference in number of Pitch Classes is always 0 (zero) !
- Therefore, a 6-note complement will always have the same interval vector as it's complement!
- True!
- Two possible situations:
  - The set is "self complementary", that is, the set and it's complement have the same prime form.
  - The set and its complement are Z-related: Two sets with the same interval vector but which can not be reduced to the same Prime Form by transposition or inversion.



Self-Complementary Set {6-30}
with 6 Pitch Classes

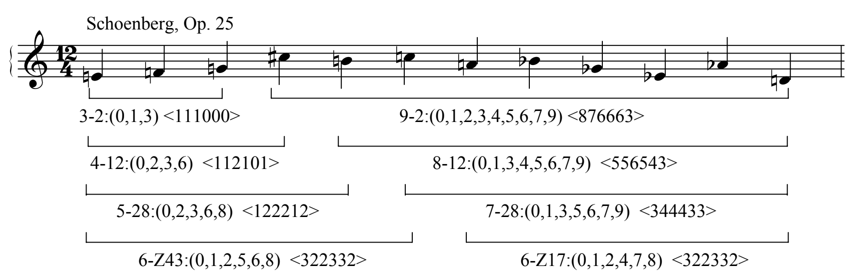


Z-Related Sets with 6 Pitch Classes

6-Z23 **6-Z45**

5.4 And 12-Tone Composition

- PC Set complements are critically important when composing music with 12-tone rows:
  - Take any 12-tone row
  - Divide it up into two pieces at any point
  - The two pieces will have similar (or exactly the same) interval content
- This is one way in which the 12-tone technique achieves harmonic cohesiveness.



Note: This is just a sample, many more combinations are possible

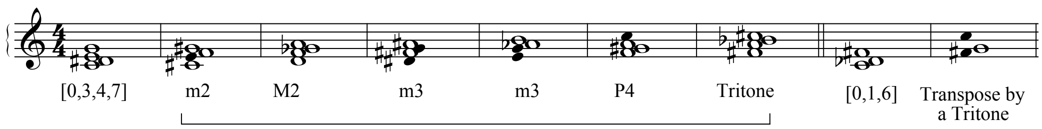
- By definition, the last 6 notes of a 12-tone row are the PC Set complement of the first 6 notes
- For more cohesiveness (i.e. more harmonic similarity), make the first and last 6 notes of the row the same PC Set - self complementary and possibly inversionally related.
- For less cohesiveness (i.e. more harmonic variety), make the first and last 6 notes of the row Z-related PC-sets

- This is the first step towards hexachordal combinatoriality: where a 12-tone row is made up of two similar halves, for example, where the 2nd half is a transposed inversion of the first half (further discussion is beyond the scope of this presentation). This is a favored technique of late Schoenberg

6 Manipulating Pitch Class Sets

6.1 By Transposition

- Take any Pitch Class Set
  - For example: [0,3,4,7] which has interval vector: <102210>
- Transpose it by any interval
- The number of common tones can be found by looking up the transposing interval in the interval vector. For example, if you transpose [0,3,4,7] by:
  - a half-step => 1 pitch class remains the same
  - a whole-step => All new pitch classes
  - a minor-third => 2 pitch classes remains the same
  - a major third => 2 pitch classes remains the same
  - a perfect 4th => 1 pitch class remains the same
  - a tritone => All new pitch classes



Shows the common tones when [0,3,4,7] is transposed up by various intervals.

- **Except**: For tritones (it would figure). When transposing by a tritone, you get <u>double</u> the number of common pitches as specified in the interval vector. For example, if you transpose [0,1,6] by a tritone, you would get two common pitch class sets, rather than one (see above for an example).
- How to use:
  - Use for common tone transposition / modulation: Transpose a PC Set around a common tone for smoother transitions.

Using common tone transposition to smoothly transpose a [0,2,6,7] (4-16:(0,1,5,7)) figure

Transposing up using two common tones

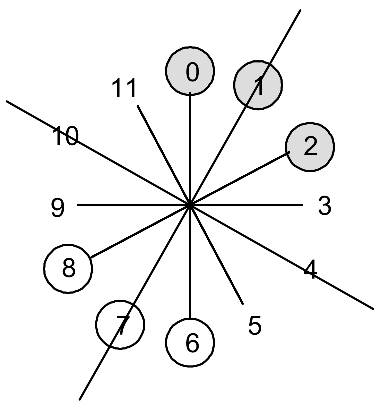
  - Alternatively, transpose with all new notes to emphasize the difference.

6.1.1 More Examples and Transpositional Symmetry

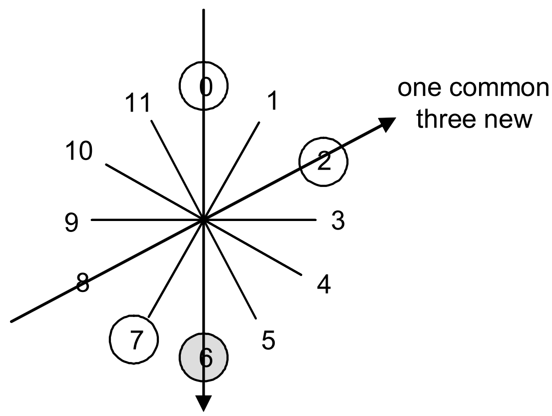
- The Diatonic Scale: 7-35:(0,1,3,5,6,8,10) has interval vector <254361>
  - Transpose the scale by a fifth or fourth (i.e. modulate to the dominant or the sub-dominant) and there will be 6 common pitch classes, and 1 new pitch class.
  - Transpose the scale by a whole step, and there will be 5 common and 2 new pitch classes. For example: C Major to D Major, or C Major to Bb major.
- The whole tone scale: 6-35:(0,2,4,6,8,10) has interval vector <060603>
  - Transpose this scale by any interval and either 1) all the pitch classes will be new or 2) all the pitch classes will be different.
  - Remember to double the value of the tritones entry (from 3 to 6).
- If any entry of the interval vector is equal to the number of pitch classes in the set, then the PC Set can be transposed to itself with all pitch classes in common. This is called "Transpositional Symmetry."
- In the Pitch Class Set Table, any PC Set with a "Count" column smaller than 12 has some Transpositional Symmetry.

6.2 By Inversion

- The best way to see if (and how) a PC Set contains common pitch classes when inverted is to visualize the PC set on a clock face, and then look for axis of symmetry.
- There is no special math involved.
- A PC Set which can invert to itself (on some axis of inversion) is said to be "Inversionally Symmetric".
- When looking at the table of all prime forms, if a PC Set has no entry in the "inversion" column, then it is inversionally symmetric



inversionally symmetric along two axis



two common two new

Alternating inversions of (0,1,6,7) with two common tones

- Looking for inversions and inversional symmetry is just another way to manipulate PC Sets to achieve new sounds.

7 Other PC Set Similarity Relations

- There are many types of other ways to judge similarity between two PC Sets.
- One way to compose a piece is to select a group of related (or strongly un-related) PC Sets and compose around them.

7.1 Ways in which PC Sets can be related to each other

- Transposition
- Inversion
- Sub-set / super-set
- Z-Relation
- Complement
- R_p, R_0, R_1, R_2 (see below)

7.2 R_p, R_0, R_1, R_2

- R_p => When two PC Sets are the same except for one different pitch class, i.e. one note different
  - Very useful for composers, this is one way to "morph" PC sets
  - But not too useful for analysis, since this relates many PC sets to many many other PC sets
- R_0 => When two PC Sets have the same number of pitch classes, but no interval vector entries in common, for example:
  - 4-2:(0,1,2,4) has interval vector <221100>
  - 4-13:(0,1,3,6) has interval vector <112011>
  - There is no interval which has the same count in both interval vectors.
  - Not a very useful measure, since it has to do with the relative strengths of the intervals, rather than the presence or total absence of intervals.
- R_1 => When two PC Sets have the same number of pitch classes, and their interval vectors are as similar as they can be without being equal
  - This will be the case when the 4 of the 6 entries in the interval vector are the same, **and** the remaining two entries are simply exchanged, for example:
  - 4-2:(0,1,2,4) has interval vector <221100>
  - 4-3:(0,1,3,4) has interval vector <212100>
  - Note the highlighted entries in the interval vector are the only ones which are different, and the two entries are merely exchanged from one to the other.
- R_2 => Just like R_1, except that the two different entries are merely an exchange of numbers. For example:
  - 5-10:(0,1,3,4,6) has interval vector <223111>
  - 5-Z12:(0,1,3,5,6) has interval vector <222121>
- Note that R_1 and R_2 are also R_p.

7.3 Other techniques for generating related PC Sets

- 12-tone: Sequence of 3, 4, 5, and 6 PC Sets in a 12-tone row
- Rotational arrays: Used by Oliver Knussen and Igor Stravinsky
- Intervallic projection to relate subsets and supersets:
  - Add notes to a PC Set by projecting up from the top note by a certain interval
  - For example: Quartal / Quintal harmony is created by projecting by adding a note to a PC set which is a perfect 4^{th} or 5^{th} above the last note added
  - Or this can be done with alternating intervals (i.e. first add a 5^{th}, then a tritone, etc)