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HIGH-TEMPERATURE CREEP AND LONG-TERM STRENGTH OF STRUCTURAL ELEMENTS UNDER CYCLIC LOADING

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We present a method for solving problems of high-temperature cyclic creep and damage accumulation in structural elements. The asymptotic expansion and averaging techniques both over the period of forced vibrations of a body and that of slowly varying loads are used for the set of equations describing the creep and damage processes in thin-walled structural elements.

Keywords: creep, long-term strength, damage, cyclic loading, thin-walled structures.

Introduction. Structural elements of up-to-date equipment operating at elevated temperatures and under the combined action of both static and slowly or rapidly varying cyclic stresses are characterized by the processes of evolution of irreversible creep deformations and accumulation of concealed damage. Many of such elements correspond to calculation schemes for thin shells of revolution. Due to a wide use of thin-walled shells of revolution in aerospace, aircraft and power engineering, the questions arise as to simulation of their stress-strain states (SSS) under cyclic loading conditions [1].

Most of the computational studies on the creep and fracture of shell structures were carried out only for the case of static loads. A number of works dealing with the evaluation of the influence of cyclic loading on the creep and long-term strength characteristics of plates and shells were published recently [2, 3]. Under actual operating conditions of structural elements, their loading is a complex process – a combination of temperature and load cycles with greatly different periods. In this connection, the development of the method for solving problems of cyclic creep and damage accumulation is of importance. This paper presents the developed method as applied to thin-walled structures consisting of shells of revolution.

Problem Statement and Solution Method. The formulated problems are solved using the finite element method (FEM). A finite element is used as a conical shell. According to the FEM approach, we write down the basic variational equality [2, 3]

$$-\int_S (N_{ij}\delta\varepsilon_{ij}^m - M_{ij}\delta\chi_{ij})dS + \int_S p\delta w dS = 0, \qquad i,\ j = 1,\ 2, \tag{1}$$

where N_{ij} are the internal membrane forces, M_{11} and M_{22} are the bending moments, M_{12} is the torque, $\delta\varepsilon_{ij}^m$ is the variation of the vector of total strains of the shell, $\delta\chi_{ij}$ is the variation of the curvature change vector, p is the load vector, and δw is the variation of displacements.

Since thin shells are under consideration in this paper, we assume that the deflection-related angles of rotation $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial \varphi}$ exceed considerably the values of the derivatives $\frac{\partial u}{\partial s}$, $\frac{\partial u}{\partial \varphi}$, $\frac{\partial v}{\partial s}$, and $\frac{\partial v}{\partial \varphi}$. Based on the adopted assumption of inextensibility and incompressibility in the direction of the shell material thickness, we obtain the geometrical relationships for the case of a nonaxisymmetric SSS in a conical shell:

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$$
\begin{cases}
\varepsilon_{11} = \varepsilon_{11}^m + z\chi_{11}, \quad \varepsilon_{22} = \varepsilon_{22}^m + z\chi_{22}, \quad \varepsilon_{12} = \varepsilon_{12}^m + 2z\chi_{12}, \\
\varepsilon_{11}^m = \varepsilon_{11}^1 + \varepsilon_{11}^n, \quad \varepsilon_{22}^m = \varepsilon_{22}^1 + \varepsilon_{22}^n, \quad \varepsilon_{12}^m = \varepsilon_{12}^1 + \varepsilon_{12}^n, \\
\varepsilon_{11}^1 = \dfrac{\partial u}{\partial s}, \quad \varepsilon_{22}^1 = \dfrac{\partial v}{r\partial s} + \dfrac{u}{r}\cos\alpha + \dfrac{w}{r}\sin\alpha, \quad \varepsilon_{12}^1 = \dfrac{\partial u}{r\partial\varphi} + \dfrac{\partial v}{\partial s} - \dfrac{v}{r}\cos\alpha, \\
\varepsilon_{11}^n = \dfrac{1}{2}\left(\dfrac{\partial w}{\partial s}\right)^2, \quad \varepsilon_{22}^n = \dfrac{1}{2r^2}\left(\dfrac{\partial w}{\partial\varphi}\right)^2, \quad \varepsilon_{12}^n = \dfrac{1}{r}\dfrac{\partial w}{\partial s}\dfrac{\partial w}{\partial\varphi}, \\
\chi_{11} = \dfrac{\partial^2 w}{\partial s^2}, \quad \chi_{22} = \dfrac{\partial^2 w}{r^2\partial\varphi^2} + \dfrac{\cos\alpha}{r}\dfrac{\partial w}{\partial s} - \dfrac{\sin\alpha}{r^2}\dfrac{\partial v}{\partial\varphi}, \\
\chi_{12} = 2\dfrac{\partial^2 w}{r\partial s\partial\varphi} - \dfrac{\partial w}{r^2\partial\varphi}\cos\alpha + \dfrac{v}{r^2}\sin\alpha\cos\alpha - \dfrac{\partial v}{\partial s}\dfrac{\sin\alpha}{r},
\end{cases}
\tag{2}
$$

where ε_{11} and ε_{22} is the linear strain of the shell in the directions x and φ, ε_{12} is the shear strain, ε_{11}^m, ε_{22}^m, and ε_{12}^m are the median surface strains, χ_{11} and χ_{22} is the change in the curvature values in the chosen directions, χ_{12} is the torsion, and $(u, v, w)^t$ is the displacement vector at the shell point.

The relations between strains and stresses in the shell under conditions of creep will be written as

$$
\sigma_{ij} = \bar{b}_{ijkl}(\varepsilon_{kl}^m - c_{kl} - \varepsilon^v) + z\bar{d}_{ijkl}(\chi_{kl} - \chi_{\bar{kl}})
\tag{3}
$$

or

$$
N_{ij} = b_{ijkl}\varepsilon_{kl}^1 + N_{ij}^n - N_{ij}^c, M_{ij} = d_{ijkl}\chi_{kl} - M_{ij}^c,
\tag{4}
$$

where

$$
\bar{b}_{ijkl} = b_{ijkl}/h, \qquad \bar{d}_{ijkl} = 12d_{ijkl}/h^3, \qquad b_{ijkl} = B\left[\delta_{ik}\delta_{jl}\frac{(1-v)}{2} + v\delta_{ij}\delta_{kl}\right],
$$

$$
d_{ijkl} = D\left[\delta_{ik}\delta_{jl}\frac{(1-\nu)}{2} + \nu\delta_{ij}\delta_{kl}\right], \qquad B = \frac{Eh}{(1-\nu^2)}, \qquad D = \frac{Eh^3}{12(1-\nu^2)},
$$

E and ν are Young's modulus and Poisson's ratio, and δ_{ij} is the Kronecker symbol.

The rest of the components of physical relationships (4) dependent on a nonlinear component of the elastic strains and irreversible creep strains are of the following form:

$$
N_{ij}^n = \int_{-h/2}^{h/2}\bar{b}_{ijkl}\varepsilon_{kl}^n dz, \qquad N_{ij}^c = \int_{-h/2}^{h/2}\bar{b}_{ijkl}c_{kl}dz, \qquad M_{ij}^c = \int_{-h/2}^{h/2}\bar{d}_{ijkl}c_{kl}zdz.
\tag{5}
$$

Equations of State. Consider the case of the action of the combined cyclic stress $\sigma = \sigma_0 + \sigma_1 + \sigma_2$, where σ_0 is the constant stress, σ_1 is the stress slowly varying during the period of the operating cycle, and σ_2 is the dynamic stress rapidly varying harmonically at high frequency [4]. The cyclic component of the total stress σ_2 is represented as

$$
\sigma_2 = \sigma_a \sin(2\pi f_2 t),
$$

where f_2 is the cyclic frequency, Hz, and σ_a is the amplitude, MPa.

In the general case, the slowly varying stress σ_1 is a function of the applied load p, the cycle period T, and the time t:

$$\sigma_1 = F(p,\ t,\ T).$$

The form of the slowly varying cycle is determined by the operating conditions of the process. Thus, for example, for an aircraft engine, these are an ascending (gain of the maximum value), constant (operating conditions) and descending (shutdown) branches. Additional stresses, for example, related to the engine boost can be imposed on the second part of the cycle. In each particular case, the shape of the slowly varying cyclic stress is predetermined. During the period of the operating cycle T, the law of variation of the stress σ_1 which results in the low-cycle creep under conditions of creep, is assumed as

$$\sigma_1 = \sigma_{\max}\left(\frac{4}{T}t - \frac{4}{T^2}t^2\right),$$

where $\sigma_{\max}$ is the maximum value of the stress σ_1.

Representing the stress σ_1 by a periodic Fourier series

$$\sigma_1 = \sigma_{\max}\left(\frac{2}{3} - \sum_{k=1}^{\infty}\frac{4}{\pi^2 k^2}\cos\left(\frac{2\pi k}{T}t\right)\right),$$

we obtain the following kind of the law for cyclic variation of the stress σ:

$$\sigma = \sigma_0\left(1 + M\left(\frac{2}{3} - \sum_{k=1}^{\infty}\frac{4}{\pi^2 k^2}\cos\left(\frac{2\pi k}{T}t\right)\right) + A\sin(2\pi f_2 t)\right),$$

where $A = \sigma_a/\sigma_0$ and $M = \sigma_{\max}/\sigma_0$ are the cycle stress ratios for rapidly and slowly varying cyclic stresses.

Let us consider the case of a simple stress state when the specimen is affected by the stress $\sigma = \sigma^0 + \sigma^1(\tau)$, where σ^0 and σ^1 are the static and cyclic components, respectively. Then, the equations of state will be represented as

$$\dot{c} = B\frac{(\sigma)^n}{(1-\omega)^k}, \tag{6}$$

$$\dot{\omega} = D\frac{(\sigma)^r}{(1-\omega)^l}, \qquad \omega(0) = \omega_0, \qquad \omega(t_*) = 1. \tag{7}$$

To describe the processes of creep and its related damage, we use the asymptotic expansion and averaging procedure at a period. Consider the asymptotic small-parameter expansion ($\mu = T/t_*$) for the unknowns of Eqs. (6) and (7) with the coefficients dependent on both the variable t (slow, or macroscopic processes) and the variable $\tau = t/\mu$ or $\xi = \tau/T$ (rapid, or microscopic processes) in the case of the action of the cyclic stress σ without due regard to the dynamic component σ_2. Then

$$c \cong c_0(t) + \mu c_1(\xi), \tag{8}$$

$$\omega = \omega_0(t) + \mu\omega_1(t,\xi), \tag{9}$$

where $c_0(t)$, $\omega_0(t)$, $c_1(\xi)$, and $\omega_1(\xi)$ are the functions corresponding to the main process of creep and damage in the slowly varying time scale and the repetitive process in the fast time scale ξ.

Using the asymptotic expansion technique for equations (6), (7) and relationships (8), (9), after averaging during the time interval of the operating cycle T for the cyclic stress $\sigma = \sigma_0 + \sigma_1$, we obtain the equations of state for the main process of the cyclic creep in the form

$$\dot{c}^0 = B\frac{(\sigma_{1e})^n}{(1-\omega^0)^k}, \tag{10}$$

$$\dot{\omega}^0 = D\frac{(\sigma_{1e})^r}{(1-\omega)^l}, \tag{11}$$

where

$$\sigma_{1e} = \langle\sigma\rangle = \sigma_0\left(\int_0^1\left(1+M\left(\frac{2}{3}-\sum_{k=1}^{\infty}\frac{4}{\pi^2k^2}\cos(2\pi k\xi)\right)\right)^s d\xi\right)^{1/s}.$$

For Eq. (10), we have $s = n$, whereas for Eq. (11), $s = r$.

Consider the process of acoustic creep under the action of the combined stress $\sigma = \sigma_0 + \sigma_1 + \sigma_2$, i.e., we impose the dynamic stress σ_2 on the obtained equivalent stress σ_{1e}. Similarly using the asymptotic small-parameter expansion method

$$\mu_1 = \frac{T_2}{t_*}, \qquad T_2 = \frac{1}{f_2},$$

and the procedure of averaging at the period T_2, the equations of state for the main process of cyclic creep can be finally obtained in the form

$$\dot{c}^0 = B\frac{(g_1g_2\sigma_0)^n}{(1-\omega^0)^k}, \tag{12}$$

$$\dot{\omega}^0 = D\frac{(g_1g_2\sigma_0)^r}{(1-\omega^0)^l}, \tag{13}$$

where

$$g_1 = \left(\int_0^1\left(1+M\left(\frac{2}{3}-\sum_{k=1}^{\infty}\frac{4}{\pi^2k^2}\cos(2\pi k\xi)\right)\right)^s d\xi\right)^{1/s}, \qquad g_2 = \left(\int_0^1 (1+A_2\sin(2\pi\xi))^s\, d\xi\right)^{1/s}, \qquad A_2 = \frac{A}{g_1}.$$

Thus, by the use of the small-parameter asymptotic expansion method twice, followed by the averaging at the periods, constitutive equations (12), (13) are obtained.

Equations of state (12), (13) are generalized to the case of the complex stress state

$$\dot{c}_{ij} = \frac{3}{2}B\frac{(g_1g_2\sigma_i^0)^{n-1}}{(1-\omega^0)^k}s_{ij}, \tag{14}$$

$$\dot{\omega} = D\frac{(g_1g_2\sigma_e^0)^r}{(1-\omega^0)^l}, \qquad \omega(0) = \omega_0, \qquad \omega(t_*) = 1, \tag{15}$$

where B, D, n, r, k, and l are constants determined from experimental data on the material creep and fracture at the given temperature T^0.

In the kinetic equation for the damage parameter, it is necessary to use the laws for the equivalent stresses σ_e obtained experimentally for different materials [1, 5]. As a rule, the best results can be obtained using the Pisarenko–Lebedev criterion [1] and three-invariant criterion [5].

Thus, the initial problem is reduced to the solution of two interrelated initial boundary value problems. The former problem corresponds to the problem of creep under the action only of the loads applied statically but with special equations of state (14), (15), the latter to the forced vibrations of elastic bodies under the action of the harmonic loading. Both of the equation systems are related by the cycle stress ratios determined in the second problem.

The proposed method for solving problems of cyclic creep and its related damage is implemented as an IBM-compatible application package. The formulated problem is solved using the finite element method.

Finite-Element Statement of the Problem. A four-nodal conical element with 28 degrees of freedom was chosen as a finite element [3]. We present the obtained resolving equations in their vector-matrix form.

Let us introduce the vector of additional forces dependent on the creep strains $\{\sigma_c\} = (N_1^c,\ N_2^c,\ S^c,\ M_1^c,\ M_2^c,\ H^c)^t$ and the vector of irreversible creep strains $\{c\} = (c_1,\ c_2,\ c_\gamma)^t$. Then the physical relationships of the problem can be presented in the following form:

$$\sigma_c = \frac{E}{1-\nu^2} \int_{-h/2}^{h/2} [M]\{c\} dz,$$

where $[M]$ is the matrix containing the physical constants of the material.

Therefore, the vector of additional forces is completely defined by the irreversible creep strains which are calculated from equations of state (14), (15).

Consider the vector of external nodal forces. The formulas for calculating the values of the components of this vector will be obtained from the condition of equality of virtual work. Then the formula for determining the generalized vector of the external nodal loads $\{P_v\}$ is of the form:

$$\{P_v\} = \sum_e \int_{S^e} [B]^t \{p\} dS,$$

where $[B]$ is the matrix of coupling the nodal displacements with the displacements across the element and $\{p\}$ is the vector of distributed loads.

The vector of the nodal forces $\{P_n\}$ determined from a nonlinear component of the elastic strains will be defined by the following formula:

$$\{P_n\} = \sum_e \int_{S^e} [D]^t [P_m] \{\varepsilon^n\} dS,$$

where $\{\varepsilon^n\}$ is the vector of nonlinear components of the elastic strains and $[P_m]$ is the matrix dependent on the physical constants of the material.

The formula for calculating the vector of the nodal forces $\{P_c\}$ determined from irreversible creep strains is of the form

$$\{P_c\} = \sum_e \int_{S^e} [D]^t [P_m] \{c_m\} dS,$$

where $\{c_m\}$ is the vector of irreversible creep strains.

The generalized vector of the nodal forces $\{P_p\}$ from the projection of the generalized forces onto the normal will be calculated by the formula

$$\{P_p\} = \sum_e \int_{S^e} [B]^t \{p^p\} dS,$$

where $\{p^p\}$ are the projections of the vector of the generalized forces onto the normal.

The use of the FEM makes it possible to reduce variational equality (1) to the following set of linear algebraic equations:

$$[K]\{\delta\} = \{P_v\} + \{P_n\} + \{P_c\} + \{P_p\}, \tag{16}$$

where $\{\delta\}$ are the nodal displacements and $[K]$ is the global stiffness matrix that can be calculated using the following formula:

$$[K] = \sum_e \int_{S^e} [D]^t [E][D] dS.$$

To describe the processes of high-temperature creep and its related damage, we use equations of state (14) and (15).

In the problems of vibrations, it is necessary to define the system mass matrix:

$$[M] = \sum_e \int_V [B]^t \rho [B] dS.$$

Then the system of resolving equations is of the form

$$([K] - \Omega^2 [M])\{q_a^k\} = \{P_a^k\}, \tag{17}$$

where $([K] - \Omega^2 [M])$ is the matrix of the "dynamic stiffness" of the system, $\{q_a^k\}$ is the vector of the amplitude values of the nodal displacements, the components of the vector $\{P_a^k\}$ are determined by the amplitude values of the harmonic component $p(t) = p_0 + p_a \sin(2\pi f_2 t)$.

The system of equations (17) is solved for $\{q_a^k\}$ by the front-end method and is used to determine the eigenfrequencies and natural vibration modes from which the values of intensities of amplitude stresses are found.

Creep and Damage of a Cylindrical Shell. As an example of calculation, we consider the creep of a cylindrical shell with rigidly fixed edges that is loaded by cyclic internal pressure

$$p(t) = p_0 + p_s\left(\frac{2}{3} - \sum_{k=1}^{\infty} \frac{4}{\pi^2 k^2} \cos\left(\frac{2\pi k}{T} t\right)\right) + p_a \sin(2\pi f_2 t). \tag{18}$$

This type of the cycle corresponds qualitatively to the pressure distribution in combustion chambers of current-technology flight vehicles [5]. The shell is made from heat-resistant nickel alloy ÉI867 deformed at a temperature of 1173 K. The physical-mechanical characteristics of the alloy are as follows: $B = 2.65 \cdot 10^{-21}$ MPa^{-n}/h, $D = 2.4 \cdot 10^{-13}$ MPa^m/h, $n = 6.7$, $m = 3.92$; and $k = l = 7.06$. The shell length $L = 0.3$ m, the median surface radius $R = 0.05$ m, wall thickness $h = 0.003$ m. A static component of the internal pressure $p_0 = 20$ MPa, the amplitude of its cyclic component varied within $(0–0.25)p_0$ for the loading frequency f_2 equal to 0.1 of the first eigenfrequency. The problem was solved using a conical finite element with a mesh of 100 finite elements.

The calculation results are presented in Figs. 1 and 2. Figure 1 illustrates the deflection plot at an initial and final instants of time along the shell generatrix under total combined loading, whereas Fig. 2 illustrates the increase of the maximum damage parameter with time on the outer surface of the shell where curve *1* corresponds to the static loading, curve *2* – to the combined action of the static component p_0 and its imposed rapidly varying component p_a, and curve *3* – to the combined loading by the law of Eq. (18).

Conclusions. The data analysis testifies to a great influence of rapidly and slowly varying cyclic stresses on the creep rate and the time of damage accumulation. Note that taking into account the slowly varying cycle stress results in significant acceleration of damage accumulation process. During 60 cycles (700 hours), the values of the

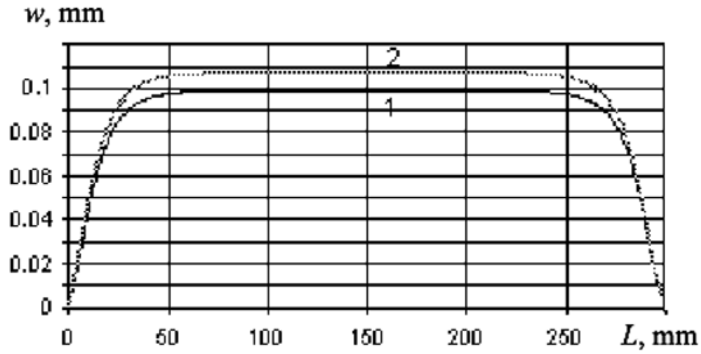

Fig. 1. Variation in the deflection at an initial (*1*) and final (*2*) instant of time along the shell generatrix.

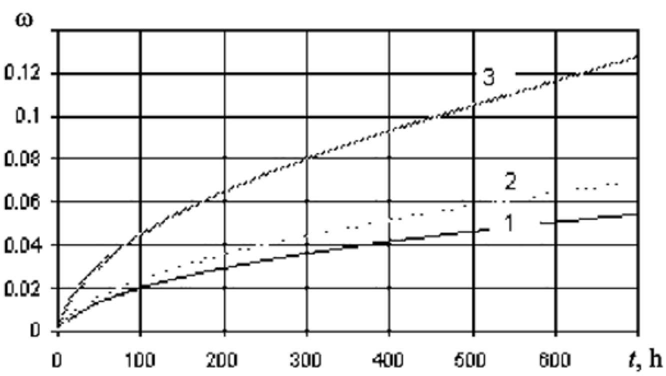

Fig. 2. Increase in the damage parameter at the point where failure occurs.

cumulative damage differ by a factor of 2.5 for static and combined loading, whereas the total time to fracture amounts to 3255 hours (279 cycles).

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