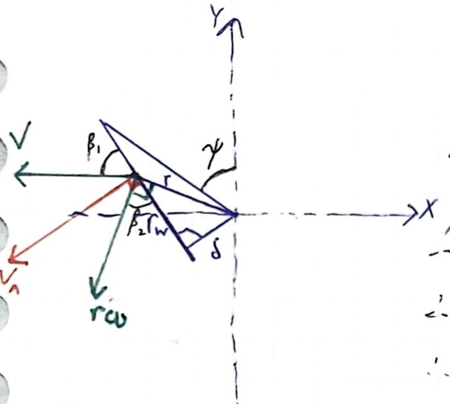


$\overline{C_{L0}}$ ✓
$\overline{C_{L\alpha}}$ ✓
$\overline{C_{mx0}}$ ✓
$\overline{C_{mx\alpha}}$ ✓

$$\beta_1 = \tan^{-1}\left(\frac{\delta}{R_w}\right) + \left(\frac{\pi}{2} - \psi\right)$$

$$\beta_2 = \frac{\pi}{2} - \tan^{-1}\left(\frac{\delta}{r_w}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{r_w}{r}\right)$$

$$\therefore V_n = V\sin\left[\frac{\pi}{2} - \left(\psi - \tan^{-1}\left(\frac{\delta}{R_w}\right)\right)\right] + r\omega \sin\left(\frac{\pi}{2} - \tan^{-1}\left(\frac{\delta}{r_w}\right)\right)$$

$$\therefore V_n = V\cos\left(\psi - \tan^{-1}\left(\frac{\delta}{R_w}\right)\right) + r\omega\left(\frac{r_w}{r}\right)$$

$$\therefore V_n = V\cos\left(\psi - \tan^{-1}\left(\frac{\delta}{R_w}\right)\right) + r_w\omega$$

$$V_n = V\left[\cos\left(\psi - \tan^{-1}\left(\frac{\delta}{R_w}\right)\right) + \frac{r_w\omega}{V}\right]$$

Let $\boxed{\chi_w = \frac{R_w\omega}{V}}$ ① and $\boxed{\xi = \left[\psi - \tan^{-1}\left(\frac{\delta}{R_w}\right)\right]}$ ②

then,

$$\boxed{V_n = V\left[\cos\xi + \left(\frac{r_w\chi_w}{R_w}\right)\right]} \quad ③$$

Solving $V_n = 0$,

$$\cos\xi = -\frac{r_w\chi_w}{R_w}$$

$$\boxed{\xi_{V_n=0} = \pi - \cos^{-1}\left(\frac{r_w\chi_w}{R_w}\right)} \quad ④$$

$$\therefore L_0^+(\xi, r_w) = \frac{1}{2}\rho V_n^2 C_{L0^+} C\, d\psi\, dr_w, \quad \text{where } d\psi = d\xi$$

$$\therefore \boxed{L_0^+(\xi, r_w) = \frac{1}{2}\rho V^2 C C_{L0^+}\left[\cos\xi + \left(\frac{r_w\chi_w}{R_w}\right)\right]^2 d\xi\, dr_w} \quad ⑤, \quad \boxed{L_0^-(\xi, r_w) = \frac{C_{L0^-}}{C_{L0^+}} L_0^+(\xi, r_w)} \quad (5\text{-}2)$$

$$\therefore \overline{L_0^+} = \frac{1}{2}\rho V^2 C C_{L0^+}\int_0^{R_w}\left\{\frac{1}{\pi}\int_0^{\pi - \cos^{-1}\left(\frac{r_w\chi_w}{R_w}\right)}\left[\cos\xi + \left(\frac{r_w\chi_w}{R_w}\right)\right]^2 d\xi\right\} dr_w$$

With similar calculations as before... ⑥

$$\boxed{\overline{L_0^+} = \frac{1}{2}\rho V^2 R_w C C_{L0^+}\left\{\frac{1}{2} + \frac{\chi_w^2}{3} + \frac{1}{2\pi\chi_w}\left[\sqrt{1-\chi_w^2} - (1-\chi_w^2)^{3/2} - \chi_w\cos^{-1}\chi_w\right] - \frac{1}{3\pi\chi_w}\left[\chi_w^3\cos^{-1}\chi_w - \frac{1}{3}\left[(2+\chi_w^2)\sqrt{1-\chi_w^2} - 2\right]\right]\right\}}$$

Similar for $\overline{L_0^-}$, then ⑦

$$\boxed{\overline{C_{L0}} = \left[\frac{1}{2} + \frac{\chi_w^2}{3}\right] + \left(1 - \frac{C_{L0^-}}{C_{L0^+}}\right)\left[\frac{1}{2\pi\chi_w}\left[\sqrt{1-\chi_w^2} - (1-\chi_w^2)^{3/2} - \chi_w\cos^{-1}\chi_w\right] - \frac{1}{3\pi\chi_w}\left[\chi_w^3\cos^{-1}\chi_w - \frac{1}{3}\left[(2+\chi_w^2)\sqrt{1-\chi_w^2} - 2\right]\right]\right]}$$

Further simplify,

$$\overline{C_{L0}} = \left[\frac{1}{2} + \frac{\chi_w^2}{3}\right] + \left(1 - \frac{C_{L0^-}}{C_{L0^+}}\right)\frac{1}{6\pi\chi_w}\left\{3\sqrt{1-\chi_w^2} - 3(1-\chi_w^2)^{3/2} - 3\chi_w\cos^{-1}\chi_w - 2\chi_w^3\cos^{-1}\chi_w + \frac{4}{3}\sqrt{1-\chi_w^2} + \frac{2}{3}\chi_w^2\sqrt{1-\chi_w^2} - \frac{4}{3}\right\}$$

$$\boxed{\overline{C_{L0}} = \left[\frac{1}{2} + \frac{\chi_w^2}{3}\right] + \frac{1}{18\pi\chi_w}\left(1 - \frac{C_{L0^-}}{C_{L0^+}}\right)\left[(4+11\chi_w^2)\sqrt{1-\chi_w^2} - 3(3+2\chi_w^2)\chi_w\cos^{-1}\chi_w - 4\right]} \quad ⑧, \quad \boxed{\overline{L_0} = \frac{1}{2}\rho V^2 R_w C C_{L0^+}\overline{C_{L0}}} \quad ⑨$$

$$L_\alpha^+(\xi, r_w) = \frac{1}{2}\rho |V_n| V_z C_{L\alpha^+} C\, d\xi\, dr_w, \quad V_z = \alpha V$$

$$\therefore \boxed{L_\alpha^+(\xi, r_w) = \frac{1}{2}\rho V^2 C\alpha C_{L\alpha^+}\left|\cos\xi + \left(\frac{r_w\chi_w}{R_w}\right)\right| d\xi\, dr_w} \quad ⑩$$

$$\therefore \boxed{\begin{aligned} \overline{L_\alpha^+} &= \frac{1}{2}\rho V^2 C\alpha C_{L\alpha^+}\int_0^{R_w}\left\{\frac{1}{\pi}\int_0^{\pi-\cos^{-1}\left(\frac{r_w\chi_w}{R_w}\right)}\left[\cos\xi + \left(\frac{r_w\chi_w}{R_w}\right)\right] d\xi\right\} dr_w \\ \overline{L_\alpha^-} &= \frac{1}{2}\rho V^2 C\alpha C_{L\alpha^-}\int_0^{R_w}\left\{\frac{1}{\pi}\int_{\pi-\cos^{-1}\left(\frac{r_w\chi_w}{R_w}\right)}^{\pi}\left[-\cos\xi - \left(\frac{r_w\chi_w}{R_w}\right)\right] d\xi\right\} dr_w \end{aligned}} \quad ⑪$$

With similar calculations, $\boxed{\overline{C_{L\alpha}} = \frac{\chi_w}{2} + \frac{1}{2\pi}\left(1 + \frac{C_{L\alpha^-}}{C_{L\alpha^+}}\right)\left[\frac{3}{2}\sqrt{1-\chi_w^2} + \frac{\sin^{-1}\chi_w}{2\chi_w} - \chi_w\cos^{-1}\chi_w\right]}$ ⑫, $\boxed{\overline{L_\alpha} = \frac{1}{2}\rho V^2 R_w C\alpha C_{L\alpha^+}\overline{C_{L\alpha}}}$ ⑬

$$M_{x0}^+(\xi, r_w) = \frac{1}{2}\rho V_n^2 C_{L0^+} C\, r\cos\left[\psi + \frac{\pi}{2} - \tan^{-1}\left(\frac{\delta}{R_w}\right) - \sin^{-1}\left(\frac{r_w}{r}\right)\right] d\psi\, dr_w$$

$$= \frac{1}{2}\rho V_n^2 C_{L0^+} C\, r\cos\left[\frac{\pi}{2} - \left(\sin^{-1}\left(\frac{r_w}{r}\right) - \xi\right)\right] d\xi\, dr_w$$

$$= \frac{1}{2}\rho V^2 C C_{L0^+}\left[\cos\xi + \left(\frac{r_w\chi_w}{R_w}\right)\right]^2 r\sin\left[\sin^{-1}\left(\frac{r_w}{r}\right) - \xi\right] d\xi\, dr_w$$

$$= \frac{1}{2}\rho V^2 C C_{L0^+}\left[\cos\xi + \left(\frac{r_w\chi_w}{R_w}\right)\right]^2 r\left\{\frac{r_w}{r}\cos\xi - \frac{\delta}{r}\sin\xi\right\} d\xi\, dr_w$$

$$= \frac{1}{2}\rho V^2 C C_{L0^+}\left[\cos\xi + \left(\frac{r_w\chi_w}{R_w}\right)\right]^2 (r_w\cos\xi - \delta\sin\xi)\, d\xi\, dr_w$$

$$\boxed{M_{x0}^+(\xi, r_w) = \left(\frac{1}{2}\rho V^2 C C_{L0^+}\left[\cos\xi + \left(\frac{r_w\chi_w}{R_w}\right)\right]^2 r_w\cos\xi\, d\xi\, dr_w\right) - L_0^+(\xi, r_w)\,\delta\sin\xi} \quad ⑭, \quad \boxed{M_{x0}^-(\xi, r_w) = \frac{C_{L0^-}}{C_{L0^+}} M_{x0}^+(\xi, r_w)} \quad (14\text{-}2)$$

$$\boxed{\overline{M_{x0}^{+}} = \frac{1}{2}\rho V^2 C C_{l0^+}\int_0^{R_w}\left\{\frac{1}{\pi}\int_0^{\pi-\cos^{-1}\left(\frac{r_wX_w}{R_w}\right)}\left[\cos\xi+\left(\frac{r_wX_w}{R_w}\right)\right]^2(r_w\cos\xi-\delta\sin\xi)\,d\xi\right\}dr_w} \tag{15}$$

$$\int_0^{R_w}\left\{\frac{1}{\pi}\int_0^{\pi-\cos^{-1}\left(\frac{r_wX_w}{R_w}\right)}\left[\cos\xi+\left(\frac{r_wX_w}{R_w}\right)\right]^2\delta\sin\xi\,d\xi\right\}dr_w = \frac{\delta}{\pi}\int_0^{R_w}\int_0^{\pi-\cos^{-1}\left(\frac{r_wX_w}{R_w}\right)}\left[\cos^2\xi+2\cos\xi\left(\frac{r_wX_w}{R_w}\right)+\left(\frac{r_wX_w}{R_w}\right)^2\right]\sin\xi\,d\xi\,dr$$

$$=\frac{\delta}{\pi}\int_0^{R_w}\int_0^{\pi-\cos^{-1}\left(\frac{r_wX_w}{R_w}\right)}\cos^2\xi\sin\xi+\sin(2\xi)\left(\frac{r_wX_w}{R_w}\right)+\left(\frac{r_wX_w}{R_w}\right)^2\sin\xi\,d\xi\,dr_w$$

$$=\frac{\delta}{\pi}\int_0^{R_w}\left[-\frac{1}{3}\cos^3\xi-\frac{1}{2}\cos(2\xi)\left(\frac{r_wX_w}{R_w}\right)-\left(\frac{r_wX_w}{R_w}\right)^2\cos\xi\right]_0^{\pi-\cos^{-1}\left(\frac{r_wX_w}{R_w}\right)}dr_w$$

$$=\frac{\delta}{\pi}\int_0^{R_w}\left[\frac{1}{3}\left(\frac{r_wX_w}{R_w}\right)^3-\frac{1}{2}\cos\left(2\cos^{-1}\left(\frac{r_wX_w}{R_w}\right)\right)\left(\frac{r_wX_w}{R_w}\right)+\left(\frac{r_wX_w}{R_w}\right)^3+\frac{1}{3}+\frac{1}{2}\left(\frac{r_wX_w}{R_w}\right)+\left(\frac{r_wX_w}{R_w}\right)^2\right]dr_w$$

$$=\frac{\delta}{\pi}\int_0^{R_w}\left[\frac{4}{3}\left(\frac{r_wX_w}{R_w}\right)^3-\left(\frac{r_wX_w}{R_w}\right)^3+\frac{1}{2}\left(\frac{r_wX_w}{R_w}\right)+\frac{1}{3}+\frac{1}{2}\left(\frac{r_wX_w}{R_w}\right)+\left(\frac{r_wX_w}{R_w}\right)^2\right]dr_w$$

$$=\frac{\delta}{\pi}\int_0^{R_w}\left[\frac{1}{3}\left(\frac{r_wX_w}{R_w}\right)^3+\left(\frac{r_wX_w}{R_w}\right)^2+\left(\frac{r_wX_w}{R_w}\right)+\frac{1}{3}\right]dr_w$$

$$=\frac{R_w\delta}{\pi}\left[\frac{1}{12}X_w^3+\frac{1}{3}X_w^2+\frac{1}{2}X_w+\frac{1}{3}\right]$$

$$\therefore\ \boxed{\overline{M_{x0}^{+}}=\frac{1}{6}\rho V^2R_w^2CC_{l0^+}\left\{X_w+\frac{1}{15\pi X_w^2}\left[2+(3X_w^4-X_w^2-2)\sqrt{1-X_w^2}\right]+\frac{2}{3\pi X_w^2}\left[1-(1-X_w^2)^{3/2}\right]-\frac{X_w}{\pi}\left[\cos^{-1}X_w-\frac{1}{3X_w^3}\left[(2+X_w^2)\sqrt{1-X_w^2}-2\right]\right]-\frac{3\delta}{R_w\pi}\left[\frac{1}{12}X_w^3+\frac{1}{3}X_w^2+\frac{1}{2}X_w+\frac{1}{3}\right]\right\}} \tag{16}$$

$$\boxed{\overline{M_{x0}^{-}}=\frac{1}{2}\rho V^2CC_{l0^-}\int_0^{R_w}\left\{\frac{1}{\pi}\int_{\pi-\cos^{-1}\left(\frac{r_wX_w}{R_w}\right)}^{\pi}\left[\cos\xi+\left(\frac{r_wX_w}{R_w}\right)\right]^2(r_w\cos\xi-\delta\sin\xi)\,d\xi\right\}dr_w} \tag{17}$$

$$\int_0^{R_w}\left\{\frac{1}{\pi}\int_{\pi-\cos^{-1}\left(\frac{r_wX_w}{R_w}\right)}^{\pi}\left[\cos\xi+\left(\frac{r_wX_w}{R_w}\right)\right]^2\delta\sin\xi\,d\xi\right\}dr_w=\frac{\delta}{\pi}\int_0^{R_w}\left[-\frac{1}{3}\cos^3\xi-\frac{1}{2}\cos(2\xi)\left(\frac{r_wX_w}{R_w}\right)-\left(\frac{r_wX_w}{R_w}\right)^2\cos\xi\right]_{\pi-\cos^{-1}\left(\frac{r_wX_w}{R_w}\right)}^{\pi}dr_w$$

$$=\frac{\delta}{\pi}\int_0^{R_w}\left[\frac{1}{3}-\frac{1}{2}\left(\frac{r_wX_w}{R_w}\right)+\left(\frac{r_wX_w}{R_w}\right)^2-\frac{1}{3}\left(\frac{r_wX_w}{R_w}\right)^3+\left(\frac{r_wX_w}{R_w}\right)^3-\frac{1}{2}\left(\frac{r_wX_w}{R_w}\right)-\left(\frac{r_wX_w}{R_w}\right)^3\right]dr_w$$

$$=\frac{\delta}{\pi}\int_0^{R_w}\left[\frac{1}{3}-\left(\frac{r_wX_w}{R_w}\right)+\left(\frac{r_wX_w}{R_w}\right)^2-\frac{1}{3}\left(\frac{r_wX_w}{R_w}\right)^3\right]dr_w$$

$$=\frac{R_w\delta}{\pi}\left[-\frac{1}{12}X_w^3+\frac{1}{3}X_w^2-\frac{1}{2}X_w+\frac{1}{3}\right]$$

$$\therefore\ \boxed{\overline{M_{x0}^{-}}=\frac{1}{6}\rho V^2R_w^2CC_{l0^-}\left\{-\frac{1}{15\pi X_w^2}\left[2+(3X_w^4-X_w^2-2)\sqrt{1-X_w^2}\right]-\frac{2}{3\pi X_w^2}\left[1-(1-X_w^2)^{3/2}\right]+\frac{X_w}{\pi}\left[\cos^{-1}X_w-\frac{1}{3X_w^3}\left[(2+X_w^2)\sqrt{1-X_w^2}-2\right]\right]-\frac{3\delta}{R_w\pi}\left[-\frac{1}{12}X_w^3+\frac{1}{3}X_w^2-\frac{1}{2}X_w+\frac{1}{3}\right]\right\}} \tag{18}$$

$$\therefore\ \overline{M_{x0}}=\overline{M_{x0}^{+}}+\overline{M_{x0}^{-}}=\frac{1}{6}\rho V^2R_w^2CC_{l0^+}\left\{X+\left(1-\frac{C_{l0^-}}{C_{l0^+}}\right)\left[\frac{1}{15\pi X_w^2}\left[2+(3X_w^4-X_w^2-2)\sqrt{1-X_w^2}\right]+\frac{2}{3\pi X_w^2}\left[1-(1-X_w^2)^{3/2}\right]-\frac{1}{\pi}\left[X_w\cos^{-1}X_w-\frac{1}{3X_w^2}\left[(2+X_w^2)\sqrt{1-X_w^2}-2\right]\right]\right]-\frac{\delta}{4R_w\pi}\left[X_w^3+4X_w^2+6X_w+4+\left(\frac{C_{l0^-}}{C_{l0^+}}\right)(-X_w^3+4X_w^2-6X_w+4)\right]\right\}$$

$$\therefore\ \overline{M_{x0}}=\frac{1}{2}\rho V^2R_w^2CC_{l0^+}\left\{\frac{X_w}{3}+\left(1-\frac{C_{l0^-}}{C_{l0^+}}\right)\frac{1}{45\pi X_w^2}\left[2+(3X_w^4-X_w^2-2)\sqrt{1-X_w^2}+10-10(1-X_w^2)\sqrt{1-X_w^2}-15X_w^3\cos^{-1}X_w+5(2+X_w^2)\sqrt{1-X_w^2}-10\right]-\frac{\delta}{12R_w\pi X_w}\left[(X_w+1)^4-1-\left(\frac{C_{l0^-}}{C_{l0^+}}\right)\left[(X_w-1)^4-1\right]\right]\right\}$$

$$\boxed{\overline{M_{x0}}=\frac{1}{2}\rho V^2R_w^2CC_{l0^+}\left\{\frac{X_w}{3}+\frac{1}{45\pi X_w^2}\left(1-\frac{C_{l0^-}}{C_{l0^+}}\right)\left[2-15X_w^3\cos^{-1}X_w+(3X_w^4+14X_w^2-2)\sqrt{1-X_w^2}\right]-\frac{\delta}{12R_w\pi X_w}\left[(X_w+1)^4-1-\left(\frac{C_{l0^-}}{C_{l0^+}}\right)\left[(X_w-1)^4-1\right]\right]\right\}} \tag{19}$$

$$\therefore\ \boxed{C_{Mx0}=\frac{X_w}{3}+\frac{1}{45\pi X_w^2}\left(1-\frac{C_{l0^-}}{C_{l0^+}}\right)\left[(3X_w^4+14X_w^2-2)\sqrt{1-X_w^2}+2-15X_w^3\cos^{-1}X_w\right]-\frac{\delta}{12R_w\pi X_w}\left[(X_w+1)^4-1-\left(\frac{C_{l0^-}}{C_{l0^+}}\right)\left[(X_w-1)^4-1\right]\right]} \tag{20}$$

$$M^+_{x\alpha}(\xi, r_w) = \frac{1}{2}\rho |V_n| V_z C_{l\alpha^+} C (r_w \cos\xi - \delta\sin\xi)\, d\xi\, dr_w$$

$$\boxed{m^+_{x\alpha}(\xi, r_w) = \frac{1}{2}\rho V^2 C\alpha C_{l\alpha^+} \left|\cos\xi + \left(\frac{r_w\chi_w}{R_w}\right)\right| (r_w\cos\xi - \delta\sin\xi)\, d\xi\, dr_w} \quad (21)$$

$$\therefore \boxed{\overline{M^+_{x\alpha}} = \frac{1}{2}\rho V^2 C\alpha C_{l\alpha^+}\int_0^{R_w}\left\{\frac{1}{\pi}\int_0^{\pi-\cos^{-1}\left(\frac{r_w\chi_w}{R_w}\right)}\left[\cos\xi + \left(\frac{r_w\chi_w}{R_w}\right)\right](r_w\cos\xi - \delta\sin\xi)\, d\xi\right\} dr_w} \quad (22)$$

$$\int_0^{R_w}\left\{\frac{1}{\pi}\int_0^{\pi-\cos^{-1}\left(\frac{r_w\chi_w}{R_w}\right)}\left[\cos\xi + \left(\frac{r_w\chi_w}{R_w}\right)\right]\delta\sin\xi\, d\xi\right\}dr_w = \frac{\delta}{\pi}\int_0^{R_w}\int_0^{\pi-\cos^{-1}\left(\frac{r_w\chi_w}{R_w}\right)}\left(\frac{1}{2}\sin(2\xi) + \left(\frac{r_w\chi_w}{R_w}\right)\sin\xi\right)d\xi\, dr_w$$

$$= \frac{\delta}{\pi}\int_0^{R_w}\left[-\frac{1}{4}(\cos^2\xi - 1) - \left(\frac{r_w\chi_w}{R_w}\right)\cos\xi\right]_0^{\pi-\cos^{-1}\left(\frac{r_w\chi_w}{R_w}\right)} dr_w$$

$$= \frac{\delta}{\pi}\int_0^{R_w}\left[-\frac{1}{4}\left(\frac{r_w\chi_w}{R_w}\right)^2 + \frac{1}{4} + \left(\frac{r_w\chi_w}{R_w}\right)^2 + \left(\frac{r_w\chi_w}{R_w}\right)\right]dr_w$$

$$= \frac{\delta}{\pi}\left[\frac{1}{4}\chi_w^2 + \frac{1}{2}\chi_w + \frac{1}{4}\right]R_w$$

$$\therefore \boxed{\overline{M^+_{x\alpha}} = \frac{1}{2}\rho V^2 R_w^2 C\alpha C_{l\alpha^+}\left\{\frac{1}{4} + \frac{1}{16\pi\chi_w}\left[(2\chi_w^2 - 1)\sqrt{1-\chi_w^2} + \frac{\sin^{-1}\chi_w}{\chi_w}\right] - \frac{1}{8\pi\chi_w}\left[\frac{\sin^{-1}\chi_w}{\chi_w} + 2\chi_w\cos^{-1}\chi_w - \sqrt{1-\chi_w^2}\right] - \frac{\delta}{4\pi R_w}(\chi_w+1)^2\right\}} \quad (23)$$

$$\boxed{\overline{M^-_{x\alpha}} = \frac{1}{2}\rho V^2 C\alpha C_{l\alpha^-}\int_0^{R_w}\left\{\frac{1}{\pi}\int_{\pi-\cos^{-1}\left(\frac{r_w\chi_w}{R_w}\right)}^{\pi}\left[-\cos\xi - \left(\frac{r_w\chi_w}{R_w}\right)\right](r_w\cos\xi - \delta\sin\xi)\, d\xi\right\}dr_w} \quad (24)$$

$$\int_0^{R_w}\left\{\frac{1}{\pi}\int_{\pi-\cos^{-1}\left(\frac{r_w\chi_w}{R_w}\right)}^{\pi}\left[\cos\xi + \left(\frac{r_w\chi_w}{R_w}\right)\right]\delta\sin\xi\, d\xi\right\}dr_w = \frac{\delta}{\pi}\int_0^{R_w}\left[-\frac{1}{4}(\cos^2\xi - 1) - \left(\frac{r_w\chi_w}{R_w}\right)\cos\xi\right]_{\pi-\cos^{-1}\left(\frac{r_w\chi_w}{R_w}\right)}^{\pi} dr_w$$

$$= \frac{\delta}{\pi}\int_0^{R_w}\left[\frac{1}{4}\left(\frac{r_w\chi_w}{R_w}\right)^2 - \frac{1}{4} + \left(\frac{r_w\chi_w}{R_w}\right) - \left(\frac{r_w\chi_w}{R_w}\right)^2\right]dr_w$$

$$= \frac{\delta}{\pi}\left[-\frac{1}{4}\chi_w^2 + \frac{1}{2}\chi_w - \frac{1}{4}\right]R_w$$

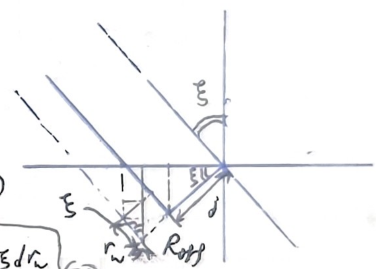
$$\therefore \boxed{\overline{M^-_{x\alpha}} = \frac{1}{2}\rho V^2 R_w^2 C\alpha C_{l\alpha^-}\left\{\frac{1}{16\pi\chi_w}\left[(2\chi_w^2 - 1)\sqrt{1-\chi_w^2} + \frac{\sin^{-1}\chi_w}{\chi_w}\right] - \frac{1}{8\pi\chi_w}\left[\frac{\sin^{-1}\chi_w}{\chi_w} + 2\chi_w\cos^{-1}\chi_w - \sqrt{1-\chi_w^2}\right] - \frac{\delta}{4\pi R_w}(\chi_w - 1)^2\right\}} \quad (25)$$

$$\therefore \overline{M_{x\alpha}} = \overline{M^+_{x\alpha}} + \overline{M^-_{x\alpha}} = \frac{1}{2}\rho V^2 R_w^2 C\alpha C_{l\alpha^+}\left\{\frac{1}{4} + \left(1 + \frac{C_{l\alpha^-}}{C_{l\alpha^+}}\right)\frac{1}{16\pi\chi_w}\left[(2\chi_w^2 + 1)\sqrt{1-\chi_w^2} - \frac{\sin^{-1}\chi_w}{\chi_w} - 4\chi_w\cos^{-1}\chi_w\right] - \frac{\delta}{4\pi R_w}\left[(\chi_w+1)^2 + \frac{C_{l\alpha^-}}{C_{l\alpha^+}}(\chi_w - 1)^2\right]\right\}$$

$$\therefore \boxed{\overline{C_{mx\alpha}} = \frac{1}{4} + \frac{1}{16\pi\chi_w}\left(1 + \frac{C_{l\alpha^-}}{C_{l\alpha^+}}\right)\left[(2\chi_w^2 + 1)\sqrt{1-\chi_w^2} - \frac{\sin^{-1}\chi_w}{\chi_w} - 4\chi_w\cos^{-1}\chi_w\right] - \frac{\delta}{4\pi R_w}\left[(\chi_w+1)^2 + \frac{C_{l\alpha^-}}{C_{l\alpha^+}}(\chi_w - 1)^2\right]} \quad (26)$$

$$\boxed{\overline{M_{x\alpha}} = \frac{1}{2}\rho V^2 R_w^2 C\alpha C_{l\alpha^+}\overline{C_{mx\alpha}}} \quad (27)$$

Offset of Lift from center line of blade $\equiv R_{off}$



$$\therefore \boxed{M^+_{Y0} = \frac{1}{2}\rho V_n^2 C_{l0^+} C\left[(R_{off}+\delta)\cos\xi + r_w\sin\xi\right]d\xi\, dr_w} \quad (28)$$

$$\therefore \boxed{M^+_{Y0} = \frac{1}{2}\rho V^2 C C_{l0^+}\left[(R_{off}+\delta)\cos\xi + r_w\sin\xi\right]\left[\cos\xi + \left(\frac{r_w\chi_w}{R_w}\right)\right]^2 d\xi\, dr_w} \quad (29)$$

$$\therefore \boxed{\overline{M^+_{Y0}} = \frac{1}{2}\rho V^2 C C_{l0^+}\int_0^{R_w}\frac{1}{\pi}\int_0^{\pi-\cos^{-1}\left(\frac{r_w\chi_w}{R_w}\right)}\left[(R_{off}+\delta)\cos\xi + r_w\sin\xi\right]\left[\cos\xi + \left(\frac{r_w\chi_w}{R_w}\right)\right]^2 d\xi\, dr_w} \quad (30)$$

$$\int_0^{R_w}\frac{1}{\pi}\int_0^{\pi-\cos^{-1}\left(\frac{r_w\chi_w}{R_w}\right)} r_w\sin\xi\left[\cos\xi + \left(\frac{r_w\chi_w}{R_w}\right)\right]^2 d\xi\, dr_w = \frac{1}{\pi}\int_0^{R_w} r_w\int_0^{\pi-\cos^{-1}\left(\frac{r_w\chi_w}{R_w}\right)}\left[\cos^2\xi\sin\xi + \sin(2\xi)\left(\frac{r_w\chi_w}{R_w}\right) + \left(\frac{r_w\chi_w}{R_w}\right)^2\sin\xi\right]d\xi\, dr_w$$

$$= \frac{1}{\pi}\int_0^{R_w}\left[-\frac{1}{3}\cos^3\xi - \cos^2\xi\left(\frac{r_w\chi_w}{R_w}\right) + \frac{1}{2}\left(\frac{r_w\chi_w}{R_w}\right) - \left(\frac{r_w\chi_w}{R_w}\right)^2\cos\xi\right]_0^{\pi-\cos^{-1}\left(\frac{r_w\chi_w}{R_w}\right)} r_w\, dr_w$$

$$= \frac{1}{\pi}\int_0^{R_w}\left[\frac{r_w}{3}\left(\frac{r_w\chi_w}{R_w}\right)^3 - r_w\left(\frac{r_w\chi_w}{R_w}\right)^3 + r_w\left(\frac{r_w\chi_w}{R_w}\right)^3 + \frac{r_w}{3} + \left(\frac{r_w\chi_w}{R_w}\right)r_w + \left(\frac{r_w\chi_w}{R_w}\right)^2 r_w\right]dr_w$$

$$= \frac{1}{\pi}\int_0^{R_w}\left[\frac{r_w}{3}\left(\frac{r_w\chi_w}{R_w}\right)^3 + \frac{r_w}{3} + \left(\frac{r_w\chi_w}{R_w}\right)r_w + \left(\frac{r_w\chi_w}{R_w}\right)^2 r_w\right]dr_w$$

$$= \frac{1}{\pi}\left[\frac{R_w^2}{15}\chi_w^3 + \frac{R_w^2}{6} + \frac{R_w^2}{3}\chi_w + \frac{R_w^2}{4}\chi_w^2\right]$$

$$= \frac{R_w^2}{\pi}\left[\frac{1}{15}\chi_w^3 + \frac{1}{4}\chi_w^2 + \frac{1}{3}\chi_w + \frac{1}{6}\right]$$

$$\therefore \boxed{\begin{aligned}\overline{M^+_{Y0}} = \frac{1}{2}\rho V^2 R_w C C_{l0^+}\Big\{(R_{off}+\delta)\Big\{\frac{\chi_w}{2} + \frac{1}{3\pi}\left[\sqrt{1-\chi_w^2} + \frac{\sin^{-1}\chi_w}{\chi_w}\right] + \frac{1}{24\pi}\left[(2\chi_w^2 - 1)\sqrt{1-\chi_w^2} + \frac{\sin^{-1}\chi_w}{\chi_w}\right] - \frac{1}{4\pi}\left[\frac{\sin^{-1}\chi_w}{\chi_w} + 2\chi_w\cos^{-1}\chi_w - \sqrt{1-\chi_w^2}\right]\Big\} \\ + \frac{R_w}{60\pi}\left[4\chi_w^3 + 15\chi_w^2 + 20\chi_w + 10\right]\Big\}\end{aligned}} \quad (31)$$

$$\boxed{\overline{M^-_{Y0}} = \frac{1}{2}\rho V^2 C C_{l0^-}\left[-(R_{off}+\delta)\cos\xi - r_w\sin\xi\right]\left[\cos\xi + \left(\frac{r_w\chi_w}{R_w}\right)\right]^2 d\xi\, dr_w} \quad (32)$$

$$\int_0^{R_w}\frac{1}{\pi}\int_{\pi-\cos^{-1}\left(\frac{r_w\chi_w}{R_w}\right)}^{\pi} -r_w\sin\xi\left[\cos\xi + \left(\frac{r_w\chi_w}{R_w}\right)\right]^2 d\xi\, dr_w = -\frac{1}{\pi}\int_0^{R_w}\left[-\frac{1}{3}\cos^3\xi - \cos^2\xi\left(\frac{r_w\chi_w}{R_w}\right) - \left(\frac{r_w\chi_w}{R_w}\right)^2\cos\xi\right]_{\pi-\cos^{-1}\left(\frac{r_w\chi_w}{R_w}\right)}^{\pi} r_w\, dr_w$$

$$= -\frac{1}{\pi}\int_0^{R_w}\left[\frac{1}{3} - \left(\frac{r_w\chi_w}{R_w}\right) + \left(\frac{r_w\chi_w}{R_w}\right)^2 - \frac{1}{3}\left(\frac{r_w\chi_w}{R_w}\right)^3 + \left(\frac{r_w\chi_w}{R_w}\right)^3 - \left(\frac{r_w\chi_w}{R_w}\right)^3\right]r_w\, dr_w$$

$$= -\frac{1}{\pi}\int_0^{R_w}\left[\frac{1}{3}r_w - \left(\frac{r_w\chi_w}{R_w}\right)r_w + \left(\frac{r_w\chi_w}{R_w}\right)^2 r_w - \frac{1}{3}\left(\frac{r_w\chi_w}{R_w}\right)^3 r_w\right]dr_w$$

$$= -\frac{1}{\pi}\left[\frac{1}{6}R_w^2 - \frac{1}{3}\chi_w R_w^2 + \frac{1}{4}\chi_w^2 R_w^2 - \frac{1}{15}\chi_w^3 R_w^2\right]$$

$$= \frac{R_w^2}{\pi}\left[\frac{1}{15}\chi_w^3 - \frac{1}{4}\chi_w^2 + \frac{1}{3}\chi_w - \frac{1}{6}\right]$$

$$\therefore \boxed{\overline{M_{Y0}^-} = -\frac{1}{2}\rho V^2 R_w c\, C_{l0^-}\left\{(R_{off}+\delta)\left\{-\frac{1}{3\pi}\left[\sqrt{1-X_w^2}+\frac{\sin^{-1}X_w}{X_w}\right]-\frac{1}{24\pi}\left[(2X_w^2-1)\sqrt{1-X_w^2}+\frac{\sin^{-1}X_w}{X_w}\right]+\frac{1}{4\pi}\left[\frac{\sin^{-1}X_w}{X_w}+2X_w\cos^{-1}X_w-\sqrt{1-X_w^2}\right]\right\} - \frac{R_w}{60\pi}\left[4X_w^3-15X_w^2+20X_w-10\right]\right\}} \quad (33)$$

$$\therefore \overline{M_{Y0}} = \overline{M_{Y0}^+}+\overline{M_{Y0}^-} = \frac{1}{2}\rho V^2 R_w c\, C_{l0^+}\left\{(R_{off}+\delta)\left\{\frac{X_w}{2}+\left(1+\frac{C_{l0^-}}{C_{l0^+}}\right)\left[\frac{1}{3\pi}\left[\sqrt{1-X_w^2}+\frac{\sin^{-1}X_w}{X_w}\right]+\frac{1}{24\pi}\left[(2X_w^2-1)\sqrt{1-X_w^2}+\frac{\sin^{-1}X_w}{X_w}\right]\right.\right.\right.$$

$$\left.\left.-\frac{1}{4\pi}\left[\frac{\sin^{-1}X_w}{X_w}+2X_w\cos^{-1}X_w-\sqrt{1-X_w^2}\right]\right]\right\}+\frac{R_w}{60\pi}\left[4X_w^3+15X_w^2+20X_w+10\right]$$

$$\left.+\frac{C_{l0^-}}{C_{l0^+}}\frac{R_w}{60\pi}\left[4X_w^3-15X_w^2+20X_w-10\right]\right\}$$

$$= \frac{1}{2}\rho V^2 R_w c\, C_{l0^+}\left\{(R_{off}+\delta)\left\{\frac{X_w}{2}+\left(1+\frac{C_{l0^-}}{C_{l0^+}}\right)\frac{1}{24\pi}\left[8\sqrt{1-X_w^2}+8\frac{\sin^{-1}X_w}{X_w}+(2X_w^2-1)\sqrt{1-X_w^2}+\frac{\sin^{-1}X_w}{X_w}-6\frac{\sin^{-1}X_w}{X_w}\right.\right.\right.$$

$$\left.\left.-12X_w\cos^{-1}X_w+6\sqrt{1-X_w^2}\right]\right\}$$

$$\left.+\frac{R_w}{60\pi}\left(1+\frac{C_{l0^-}}{C_{l0^+}}\right)\left[4X_w^3+20X_w\right]+\frac{R_w}{60\pi}\left(1-\frac{C_{l0^-}}{C_{l0^+}}\right)\left[15X_w^2+10\right]\right\}$$

$$= \frac{1}{2}\rho V^2 R_w c\, C_{l0^+}\left\{(R_{off}+\delta)\left\{\frac{X_w}{2}+\left(1+\frac{C_{l0^-}}{C_{l0^+}}\right)\frac{1}{24\pi}\left[(2X_w^2+13)\sqrt{1-X_w^2}+3\frac{\sin^{-1}X_w}{X_w}-12X_w\cos^{-1}X_w\right]\right\}\right.$$

$$\left.+\frac{R_w}{\pi 60}\left[\left(1+\frac{C_{l0^-}}{C_{l0^+}}\right)(4X_w^3+20X_w)+\left(1-\frac{C_{l0^-}}{C_{l0^+}}\right)(15X_w^2+10)\right]\right\}$$

$$\therefore \boxed{\overline{C_{mY0}} = \frac{X_w}{2}+\frac{1}{24\pi}\left(1+\frac{C_{l0^-}}{C_{l0^+}}\right)\left[(2X_w^2+13)\sqrt{1-X_w^2}+\frac{3\sin^{-1}X_w}{X_w}-12X_w\cos^{-1}X_w\right]+\frac{R_w}{60\pi(R_{off}+\delta)}\left[\left(1+\frac{C_{l0^-}}{C_{l0^+}}\right)(4X_w^3+20X_w)+\left(1-\frac{C_{l0^-}}{C_{l0^+}}\right)(15X_w^2+10)\right]} \quad (34)$$

$$\boxed{\overline{M_{Y0}} = \frac{1}{2}\rho V^2 R_w c\, C_{l0^+}(R_{off}+\delta)\,\overline{C_{mY0}}} \quad (35)$$

$$M_{Y\alpha}^+(\xi, r_w) = \frac{1}{2}\rho |V_n| V_z C_{l\alpha^+} c\left[(R_{off}+\delta)\cos\xi + r_w\sin\xi\right] d\xi\, dr_w$$

$$\boxed{M_{Y\alpha}^+(\xi, r_w) = \frac{1}{2}\rho V^2 c\,\alpha\, C_{l\alpha^+}\left[(R_{off}+\delta)\cos\xi + r_w\sin\xi\right]\left|\cos\xi+\left(\frac{r_w X_w}{R_w}\right)\right| d\xi\, dr_w} \quad (36)$$

$$\therefore \boxed{\overline{M_{Y\alpha}^+} = \frac{1}{2}\rho V^2 c\,\alpha\, C_{l\alpha^+}\int_0^{R_w}\frac{1}{\pi}\int_0^{\pi-\cos^{-1}\left(\frac{r_w X_w}{R_w}\right)}\left[(R_{off}+\delta)\cos\xi+r_w\sin\xi\right]\left[\cos\xi+\left(\frac{r_w X_w}{R_w}\right)\right]d\xi\, dr_w} \quad (37)$$

$$\int_0^{R_w}\frac{1}{\pi}\int_0^{\pi-\cos^{-1}\left(\frac{r_w X_w}{R_w}\right)} r_w\sin\xi\left[\cos\xi+\left(\frac{r_w X_w}{R_w}\right)\right]d\xi\, dX_w = \frac{1}{\pi}\int_0^{R_w} r_w\int_0^{\pi-\cos^{-1}\left(\frac{r_w X_w}{R_w}\right)}\sin\xi\cos\xi+\left(\frac{r_w X_w}{R_w}\right)\sin\xi\; d\xi\, dr_w$$

$$= \frac{1}{\pi}\int_0^{R_w} r_w\left[-\frac{\cos^2\xi}{2}-\left(\frac{r_w X_w}{R_w}\right)\cos\xi\right]_0^{\pi-\cos^{-1}\left(\frac{r_w X_w}{R_w}\right)} dr_w$$

$$= \frac{1}{\pi}\int_0^{R_w} r_w\left[-\frac{1}{2}\left(\frac{r_w X_w}{R_w}\right)^2+\left(\frac{r_w X_w}{R_w}\right)^2+\frac{1}{2}+\left(\frac{r_w X_w}{R_w}\right)\right]dr_w$$

$$= \frac{1}{\pi}\int_0^{R_w}\frac{1}{2}\left(\frac{r_w X_w}{R_w}\right)^2 r_w+\left(\frac{r_w X_w}{R_w}\right)r_w+\frac{1}{2}r_w\, dr_w$$

$$= \frac{1}{\pi}\left[\frac{1}{8}R_w^2 X_w^2+\frac{1}{3}R_w^2 X_w+\frac{1}{4}R_w^2\right]$$

$$= \frac{R_w^2}{\pi}\left[\frac{1}{8}X_w^2+\frac{1}{3}X_w+\frac{1}{4}\right]$$

$$\therefore \boxed{\overline{M_{Y\alpha}^+} = \frac{1}{2}\rho V^2 R_w c\,\alpha\, C_{l\alpha^+}(R_{off}+\delta)\left\{\frac{1}{2}+\frac{1}{6\pi X_w}\left[1-(1-X_w^2)^{3/2}\right]-\frac{1}{2\pi X_w}\left[1+X_w\cos^{-1}X_w-\sqrt{1-X_w^2}\right]+\frac{R_w}{24\pi}\left[3X_w^2+8X_w+6\right]\frac{1}{R_{off}+\delta}\right\}} \quad (38)$$

$$\boxed{\overline{M_{Y\alpha}^-} = \frac{1}{2}\rho V^2 c\,\alpha\, C_{l\alpha^-}\int_0^{R_w}\frac{1}{\pi}\int_{\pi-\cos^{-1}\left(\frac{r_w X_w}{R_w}\right)}^{\pi}\left[-(R_{off}+\delta)\cos\xi-r_w\sin\xi\right]\left[-\cos\xi-\left(\frac{r_w X_w}{R_w}\right)\right]d\xi\, dr_w} \quad (39)$$

$$\int_0^{R_w}\frac{1}{\pi}\int_{\pi-\cos^{-1}\left(\frac{r_w X_w}{R_w}\right)}^{\pi} r_w\sin\xi\left[\cos\xi+\left(\frac{r_w X_w}{R_w}\right)\right]d\xi\, dr_w = \frac{1}{\pi}\int_0^{R_w} r_w\left[-\frac{\cos^2\xi}{2}-\left(\frac{r_w X_w}{R_w}\right)\cos\xi\right]_{\pi-\cos^{-1}\left(\frac{r_w X_w}{R_w}\right)}^{\pi} dr_w$$

$$= \frac{1}{\pi}\int_0^{R_w} r_w\left[-\frac{1}{2}+\left(\frac{r_w X_w}{R_w}\right)+\frac{1}{2}\left(\frac{r_w X_w}{R_w}\right)^2-\left(\frac{r_w X_w}{R_w}\right)^2\right]dr_w$$

$$= -\frac{1}{\pi}\left[\frac{1}{8}R_w^2 X_w^2-\frac{1}{3}R_w^2 X_w+\frac{1}{4}R_w^2\right]$$

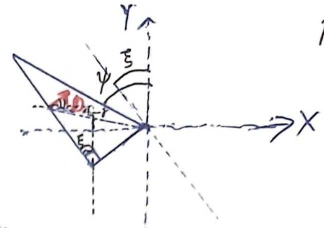
$$= -\frac{R_w^2}{\pi}\left[\frac{1}{8}X_w^2-\frac{1}{3}X_w+\frac{1}{4}\right]$$

$$\therefore \boxed{\overline{M_{Y\alpha}^-} = -\frac{1}{2}\rho V^2 R_w c\,\alpha\, C_{l\alpha^-}(R_{off}+\delta)\left\{\frac{1}{6\pi X_w}\left[1-(1-X_w^2)^{3/2}\right]-\frac{1}{2\pi X_w}\left[1+X_w\cos^{-1}X_w-\sqrt{1-X_w^2}\right]+\frac{R_w}{24\pi}\left[3X_w^2-8X_w+6\right]\frac{1}{R_{off}+\delta}\right\}} \quad (40)$$

$$\therefore \overline{C_{mY\alpha}} = \frac{1}{2}+\frac{1}{6\pi X_w}\left(1-\frac{C_{l\alpha^-}}{C_{l\alpha^+}}\right)\left[1-(1-X_w^2)\sqrt{1-X_w^2}-3-3X_w\cos^{-1}X_w+3\sqrt{1-X_w^2}\right]+\frac{R_w}{24\pi(R_{off}+\delta)}\left[\left(1-\frac{C_{l\alpha^-}}{C_{l\alpha^+}}\right)(3X_w^2+6)+\left(1+\frac{C_{l\alpha^-}}{C_{l\alpha^+}}\right)8X_w\right]$$

$$\boxed{\overline{C_{mY\alpha}} = \frac{1}{2}+\frac{1}{6\pi X_w}\left(1-\frac{C_{l\alpha^-}}{C_{l\alpha^+}}\right)\left[(4-X_w^2)\sqrt{1-X_w^2}-2-3X_w\cos^{-1}X_w\right]+\frac{R_w}{24\pi(R_{off}+\delta)}\left[\left(1-\frac{C_{l\alpha^-}}{C_{l\alpha^+}}\right)(3X_w^2+6)+\left(1+\frac{C_{l\alpha^-}}{C_{l\alpha^+}}\right)(8X_w)\right]} \quad (41)$$

$$\boxed{\overline{M_{Y\alpha}} = \frac{1}{2}\rho V^2 R_w c\,\alpha\, C_{l\alpha^+}(R_{off}+\delta)\,\overline{C_{mY\alpha}}} \quad (42)$$



$$D_0^+(\xi, r_w) = \frac{1}{2}\rho V_n^2 C_{d0^+} C \cos\xi \, d\xi \, dr_w$$
$$= \frac{1}{2}\rho V^2 C \, C_{d0^+} \left[\cos\xi + \left(\frac{r_w X_w}{R_w}\right)\right] \cos\xi \, d\xi \, dr_w$$

$$\therefore \overline{D_0^+} = \frac{1}{2}\rho V^2 C \, C_{d0^+} \int_0^{R_w} \frac{1}{\pi} \int_0^{\pi - \cos^{-1}\left(\frac{r_w X_w}{R_w}\right)} \left[\cos\xi + \left(\frac{r_w X_w}{R_w}\right)\right] \cos\xi \, d\xi \, dr_w$$

As 2020 paper,

$$\therefore \overline{D_0^+} = \frac{1}{2}\rho V^2 R_w C \, C_{d0^+} \left\{ \frac{X_w}{2} + \frac{1}{24\pi}\left[8\sqrt{1-X_w^2} + \frac{8\sin^{-1}X_w}{X_w} + (2X_w^2-1)\sqrt{1-X_w^2} + \frac{\sin^{-1}X_w}{X_w} - \frac{6\sin^{-1}X_w}{X_w} - 12 X_w \cos^{-1}X_w + 6\sqrt{1-X_w^2} \right] \right\}$$

$$\boxed{\overline{D_0^+} = \frac{1}{2}\rho V^2 R_w C \, C_{d0^+} \left\{ \frac{X_w}{2} + \frac{1}{24\pi}\left[(2X_w^2+13)\sqrt{1-X_w^2} + \frac{3\sin^{-1}X_w}{X_w} - 12 X_w \cos^{-1}X_w \right] \right\}} \quad (43)$$

$\therefore$ Similarly,

$$\boxed{\overline{D_0^-} = \frac{1}{2}\rho V^2 R_w C \, C_{d0^-} \left\{ \frac{1}{24\pi}\left[(2X_w^2+13)\sqrt{1-X_w^2} + \frac{3\sin^{-1}X_w}{X_w} - 12 X_w \cos^{-1}X_w \right] \right\}} \quad (44)$$

$$\therefore \boxed{\overline{C_{D0}} = \frac{X_w}{2} + \frac{1}{24\pi}\left(1 + \frac{C_{d0^-}}{C_{d0^+}}\right)\left[(2X_w^2+13)\sqrt{1-X_w^2} + \frac{3\sin^{-1}X_w}{X_w} - 12 X_w \cos^{-1}X_w \right]} \quad (45)$$

$$\boxed{\overline{D_0} = \frac{1}{2}\rho V^2 R_w C \, C_{d0^+} \overline{C_{D0}}} \quad (46)$$

$$M_{z0}^+(\xi, r_w) = \frac{1}{2}\rho V_n^2 C_{d0^+} C \, r \sin\gamma \, d\xi \, dr_w$$
$$= \frac{1}{2}\rho V^2 C \, C_{d0^+} \left[\cos\xi + \left(\frac{r_w X_w}{R_w}\right)\right]^2 r_w \, d\xi \, dr_w$$

$$\therefore \overline{M_{z0}^+} = \frac{1}{2}\rho V^2 C \, C_{d0^+} \int_0^{R} \frac{1}{\pi} \int_0^{\pi - \cos^{-1}\left(\frac{r_w X_w}{R_w}\right)} \left[\cos\xi + \left(\frac{r_w X_w}{R_w}\right)\right]^2 r_w \, d\xi \, dr_w$$

$\therefore$ As in 2020 paper,

$$\overline{M_{z0}^+} = \frac{1}{2}\rho V^2 R_w^2 C \, C_{d0^+} \left\{ \frac{1}{4} + \frac{X_w^2}{4} + \frac{1}{32\pi X_w}\left[8(2X_w^2-1)\sqrt{1-X_w^2} + \frac{8\sin^{-1}X_w}{X_w} - \frac{4\sin^{-1}X_w}{X_w} - 8X_w\cos^{-1}X_w + 4\sqrt{1-X_w^2} - 8X_w^3\cos^{-1}X_w + (2X_w^2+3)\sqrt{1-X_w^2} - \frac{3\sin^{-1}X_w}{X_w} \right] \right\}$$

$$\boxed{\overline{M_{z0}^+} = \frac{1}{2}\rho V^2 R_w^2 C \, C_{d0^+} \left\{ \frac{1}{4} + \frac{X_w^2}{4} + \frac{1}{32\pi X_w}\left[(14X_w^2+1)\sqrt{1-X_w^2} - \frac{\sin^{-1}X_w}{X_w} - 8(X_w^3+X_w)\cos^{-1}X_w \right] \right\}} \quad (47)$$

Similarly,

$$\boxed{\overline{M_{z0}^-} = \frac{1}{2}\rho V^2 R_w^2 C \, C_{d0^-} \left\{ \frac{1}{32\pi X_w}\left[(14X_w^2+1)\sqrt{1-X_w^2} - \frac{\sin^{-1}X_w}{X_w} - 8(X_w^3+X_w)\cos^{-1}X_w \right] \right\}} \quad (48)$$

$$\therefore \boxed{\overline{C_{Mz0}} = \frac{1}{4} + \frac{X_w^2}{4} + \frac{1}{32\pi X_w}\left(1 + \frac{C_{d0^-}}{C_{d0^+}}\right)\left[(14X_w^2+1)\sqrt{1-X_w^2} - \frac{\sin^{-1}X_w}{X_w} - 8(X_w^3+X_w)\cos^{-1}X_w \right]} \quad (49)$$

$$\boxed{\overline{M_{z0}} = \frac{1}{2}\rho V^2 R_w^2 C \, C_{d0^+} \overline{C_{Mz0}}} \quad (50)$$

$$D_\alpha^+(\xi, r_w) = \frac{1}{2}\rho V^2 C\alpha \, C_{d\alpha^+} \left| \cos\xi + \left(\frac{r_w X_w}{R_w}\right) \right| \cos\xi \, d\xi \, dr_w$$

$$\therefore \boxed{\overline{D_\alpha^+} = \frac{1}{2}\rho V^2 R_w C\alpha \, C_{d\alpha^+} \left\{ \frac{1}{2} + \frac{1}{2\pi X_w}\left[\frac{2}{3} + X_w\cos^{-1}X_w - \sqrt{1-X_w^2} + \frac{1}{3}(1-X_w^2)^{3/2} \right] \right\}} \quad (51)$$

$$\therefore \boxed{\overline{D_\alpha^-} = \frac{1}{2}\rho V^2 R_w C\alpha \, C_{d\alpha^-} \left\{ -\frac{1}{2\pi X_w}\left[\frac{2}{3} + X_w\cos^{-1}X_w - \sqrt{1-X_w^2} + \frac{1}{3}(1-X_w^2)^{3/2} \right] \right\}} \quad (52)$$

$$\therefore \boxed{\overline{C_{D\alpha}} = \frac{1}{2} + \left(1 - \frac{C_{d\alpha^-}}{C_{d\alpha^+}}\right)\frac{1}{2\pi X_w}\left[\frac{2}{3} + X_w\cos^{-1}X_w - \sqrt{1-X_w^2} + \frac{1}{3}(1-X_w^2)^{3/2} \right]} \quad (53)$$

$$\boxed{\overline{D_\alpha} = \frac{1}{2}\rho V^2 R_w C\alpha \, C_{d\alpha^+} \overline{C_{D\alpha}}} \quad (54)$$

$$M_{z\alpha}^+(\xi, r_w) = \frac{1}{2}\rho V^2 C\alpha \, C_{d\alpha^+} \left| \cos\xi + \left(\frac{r_w X_w}{R_w}\right) \right| r_w \, d\xi \, dr_w$$

$$\therefore \overline{C_{Mz\alpha}} = \frac{X_w}{3} + \left(1 - \frac{C_{d\alpha^-}}{C_{d\alpha^+}}\right)\frac{1}{3\pi X_w^2}\left[1 - (1-X_w^2)\sqrt{1-X_w^2} - X_w^3\cos^{-1}X_w + \frac{1}{3}(X_w^2+2)\sqrt{1-X_w^2} - \frac{2}{3} \right]$$

$$\boxed{\overline{C_{Mz\alpha}} = \frac{X_w}{3} + \frac{1}{9\pi X_w^2}\left(1 - \frac{C_{d\alpha^-}}{C_{d\alpha^+}}\right)\left[(4X_w^2-1)\sqrt{1-X_w^2} - 3X_w^3\cos^{-1}X_w + 1 \right]} \quad (55)$$

$$\boxed{\overline{M_{z\alpha}} = \frac{1}{2}\rho V^2 R_w^2 C\alpha \, C_{d\alpha^+} \overline{C_{Mz\alpha}}} \quad (56)$$