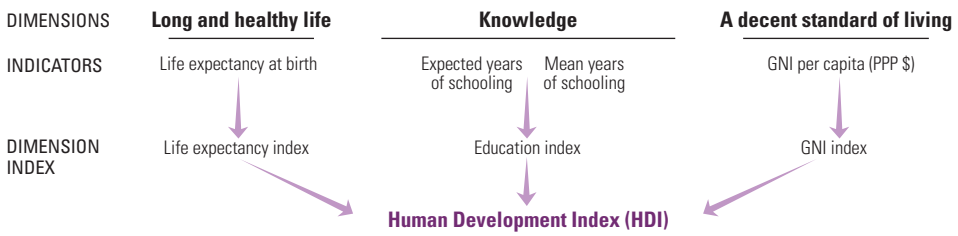
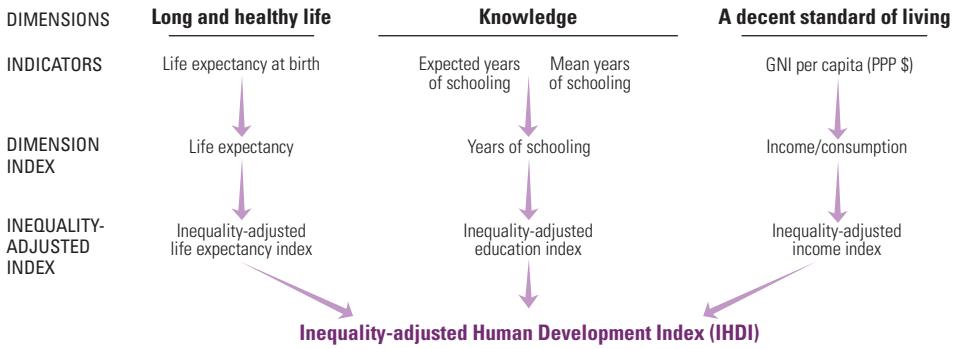
# Technical notes

## Calculating the human development indices—graphical presentation

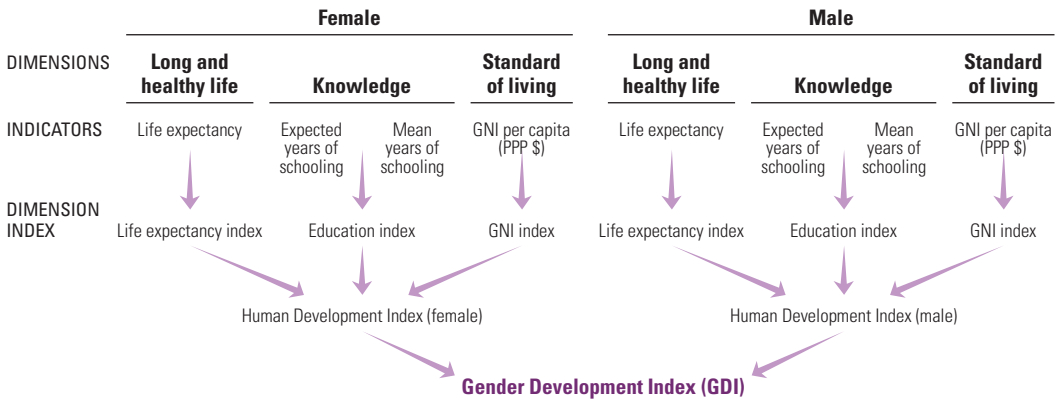
### Human Development Index (HDI)



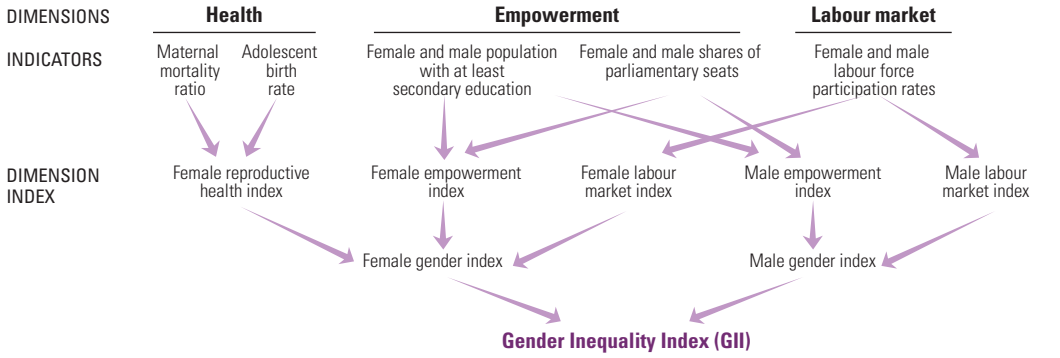

### Inequality-adjusted Human Development Index (IHDI)



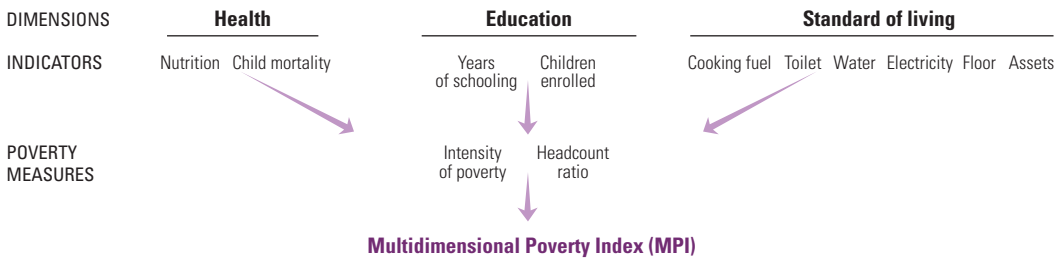

### Gender Development Index (GDI)




### Gender Inequality Index (GII)




### Multidimensional Poverty Index (MPI)



## Technical note 1. Human Development Index

The Human Development Index (HDI) is a summary measure of achievements in three key dimensions of human development: a long and healthy life, access to knowledge and a decent standard of living. The HDI is the geometric mean of normalized indices for each of the three dimensions.

## Data sources

- Life expectancy at birth: UNDESA (2017).
- Expected years of schooling: UNESCO Institute for Statistics (2018), ICF Macro Demographic and Health Surveys, United Nations Children's Fund (UNICEF) Multiple Indicator Cluster Surveys and OECD (2017).
- Mean years of schooling: UNESCO Institute for Statistics (2018), Barro and Lee (2016), ICF Macro Demographic and Health Surveys, UNICEF Multiple Indicator Cluster Surveys and OECD (2017).
- GNI per capita: World Bank (2018), IMF (2018) and United Nations Statistics Division (2018).

## Steps to calculate the Human Development Index

There are two steps to calculating the HDI.

### Step 1. Creating the dimension indices

Minimum and maximum values (goalposts) are set in order to transform the indicators expressed in different units into indices between 0 and 1. These goalposts act as the "natural zeros" and "aspirational targets," respectively, from which component indicators are standardized (see equation 1 below). They are set at the following values:

| Dimension | Indicator | Minimum | Maximum |
|---|---|---|---|
| Health | Life expectancy (years) | 20 | 85 |
| Education | Expected years of schooling (years) | 0 | 18 |
| | Mean years of schooling (years) | 0 | 15 |
| Standard of living | Gross national income per capita (2011 PPP $) | 100 | 75,000 |

The justification for placing the natural zero for life expectancy at 20 years is based on historical evidence that no country in the 20th century had a life expectancy of less than 20 years (Maddison, 2010; Oeppen and Vaupel, 2002; Riley, 2005). Maximum life expectancy is set at 85, a realistic aspirational target for many countries over the last 30 years. Due to constantly improving living conditions and medical advances, life expectancy has already come very close to 85 years in several economies: Hong Kong, China (Special Administrative Region) (84.1 years) and Japan (83.9 years).

Societies can subsist without formal education, justifying the education minimum of 0 years. The maximum for expected years of schooling, 18, is equivalent to achieving a master's degree in most countries. The maximum for mean years of schooling, 15, is the projected maximum of this indicator for 2025.

The low minimum value for gross national income (GNI) per capita, $100, is justified by the considerable amount of unmeasured subsistence and nonmarket production in economies close to the minimum, which is not captured in the official data. The maximum is set at $75,000 per capita. Kahneman and Deaton (2010) have shown that there is virtually no gain in human development and well-being from annual income per capita above $75,000. Currently, only four countries (Brunei Darussalam, Liechtenstein, Qatar and Singapore) exceed the $75,000 income per capita ceiling.

Having defined the minimum and maximum values, the dimension indices are calculated as:

$$\text{Dimension index} = \frac{\text{actual value} - \text{minimum value}}{\text{maximum value} - \text{minimum value}}. \qquad (1)$$

For the education dimension, equation 1 is first applied to each of the two indicators, and then the arithmetic mean of the two resulting indices is taken. Using the arithmetic mean allows perfect substitutability between mean years of schooling and expected years of schooling. Many developing countries have low school attainment among adults but are eager to achieve universal primary and secondary school enrolment.

Because each dimension index is a proxy for capabilities in the corresponding dimension, the transformation function from income to capabilities is likely to be concave (Anand and Sen 2000)—that is, each additional dollar of income has a smaller effect on expanding capabilities. Thus for income, the natural logarithm of the actual, minimum and maximum values is used.

### Step 2. Aggregating the dimensional indices to produce the Human Development Index

The HDI is the geometric mean of the three dimensional indices:

$$HDI = (I_{Health} \cdot I_{Education} \cdot I_{Income})^{1/3}$$

*Example: Egypt*

| Indicator | Value |
|---|---|
| Life expectancy at birth (years) | 71.7 |
| Expected years of schooling (years) | 13.1 |
| Mean years of schooling (years) | 7.2 |
| Gross national income per capita (2011 PPP $) | 10,355 |

**Note:** Values are rounded.

$$\text{Health index} = \frac{71.661 - 20}{85 - 20} = 0.7948$$

$$\text{Expected years of schooling index} = \frac{13.0898 - 0}{18 - 0} = 0.7272$$

$$\text{Mean years of schooling index} = \frac{7.218 - 0}{15 - 0} = 0.4812$$

$$\text{Education index} = \frac{0.4812 + 0.7272}{2} = 0.6042$$

$$\text{Income index} = \frac{\ln(10{,}355) - \ln(100)}{\ln(75{,}000) - \ln(100)} = 0.7009$$

$$\text{Human Development Index} = (0.7948 \cdot 0.6042 \cdot 0.7009)^{1/3} = 0.696$$

## Methodology used to express income

The World Bank's 2018 World Development Indicators database contains estimates of GNI per capita in constant 2011 purchasing power parity (PPP) terms for many countries. For countries missing this indicator (entirely or partly), the Human Development Report Office (HDRO) calculates it by converting GNI per capita from current to constant terms using two steps. First, the value of GNI per capita in current terms is converted into PPP terms for the base year (2011). Second, a time series of GNI per capita in constant 2011 PPP terms is constructed by applying the real growth rates to the GNI per capita in PPP terms for the base year. The real growth rate is implied by the ratio of the nominal growth of GNI per capita in current local currency terms to the GDP deflator.

For several countries without a value of GNI per capita in constant 2011 PPP terms for 2017 reported by the World Bank, the International Monetary Fund (IMF)–projected real growth rates of GDP are applied to the most recent GNI values in constant PPP terms. The IMF-projected growth rates are calculated based on local currency terms and constant prices rather than in PPP terms. This avoids mixing the effects of the PPP conversion with those of real growth of the economy.

Official PPP conversion rates are produced by the International Comparison Program, whose surveys periodically collect thousands of prices of matched goods and services in many countries. The last round of this exercise refers to 2011 and covered 199 countries.

## Estimating missing values

For a small number of countries missing one of the four indicators, the HDRO estimated the missing values using cross-country regression models.

In this Update expected years of schooling were estimated for Bahamas, Dominica, Equatorial Guinea, Haiti, Libya, Papua New Guinea, Tonga, Trinidad and Tobago, and Vanuatu. Mean years of schooling were estimated for Eritrea, Grenada and Saint Kitts and Nevis.

## Country groupings

The 2014 Human Development Report introduced fixed cutoff points for four categories of human development achievements. The cutoff points ($COP$) are the HDI values calculated using the quartiles ($q$) from the distributions of the component indicators (life expectancy at birth [$LE$], expected years of schooling [$EYS$], mean years of schooling [$MYS$] and GNI per capita [$GNIpc$]):

$$COP_q = HDI\,(LE_q, EYS_q, MYS_q, GNIpc_q),\ q = 1,2,3.$$

For example, LE1, LE2 and LE3 denote three quartiles of the distribution of life expectancy across countries. The resulting HDI values are averaged over 2004–2013.

This Update keeps the same cutoff points of the HDI for grouping countries that were introduced in the 2014 Report:

| | |
|---|---|
| Very high human development | 0.800 and above |
| High human development | 0.700–0.799 |
| Medium human development | 0.550–0.699 |
| Low human development | Below 0.550 |