CAPM_MVO

July 15, 2025

[ ]:
```python
import pandas as pd
import numpy as np
from sklearn.linear_model import LinearRegression
from pathlib import Path
from pypfopt.expected_returns import mean_historical_return
from pypfopt.risk_models import risk_matrix
import os
```

1 Load Data

[ ]:
```python
tickers = ['AAPL', 'AMZN', 'BA', 'COST', 'JNJ', 'NVDA', 'TMO', 'TSLA', 'VLO']
data_dir = "Data"
features = ['close']

all_data = {}
min_length = float('inf')


for stock in tickers:
    df = pd.read_csv(os.path.join(data_dir, f"{stock}_daily_aggregated.csv"))
    df['log_return'] = np.log(df['close'] / df['close'].shift(1))


    #
    df['date'] = pd.to_datetime(df['date'])
    #
    df = df.dropna().reset_index(drop=True)

    all_data[stock] = df
    min_length = min(min_length, len(df))  #

for stock in tickers:
    all_data[stock] = all_data[stock].tail(min_length).reset_index(drop=True)
    all_data[stock] = all_data[stock][["date","close"]]
```

[ ]:
```python
close_price_list = []
```

```python
for ticker, df in all_data.items():
    df = df[['date', 'close']].copy()
    df['date'] = pd.to_datetime(df['date'])
    df.set_index('date', inplace=True)
    df.rename(columns={'close': ticker}, inplace=True)
    close_price_list.append(df)


close_df = pd.concat(close_price_list, axis=1)
close_df = close_df.sort_index()
close_df
```

[ ]:

```
                AAPL      AMZN        BA       COST       JNJ      NVDA  \
date
2021-01-05  127.7365  160.7550  211.3000   356.6557  139.3333   13.3759
2021-01-06  123.9804  157.1575  211.2400   351.2073  140.6264   12.6792
2021-01-07  127.4439  158.4420  210.3000   349.8499  141.3654   13.2540
2021-01-08  128.9268  159.0995  209.9900   351.1670  140.7848   13.2473
2021-01-11  125.8050  155.7285  205.8400   345.5120  140.1954   13.5899
…                …         …         …          …         …         …
2025-05-23  195.2694  200.7890  193.6696  1007.3350  151.3065  131.2600
2025-05-27  200.2654  205.9100  201.1800  1018.0000  153.2100  135.2500
2025-05-28  200.3800  205.6600  201.9000  1014.5100  152.0325  141.7700
2025-05-29  199.8750  205.6500  207.9100  1010.3360  153.9488  138.6950
2025-05-30  200.6500  205.1200  207.2600  1041.0250  155.4900  134.9900

                 TMO      TSLA       VLO
date
2021-01-05  472.7761  249.7867   48.7026
2021-01-06  480.1886  254.2500   49.4707
2021-01-07  495.1521  276.4833   50.0277
2021-01-08  506.9982  293.2667   48.7109
2021-01-11  511.3527  270.4600   49.5382
…                …         …         …
2025-05-23  393.6600  339.9900  128.9000
2025-05-27  405.4000  362.7800  130.7100
2025-05-28  404.0000  361.8500  127.0100
2025-05-29  402.5300  357.3600  126.0500
2025-05-30  409.9000  346.3700  128.9700

[1106 rows x 9 columns]
```

[ ]:

```python
sp = pd.read_csv("sp500.csv")
sp.date = pd.to_datetime(sp.date)
market_close_df = sp[["date","close"]]
market_close_df.set_index("date", inplace = True)
market_close_df
```

[ ]:
```
             close
date
2021-01-05  371.33
2021-01-06  373.55
2021-01-07  379.10
2021-01-08  381.26
2021-01-11  378.69
…              …
2025-05-23  579.11
2025-05-27  591.15
2025-05-28  587.73
2025-05-29  590.05
2025-05-30  589.39

[1106 rows x 1 columns]
```

[ ]:
```
rf_df = pd.read_csv("rf.csv")
rf_df.drop(columns = "Unnamed: 0", inplace = True)
daily_rfr = rf_df.set_index("date")
daily_rfr.index = pd.to_datetime(daily_rfr.index)
daily_rfr
```

[ ]:
```
            rf_daily
date
2021-01-04  0.000004
2021-01-05  0.000004
2021-01-06  0.000004
2021-01-07  0.000004
2021-01-08  0.000003
…                …
2025-06-12  0.000177
2025-06-13  0.000177
2025-06-16  0.000176
2025-06-17  0.000175
2025-06-18  0.000175

[1163 rows x 1 columns]
```

2 Find Tangency Portfolio using CAPM Based Expected Return

[ ]:
```
from sklearn.linear_model import LinearRegression
from pypfopt.risk_models import sample_cov
from pypfopt.efficient_frontier import EfficientFrontier

# Ensure sorted dates
close_df = close_df.sort_index()
market_close_df = market_close_df.sort_index()
```

```python
stocks = close_df.columns.tolist()

window = 252
rebalance_every = 21
dynamic_weights = {}

for t in range(window, len(close_df) - 1):
    current_date = close_df.index[t]

    # === Risk-Free Rate (annualized from daily) ===
    rf_row = daily_rfr.loc[daily_rfr.index.normalize() == current_date, "rf_daily"]
    if not rf_row.empty:
        risk_free_rate = (1 + rf_row.values[0]) ** 252 - 1
    else:
        risk_free_rate = 0.0

    # === Rebalance only every N days ===
    if (t - window) % rebalance_every != 0:
        if dynamic_weights:
            dynamic_weights[current_date] = dynamic_weights[list(dynamic_weights.keys())[-1]]
        continue

    # === Get window data ===
    window_data = close_df.iloc[t - window:t]
    market_window = market_close_df.iloc[t - window:t]
    if window_data.isnull().values.any() or market_window.isnull().values.any():
        continue

    # === Compute returns ===
    asset_returns = window_data.pct_change().dropna()
    market_returns = market_window.pct_change().dropna()

    # === Align dates ===
    common_index = asset_returns.index.intersection(market_returns.index)
    asset_returns = asset_returns.loc[common_index]
    market_returns = market_returns.loc[common_index]
    rf_daily_window = daily_rfr.loc[common_index].fillna(0)

    # === Excess returns ===
    excess_market = market_returns["close"] - rf_daily_window["rf_daily"]
    excess_assets = asset_returns.sub(rf_daily_window["rf_daily"], axis=0)

    # === Estimate beta via linear regression ===
    betas = {}
    for asset in excess_assets.columns:
```

```
        X = excess_market.values.reshape(-1, 1)
        y = excess_assets[asset].values.reshape(-1, 1)
        reg = LinearRegression().fit(X, y)
        betas[asset] = reg.coef_[0][0]

    # === Compute CAPM expected returns ===
    mu_market = market_returns["close"].mean() * 252
    rf_annual = (1 + rf_daily_window["rf_daily"].mean()) ** 252 - 1

    mu_capm = pd.Series({
        asset: rf_annual + beta * (mu_market - rf_annual)
        for asset, beta in betas.items()
    })

    # === Covariance matrix ===
    S = sample_cov(window_data, frequency=252)

    # === Align assets ===
    common_assets = mu_capm.index.intersection(S.columns)
    mu_capm = mu_capm.loc[common_assets]
    S = S.loc[common_assets, common_assets]

    # Fallback to historical mean return if no CAPM asset beats r_f
    if (mu_capm - risk_free_rate <= 0).all():
        print(f" {current_date.date()}: CAPM returns too low. Using historical␣
↪mean return instead.")
        mu_hist = window_data.pct_change().mean() * 252
        mu_hist = mu_hist.loc[common_assets]
        mu_used = mu_hist
    else:
        mu_used = mu_capm

    # === Portfolio optimization ===
    try:
        ef = EfficientFrontier(mu_used, S, weight_bounds=(-1.5, 1.5))
        ef.max_sharpe(risk_free_rate=risk_free_rate)
        cleaned_weights = ef.clean_weights()
        dynamic_weights[current_date] = pd.Series(cleaned_weights)
    except Exception as e:
        print(f" Optimization failed on {current_date.date()} using max Sharpe:
↪ {e}")
        try:
            # Fallback: global minimum variance portfolio
            ef = EfficientFrontier(None, S, weight_bounds=(-1.5, 1.5))
            ef.min_volatility()
            cleaned_weights = ef.clean_weights()
            dynamic_weights[current_date] = pd.Series(cleaned_weights)
```

```
            print(f"  Fallback to GMV succeeded on {current_date.date()}")
        except Exception as e2:
            print(f"  Fallback to GMV also failed on {current_date.date()}:␣
↪{e2}")
            continue

# === Final weights DataFrame ===
weights_df = pd.DataFrame(dynamic_weights).T
weights_df = weights_df.reindex(close_df.index).fillna(method='ffill').fillna(0)
```

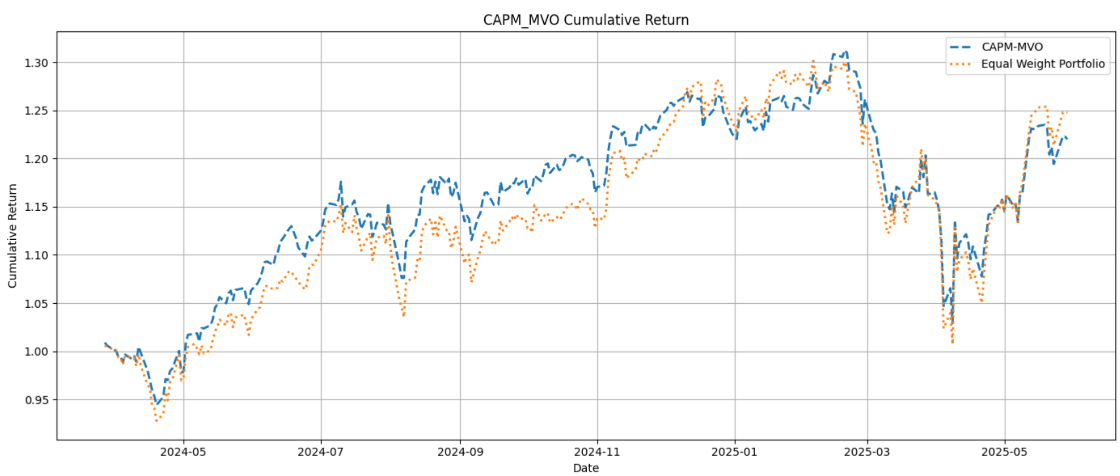
```
2022-06-06: CAPM returns too low. Using historical mean return instead.
2022-07-07: CAPM returns too low. Using historical mean return instead.
2022-08-05: CAPM returns too low. Using historical mean return instead.
2022-09-06: CAPM returns too low. Using historical mean return instead.
2022-10-05: CAPM returns too low. Using historical mean return instead.
2022-11-03: CAPM returns too low. Using historical mean return instead.
2022-12-05: CAPM returns too low. Using historical mean return instead.
2023-01-05: CAPM returns too low. Using historical mean return instead.
2023-02-06: CAPM returns too low. Using historical mean return instead.
2023-03-08: CAPM returns too low. Using historical mean return instead.
2023-04-06: CAPM returns too low. Using historical mean return instead.
2023-05-08: CAPM returns too low. Using historical mean return instead.
```

```
/var/folders/w_/wzxdnvq13mxbxkhjgg1mty_80000gn/T/ipykernel_24774/3722444315.py:1
05: FutureWarning: DataFrame.fillna with 'method' is deprecated and will raise
in a future version. Use obj.ffill() or obj.bfill() instead.
  weights_df =
weights_df.reindex(close_df.index).fillna(method='ffill').fillna(0)
```

3 Output and Evaluation

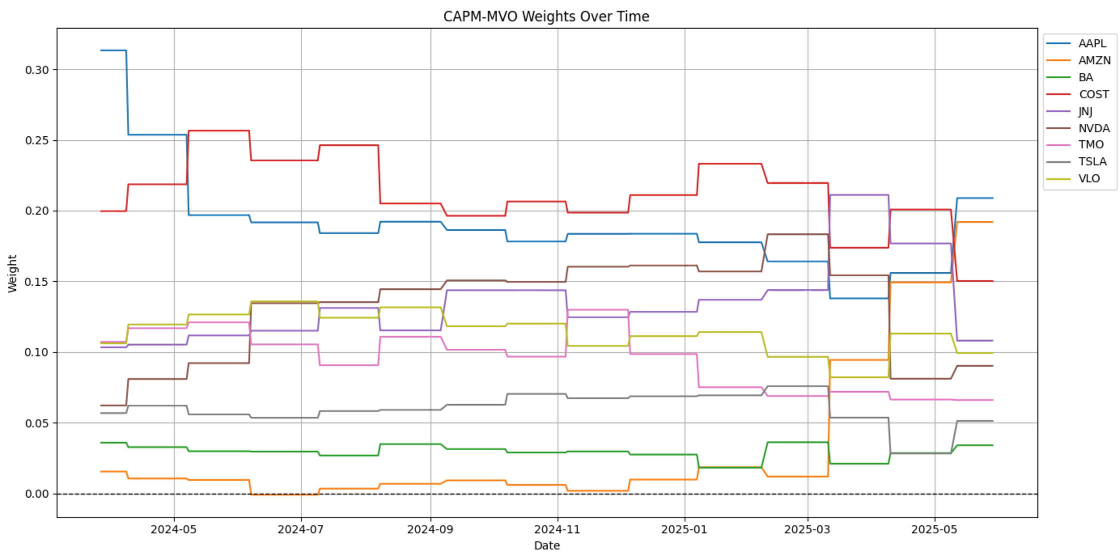
[ ]:
```
import matplotlib.pyplot as plt

# 1. Calculate daily returns
returns_df = close_df.pct_change().fillna(0)

# 2. Align weights and returns
start_date = pd.to_datetime("2024-03-27")
end_date = pd.to_datetime("2025-05-29")
returns_df = returns_df.loc[start_date:end_date]
weights_df_eval = weights_df.loc[start_date:end_date]

# Match dates and ensure weights & returns align
common_dates = returns_df.index.intersection(weights_df_eval.index)
returns_df = returns_df.loc[common_dates]
weights_df_eval = weights_df_eval.loc[common_dates]
```

```
# 3. Compute daily portfolio returns
# (row-wise dot product of weights and asset returns)
port_returns = (weights_df_eval * returns_df).sum(axis=1)
equal_weights = np.ones(len(tickers)) / len(tickers)
equal_returns = (returns_df.values * equal_weights).sum(axis=1)


# 4. Compute cumulative returns
cumulative_returns = (1 + port_returns).cumprod()
cumulative_equal = (1 + equal_returns).cumprod()

# 5. Plot cumulative return
plt.figure(figsize=(14, 6))
plt.plot(cumulative_returns.index, cumulative_returns, label='CAPM-MVO',␣
↪linestyle='--', linewidth=2)
plt.plot(cumulative_returns.index, cumulative_equal, label='Equal Weight␣
↪Portfolio', linestyle=':', linewidth=2)
plt.title("CAPM_MVO Cumulative Return")
plt.xlabel("Date")
plt.ylabel("Cumulative Return")
plt.grid(True)
plt.legend()
plt.tight_layout()
plt.show()
```



[ ]:
```
# 6. Plot weights over time (line plot for long/short portfolios)
import matplotlib.pyplot as plt
plt.figure(figsize=(14, 7))
for col in weights_df_eval.columns:
    plt.plot(weights_df_eval.index, weights_df_eval[col], label=col)
```

```
plt.axhline(0, color='black', linewidth=1, linestyle='--')
plt.title("CAPM-MVO Weights Over Time")
plt.xlabel("Date")
plt.ylabel("Weight")
plt.legend(loc='upper left', bbox_to_anchor=(1.0, 1.0))
plt.grid(True)
plt.tight_layout()
plt.show()
```



[ ]:
```
def calculate_metrics(returns, var_conf_level=0.95):
    returns = pd.Series(returns)
    cumulative = (1 + returns).cumprod()
    total_return = cumulative.iloc[-1] - 1
    annualized_return = (1 + total_return) ** (252 / len(returns)) - 1
    volatility = returns.std() * np.sqrt(252)
    sharpe_ratio = annualized_return / volatility if volatility > 0 else 0
    max_drawdown = (cumulative / cumulative.cummax() - 1).min()

    # Historical Value at Risk (e.g., 5% worst return)
    var_percentile = 100 * (1 - var_conf_level)
    value_at_risk = np.percentile(returns, var_percentile)

    return total_return, annualized_return, volatility, sharpe_ratio,
 ↪max_drawdown, value_at_risk
```

[ ]:
```
# Calculate metrics
port_metrics = calculate_metrics(port_returns)
```

```python
# Unpack results
total_return, annualized_return, volatility, sharpe_ratio, max_drawdown, value_at_risk = port_metrics

# Print results nicely
print("\n Portfolio Performance Metrics:")
print(f"Total Return:        {total_return:.2%}")
print(f"Annualized Return:   {annualized_return:.2%}")
print(f"Volatility:          {volatility:.2%}")
print(f"Sharpe Ratio:        {sharpe_ratio:.4f}")
print(f"Max Drawdown:        {max_drawdown:.2%}")
print(f"Value at Risk (5%):  {value_at_risk:.2%}")
```

```
 Portfolio Performance Metrics:
Total Return:        21.99%
Annualized Return:   18.58%
Volatility:          22.03%
Sharpe Ratio:        0.8432
Max Drawdown:        -21.59%
Value at Risk (5%):  -2.04%
```