OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT **4771/01**

MATHEMATICS (MEI)

Decision Mathematics 1

TUESDAY 22 JANUARY 2008

Afternoon

Time: 1 hour 30 minutes

Additional materials: Printed Answer Book (Enclosed)
MEI Examination Formulae and Tables (MF2)

CUP/T38511

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Printed Answer Book.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

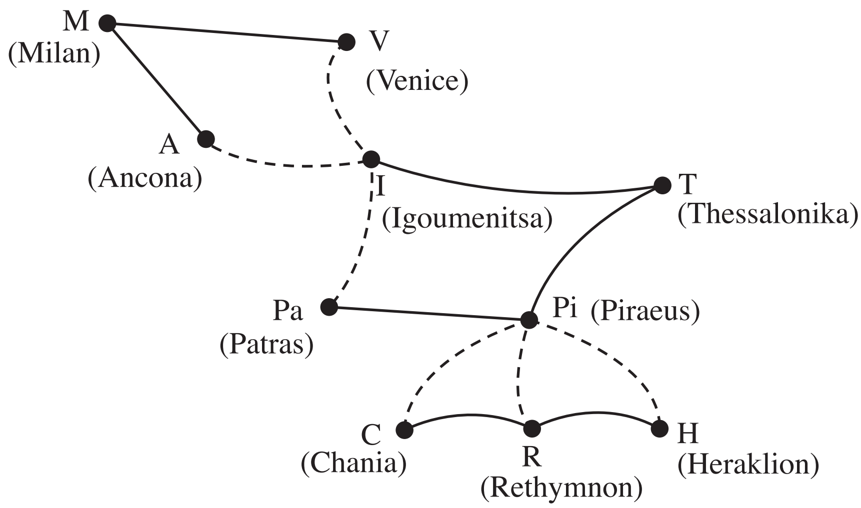
- The number of marks for each question is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is **72**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **7** printed pages and **1** blank page.

Answer all the questions in the printed answer book provided.

Section A (24 marks)

1 The graph shows routes that are available to an international lorry driver. The solid arcs represent motorways and the broken arcs represent ferry crossings.



(i) Give a route from Milan to Chania involving exactly two ferry crossings.

How many such routes are there? [2]

(ii) Give a route from Milan to Chania involving exactly three ferry crossings.

How many such routes are there? [2]

(iii) Give a route from Milan to Chania using as many ferry crossings as possible, without repeating any arc. [1]

(iv) Give a route leaving Piraeus and finishing elsewhere which uses every arc once and only once. [3]

2 Consider the following linear programming problem.

Maximise $P = 6x + 7y$

subject to
$2x + 3y \leqslant 9$
$3x + 2y \leqslant 12$
$x \geqslant 0$
$y \geqslant 0$

(i) Use a graphical approach to solve the problem. [6]

(ii) Give the optimal values of x, y and P when x and y are integers. [2]

3 The following algorithm (J. M. Oudin, 1940) claims to compute the date of Easter Sunday in the Gregorian calendar system.
The algorithm uses the year, y, to give the month, m, and day, d, of Easter Sunday.
All variables are integers and **all remainders from division are dropped**. For example, 7 divided by 3 is 2 remainder 1. The remainder is dropped, giving the answer 2.

$c = y / 100$

$n = y - 19 \times (y / 19)$

$k = (c - 17) / 25$

$i = c - (c / 4) - (c - k) / 3 + (19 \times n) + 15$

$i = i - 30 \times (i / 30)$

$i = i - (i / 28) \times (1 - (i / 28) \times (29 / (i + 1)) \times ((21 - n) / 11))$

$j = y + (y / 4) + i + 2 - c + (c / 4)$

$j = j - 7 \times (j / 7)$

$p = i - j$

$m = 3 + (p + 40) / 44$

$d = p + 28 - 31 \times (m / 4)$

For example, for 2008:

$y = 2008$

$c = 2008/100 = 20$

$n = 2008 - 19 \times (2008/19) = 2008 - 19 \times (105) = 13$, etc.

Complete the calculation for 2008. [8]

Section B (48 marks)

4 In a population colonizing an island 40% of the first generation (parents) have brown eyes, 40% have blue eyes and 20% have green eyes. Offspring eye colour is determined according to the following rules.

	Eye colours of parents	Eye colour of offspring
1	both brown	brown
2	one brown and one blue	50% brown and 50% blue
3	one brown and one green	blue
4	both blue	25% brown, 50% blue and 25% green
5	one blue and one green	50% blue and 50% green
6	both green	green

(i) Give an efficient rule for using 1-digit random numbers to simulate the eye colour of a parent randomly selected from the colonizing population. [3]

(ii) Give an efficient rule for using 1-digit random numbers to simulate the eye colour of offspring born of parents both of whom have blue eyes. [4]

The table in your answer book shows an incomplete simulation in which parent eye colours have been randomly selected, but in which offspring eye colours remain to be determined or simulated.

(iii) Complete the table using the given random numbers where needed. (You will need your own rules for cases 2 and 5.)
Each time you use a random number, explain how you decide which eye colour for the offspring. [9]

5 The table shows some of the activities involved in building a block of flats. The table gives their durations and their immediate predecessors.

Activity		Duration (weeks)	Immediate Predecessors
A	Survey sites	8	–
B	Purchase land	22	A
C	Supply materials	10	–
D	Supply machinery	4	–
E	Excavate foundations	9	B, D
F	Lay drains	11	B, C, D
G	Build walls	9	E, F
H	Lay floor	10	E, F
I	Install roof	3	G
J	Install electrics	5	G

(i) Draw an activity on arc network for these activities. [4]

(ii) Mark on your diagram the early and late times for each event. Give the minimum completion time and the critical activities. [6]

Each of the tasks E, F, H and J can be speeded up at extra cost. The maximum number of weeks by which each task can be shortened, and the extra cost for each week that is saved, are shown in the table below.

Task	E	F	H	J
Maximum number of weeks by which task may be shortened	3	3	1	3
Cost per week of shortening task (in thousands of pounds)	30	15	6	20

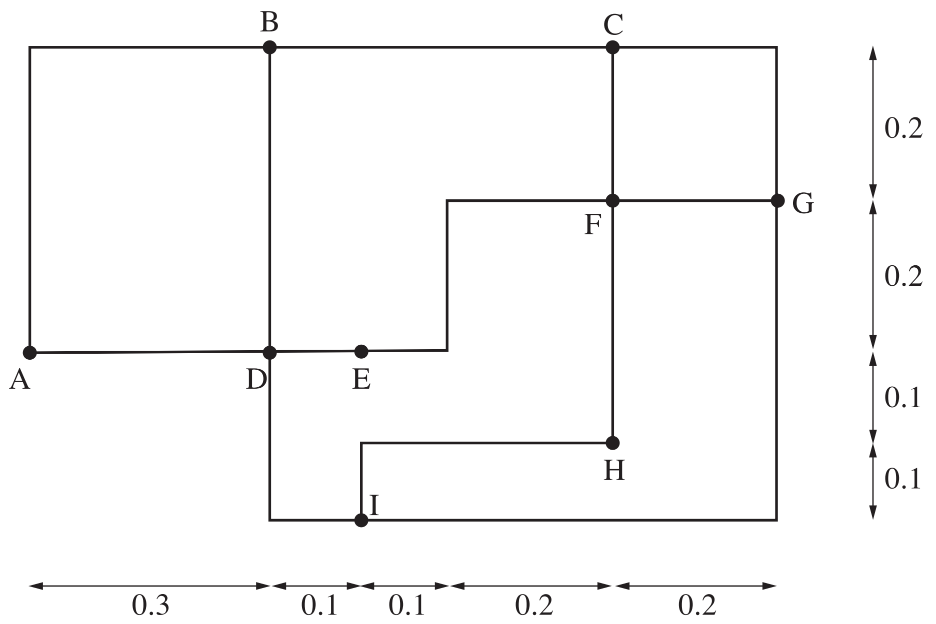
(iii) Find the new shortest time for the flats to be completed. [1]

(iv) List the activities which will need to be speeded up to achieve the shortest time found in part **(iii)**, and the times by which each must be shortened. [4]

(v) Find the total extra cost needed to achieve the new shortest time. [1]

[Question 6 is printed overleaf.]

6 The diagram shows routes between points in a town. The distances are in kilometres.



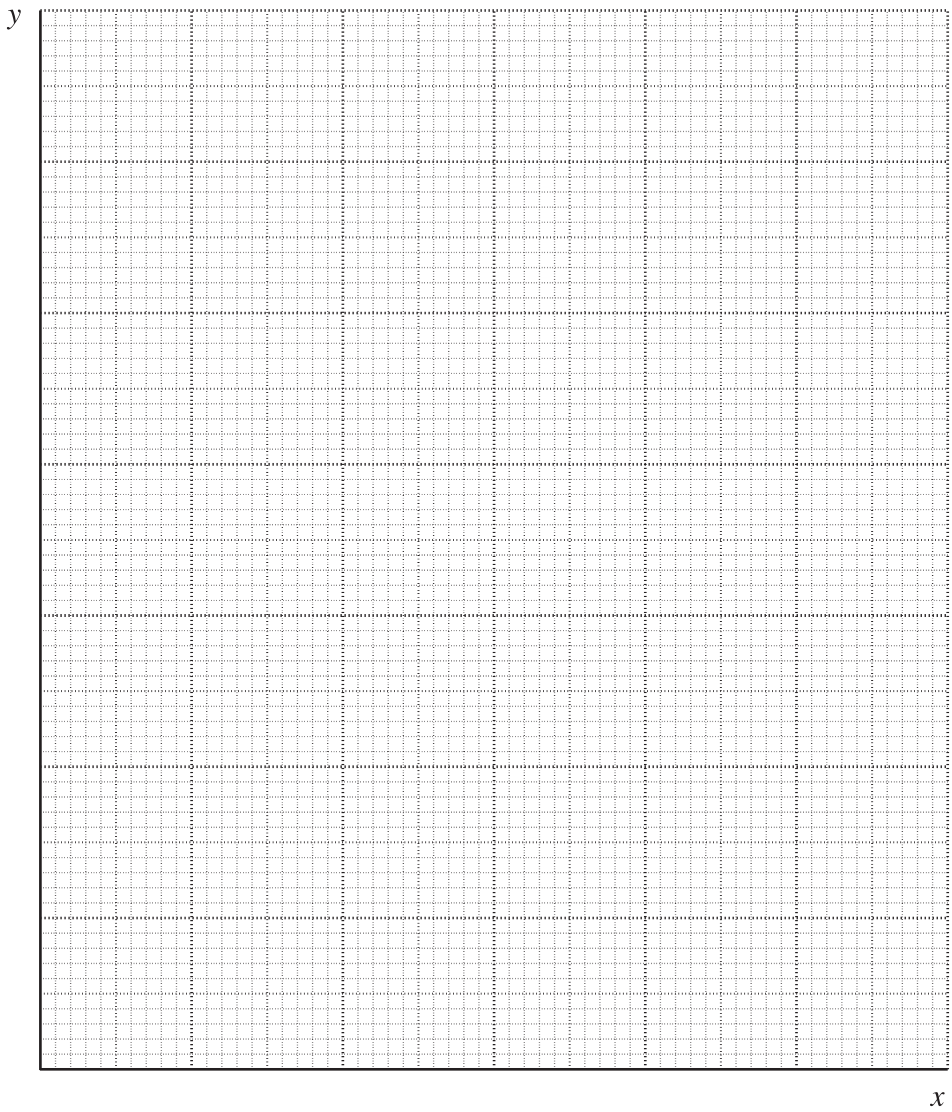
(i) Use an appropriate algorithm to find a set of connecting arcs of minimum total length. Indicate your connecting arcs on the copy of the diagram in your answer book, and give their total length. [7]

(ii) Give the name of the algorithm you have used, and describe it briefly. [2]

(iii) Using the second diagram in your answer book, apply Dijkstra's algorithm to find the shortest distances from A to each of the other points.

List the connections that are used, and give their total length. [7]

2 (i)



(ii)

3 $y = 2008$

$c = 2008/100 = 20$

$n = 2008 - 19 \times (2008/19) = 2008 - 19 \times (105) = 13$

$k =$

$i =$

$i =$

$i =$

$j =$

$j =$

$p =$

$m =$

$d =$

So Easter Sunday, 2008 will be on ....

4 (i)

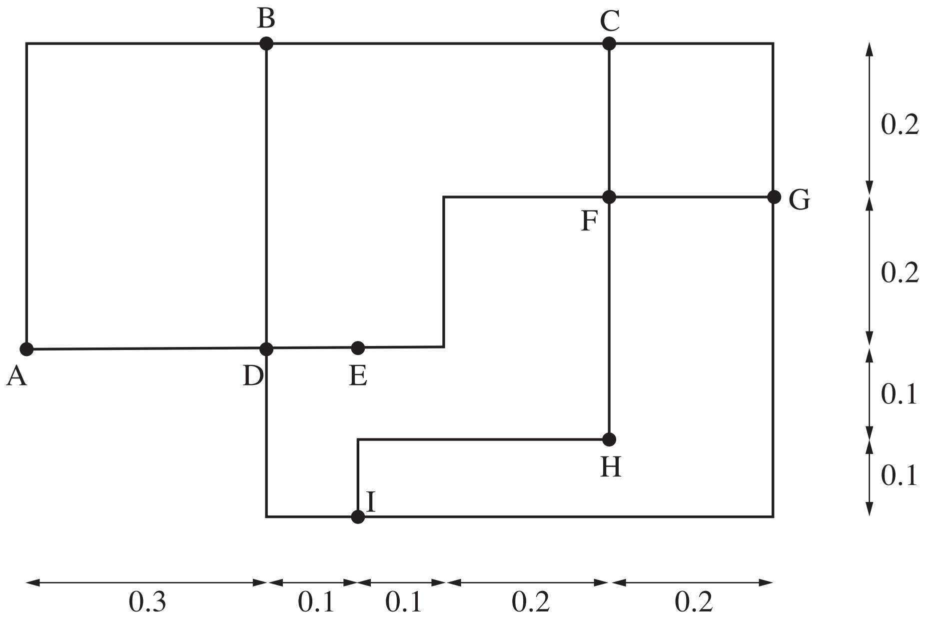
(ii)

(iii) Random numbers: 3 9 0 1 3 5 8 8 0 9 7

Eye colours

Parent 1	brown	brown	brown	blue	brown	green	blue	green	brown	brown
Parent 2	brown	blue	brown	blue	brown	blue	brown	green	brown	green
Offspring										

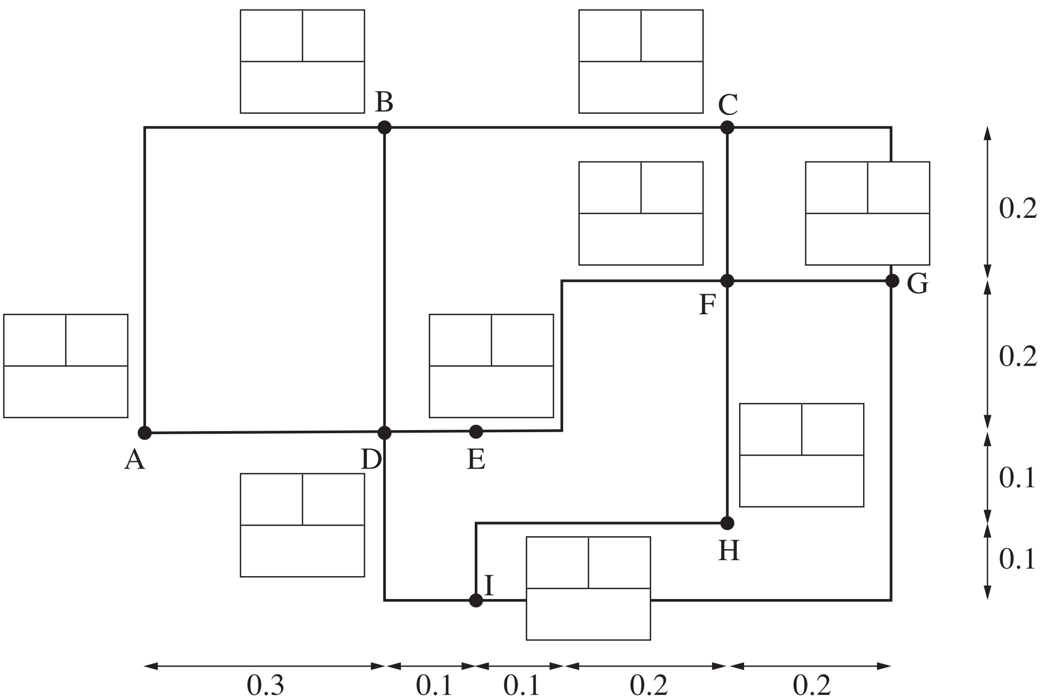
6 (i)



Total length of minimum connector: ....

(ii)

(iii)



Arcs used: ....

Total length of arcs used: ....

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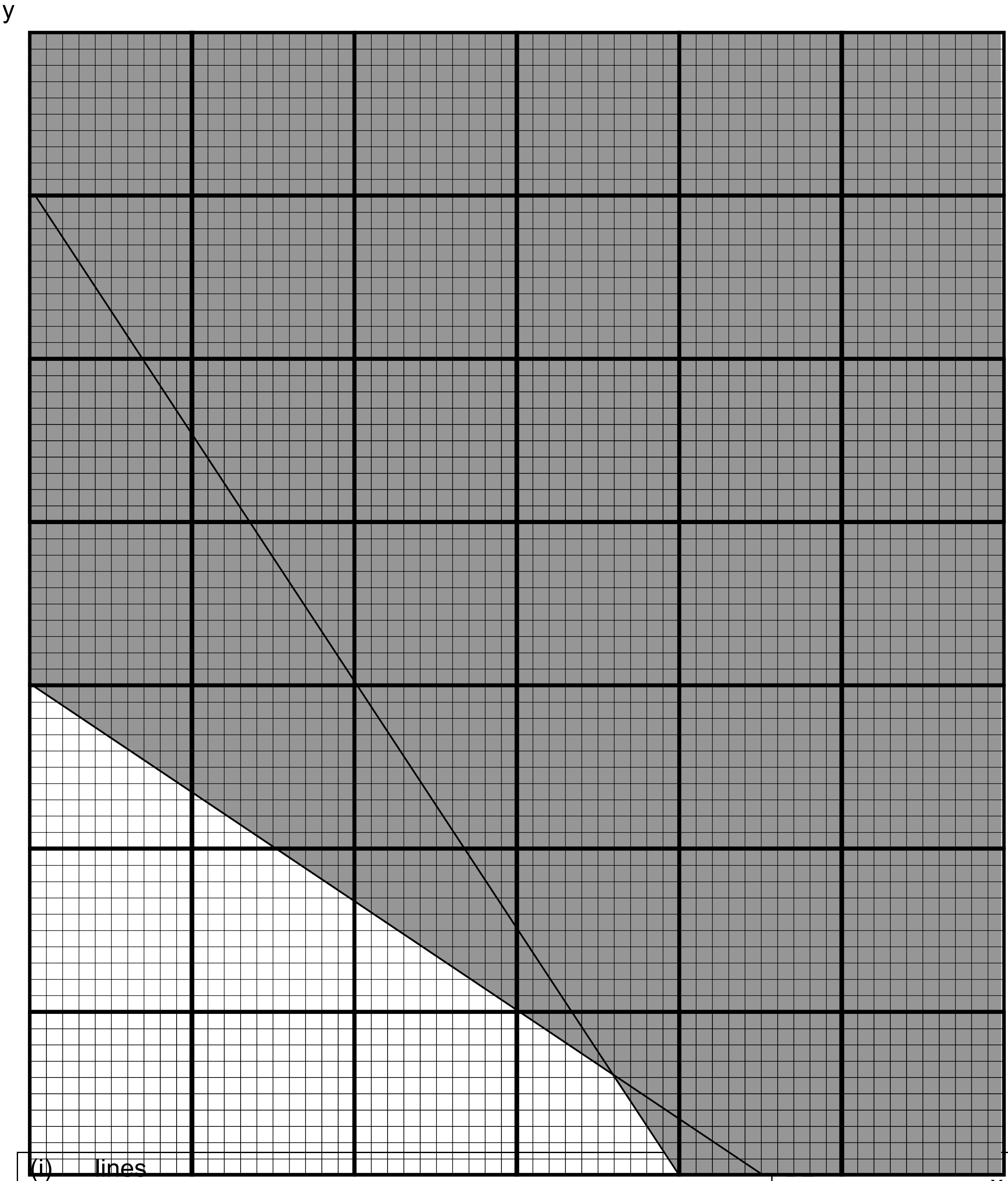
1

(i)	6 routes M→A→I→T→Pi→C M→A→I→T→Pi→R→C M→A→I→T→Pi→H→R→C M→V→I→T→Pi→C M→V→I→T→Pi→R→C M→V→I→T→Pi→H→R→C	B1 B1	
(ii)	6 routes M→A→I→Pa→Pi→C M→A→I→Pa→Pi→R→C M→A→I→Pa→Pi→H→R→C M→V→I→Pa→Pi→C M→V→I→Pa→Pi→R→C M→V→I→Pa→Pi→H→R→C	B1 B1	
(iii)	M→V→I→Pa→Pi→H→R→Pi→C A R→H	B1	
(iv)	e.g. P→T→I→V→M→A→I→Pa→P→H→R→C→P→R	 M1 A2	 ends at R (–1 each error/omission)

2.



(i)	lines		
	shading	B1	
	(3.6, 0.6)	B1	graph or sim. eqns
	25.8 at (3.6, 0.6) versus 21 and 24 (or profit line)	M1 A1	
(ii)	25 at (3, 1)	B1 B1	

3.

y = 2008 c = 2008/100 = 20 n = 2008 – 19 x (2008/19) = 2008 – 19 x (105) = 13	
k = 3/25 = 0	B1
i = 20 – 5 – 20 / 3 + 19 × 13 + 15 = 271 i = 1	B1
i = 1 – 0 = 1 j = 2008 + 502 + 1 + 2 – 20 + 5 = 2498	B1
j = 6	B1
p = – 5	B1
m = 3	B1
d = 23	B1
So 23rd March	B1

4.

<table>
<tr><td>

(i) e.g. 0–3→brown
4–7→blue
8–9→green

(ii) e.g. 0–1→brown
2–5→blue
6–7→green
8–9→reject

(iii) e.g.

Eye colours

Parent 1	brown	brown	brown	blue
Parent 2	brown	blue	brown	blue
Offspring	brown	brown	brown	brown

brown	green	blue	green	brown	brown
brown	blue	brown	green	brown	green
brown	blue	brown	green	brown	blue

</td><td>

M1
A1 proportions OK
A1 efficient

M1 some rejected
A2 proportions OK
(–1 each error)
A1 efficient

B1 br/br→br (4 times)
B1 br/gr→bl
B1 gr/gr→gr

M1 br/bl rule
A1 application
A1 application

B1 bl/bl application

M1 gr/bl rule
A1 application

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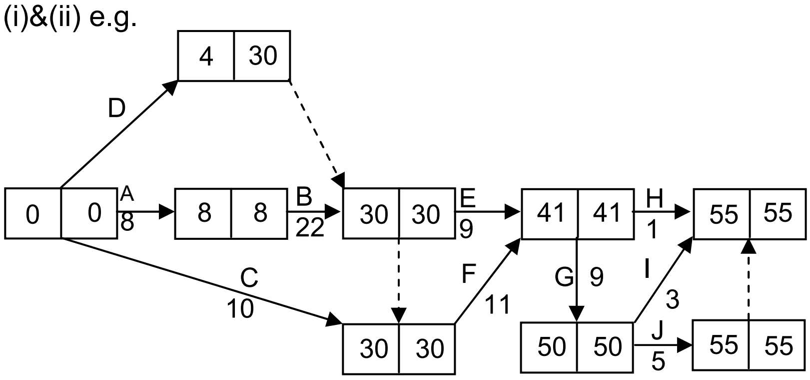
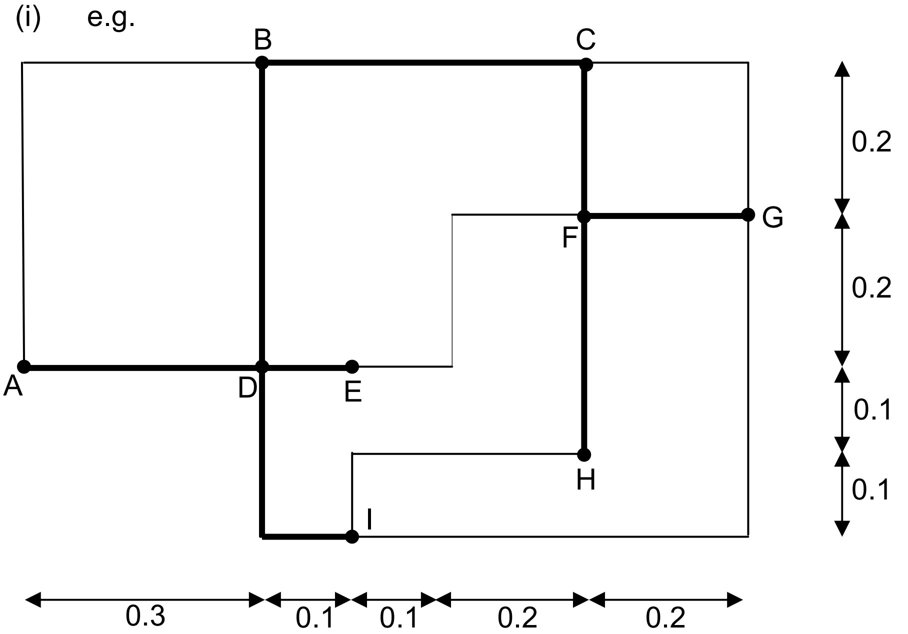
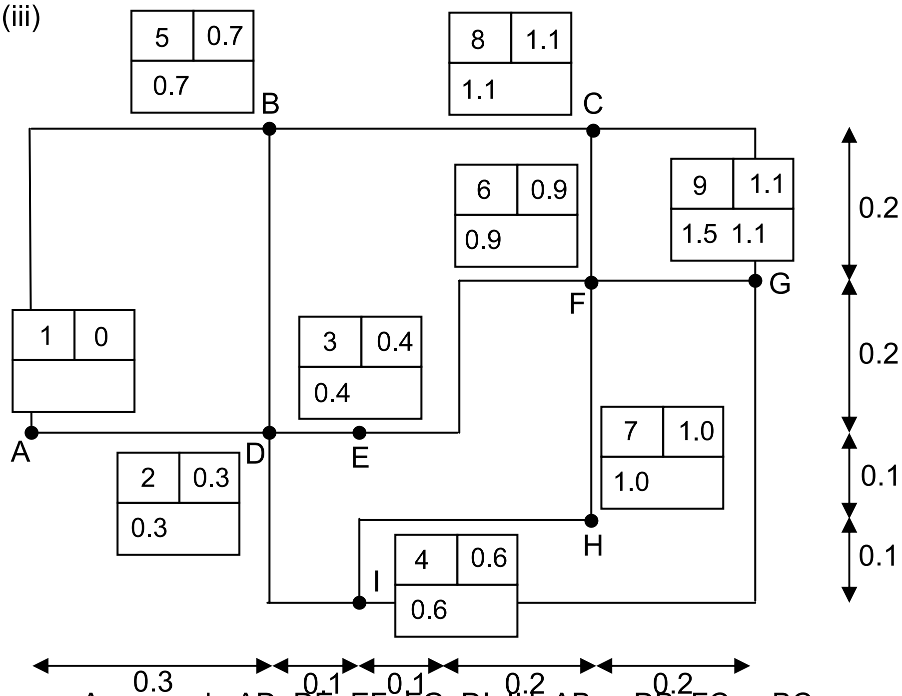
</table>

5.

(i)&(ii) e.g.  	M1 sca (activity on arc) A1 dummy activities + E and F A1 A, B, C, D A1 G, H, I, J M1 forward pass A1 M1 backward pass A1
time – 55 weeks critical – A; B; F; G; J	B1 cao B1 cao
(iii) 50 weeks (49 weeks if G crashed rather than H)	B1
(iv) E – 1 week F – 3 weeks J – 2 weeks (G − 1 week, if crashed)	 M1 A3
(v) £115000 (£121000)	A1

6.

(i) e.g. 	M1 connecting tree A1 DE A1 FC, FG A1 AD, DI, FH A1 2 of length 0.4
Total length = 2.2 km	M1 A1
(ii) Prim: connect in <u>nearest</u> to <u>connected set</u> Kruskal: <u>Shortest</u> arc s.t. <u>no cycles</u>	M1 name A1 description
(iii) 	M1 Dijkstra A1 working values (see vertex G) A1 order of labelling A1 labels
Arcs used: AD, DE, EF, FG, DI, IH, AB or DB, FC or BC Total length = 2.7 km (AB&FC) or 2.9 km (AB&BC) or 2.4 km (DB&FC) or 2.6 km (DB&BC)	M1 arcs counted only once A1 A1